深蓝学院
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Sample-based path finding

Lecture 3

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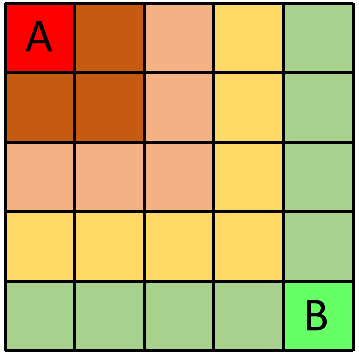
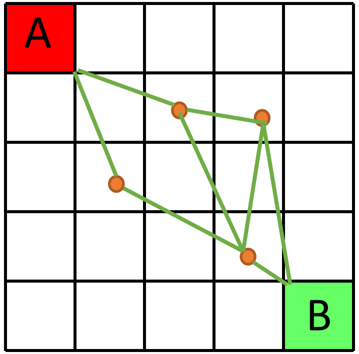
Preliminaries

Sampling Based-Planners

- Do not attempt to explicitly construct the C-Space and its boundaries
- Simply need to know if a single robot configuration is in collision
- Exploits simple tests for collision with full knowledge of the space
- Collision detection is a separate module- can be tailored to the application
- As collision detection improves, so do these algorithms
- Different approaches for single-query and multi-query requests





Preliminaries

Notion of Completeness in Planning

- Complete Planner: always answers a path planning query correctly in bounded time
- Probabilistic Complete Planner: if a solution exists, planner will eventually find it, using random sampling (e.g. Monte Carlo sampling)
- Resolution Complete Planner: same as above but based on a deterministic sampling (e.g sampling on a fixed grid).

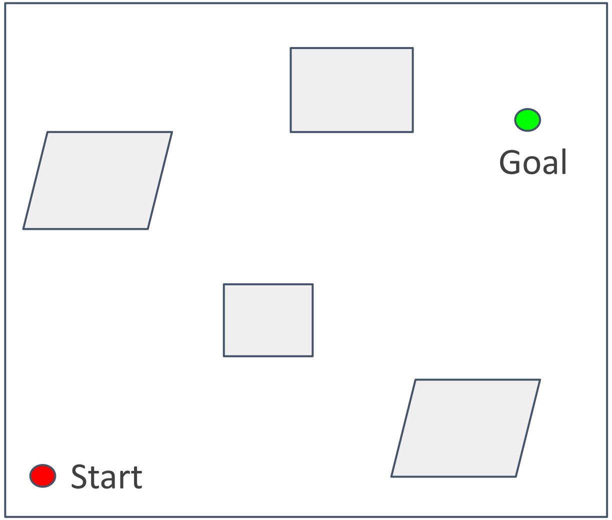
Content

Probabilistic Road Map

Probabilistic Road Map

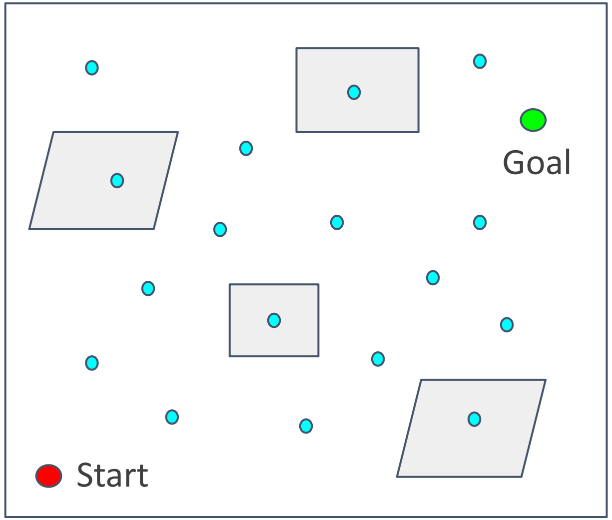
What is PRM?

- A graph structure
- Divide planning into two phases:
  - Learning phase:
  - Query phase:
- Checking sampled configurations and connections between samples for collision can be done efficiently.
- A relatively small number of milestones and local paths are sufficient to capture the connectivity of the free space



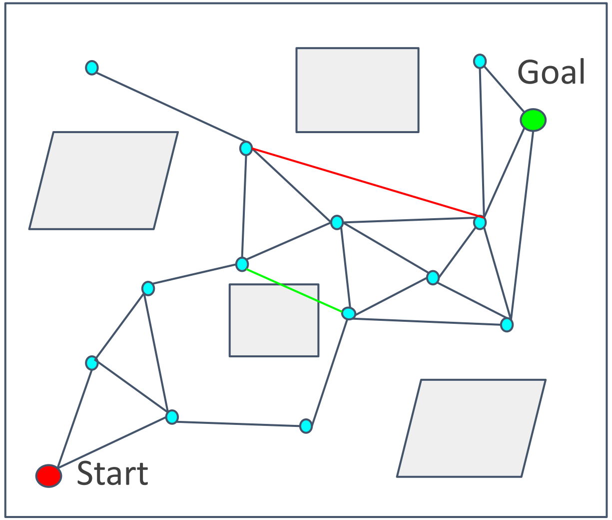
Probabilistic Road Map

- Learning phase:
  - Sample N points in C-space
  - Delete points that are not collision-free
- Detect the c-space with random points



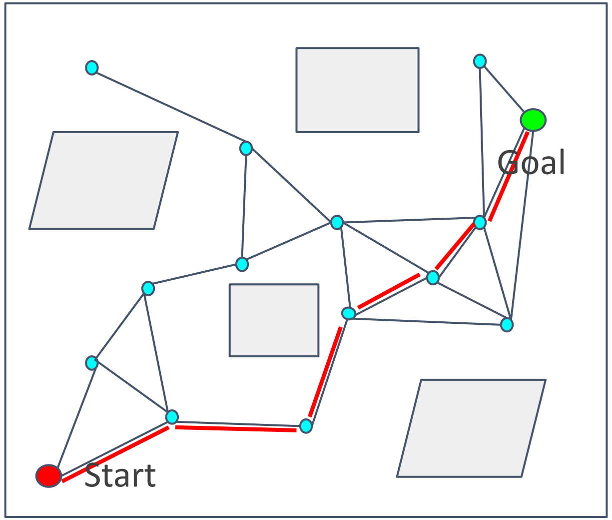
Probabilistic Road Map

- Learning phase:
  - Connect to nearest points and get collision-free segments.
  - Delete segments that are not collision free.



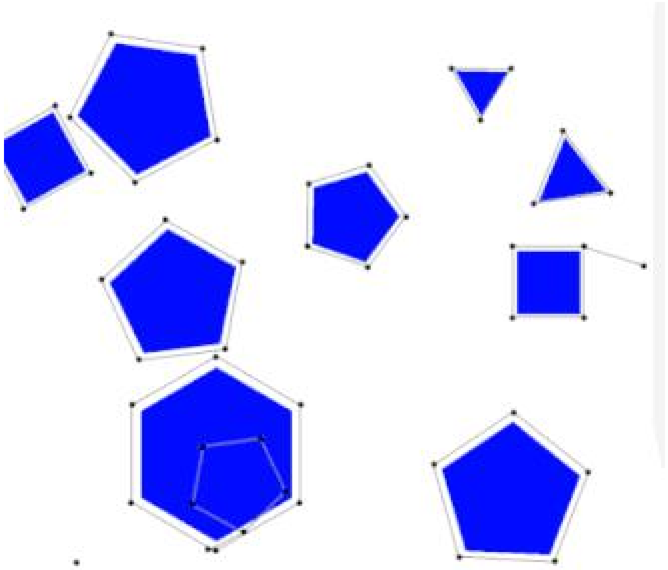
Probabilistic Road Map

- Query phase:
  - Search on the road map to find a path from the start to the goal (using Dijkstra's algorithm or the A* algorithm).
  - Road map is now similar with the grid map (or a simplified grid map)

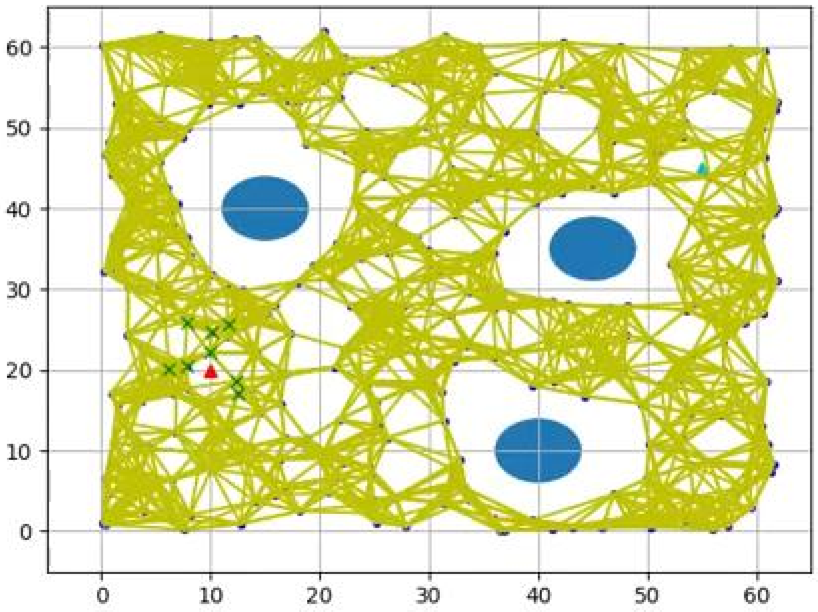


Probabilistic Road Map

Learning phase[1]



Query phase[2]



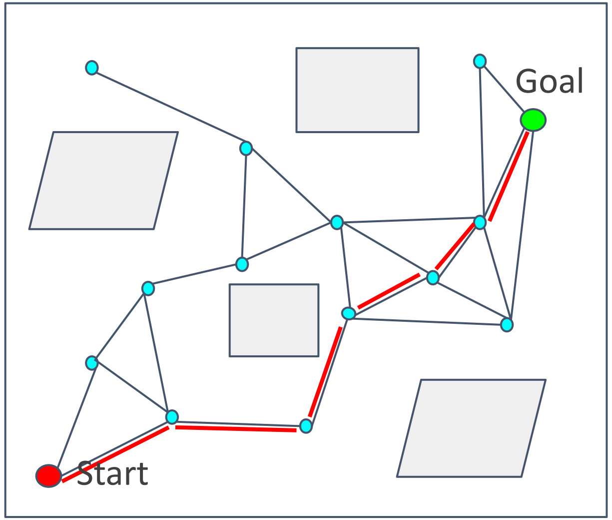
[1] https://en.wikipedia.org/wiki/Probabilistic_roadmap

[2] https://www.youtube.com/watch?v=8Dln3sS_p4Q

Probabilistic Road Map

Pros and Cons

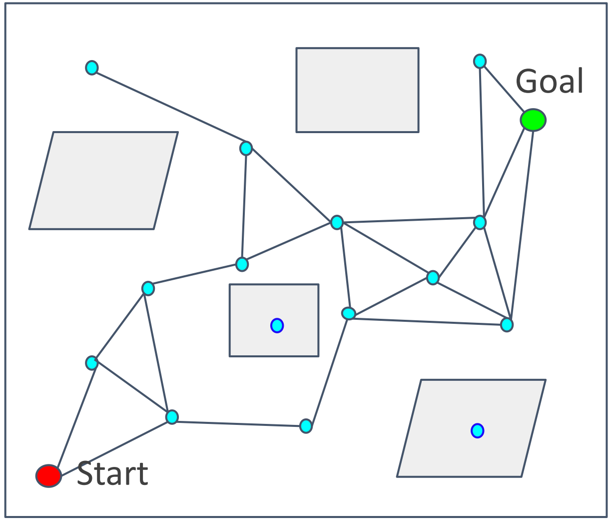
- Pros
  - Probabilistically complete
- Cons
  - Required to solve 2 point boundary value problem
  - Build graph over state space but no particular focus on generating a path
  - Not efficient



Probabilistic Road Map

Note: towards improving efficiency

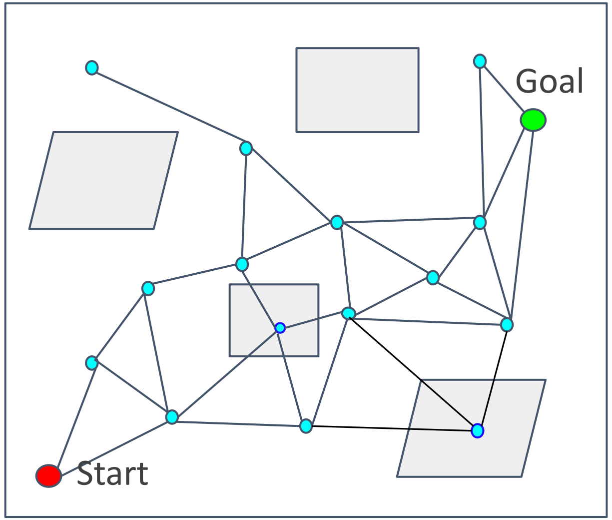
- Lazy collision-checking
  - Collision-checking process is time-consuming, especially in complex or high-dimensional environments.
  - Sample points and generate segments without considering the collision (Lazy).



Probabilistic Road Map

Lazy collision-checking

Sample points and generate segments without considering the collision (Lazy).

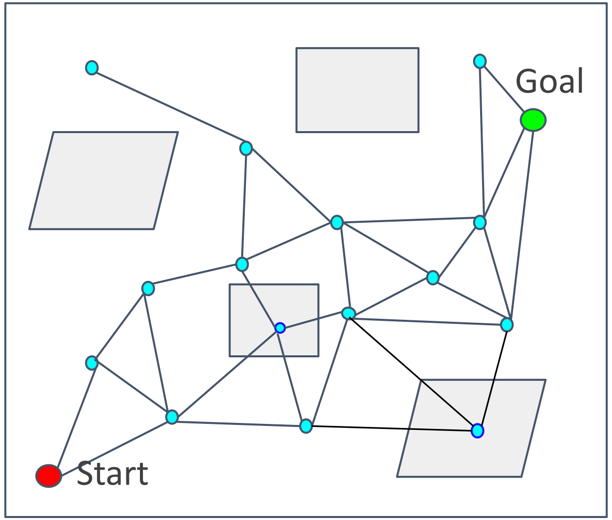


Probabilistic Road Map

Lazy collision-checking

Collision-checking if necessary:

Find a path on the road map generated without collision-checking

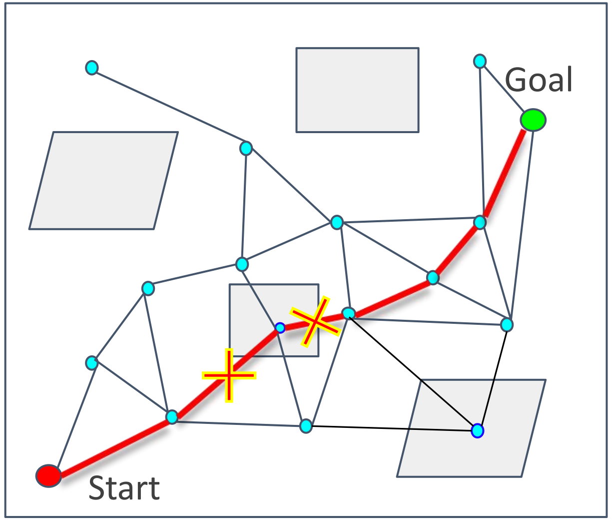


Probabilistic Road Map

Lazy collision-checking

Collision-checking if necessary:

Delete the corresponding edges and nodes if the path is not collision free.

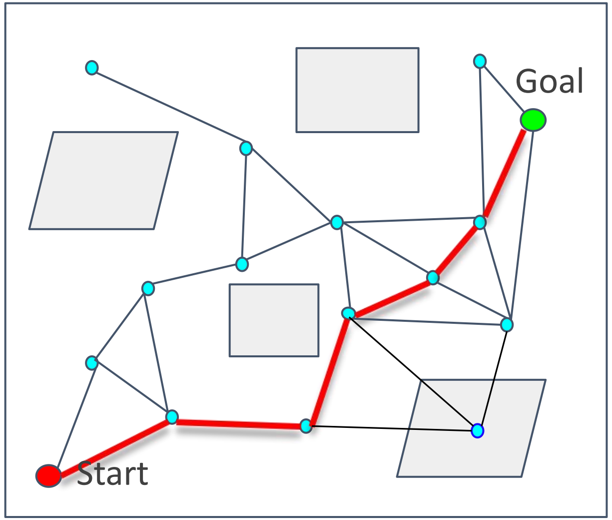


Probabilistic Road Map

Lazy collision-checking

Collision-checking if necessary:

- Delete the corresponding edges and nodes if the path is not collision free.
- Restart path finding.



Probabilistic Road Map

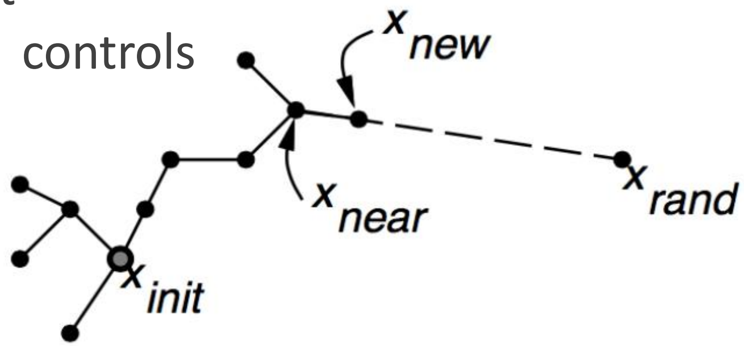
Note:

- Learning phase
- Query phase
- Lazy collision-checking

Rapidly-exploring Random Tree

Rapidly-exploring Random Trees

- Build up a tree through generating “next states” in the tree by executing random controls



Rapidly-exploring Random Trees

Algorithm 1: RRT Algorithm

Input: $\mathcal{M}$,x_{init},x_{goal}
Result: A path Γ from x_{init} to x_{goal}
$\mathcal{T}$.init();
for $i = 1$ *to* n **do**
  $x_{rand} \leftarrow Sample(\mathcal{M})$;
  $x_{near} \leftarrow Near(x_{rand}, \mathcal{T})$;
  $x_{new} \leftarrow Steer(x_{rand}, x_{near}, StepSize)$;
  $E_i \leftarrow Edge(x_{new}, x_{near})$;
  if *CollisionFree*($\mathcal{M}$, E_i) **then**
    $\mathcal{T}.addNode(x_{new})$;
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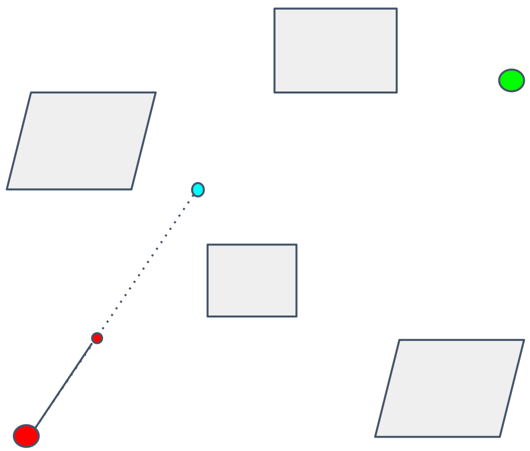
  if $x_{new} = x_{goal}$ **then**
    Success();



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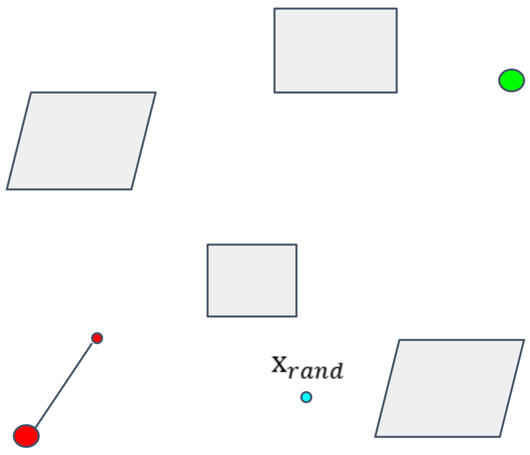

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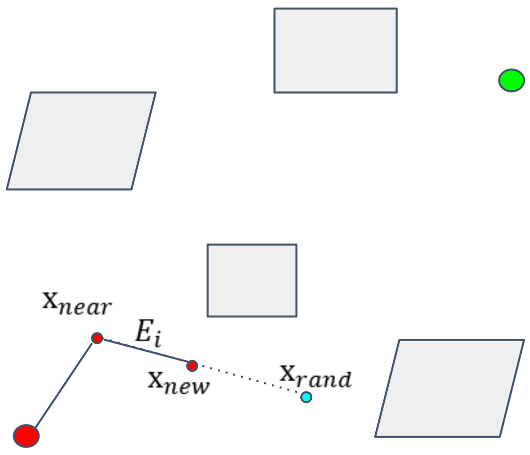

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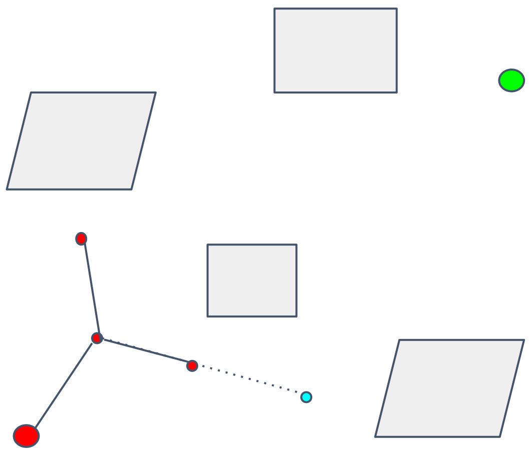
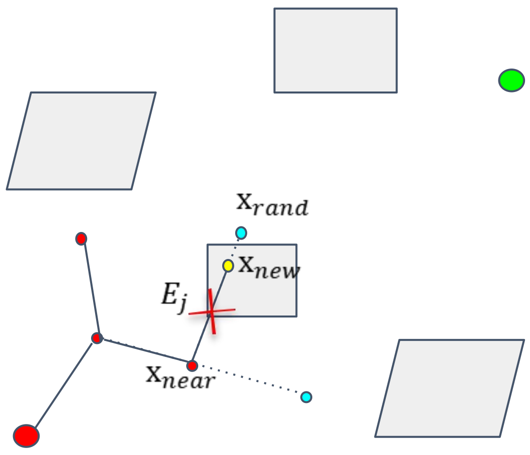


Rapidly-exploring Random Trees

Algorithm 1: RRT Algorithm

Input: $\mathcal{M}$, x_{init}, x_{goal}
Result: A path Γ from x_{init} to x_{goal}

```
T.init();
for i = 1 to n do
    x_rand ← Sample(M) ;
    x_near ← Near(x_rand, T);
    x_new ← Steer(x_rand, x_near, StepSize);
    E_i ← Edge(x_new, x_near);
    if CollisionFree(M, E_i) then
        T.addNode(x_new);
        T.addEdge(E_i);
    if x_new = x_goal then
        Success();
```



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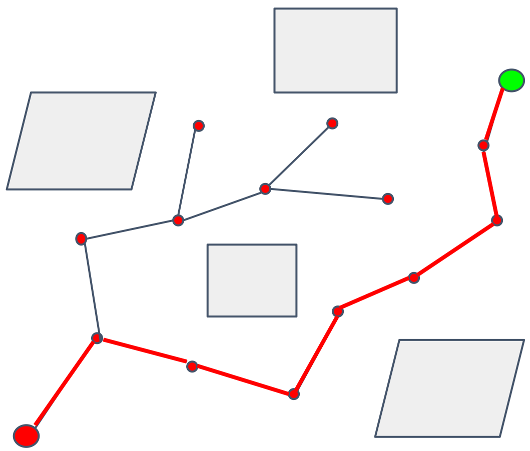
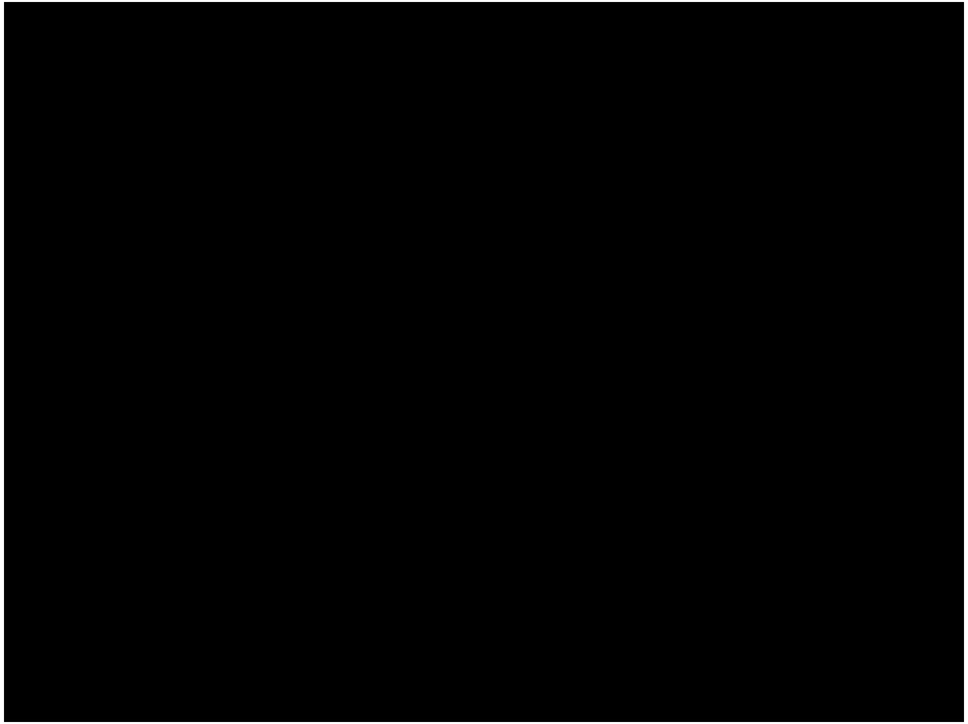


Rapidly-exploring Random Trees

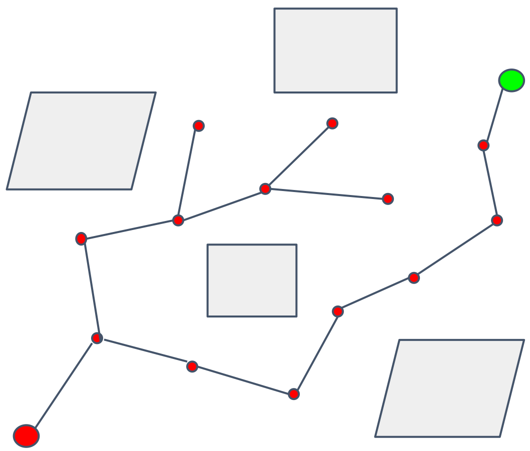
Demonstration of RRT[1]



[1] https://www.youtube.com/watch?v=pOFtvZ_qVsA

Rapidly-exploring Random Trees

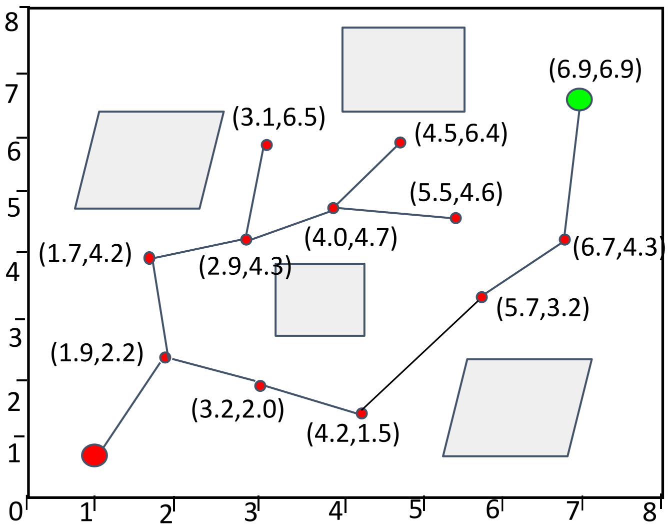
- Pros:
  - Aims to find a path from the start to the goal
  - More target-oriented than PRM
- Cons:
  - Not optimal solution
  - Not efficient(leave room for improvement)
  - Sample in the whole space

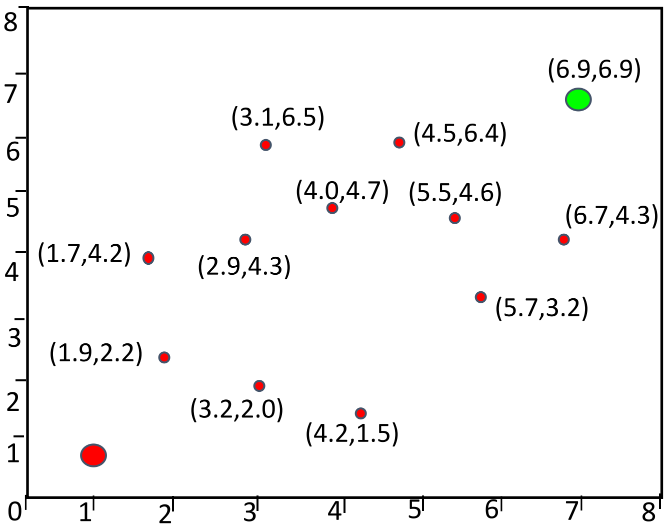


Rapidly-exploring Random Trees

Note: towards improving efficiency

Kd-tree

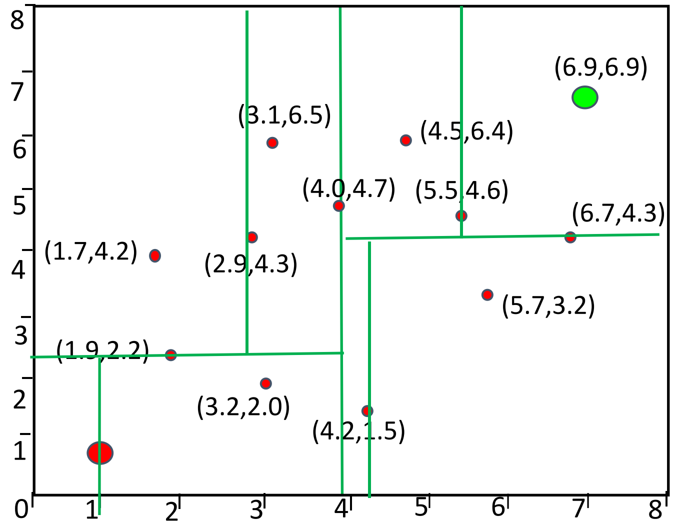


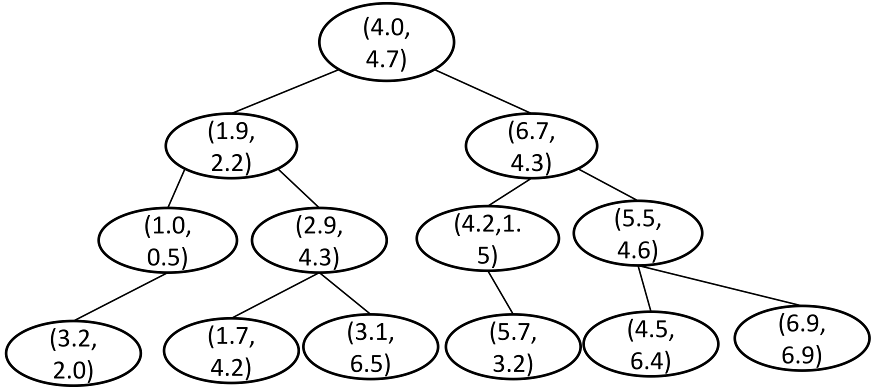


Rapidly-exploring Random Trees

Note: towards path planning efficiency

- Kd-tree



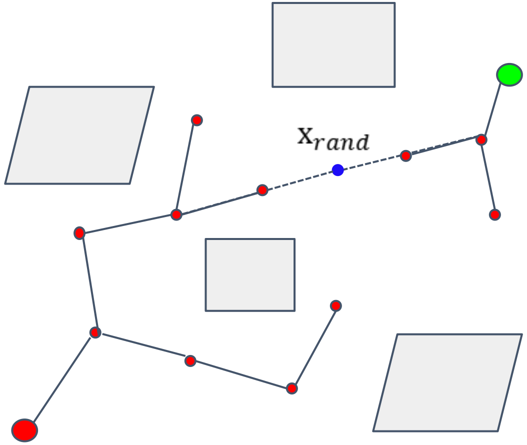


- Other variants: Spatial grid, hill climbing,etc
- 参考：https://blog.csdn.net/junshen1314/article/details/51121582

Rapidly-exploring Random Trees

Note: towards improving efficiency

Bidirectional RRT / RRT Connect

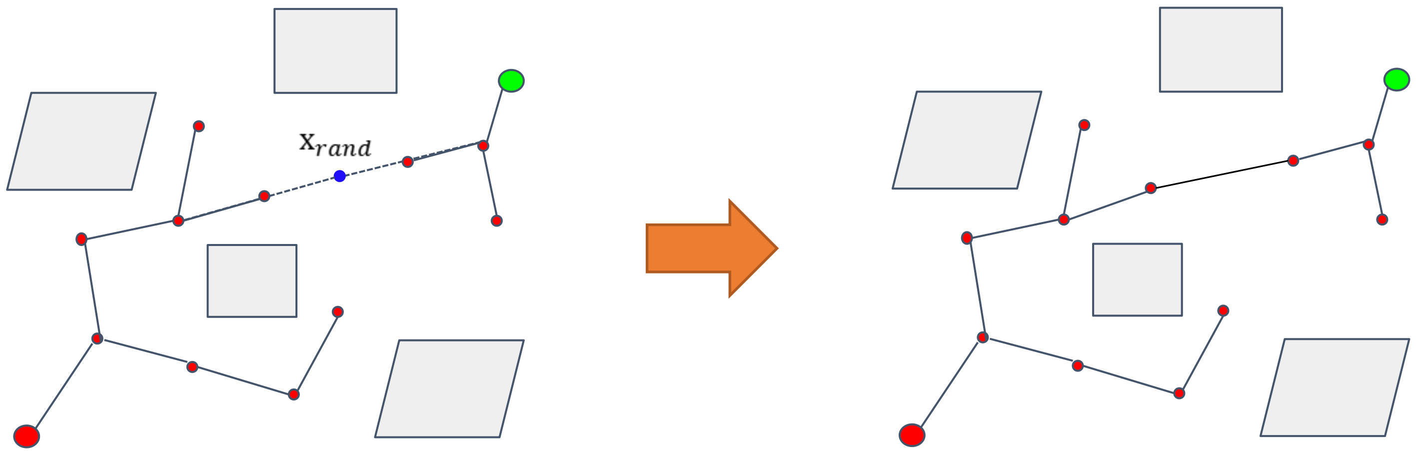


- Grow a tree from both the start point and the goal point
- Path finding when two trees are connected

Rapidly-exploring Random Trees

Note: towards improving efficiency

Bidirectional RRT / RRT Connect



Rapidly-exploring Random Trees

Note: towards path planning efficiency

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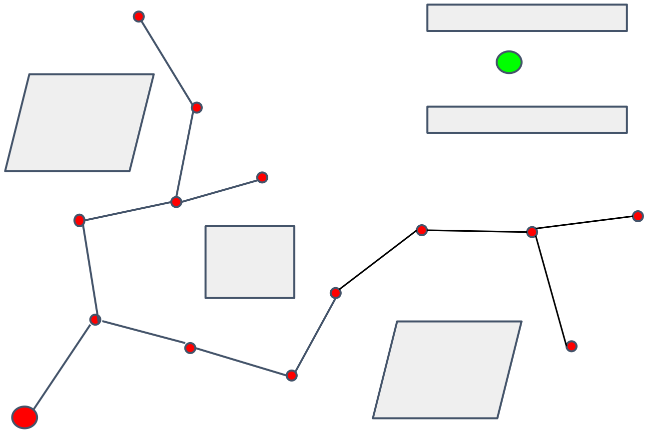
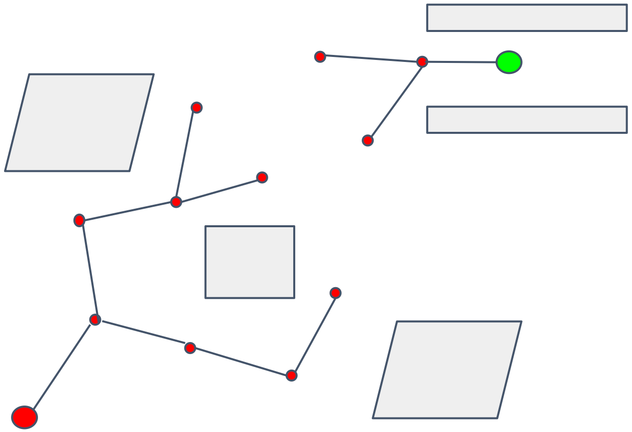




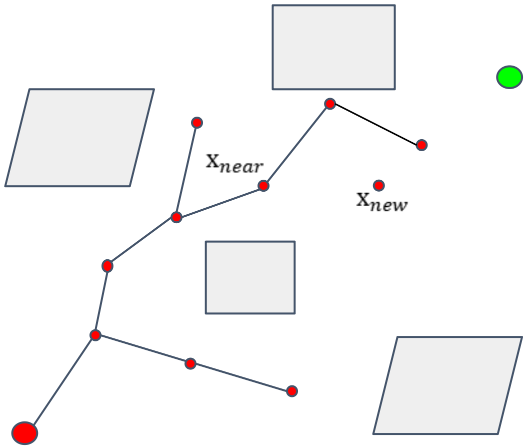
Rapidly-exploring Random Trees

- Incrementally build
- Rapidly searching
- Key functions: Sampling, Nearest, Collision-checking

Optimal sampling-based path planning methods

Optimal sampling-based path planning methods

Rapidly-exploring Random Tree*



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  $X_{near} \leftarrow NearC(\mathcal{T}, x_{new})$;
  $x_{min} \leftarrow ChooseParent(X_{near}, x_{near}, x_{new})$;
  $\mathcal{T}.addNodEdge(x_{min}, x_{new})$;
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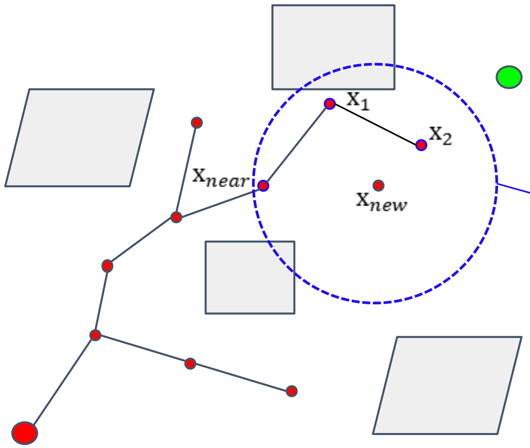
Optimal sampling-based path planning methods

Rapidly-exploring Random Tree*



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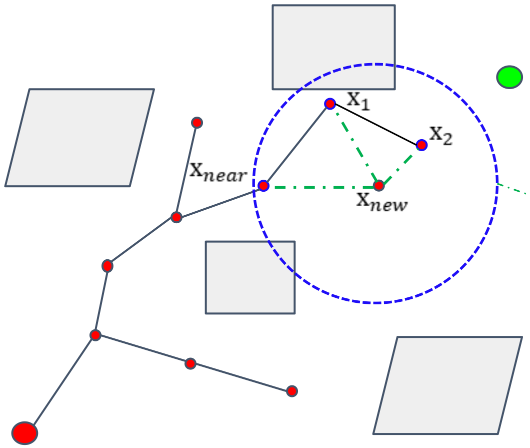

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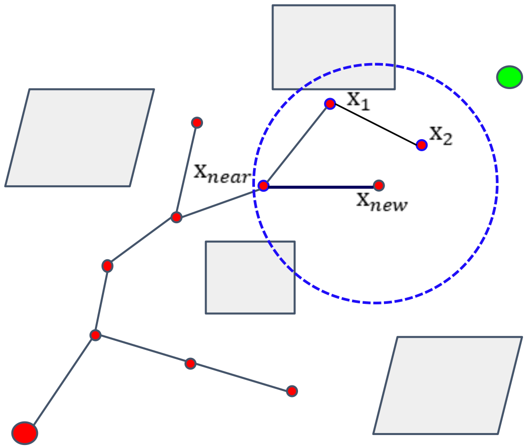

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Optimal sampling-based path planning methods

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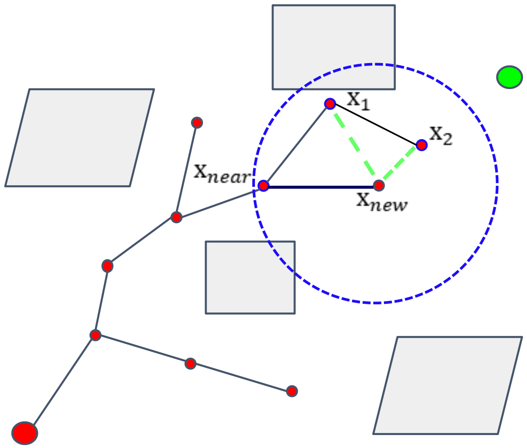

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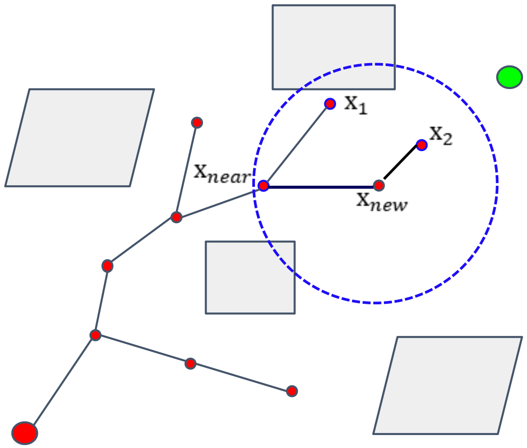

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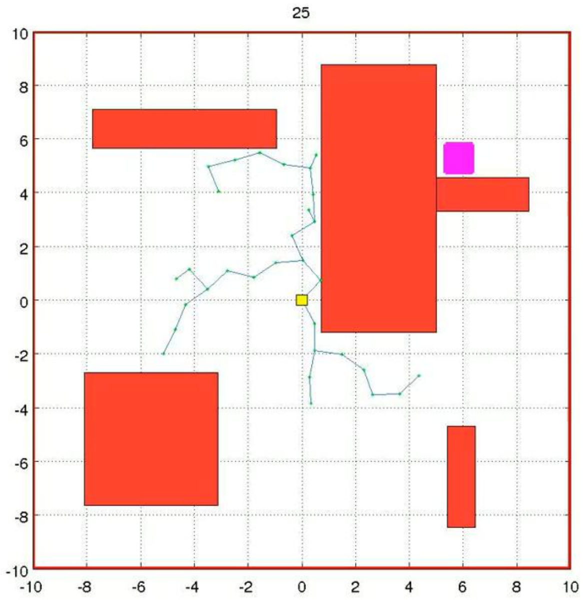
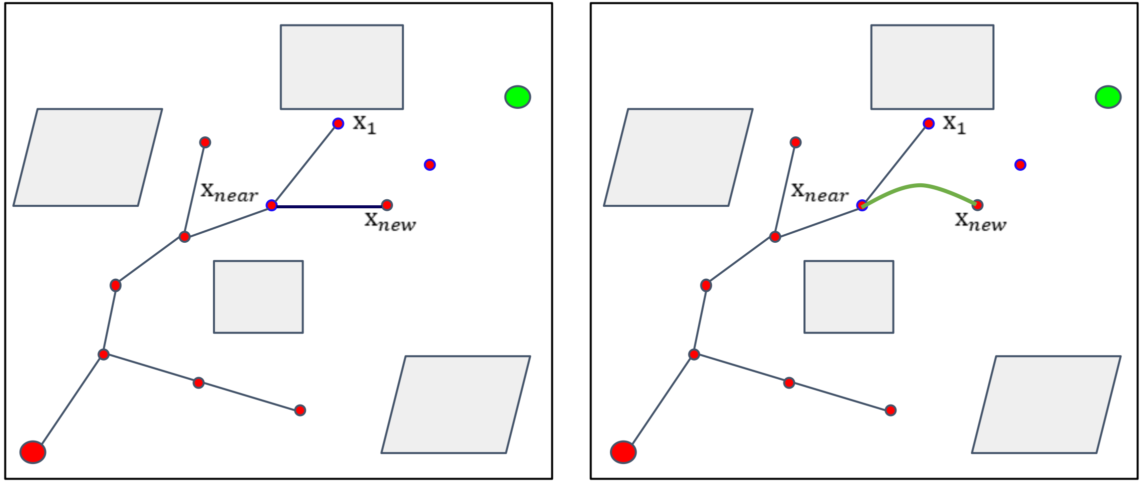


https://www.youtube.com/watch?v=YKiQTJpPFkA

Optimal sampling-based path planning methods

Kinodynamic-RRT*

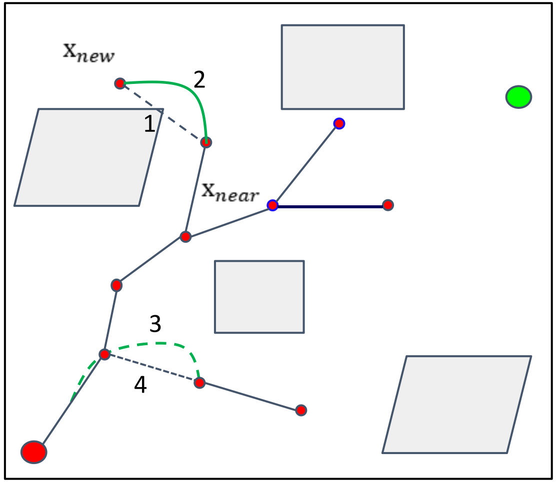


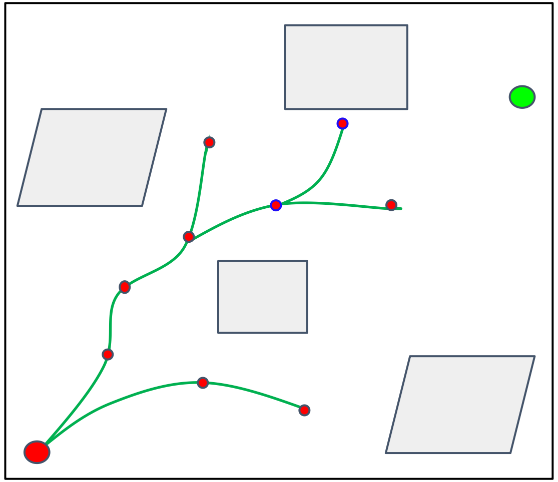
Change Steer() function to fit with motion or other constraints in robot navigation.

[Kinodynamic RRT*: Optimal Motion Planning for Systems with Linear Differential Constraints]

Optimal sampling-based path planning methods

Kinodynamic-RRT*

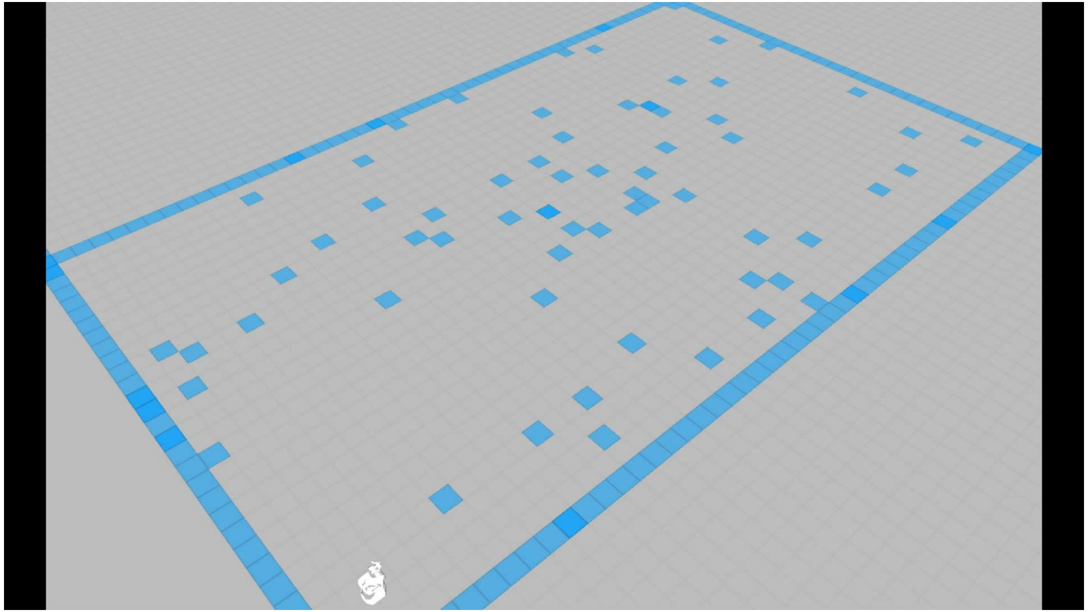




Change **Steer()** function to fit with motion or other constraints in robot navigation.

Optimal sampling-based path planning methods

Kinodynamic-RRT*

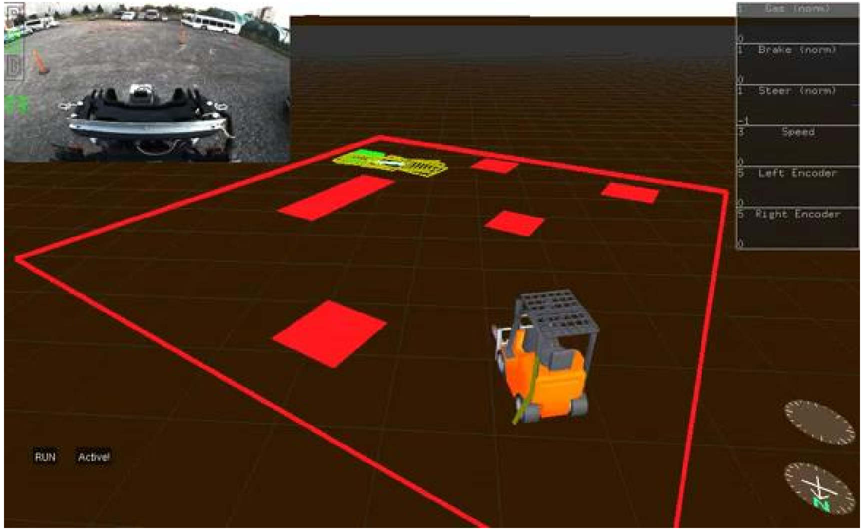


Change Steer() function to fit with motion or other constraints in robot navigation.

[1] https://www.youtube.com/watch?v=RB3g_GP0-dU

Optimal sampling-based path planning methods

Anytime-RRT*



Keep optimizing the leaf RRT tree when the robot executes the current trajectory Anytime Fashion

[Anytime Motion Planning using the RRT*](#)

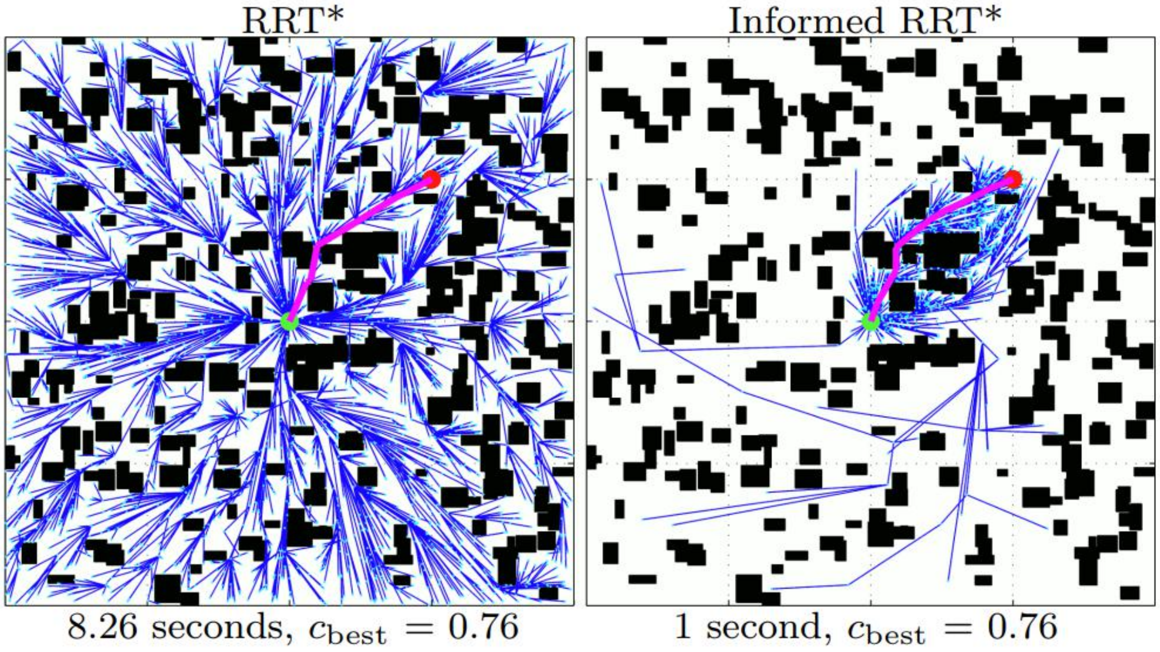
Optimal sampling-based path planning methods

- Rewire function
- RRT*
- Kino-dynamic RRT*
- Anytime RRT*

Advanced Sampling-based Methods

Advanced Sampling-based Methods

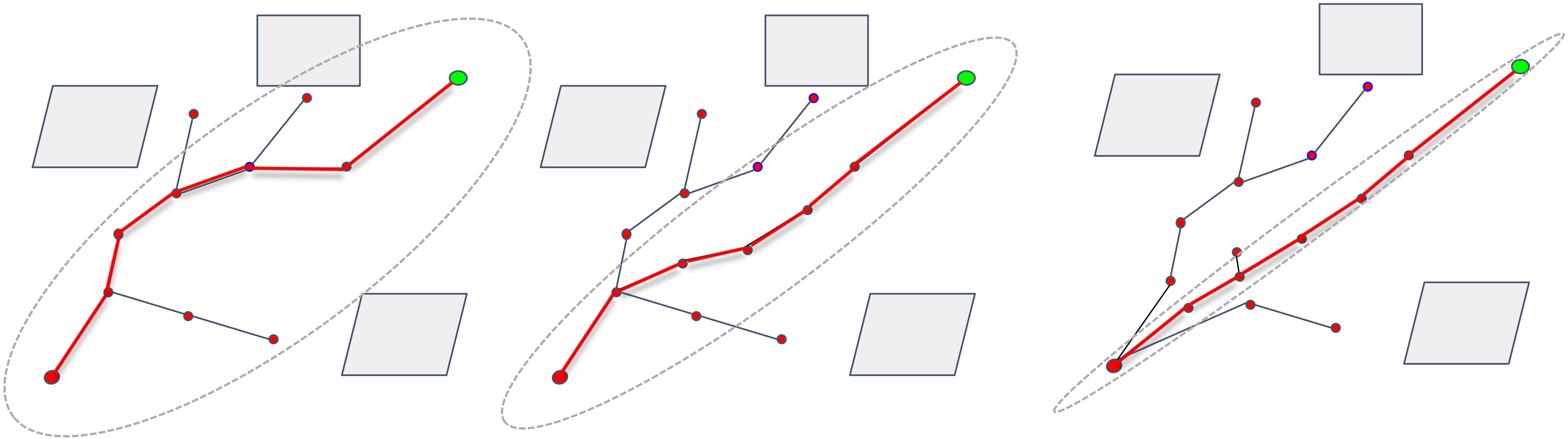
Informed RRT*



[Informed RRT*: Optimal sampling-based path planning focused via direct sampling of an admissible ellipsoidal heuristic]()

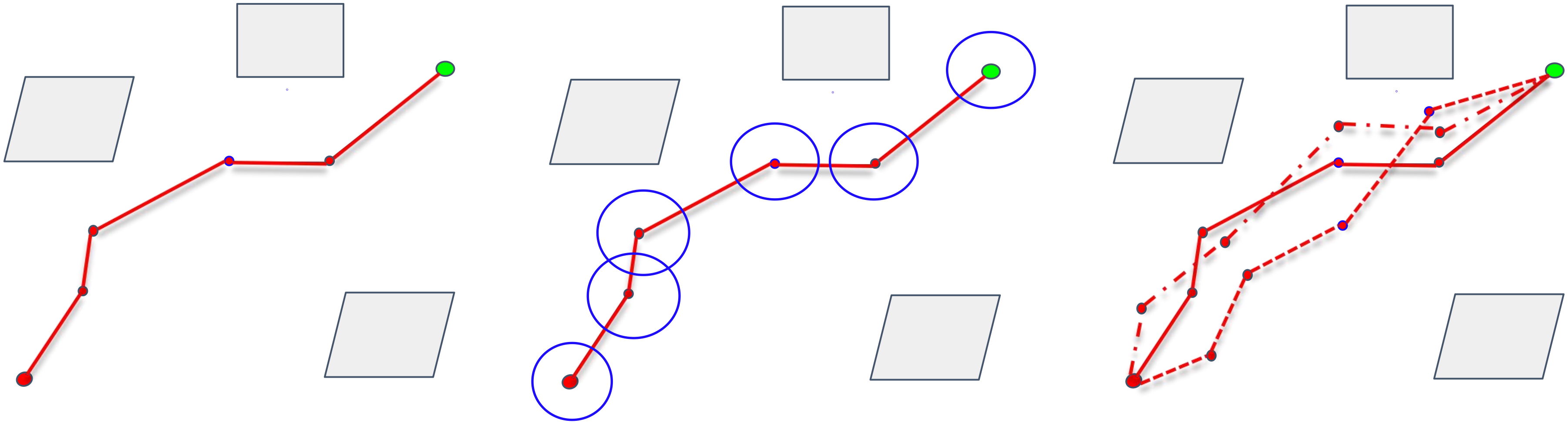
Advanced Sampling-based Methods

Informed RRT*



Advanced Sampling-based Methods

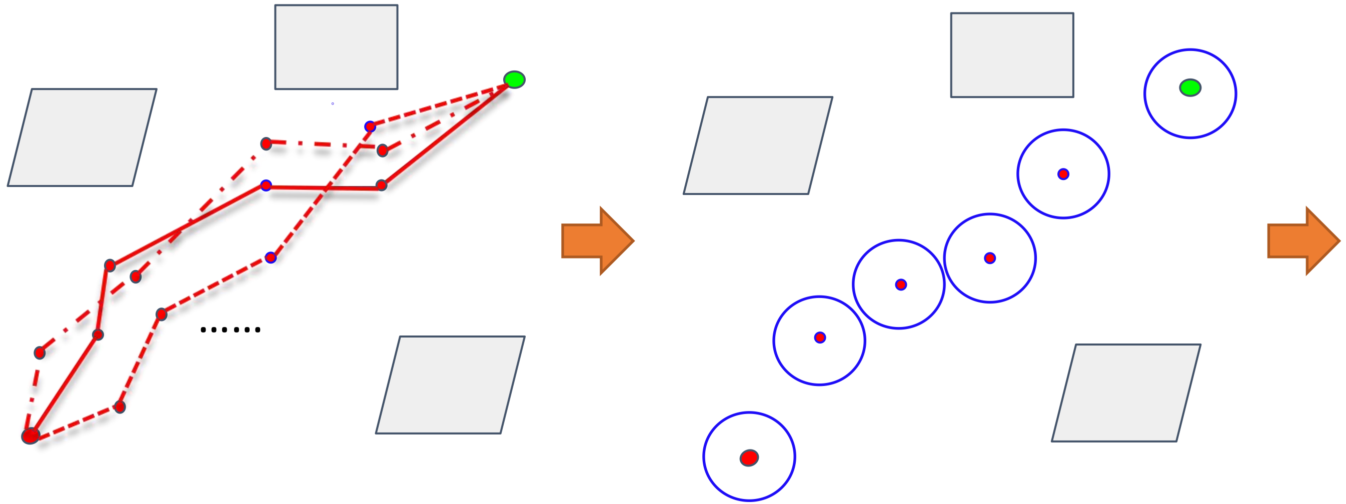
Cross-entropy motion planning



[Cross-entropy motion planning]()

Advanced Sampling-based Methods

Cross-entropy motion planning



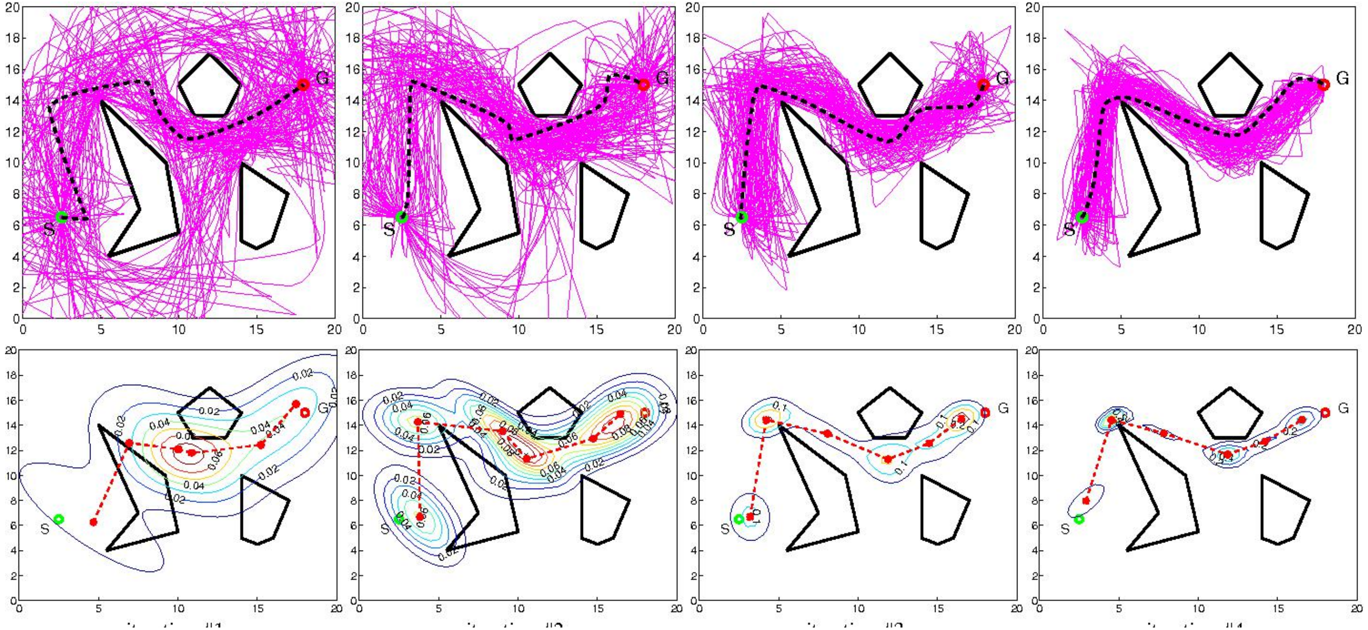
Next sample round

Select elite paths

Update sample region

Advanced Sampling-based Methods

Cross-entropy motion planning



Link of implementation on github:

Advanced Sampling-based Methods

Other variants

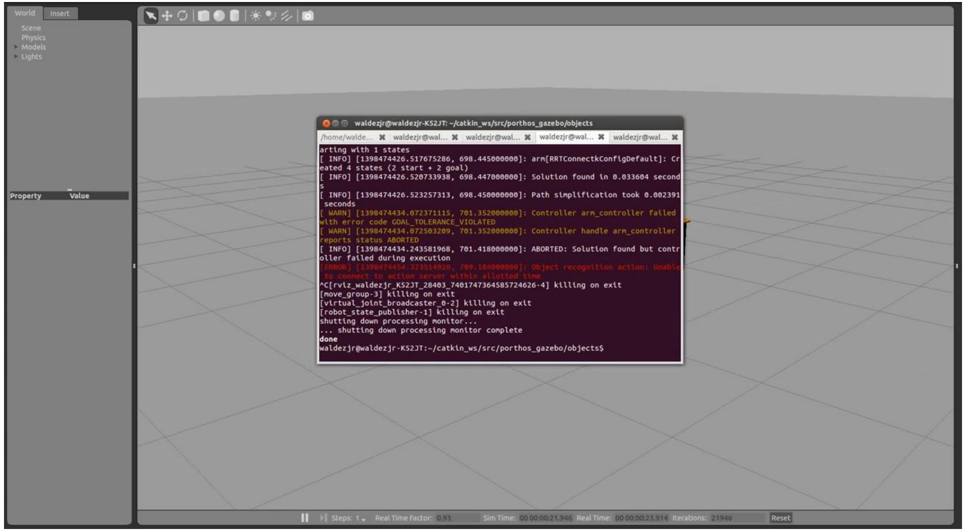
- Lower Bound Tree RRT (LBTRRT)[a]
- Sparse Stable RRT[b]
- Transition-based RRT (T-RRT)[c]
- Vector Field RRT[d]
- Parallel RRT (pRRT)[e]
- Etc.[f]

[1] An Overview of the Class of Rapidly-Exploring Random Trees
[2] http://msl.cs.uiuc.edu/rrt/
[a] https://arxiv.org/pdf/1308.0189.pdf
[b] http://pracsyslab.org/sst_software
[c] http://homepages.laas.fr/jcortes/Papers/jaillet_aaaiWS08.pdf
[d] https://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=6606360
[e] https://robotics.cs.unc.edu/publications/Ichnowski2012_IROS.pdf
[f] https://github.com/zychaoqun

Implementation

Implementation

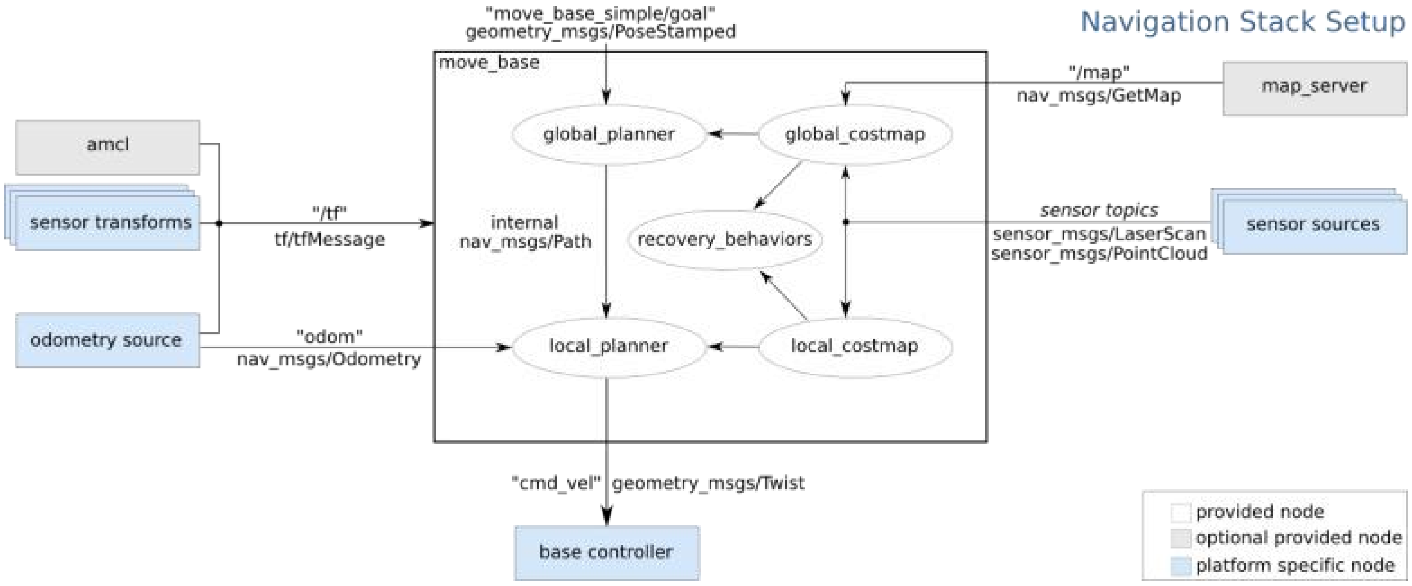
- Open Motion Planning Library [1]
- Moveit with ROS [2]
- Tutorials[3]



[1] https://ompl.kavrakilab.org/
[2] https://moveit.ros.org/
[3] https://industrial-training-master.readthedocs.io/en/melodic/_source/session4/Motion-Planning-CPP.html

Implementation

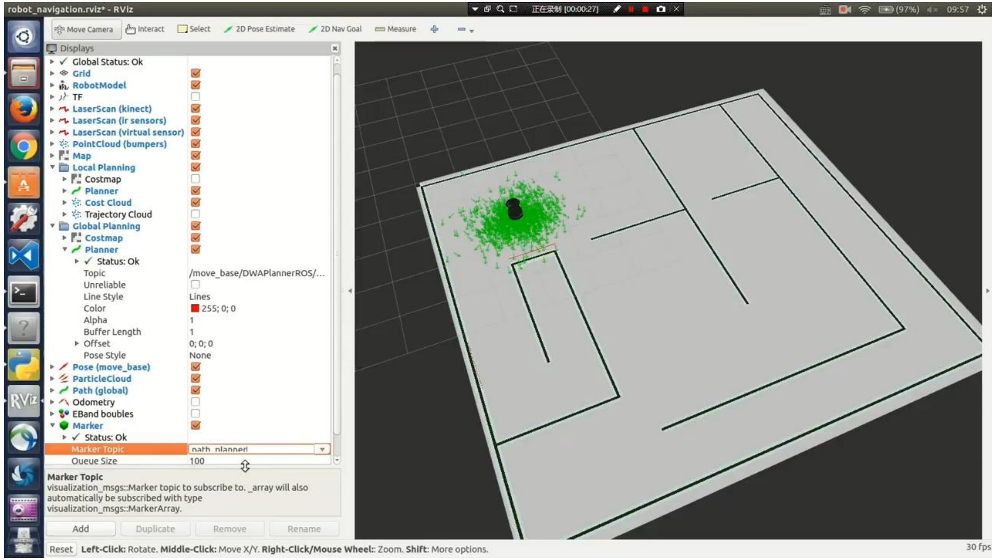
- Navigation stick - ROS



http://wiki.ros.org/move_base

Implementation

- Navigation stick - ROS
  - Global planner
    A*,D*, RRTs,etc
  - Local planner
    Dwa,eband, Teb,etc



Video demonstration of RRT implemented on ROS [1]

[1] https://youtu.be/FsZ9b6fsQUg

Homework

- Implementation of RRT
  - You can either use MATLAB or C++
  - Hints: write RRT as a global planner in ROS
- Bonus: Implementation of Informed-RRT*

Q&A

结语

感谢各位聆听!

Thanks for Listening