EECE 5644 HW1 Report

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Q1.1.1:

The minimum expected risk classification rule in the form of a likelihood-ratio test:

$$\frac{p(x|L=1)}{p(x|L=0)} \underset{D=0}{\overset{D=1}{\gtrless}} \frac{q_0(\lambda_{10}-\lambda_{00})}{q_1(\lambda_{01}-\lambda_{11})} \quad \text{take } \frac{q_0(\lambda_{10}-\lambda_{00})}{q_1(\lambda_{01}-\lambda_{11})} \text{ as threshold } T.$$

$$\Rightarrow \ln(p(x|L=1)) - \ln(p(x|L=0)) \underset{D=0}{\overset{D=1}{\gtrless}} \tau$$

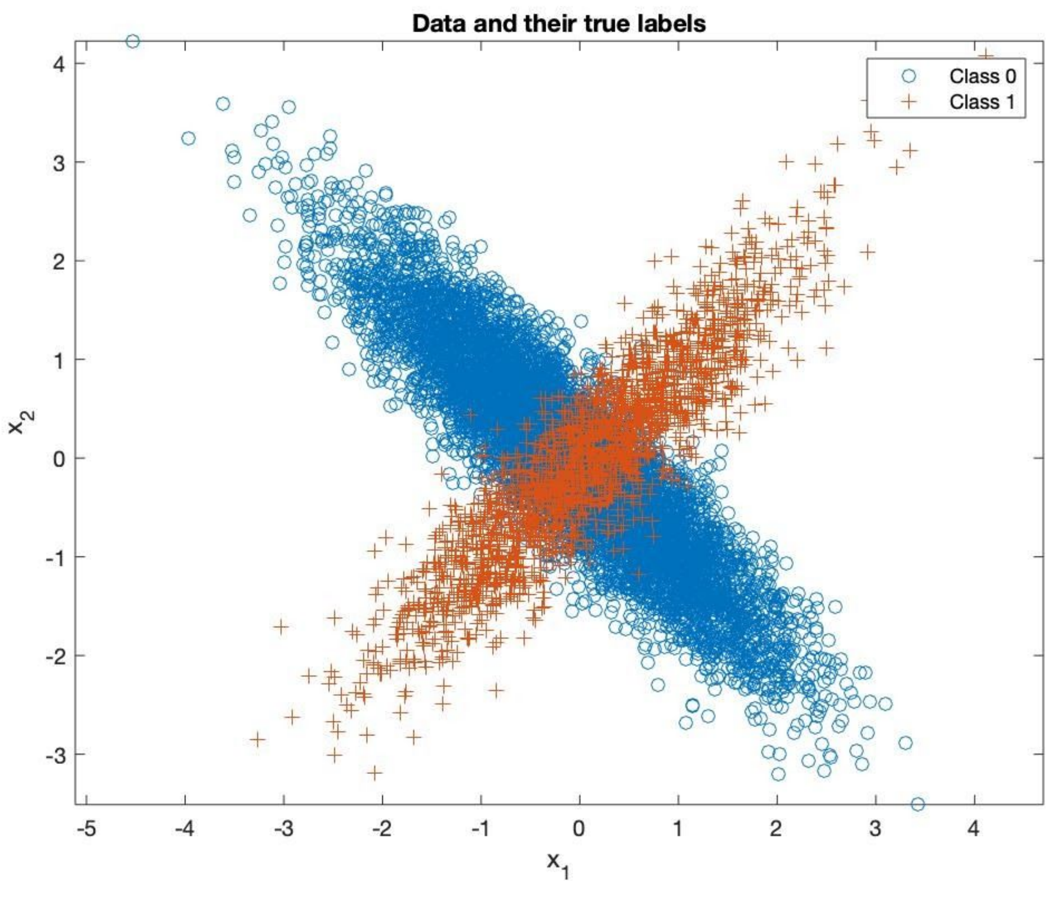

Figure 1. Scatter plot of 10000 Samples from class 0 and class 1

Q1.1.2:

The classifier from Q1.1.1 was implemented by Matlab. The thresholds were generated by the scores from my classifier $\ln(P_1(x)) - \ln(P_0(x))$. I sorted this array, subtracted a very small value(0.05) to the first value of this array, added a very small value(0.05) to the last value of this array. The rest values in this array are generated by the average between two scores. Then I construct 3 new arrays, “fp”, “tp” and “perror”. “fp” is to store the false positive probability P(D=1|L=0), “tp” is to store the true positive probability P(D=1|L=1) and “perror” is to store the error probability (P(D=0|L=1)*P(1) + P(D=1|L=0)*P(0)) for each threshold.

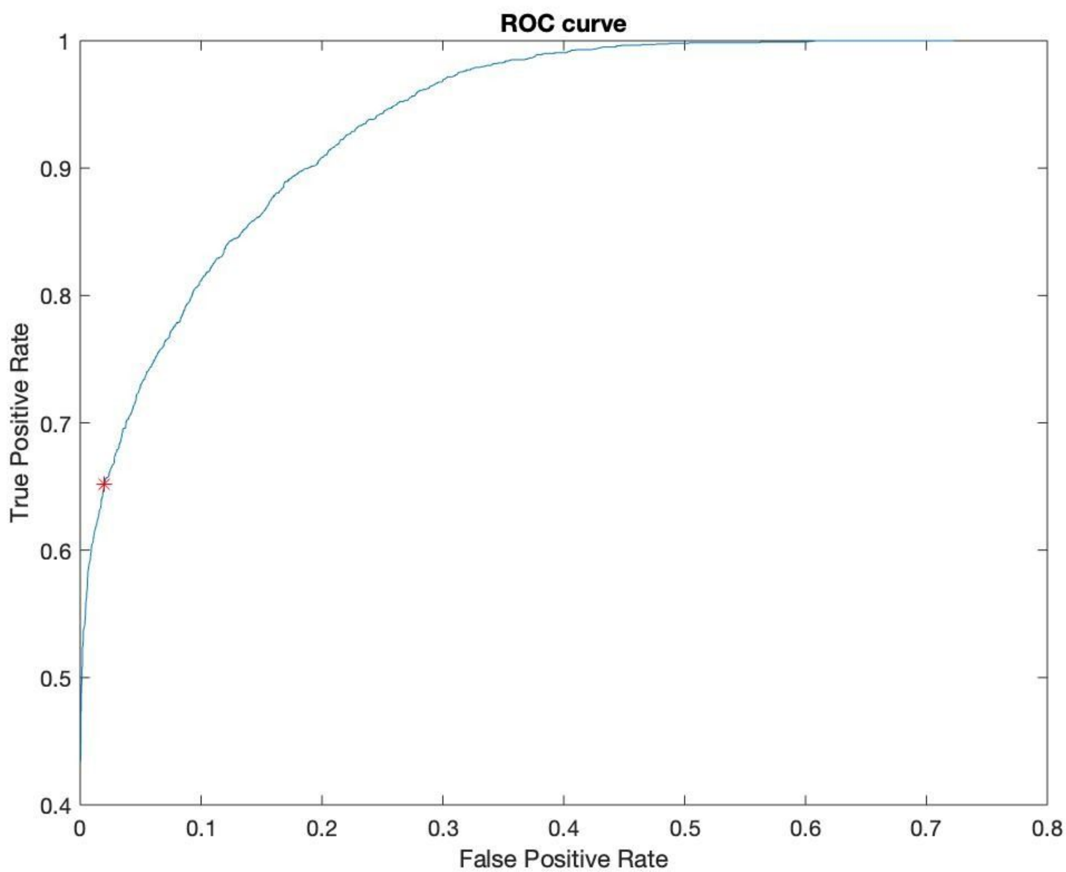

Figure 2. ROC curve of the minimum expected risk classifier.

Q1.1.3:

The true positive value and false-positive value of this point are already in Figure2.

```
Question 1 Problem 1
    "Min Perror is "    "0.0858"

    "Best threshold is "    "3.4452"

    "tp is "    "0.65157"

    "fp is "    "0.019772"
```

Q1.2.1:
The minimum expected risk classification rule in the form of a likelihood-ratio test for naive Bayesian approximation:

$$\frac{p_{NB}(x|L=1)}{p_{NB}(x|L=0)} \underset{D=0}{\overset{D=1}{\gtrless}} \frac{q_0(\lambda_{10}-\lambda_{00})}{q_1(\lambda_{01}-\lambda_{11})} \quad \text{treat} \quad \frac{q_0(\lambda_{10}-\lambda_{00})}{q_1(\lambda_{01}-\lambda_{11})} \text{ as threshold } \tau.$$

If $\Sigma_0 = \Sigma_1$. $\ln(p_{NB}(x|L=1)) - \ln(p_{NB}(x|L=0)) \underset{D=0}{\overset{D=1}{\gtrless}} \tau$

$$-2(m_0 - m_1)^T \Sigma^{-1} x + p \underset{D=0}{\overset{D=1}{\gtrless}} 0 \quad \Rightarrow \text{linear.}$$

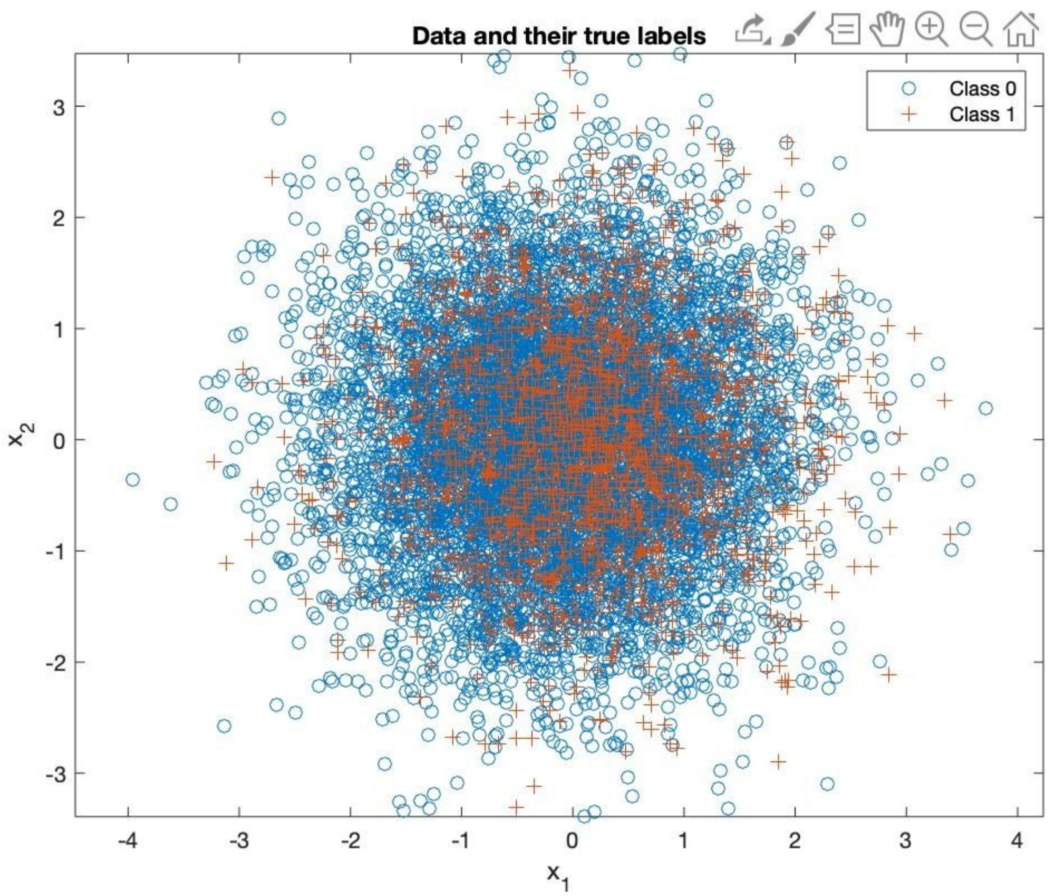

Figure 3. Scatter plot of 10000 Samples from class 0 and class 1 with covariance as the identity matrix

Q1.2.2:
The steps to generate the ROC curve are the same as Q1.1.2.

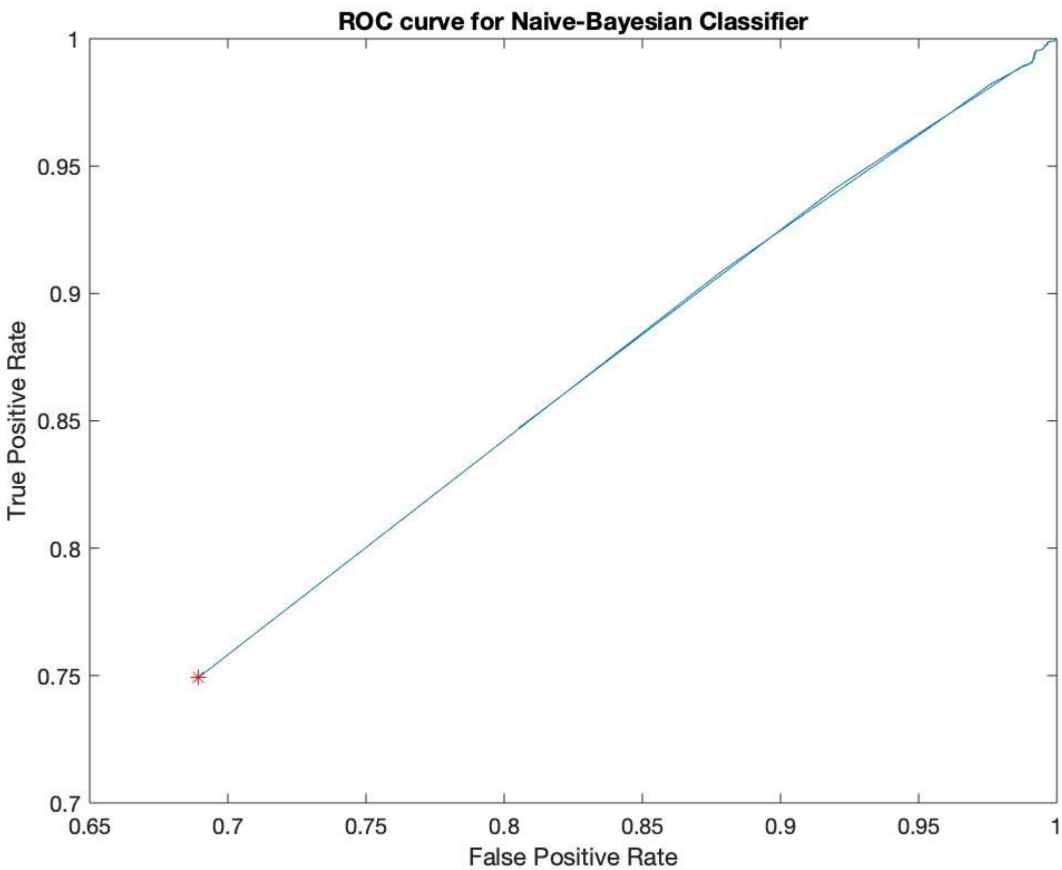

Figure 4. ROC curve of the minimum expected risk classifier.

Q1.2.3:
The true positive value and false-positive value of this point are already in Figure2.

```
Question 1 Problem 2
    "Min Perror is "    "0.6012"

    "Best threshold is "    "0.89193"

    "tp is "    "0.74913"

    "fp is "    "0.68928"
```

Q1.3.1:
Determine the Fisher LDA projection vector W_{LDA}:

$$S_W = (X-\mu_1)(X-\mu_1)^T + (X-\mu_2)(X-\mu_2)^T$$

$$S_B = (\mu_1-\mu_2)(\mu_1-\mu_2)^T$$

$$J(w) = \frac{w^T S_B w}{w^T S_W w}$$

$$w^* = \arg\max J(w) = \arg\max \left(\frac{w^T S_B w}{w^T S_W w}\right) = S_W^{-1}(\mu_1-\mu_2)$$

Threshold : $w_0 = \frac{1}{2} w^T(\mu_1+\mu_2)$

$$w^T * x \underset{D=2}{\overset{D=1}{\gtrless}} w_0$$

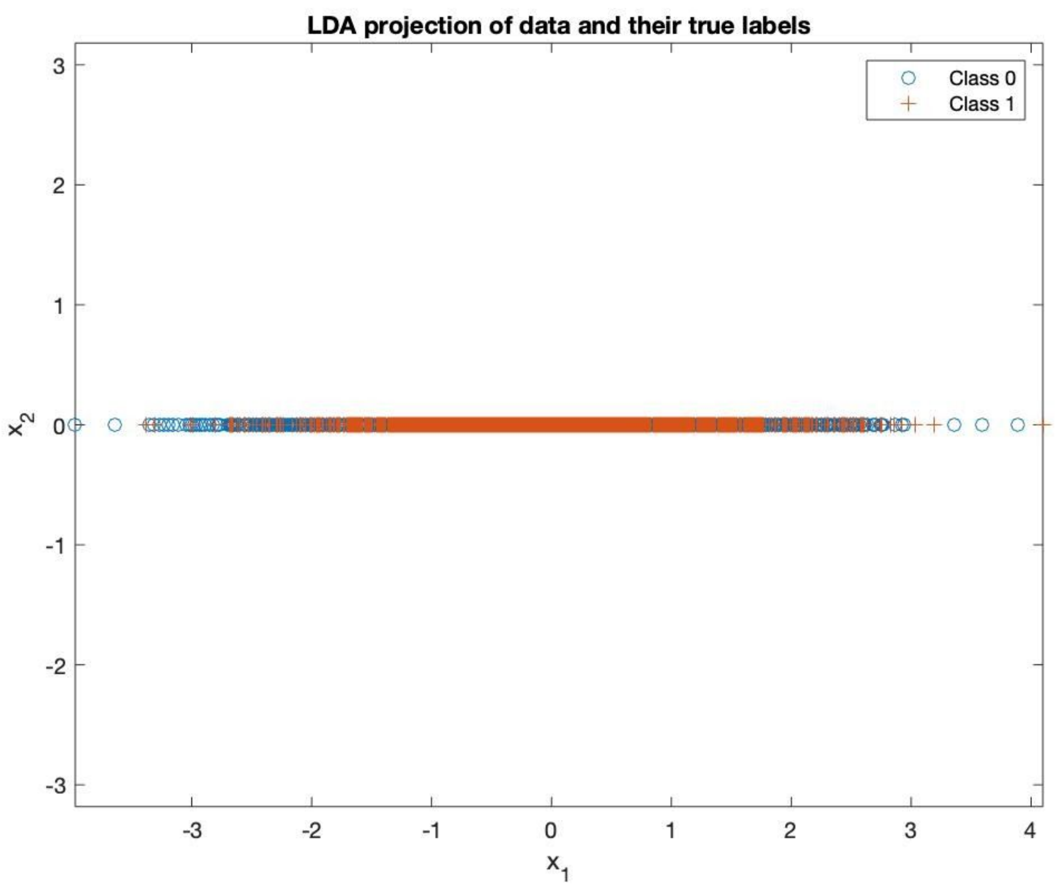

Figure 5. Scatter plot of 10000 Samples on LDA projection

Q1.3.2:

The steps to generate the ROC curve are the same as Q1.1.2. However, instead of using the scores generated by the classifier form Q1.1 and Q1.2, use the scores generated by classification rule w^T_{LDA}.

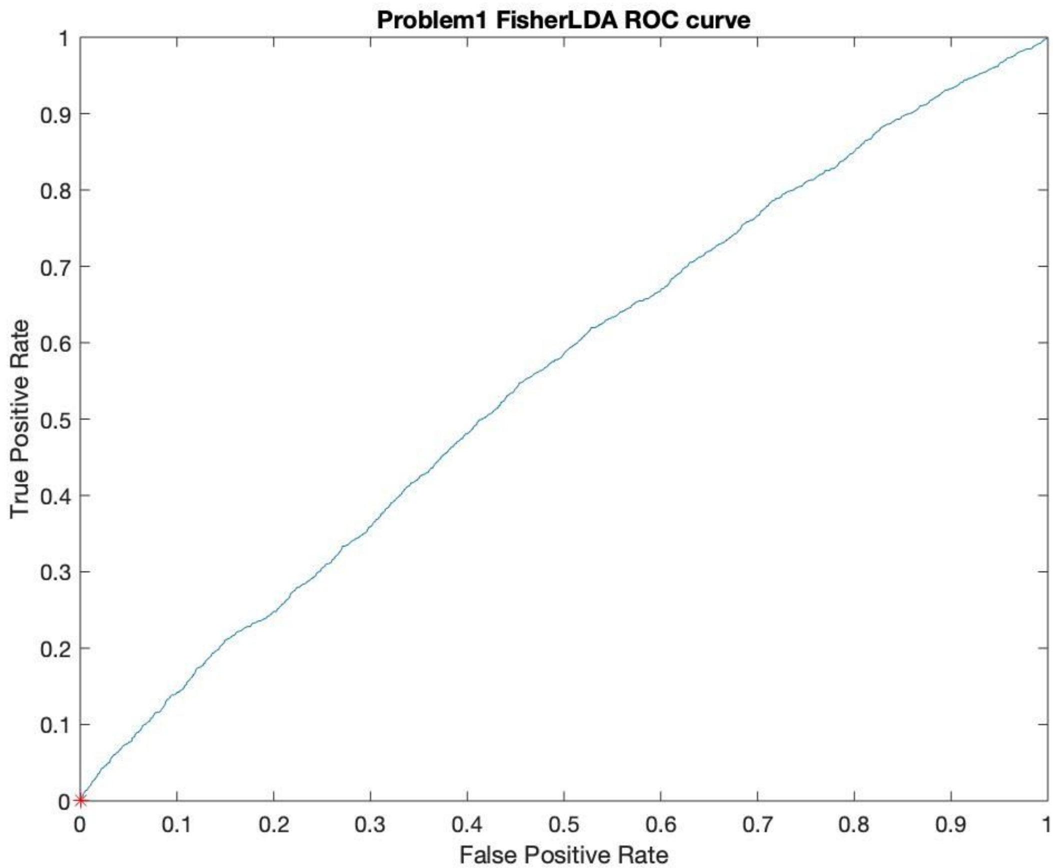

Figure 6. ROC curve of the Fisher LDA classification rule.

Q1.3.3:

The true positive value and false-positive value of this point are already in Figure2.

```
Question 1 Problem 1_FisherLDA
    "Min Perror is "    "0.2006"

    "Best threshold is "    "2.9196"

    "tp is "    "0.0034843"

    "fp is "    "0.00050056"
```

Q1.4.1:
Determine the Fisher LDA projection vector W_{LDA}:

Same as Q1.3.1

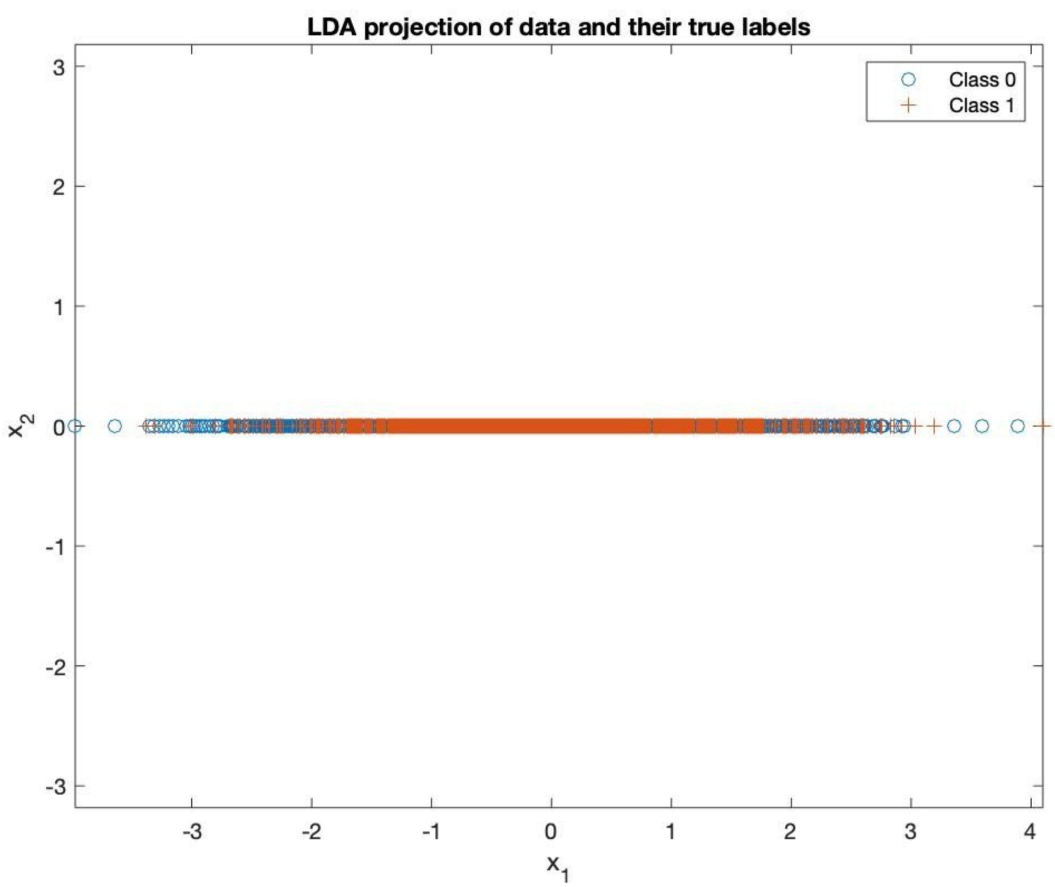

Figure 7. Scatter plot of 10000 Samples on LDA projection

Q1.3.2:
The steps to generate the ROC curve are the same as Q1.1.2. However, instead of using the scores generated by the classifier form Q1.1 and Q1.2, use the scores generated by classification rule w^T_{LDA}.

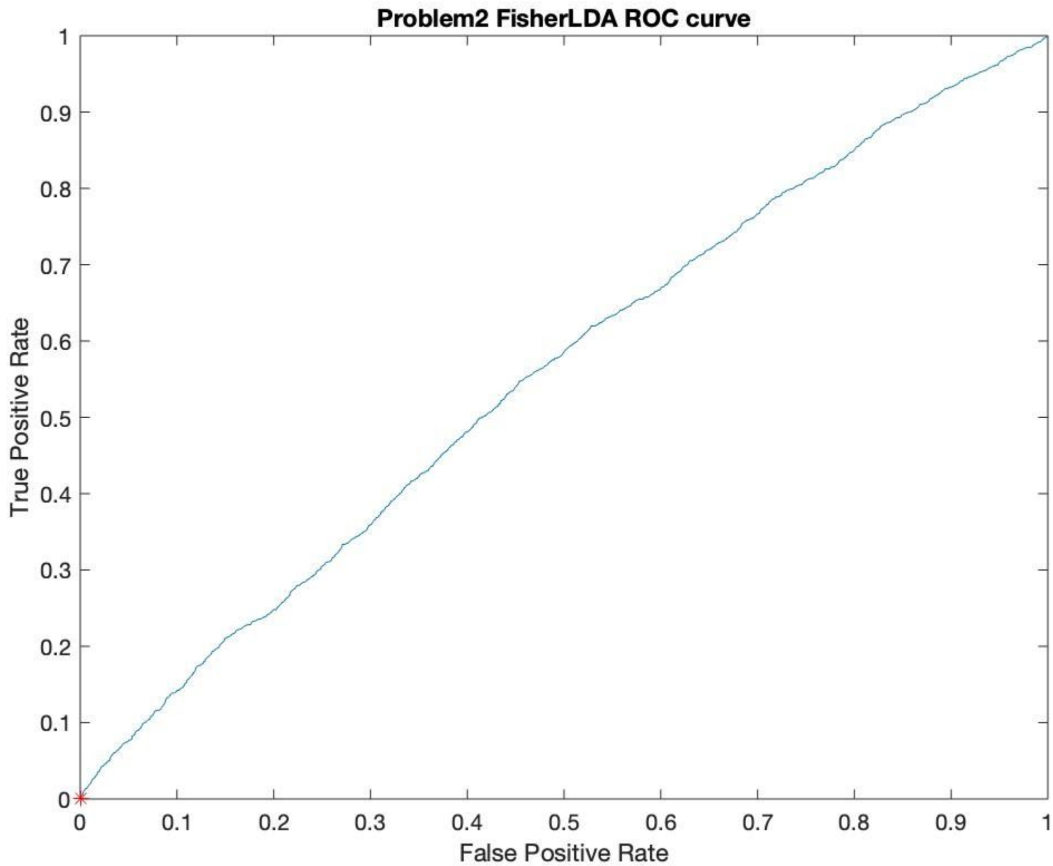

Figure 8. ROC curve of the Fisher LDA classification rule.

Q1.3.3:
The true positive value and false-positive value of this point are already in Figure2.

```
Question 1 Problem 2_FisherLDA
    "Min Perror is "    "0.2006"

    "Best threshold is "    "2.9196"

    "tp is "    "0"

    "fp is "    "0"
```

$$\frac{P(x|L=1)}{P(x|L=0)} = \frac{\alpha_1^1 g(x|m_1^1 C_1^1) + \alpha_2^1 g(x|m_2^1 C_2^1)}{\alpha_1^0 g(x|m_1^0 C_1^0) + \alpha_2^0 g(x|m_2^0 C_2^0)} \underset{D=0}{\overset{D=1}{\gtrless}} \gamma$$

Q2.1:
Class 0 prior: [0.4, 0.6]
Class 0 Gaussian1: mean = [3;5], covariance = [12 3; 3 6]
Class 0 Gaussian2: mean = [2;1], covariance = [3 -2; -2 9]
Class 1 prior: [0.6, 0.4]
Class 1 Gaussian1: mean = [0;2], covariance = [4 -2; -2 10]
Class 1 Gaussian2: mean = [3;4], covariance = [10 2; 2 4]

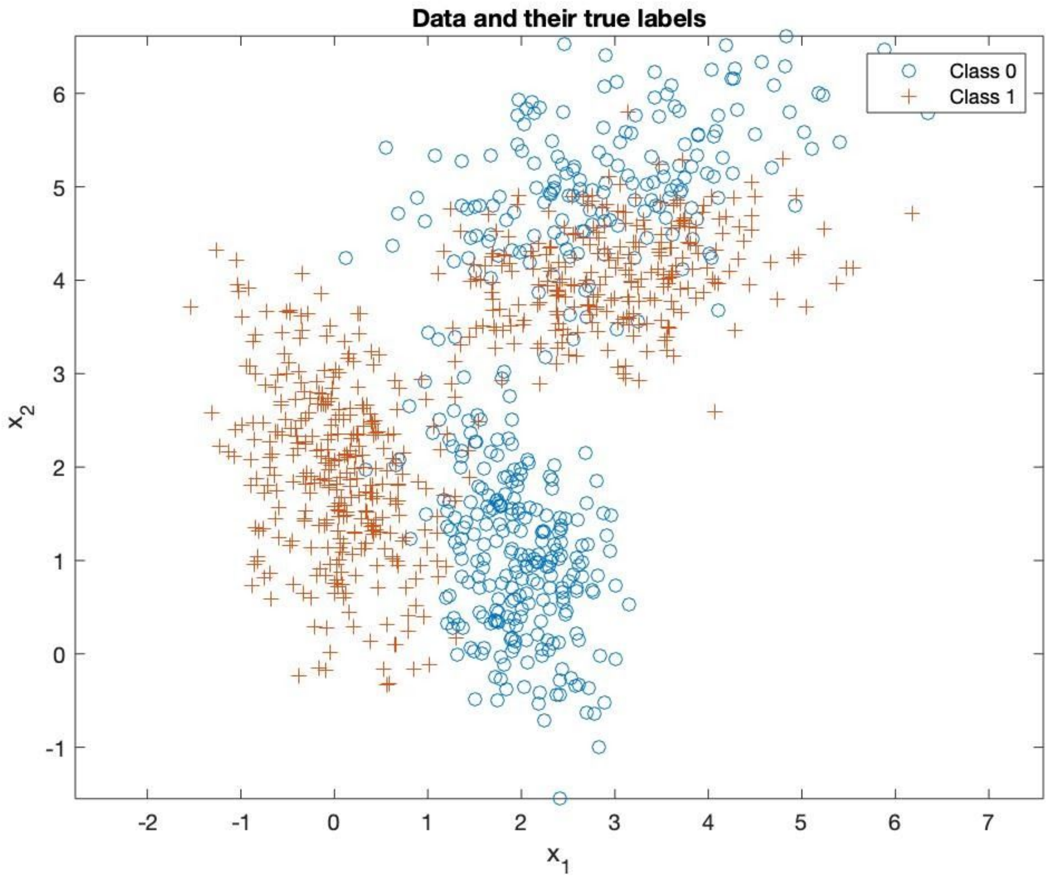

Figure 9. Scatter plot of 1000 Samples from class 0 and class 1

Q2.2:
Determine the minimum-P(error) classification rule:

$$D(x) = \arg\min R(D=i|x) \quad i \in \{0,1\}$$

$$= \arg\min \sum_{j=0}^{1} \lambda_{ij} P_j(x)/q_j \quad i \in \{0,1\}$$

where q is the perior.

$\lambda_{11} . \lambda_{00} = 1$

$\lambda_{10} . \lambda_{01} = 0$

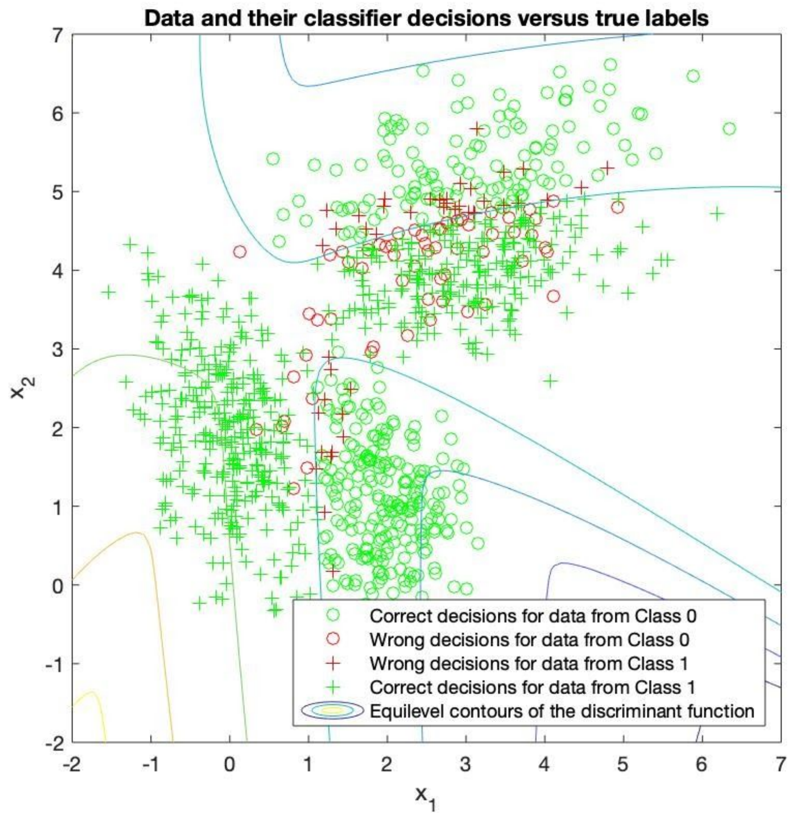

Figure 10. Scatter plot of 1000 data with the decision boundary

The estimate of the smallest probability of error: “0.1040”

Q3:
Determine the classification rule that achieves the minimum probability of error:

likelihood ratio base equation

$$\frac{p(x|L=1)}{p(x|L=0)} \underset{L=0}{\overset{L=1}{\gtrless}} \gamma \frac{p(L=0)}{p(L=1)}$$

Smallest error probability achievable by this classifier:

github: github.com/Alanbition/EECE5644

$$p(\text{error}) = \int_{-\infty}^{\infty} p(\text{error}|x)\,p(x)\,dx$$

$$= \int_{-\infty}^{0} p(L=1|x)\,p(x)\,dx + \int_{0}^{\infty} p(L=0|x)\,p(x)\,dx$$

$$= \int_{-\infty}^{0} p(x|L=1)\,p(L=1)\,dx + \int_{0}^{\infty} p(x|L=0)\,p(L=0)\,dx = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{0} e^{-\frac{(x-2)^2}{2}}\,dx + \frac{1}{\sqrt{2\pi}}\int_{0}^{\infty} e^{-\frac{(x+2)^2}{2}}\,dx$$