

CS 5012

Module 0 Homework

Alanna Hazlett (uwa6xv)

September 2nd, 2024

Problem 1

Given the predicates and their meanings and utilizing real numbers give the truth values for each proposition.

$P(x,y): x > y$

$Q(x,y): x \leq y$

$R(x): x - 7 = 2$

$S(x): x > 9$

(i.) $(\exists x)R(x)$

Do there exist numbers where $x - 7 = 2$?

True

(ii.) $(\forall y)[\neg S(y)]$

Are all real numbers not greater than 9?

False

(iii.) $(\forall x)(\exists y)P(x, y)$

For all Xs are there Ys where $x > y$?

True

(vi.) $(\exists y)(\forall x)Q(x, y)$

Does there exist any y for all x, such that $x \leq y$?

True

(v.) $(\forall x)(\forall y)[P(x, y) \vee Q(x, y)]$

For all Xs and all Ys is $x > y$ or $x \leq y$?

True

$$(vi.) (\exists_x)S(x) \wedge \neg(\forall_x)R(x)$$

For some x, is $x > 9$ and not all Xs $x - 7 = 2$?

True

$$(vii.) (\exists_y)(\forall_x)[S(y) \wedge Q(x, y)]$$

For some y and all Xs is $y > 9$ and $x \leq y$?

False

$$(viii.) (\forall_x)(\forall_y)[R(x) \wedge S(y) \rightarrow Q(x, y)]$$

For all Xs and all Ys is $x - 7 = 2$ and $y > 9$ true, implying that $x \leq y$?

If $x - 7 = 2$ and $y > 9$ then $x \leq y$?

True

Problem 2

Which sentence has the form $(p \wedge q) \rightarrow r$?

c.) If it rains and it snows then flooding will result

Problem 3

Which formula represents the sentence "If there is no fruit in the market then the farmers didn't plant."

p = There are no fruit in the market

q = Farmers didn't plant fruit trees

r = Farmers didn't water the trees

b.) $p \rightarrow q \vee r$

Problem 4

Show $[p \wedge (p \rightarrow q)] \rightarrow q$ is a tautology

$$[p \wedge (\neg p \vee q)] \rightarrow q$$

$$[(p \wedge \neg p) \vee (p \wedge q)] \rightarrow q$$

$$[(F) \vee (p \wedge q)] \rightarrow q$$

$$(p \wedge q) \vee q$$

$$\neg(p \wedge q) \vee q$$

$$\neg p \vee \neg q \vee q$$

$$\neg p \vee T$$

$$True$$

Problem 5

Argue that Set A and Set A' (complement of A) are disjoint.

Disjoint means that they have no common element.

The complement of a set is the set of elements not found in the other set. For this situation Set A = {2,3,4,5} and it's complement will be a set of numbers not found in Set A, A'={ } an empty set. These two sets will have no common elements, meaning they are disjoint.

Problem 6

Which is a one-to-one function?

e.) {(1,2),(2,4),(3,6),(4,8)}

Problem 7

Let $U = \{x: x \text{ is an integer and } 2 \leq x \leq 10\}$. In each case determine whether $A \subseteq B$, $B \subseteq A$, neither, or both.

(i.) $A = \{x: x \text{ is odd}\} = \{3,5,7,9\}$ and $B = \{x: x \text{ is a multiple of 3}\} = \{3,6,9\}$

neither

(ii.) $A = \{x: x \text{ is even}\} = \{2,4,6,8,10\}$ and $B = \{x: x^2 \text{ is even}\} = \{2,4,6,8,10\}$

both

(iii.) $A = \{x: x \text{ is even}\} = \{2,4,6,8,10\}$ and $B = \{x: x \text{ is a power of 2}\}$

both

(iv.) $A = \{x: 2x+1 > 7\} = \{4,5,6,7,8,9,10\}$ and $B = \{x: x^2 > 20\} = \{5,6,7,8,9,10\}$

$B \subseteq A$

(v.) $A = \{x: \sqrt{x} \in \mathbb{Z}\} = \{4,9\}$ and $B = \{x: x \text{ is a power of 2 or 3}\} = \{2,3,4,6,8,9,10\}$

$$A \subseteq B$$

(vi.) $A = \{x: \sqrt{x} \leq 2\} = \{2,3,4\}$ and $B = \{x: x \text{ is a perfect square}\} = \{4,9\}$

neither

(vii.) $A = \{x: x^2 - 3x + 2 = 0\} = \{2\}$ and $B = \{x: x + 7 \text{ is a perfect square}\} = \{2,9\}$

$$A \subseteq B$$

I pledge that I have neither given nor received help on this assignment. : Alanna Hazlett

In []: