

## Error analysis

Construct a matrix  $\mathbf{V}$  with the following elements:

$$V_{ij} = \frac{(-1)^{i+j}}{i+2j}, \quad i, j = 1, \dots, N$$

and let  $\mathbf{r}$  be the first column vector of  $\mathbf{V}$ .

For  $N = 2, \dots, 10$ , compute the condition number of  $V$  and solve

$$\mathbf{V}\mathbf{x} = \mathbf{r}$$

using LUP-decomposition (you can use `scipy.linalg.solve` function or the LUP/forward substitution/backward substitution function in the repository). Of course, the exact solution is  $\mathbf{x} = \mathbf{e}_1$  (verify this by hand!). Compute the relative residual and error for all matrix sizes  $N$ .

Plot the relative error along with the maximal relative error (according to the upper bound in lecture 7) on a semilogarithmic scale (use `matplotlib`).

Answer the following questions:

1. What happens to the condition number for increasing  $N$ ? Is that bad news or good news when we are solving linear systems?
2. What happens to the relative errors and residuals for increasing  $N$ ?
3. Up to what matrix size does the numerically obtained solution make sense?