Assignment 1

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Problem Statement

In the system shown in Figure , the inertia, J, of radius, r, is constrained to move only about the stationary axis A. A viscous damping force of translational value f_v exists between the bodies J and M. If an external force, f(t), is applied to the mass, find the transfer function, $G(s) = \frac{Q(s)}{F(s)}$.

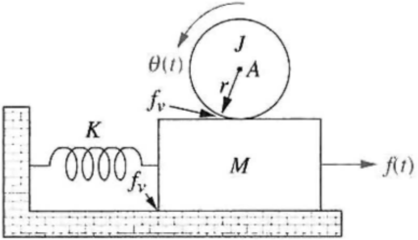


FIGURE P2.25

Analyzing the given system

In the given system, there are 2 bodies whose motion must be analysed.

- Body 1- Body of mass M which is connected to the spring (spring constant K) - This body has only translational motion and no rotational motion.
- Body 2- Body of inertia J which is constrained to move about stationary axis A - This body has rotational motion and no translational motion.

We are already given that the viscous drag force is f_v in *translational domain*.
And force is applied on $Body - 1$, transfer function which must be found is $G(s) = \frac{Q(s)}{F(s)}$.

Forces on body 1

Let us assume that displacement of Body-1 $= x(t)$ whose Laplace transform is $X(s)$. All the forces are written in Laplace domain. Force on body-1 due to its own motion.(Body-1 has translational motion)

- Force due to its mass : $Ms^2X(s)$ (opposite to the direction of motion)
- Force due to spring : $KX(s)$(opposite to the direction of motion)
- Force due to viscous drag force : $f_v sX(s)$(opposite to the direction of motion)
- Force acting on the body-1 : $\mathcal{L}(f(t)) =$ F(s)(in the direction of motion)

Force on body-1 due to the motion of body-2.
Force due to viscous drag is given by $f_v v(t) = f_v \frac{dx(t)}{dt}$
Velocity of body-2 at the point of contact with body-1
$v(t) = r\frac{d\Theta(t)}{dt}$
Thus force $= f_v r \frac{d\Theta(t)}{dt}$
Laplace transform $= \mathcal{L}(f_v r \frac{d\Theta(t)}{dt}) = f_v rs\Theta(s)$(in the direction of motion)

Forces on body-2

Body-2 has angular displacement $= \Theta(t)$ whose Laplace transform is $\mathcal{L}(\Theta(t)) = \Theta(s)$. All torques are written in Laplace domain. Torque on body-2 because of its own motion. (Body-2 has rotational motion).

- Torque due to its inertia : $Js^2\Theta(s)$(opposite to the direction of motion)
- Torque due to viscous drag :
  Torque $= \mathbf{r} x \mathbf{F}$
  Force due to viscous drag $= f_v v(t)$ where $v(t) = r\frac{d\Theta}{dt}$
  Torque $= rf_v r\frac{d\Theta}{dt}$
  Laplace transform $= \mathcal{L}(rf_v r\frac{d\Theta}{dt}) = f_v r^2 s\Theta(s)$(opposite to the direction of motion)

Torque on body-2 because of body-1
Torque $= \mathbf{r} x \mathbf{F}$
Force due to viscous drag $= f_v v(t) = f_v \frac{dx(t)}{dt}$
Torque $= r f_v \frac{dx(t)}{dt}$
Laplace Transform $= \mathcal{L}(r f_v \frac{dx(t)}{dt}) = f_v r s X(s)$(in the direction of motion)

For body-1 :

$$(Ms^2 + K + f_v s)X(s) - F(s) - f_v rs\Theta(s) = 0 \tag{2.1}$$

$$(Ms^2 + K + f_v s)X(s) - f_v rs\Theta(s) = F(s) \tag{2.2}$$

For body-2 :

$$(Js^2 + f_v r^2 s)\Theta(s) - f_v rsX(s) = 0 \tag{2.3}$$

$$-f_v rsX(s) + (Js^2 + f_v r^2 s)\Theta(s) = 0 \tag{2.4}$$

We have to solve for $G(s) = \frac{\Theta(s)}{F(s)}$

We can write the equations using this formula too.
[Sum of impedances connected to the motion of body-1]X(s) - [Sum of impedances between body-1 and body-2]Θ(s) = Force acting on body-1
- [Sum of impedances between body-1 and body-2]X(s)+ [Sum of impedances connected to the motion of body-2]Θ(s) = Torque acting on body-2

We need to solve for $G(s) = \frac{\Theta(s)}{F(s)}$
Using the formula (Co-factor matrix), we can find $\Theta(s)$ in terms of F(s):-

$$\Theta(s) = \frac{\begin{vmatrix} Ms^2 + 2f_v s + K & F(s) \\ -f_v rs & 0 \end{vmatrix}}{\begin{vmatrix} Ms^2 + 2f_v s + K & -f_v rs \\ -f_v rs & Js + f_v r^2 s \end{vmatrix}}$$

On solving, we get

$$\Theta(s) = \frac{f_v rsF(s)}{JMs^3 + (2Jf_v + Mf_v r^2)s^2 + (JK + f_v^2 r^2)s + Kf_v r^2} \quad (2.5)$$

Transform Function

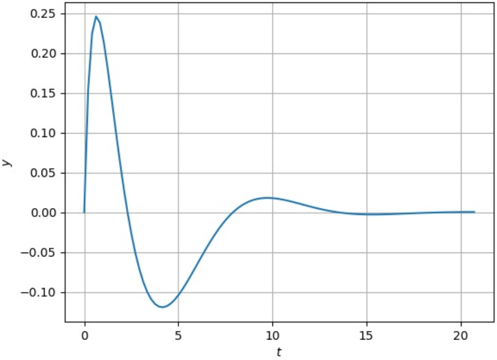
$$G(s) = \frac{\Theta(s)}{F(s)} = \frac{f_v rs}{JMs^3 + (2Jf_v + Mf_v r^2)s^2 + (JK + f_v^2 r^2)s + Kf_v r^2} \tag{2.6}$$

Plot of Transfer function

Code in github plots the following fig.

The following plot shows the impulse response of transform function when J = 1; r = 1; M = 1; K= 1; $f_v = 1$;

Transform function $= \frac{s}{s^3+3s^2+2s+1}$



Plot of Transfer function

Code in github plots the following fig.

The following plot shows the step response of transform function when J = 1; r = 1; M = 1; K= 1; $f_v = 1$;

Transform function $= \frac{s}{s^3+3s^2+2s+1}$