Title: Speed of Sound Lab

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Abstract

From the experiment, the speed of sound through Argon was found to be 325 m/s with an experimental heat capacity, $\frac{Cv}{nR}$, to be 1.424 and a theoretical heat capacity to be $\frac{3}{2}$. For N_2, it was found to be 348 m/s with an experimental heat capacity of 2.714 and a theoretical heat capacity of $\frac{5}{2}$. For CO_2 it was found to be 269 m/s with an experimental heat capacity of 3.513 and a theoretical heat capacity of $\frac{7}{2}$. It was found that CO_2 only had the three major contributions from the four vibrational modes. The three major contributions were two of the degenerate modes at 667cm^{-1} with 0.90 total, and from 1388cm^{-1} with 0.0554. The final mode at 2349cm^{-1} only had a 0.0015 contribution, which didn't change the final outcome drastically.

Keywords: speed of sound, heat capacity, contribution, degenerate modes, vibrational mode.

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Introduction

The goal of this experiment was to confirm the speed of sound in air and in three other gases, Argon, Nitrogen, and Carbon Dioxide. To find the speed of sound in these gases, equation 1 is used:

$$\left(\frac{Measure_2 - Measure_1}{100}\right) meters * frequency = speed \tag{1}$$

where $measure_1$ and $measure_2$ are the distances on the ruler from 45 degrees to 45 degrees of the line on the oscilloscope in centimeters. This yielded the speed of sound in the gas and the average between the frequencies were found using equation 2:

$$\frac{\sum_{1}^{n} Value_n}{n} \tag{2}$$

with $value_n$ being the value of the the trial over the amount of entities. Using the experimental speed of sound through these gases, an experimental unitless heat capacity can be calculated to be compared with the theoretical heat capacity. To calculate the theoretical heat capacity, the degrees of freedom of the molecule is used. For the translational and rotational degrees of freedom contribution, equation 3 is used:

$$\frac{Cv}{nR} = \frac{i}{2} \tag{3}$$

with $\frac{Cv}{nR}$ being the unitless heat capacity, and i is the degrees of freedom from the translational and rotational modes a molecule has. To determine the contribution from the last mode, the vibrational mode, equation 4 is used:

$$\frac{Cv}{nR} = \frac{X^2 e^x}{(e^x - 1)^2}\ ;\ X = \frac{hc\omega}{k_B T} \tag{4}$$

with h being Planck's constant, c is the speed of light in centimeters, ω is the wavenumber of the stretch in the vibrational mode, k_B is the Boltzmann constant and T is the temperature. Finding the contribution yields the possibility of finding the total heat capacity which is just the sum of all the mode contributions. To find the experimental heat capacity, equation 5 is used:

$$\frac{Cv}{nR} = \frac{RT}{c^2 M - RT} \tag{5}$$

with R being the Rydberg constant, T being the temperature in Kelvin, c being the speed of sound, and M being the mass of the molecule in kilograms. Equation 5 is found by solving for $\frac{Cv}{nR}$ using equations 6 and 7:

$$C_p - C_v = nR \tag{6}$$

$$c = \sqrt{\frac{\gamma RT}{M}} \tag{7}$$

with γ being the ratio of C_p over C_v.

Methods

In the experiment, we measured the speed of sound within four different gases. The first gas that we measured was for air and as an introduction to what we were going to do for the later samples. The machine was turned on and the first frequency, 1,200 Hz, was set out of the two to be done. To measure the half wavelength for the gas environment, the first push/pull of the piston was placed so that the oscilloscope created a line that was 45 degrees. The distance that the piston changed was checked by all members of the group and gave an agreed answer. The next push/pull of the piston was placed so that the oscilloscope created a similar line that was -45 degrees. Again, the measurement was checked by the whole group until an agreed answer was chosen. This happened again for the 1,500 Hz to make sure that the machine operated correctly. After the correct value was obtained for the gas, the next gas was pumped into the system, with time passing to let the gas settle in the tube, and the same process was done again. After all gases were measured, the machine was turned off and the area was cleaned up of any mess if any.

Results

TABLE I: Experimental and Calculated Values

Gas	Speed of Sound exp (m/s)	C_v / nR exp	C_v / nR theoretical
Ar	325	1.424	3/2
N_2	348	2.714	5/2
CO_2	269	3.513	7/2

Sample Calculations for CO_2: (rest of the gases are the same calculations)

Speed of Sound:

$$\left(\frac{Measure_2 - Measure_1}{100}\right) meters * frequency = speed$$

1,200 Hz:

$$\frac{47.0cm - 35.8cm}{100cm} * 1,200Hz = 268.8\ m/s$$

1,500 Hz:

$$\frac{38.9cm - 29.9cm}{100cm} * 1,500Hz = 270\ m/s$$

Average speed of sound:

$$\frac{\sum_{1}^{n} Value_n}{n}$$

From Experiment:

$$\frac{(268.8 + 270)}{2} = 269.4\ m/s$$

Finding theoretical heat capacity:

$$\frac{Cv}{nR} = \frac{i}{2}$$

where i is degrees of freedom of molecule, CO_2 has 9, 3 for translational, 2 for rotational, and 4 for vibrational.

From translational and rotational,

$$\frac{Cv}{nR} = \frac{(3+2)}{2} = \frac{5}{2}$$

To determine the contribution via vibrational nodes, Einstein's equation, equation 4, is used from the introduction:

$$\frac{Cv}{nR} = \frac{X^2 e^x}{(e^x - 1)^2}\ ;\ X = \frac{hc\omega}{k_B T}$$

Since there are four vibrational modes, calculations need to be done for all vibrational nodes. There are 2 degenerate bending modes at 667cm^{-1}, 1 symmetric stretch at 1388cm^{-1}, and 1 asymmetric stretch at 2349cm^{-1}.
For 667cm^{-1}: (same for the others)

$$\frac{Cv}{nR} = \frac{X^2 e^x}{(e^x-1)^2}\ ;\ X = \frac{(6.626*10^{-34}\frac{m^2*kg}{s})*(2.998*10^{10}cm/s)*(667cm^{-1})}{(1.3806*10^{-23}\frac{m^2*kg}{s^{-2}*K})*(300K)}$$

Sub X for the equation and get 0.45 for ONE contribution of the 667cm^{-1} vibrational mode. To find the total, we sum the contributions from all four vibrational modes and the total we had for the translational and vibrational modes.

$$\frac{Cv}{nR} = \frac{5}{2} + 2 * (0.45) + 0.0554 + 0.0015 = 3.46$$

which is about 3.5.

For experimental, we use equation 5 from the introduction:

$$\frac{Cv}{nR} = \frac{RT}{c^2M-RT} = \frac{8.3145\ \frac{J}{K*mol} * 298.15K}{(269\ m/s)^2*(0.04401\frac{kg}{mol})-(8.3145\ \frac{J}{K*mol} * 298.15K)} = 3.513$$

which is about the value that was theoretically calculated.

Conclusion

From the results of the experiment, the theoretical and experimental heat capacities were not too far off from each other. For Argon, the theoretical was 1.424 which is about the 1.5 theoretical that was calculated. For Nitrogen, it was 2.714, which is also about the 2.5 that was calculated. Lastly, for Carbon Dioxide, it was 3.513 which is about the 3.5 calculated including the contribution of the four vibrational modes. The experimental heat capacity of each gas was found to be just about the theoretical heat capacities calculated from equation 5 from the introduction.

Using another equation, equation 7, it can be seen that the speed of sound through a given gas is inversely proportionally squared to the mass of the gas in kg. This experiment shows that the speed of sound increases from CO_2 to Ar to N_2. This result aligns with the theory of the equation showing how the heaviest gas has the slowest speed of sound and the lightest has the fastest speed of sound. From slowest to fastest, Carbon Dioxide had a value of 269 m/s, Argon had a value of 325 m/s, and Nitrogen had a value of 348 m/s. Comparing to the online source[1] at 20 degrees C versus the 25 degrees C in the lab, Carbon Dioxide had a speed of sound of 267 m/s, Argon had a value of 319 m/s, and Nitrogen had a value of 349 m/s. These values are about the same from experiment and from a documented resource.

References

1. https://pages.mtu.edu/~suits/SpeedofSoundOther.html (accessed Feb 13, 2020).

Discussion Questions

1. **1.5 points** In column two of Table 1 enter the average of the speed of sound you measured during the lab. Calculate the molar heat capacity at constant volume divided by R, C_v/nR, from your measured speed of sound data. Enter this in column three of Table 1.
2. **2.5 points** Using the Einstein model calculated the contribution from each vibrational mode to the heat capacity, and enter your new value for C_v/nR in the last column of Table 1. Don't forget to include the contribution to the heat capacity from translational and rotational modes.
3. **2.0 points** At 300 Kelvin what is the speed of sound through the noble gas Krypton. Krypton has a molar mass of 83.8 g/mol. Show all your calculations.

Since Krypton is a single atom, it can be assumed that it has the same degrees of freedom like Argon that was tested above. Argon's C_v/nR was found to be 3/2 so Krypton's should be the same. Using equation 5 from the Introduction, we can rearrange the equation to find c.

$$c = \sqrt{\frac{\frac{2}{3RT}+RT}{M}}$$

This rearrangement let's us find c and when each value plugged in:

$$c = \sqrt{\frac{\frac{2}{3(8.3145\ \frac{J}{mol*K}*300K)}+(8.3145\ \frac{J}{mol*K}*300K)}{.0838\,\frac{kg}{mol}}}$$

This yields 172.53 m/s which is about 173 m/s for the speed of sound through this gas.

4. **2.0 points** Consider the following relation: $C_p - C_v = nR$. It is obvious that nR is a positive number which means that $C_p - C_v$ must be positive, thus C_p is greater than C_v. Give a physical rationalization for this.

Both C_p and C_v are specific heats which is the amount of heat required to raise a system's temperature by a specific amount. Since C_p is for constant pressure, increasing the heat for the system both increases the volume and the internal energy of the system. For C_v, volume is kept constant which means that only the internal energy increases in the system with added heat. A higher specific heat is required if there is energy being acted upon two items rather than one item.

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