19 - Oscillations

Q-1) What are oscillations?

> When a body oscillates, the same set of movements are repeated again and again.
One such set of movements is one complete oscillation.

* **Amplitude (x_0) :**
the maximum displacement of an oscillating particle from the equilibrium position.

* **Time period (T) :**
the time taken by an oscillating particle to complete one oscillation.

* **Frequency (f) :** the no. of oscillations made by the particle in one second (per unit time) $f = 1/T$

* **Phase :**
describes the point that an oscillating mass has reached within the complete cycle of an oscillation.

Q-2) What are free oscillations?

> A body, which once disturbed and left to itself performs oscillations called free oscillations.
The frequency of free oscillations is called the *natural frequency*.

Q-3) What are forced oscillations?

> A body can be made to vibrate at a desired frequency by continuously applying a force of that frequency. Such oscillations are called forced oscillations

The frequency is called the applied frequency

Q-4) What is simple harmonic motion? (SHM)

> SHM is the motion of an object in which ~~the~~ its acceleration is directly proportional to its displacement from ~~the~~ its equilibrium position, and is directed towards the equilibrium position.

$$a \propto -x \quad \text{OR} \quad a = -\omega^2 x$$

(ω: angular velocity; a: acceleration; x: displacement)

$\therefore \omega = 2\pi f$

i.e:

* a is directed towards the mean position.
* x is away from the mean position.
* a and x are in opposite directions.

Equations:

displacement : $x = x_0 \sin \omega t$

velocity : $v = v_0 \cos \omega t$

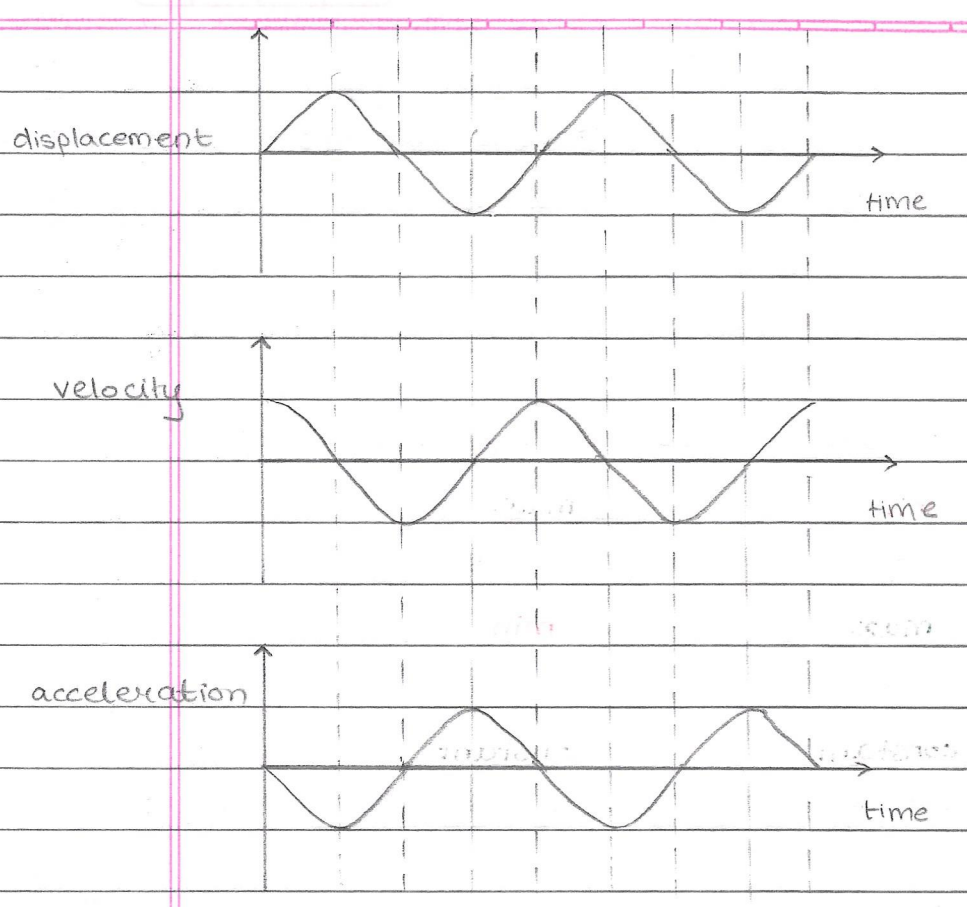
acceleration : $a = -\omega^2 x_0 \sin \omega t$

* if x is in terms of cos, the swap the cos and sin for others too.

$\therefore v_0 = \omega x_0 \quad \therefore \omega = 2\pi f$

$\therefore$ either : $v = v_0 \cos \omega t$

: $v = \pm \omega \sqrt{x_0^2 - x^2}$



Phase difference :

displacement - velocity : 90° ($\pi/2$)

displacement - acceleration : 180° (π)

velocity - acceleration : 90° ($\pi/2$)

Q-5) Energy of Oscillations.

PE derivation :

① $F = ma = m\omega^2 x$

② average $F = \frac{1}{2} m\omega^2 x$

③ $P.E = F \times x = \frac{1}{2} m\omega^2 x^2$

> $K.E = \frac{1}{2} m\omega^2 (x_0^2 - x^2)$

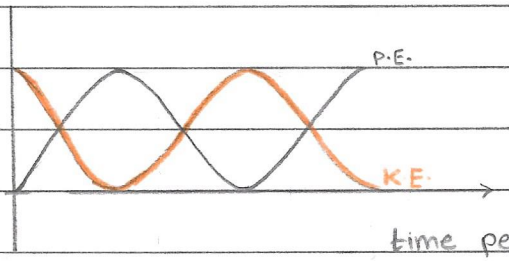
> $PE = \frac{1}{2} m\omega^2 x^2$

> Total energy $= PE + KE$

$= \frac{1}{2} m\omega^2 x_0^2$

$= \frac{1}{2} m 4\pi^2 f^2 x_0^2$

$\therefore$ Total energy $\propto$ Frequency^2



* k.e is max when p.e is zero & vice versa.

	Mean position	Extreme position
P.E.	min (0)	max ($\frac{1}{2}m\omega^2x^2$)
K.E.	max ($\frac{1}{2}m\omega^2x_0^2$)	min (0)
Total Energy	constant ($\frac{1}{2}m\omega^2x_0^2$)	constant ($\frac{1}{2}m\omega^2x_0^2$)

Q-6) What is damping?

> It's a phenomen in which the amplitude of the oscillations gradually decreases to zero due to the resistive forces of the medium (eg: air)
The energy of the oscillations becomes the internal energy of the surrounding air.
Hence, the amplitude of the vibrations decreases but the frequency stays the same.

* Lightly damped:

oscillations of decreasing amplitude occur.

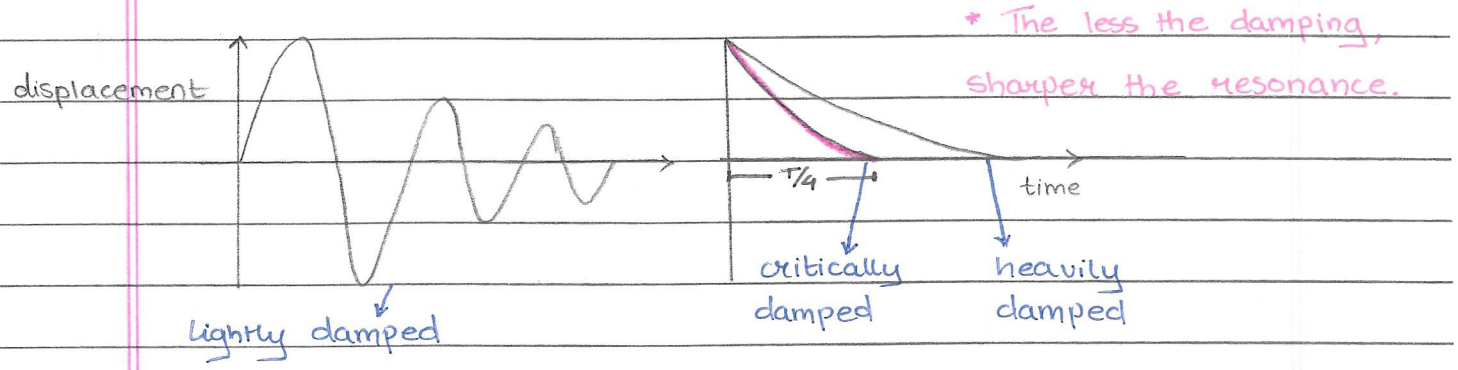
* Critically damped:

when time taken for the displacement to become zero is a minimum ($T/4$), the system is critically damped.
→ no oscillations occur.

* **Heavily damped:**

System returns very slowly to its equilibrium position

→ no oscillations occur.



* The less the damping, sharper the resonance.

Q-7) What is resonance?

> Resonance is a particular case of forced vibrations in which the applied frequency is equal to the natural frequency of the oscillating body.
Here the body oscillates with maximum amplitude.

Good effects

→ determine velocity of sound in air by resonance column.
→ electrical resonance in TV and radio stations / channels.
→ musical instruments.
→ MRI scans
→ microwaves ; frequency matches that of the H_2O molecules
∴ they absorb microwave radiation, ⋆ heat the food.

Bad effects

→ oscillating bridges
→ destruction during earthquakes.