Finite Elements: revision checklist

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For the exam, you should be able to:

- Formulate one-dimensional boundary value problems as variational problems, with homogeneous and non-homogeneous Dirichlet and Neumann boundary conditions.
- Construct Ritz-Galerkin approximations for one-dimensional variational problems, and reformulate them as matrix-vector systems.
- Define the P1 space and corresponding nodal basis for one-dimensional variational problems.
- Define the one-dimensional P1 interpolant.
- Develop a first-order error estimate for the 1D P1 interpolant in the energy norm.
- Use Galerkin orthogonality to derive a first order error estimate for the example 1D Ritz-Galerkin approximation in the energy norm.
- Understand the duality argument to make L^2 estimates for the example 1D Ritz-Galerkin approximation in the energy norm.
- Recall the general definition of a Finite Element.
- Define the nodal basis, and compute it in simple 1D examples.
- Recall definitions for 1D Lagrange elements on intervals, and 2D Lagrange elements on triangles.
- Recall the definition for a dual basis "determining" a finite element, and it's equivalence to the nodal basis being a basis for P.
- Recall the result about polynomials vanishing on a hyperplane, and use it to prove that dual bases determine finite elements for the case of triangular Lagrange elements, cubic Hermite elements, and other similar examples.
- Define the local interpolant for a finite element, and show that it is a linear operator, and a projection.
- Define the global interpolant for a triangulation with finite elements defined on each triangle, and show that the Lagrange elements are C^0. Understand the continuity of other examples of elements.
- Recall the definition of the L^P norm for $1 \leq p < \infty$, and $p = \infty$.
- Understand the definition of L^p spaces as sets of equivalence classes.
- Recall Minkowski's inequality, and the Schwarz inequality.
- Recall the definition of a Banach space, and that L^p is a Banach space.
- Define C_0^∞, L^1_{loc}, and use them to define a weak derivative.
- Use the definition to compute weak derivatives of continuous functions, and finite element functions.
- Recall the Sobolev norms, semi-norms and spaces, and recall that $W_p^k(\Omega)$ is a Banach space.
- Recall Sobolev's inequality (and the corollary given in lectures).

- Recall the trace theorem.
- Define inner-product spaces, and recall that L^2 and $W_2^k = H^k$ are inner product spaces.
- Recall Schwarz's inequality for inner product spaces.
- Understand how to use an inner-product space as a normed space.
- Recall the definition of a Hilbert space, and recall that L^2 and H^k are Hilbert spaces.
- Define the dual of a Hilbert space, and recall the Riescz Representation theorem.
- Define a symmetric variational problem using the definitions of bounded and coercive symmetric bilenear forms. Understand that such as form defines a Hilbert space.
- Use the Riescz Representation theorem to show that symmetric variational problems have a unique solution.
- Define the Ritz-Galerkin approximation to symmetric variational problems and show that they have a unique solution.
- Derive Galerkin orthogonality for Ritz-Galerkin approximations, and use them to obtain error estimates in the energy norm.
- Define non-symmetric variational problems, and recall the Lax-Milgram theorem for non-symmetric problems and their Ritz-Galerkin approximations.
- Recall Cea's Lemma, and the proof.
- Use approximation theory results to obtain convergence estimates for non-symmetric variational problems via Cea's Lemma.
- Formulate Poisson's equation as a symmetric variational problem, with Dirichlet and Neumann's boundary conditions.
- Recall results on integration by parts for H^1 spaces.
- Use Poincaré-Friedrich inequality to derive a coercivity result for Poisson's equation with Dirichlet boundary conditions.
- Show that the Ritz-Galerkin approximation for Poisson's equation converges.