Stat 6021: Homework Set 11 Solutions

2. (a) $\hat{\beta}_3 = 0.43397$. A few interpretations:
   - The estimated difference in log odds between males and females is 0.43397, for given value of age and awareness.
   - The estimated odds ratio of getting a flu shot between males and females is $\exp(0.43397) = 1.543373$, for given value of age and awareness.
   - The estimated odds of getting a flu shot for males is 1.543373 times the estimated odds of getting a flu shot for females, for given value of age and awareness.

   (b) The Wald test for β_3 is

$$\begin{aligned} H_0 &: \beta_3 = 0. \\ H_a &: \beta_3 \neq 0. \end{aligned}$$

   The test statistic is

$$\begin{aligned} Z &= \frac{\hat{\beta}_3}{s\{\hat{\beta}_3\}} \\ &= \frac{0.43397}{0.52179} \\ &= 0.832. \end{aligned}$$

   The corresponding p-value is $2 \times P(Z > 0.832) = 0.4054089$ using `2*(1-pnorm(0.832))`. We fail to reject the null hypothesis. Gender is not a significant predictor of the probability of getting a flu shot, given age and awareness.

   (c) The 95% CI is $\hat{\beta}_j \pm Z_{1-\alpha/2} se(\hat{\beta}_j)$:

$$0.43397 \pm 1.96 \times 0.52179 = (-0.5887, 1.4567).$$

   $Z_{1-\alpha/2}$ found by `qnorm(1-0.05/2)` which gives 1.96.
   We have 95% confidence the odds of getting a flu shot for males is between $(\exp -0.5887, \exp 1.4567) = (0.5550, 4.2917)$ times the odds of getting a flu shot for females, for given value of age and awareness.

(d) The conclusions are consistent since we failed to reject the null hypothesis that $\beta_3 = 0$ and 0 lies in the 95% CI.

(e) The hypotheses are

$$\begin{aligned} H_0 &: \beta_1 = \beta_3 = 0. \\ H_a &: \text{at least one } \beta_k \text{ above is not zero.} \end{aligned}$$

The test statistic is

$$\begin{aligned} \Delta G^2 &= \text{Residual deviance of full model} - \text{Residual deviance of reduced model} \\ &= 113.20 - 105.09 \\ &= 8.11. \end{aligned}$$

The corresponding p-value is $P(8.11 > \chi^2_2) = 0.01733548$ using `1-pchisq(8.11,2)`. We cannot drop both age and gender from the model.

(f) The estimated log odds are

$$\begin{aligned} \log\left(\frac{\pi}{1-\pi}\right) &= -1.1772 + 0.0728 \times 70 - 0.0990 \times 65 + 0.4340 \\ &= -2.0822. \end{aligned}$$

Therefore, the estimated odds are $\exp(-2.0822) = 0.1247$. The estimated probability is $\frac{0.1247}{1+0.1247} = 0.1108$.