University of Pittsburgh

Algorithms and Data Structures 2
CS 1501

Spring 2023

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(Slides are adapted from Dr. Ramirez's and Dr. Farnan's CS1501 slides.)

Announcements

- Upcoming Deadlines
  - Homework 3: this Friday @ 11:59 pm
  - Lab 2: Tuesday 2/7 @ 11:59 pm
  - Assignment 1: Friday 2/17 @ 11:59 pm
- Please make your Piazza posts public as much as possible

Previous lecture

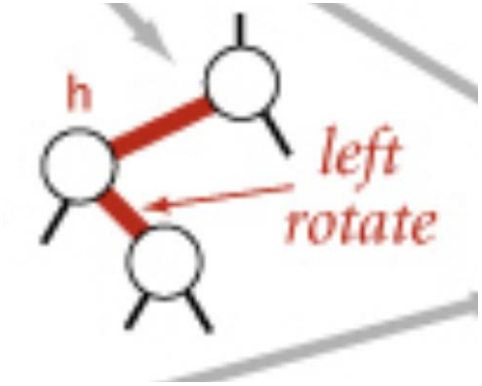
- Red-Black BST (self-balancing BST)
  - definition and basic operations
  - delete
  - runtime of operations
- Turning recursive tree traversals to iterative

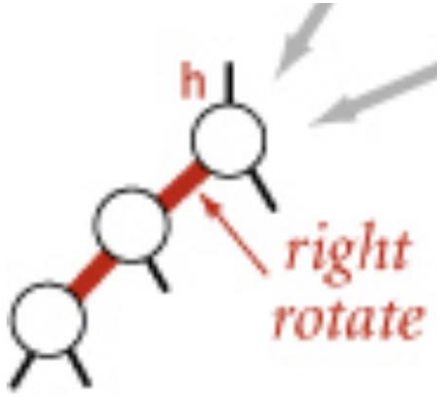
Today …

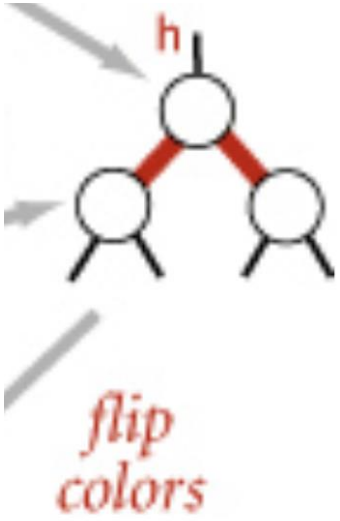
- Red-Black BST (self-balancing BST)
  - add
  - delete
  - runtime of operations
- Turning recursive tree traversals to iterative

Fixing Violations using Recursion

- Violations are checked and corrected as we climb back up the tree.
- The code for violation corrections is after the recursive calls.
- Violations are detected and corrected in the following order (*h* is the current node)







Adding to a RB-BST: Example

- Let's add the numbers:
- 1, 2, 3, 4, 5, 6, 7

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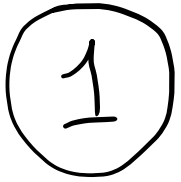

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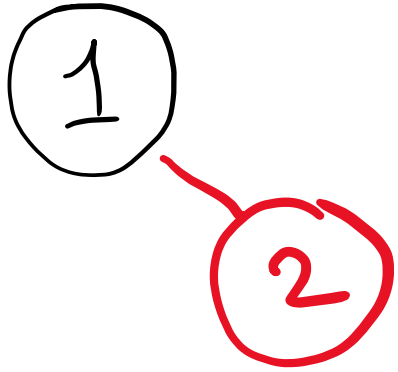

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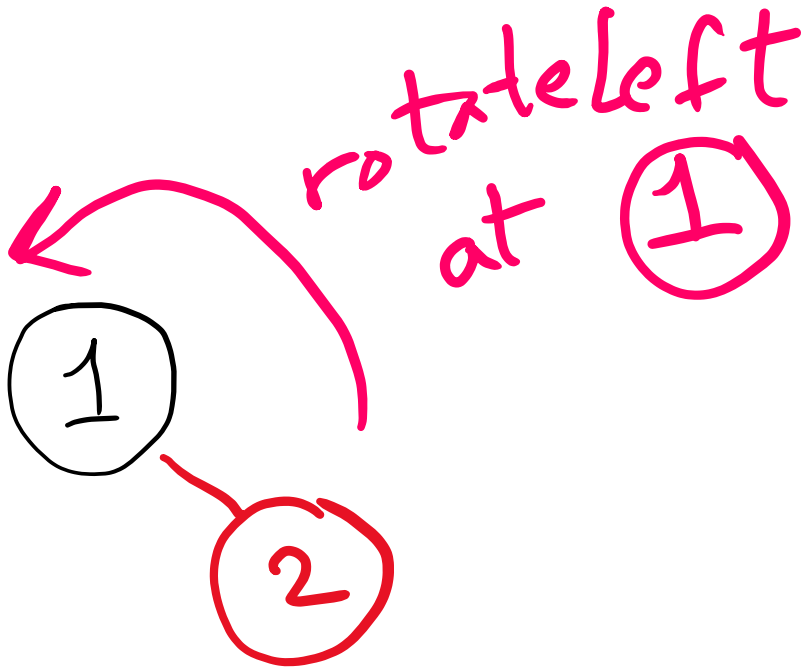

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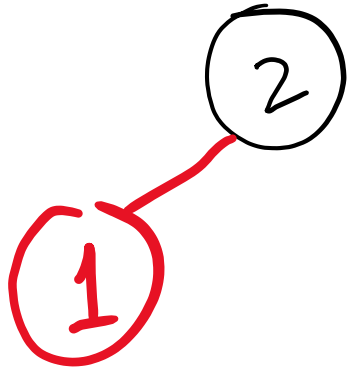

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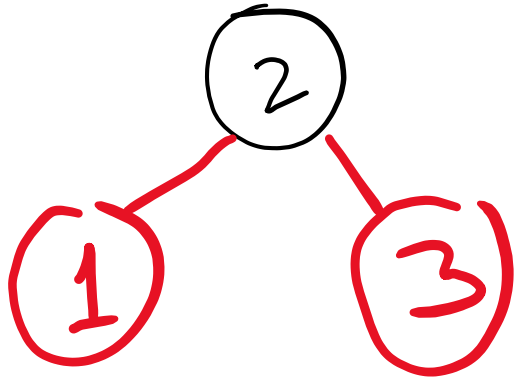

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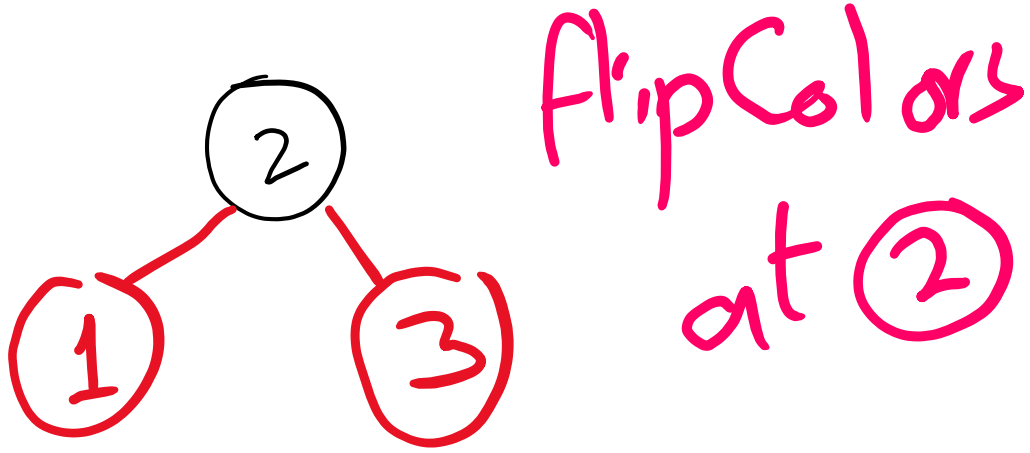

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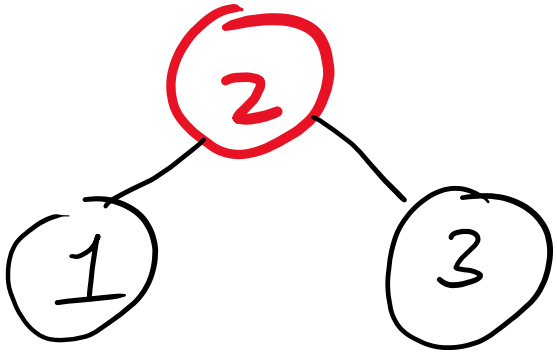

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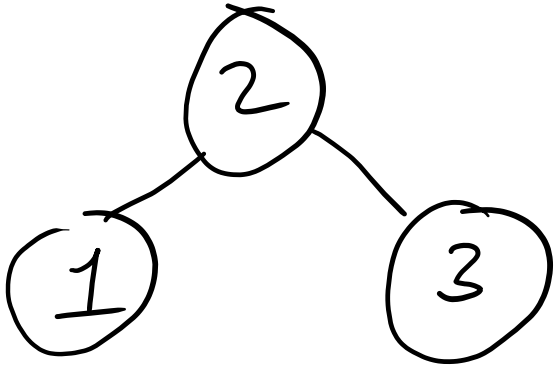

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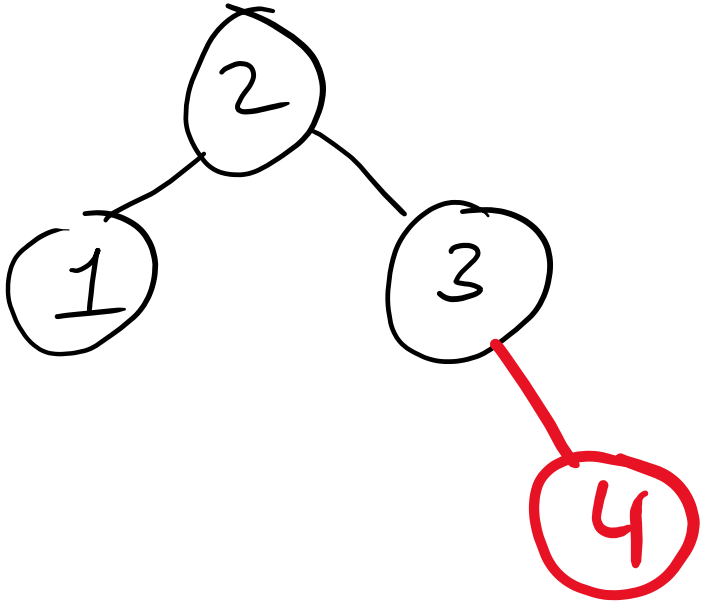

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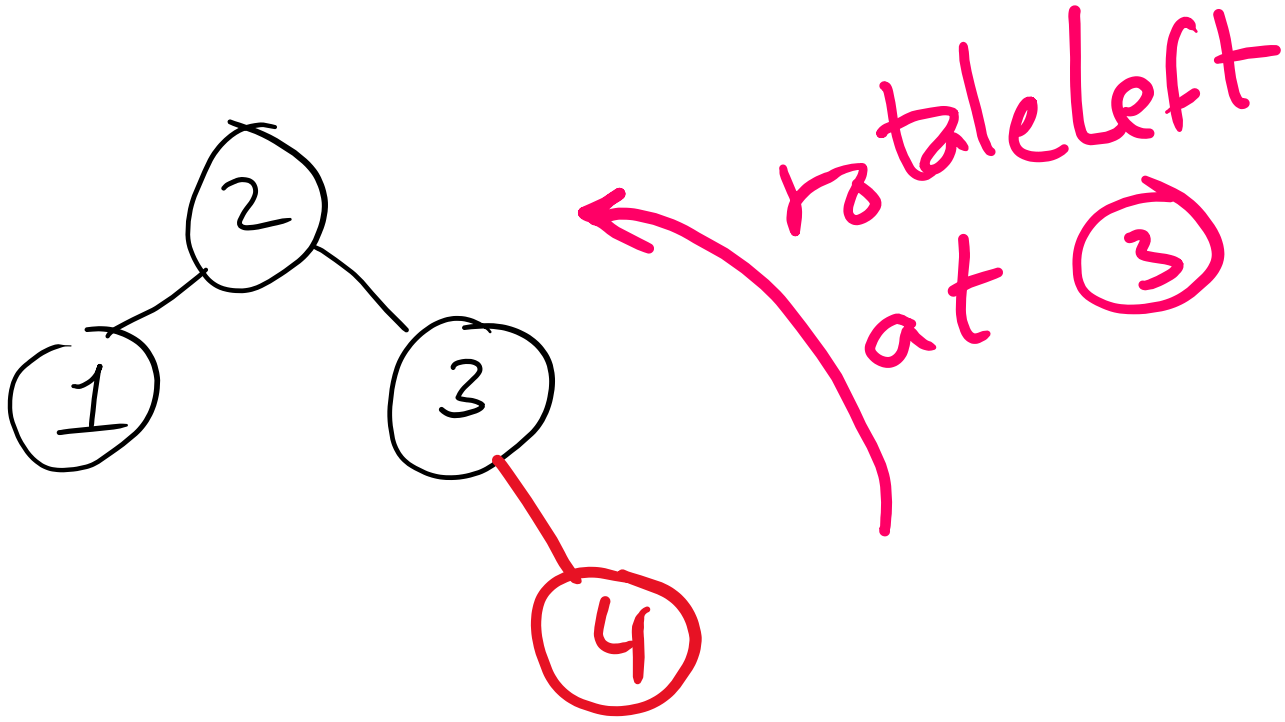

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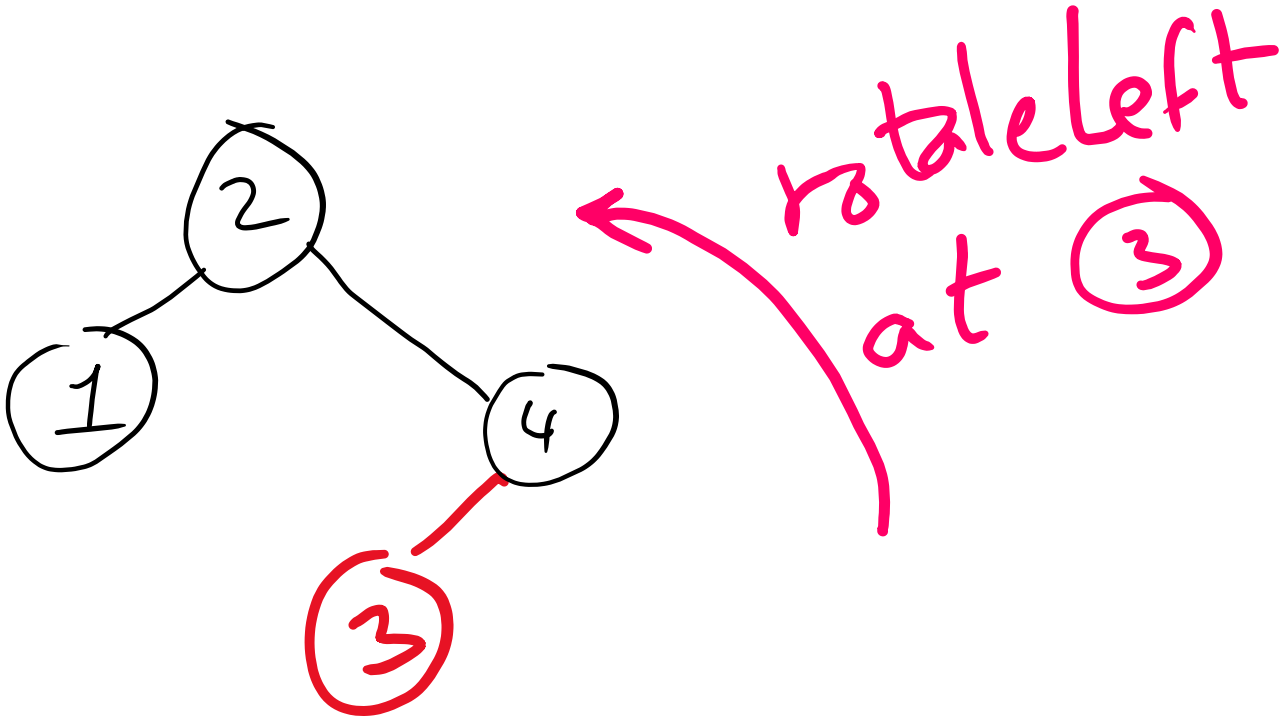

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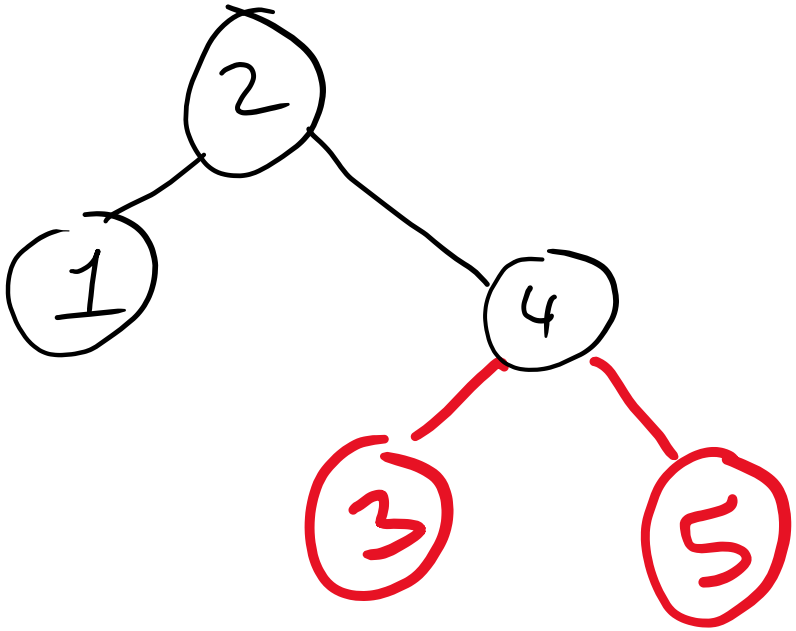

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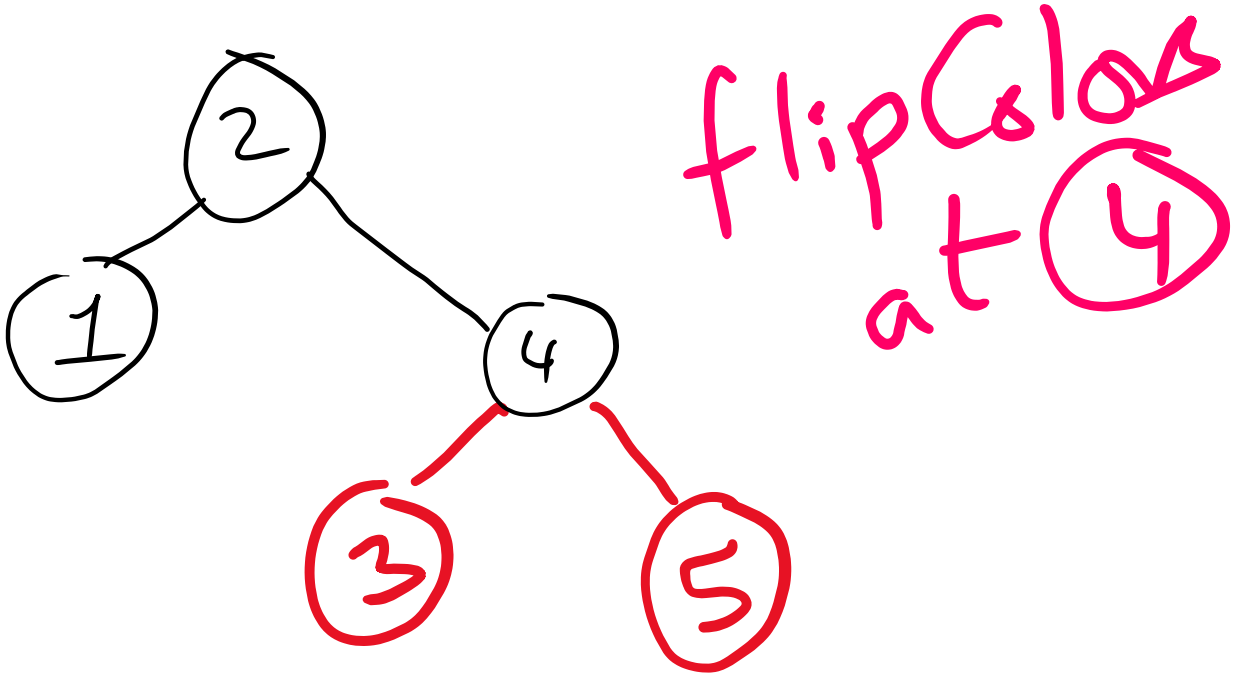

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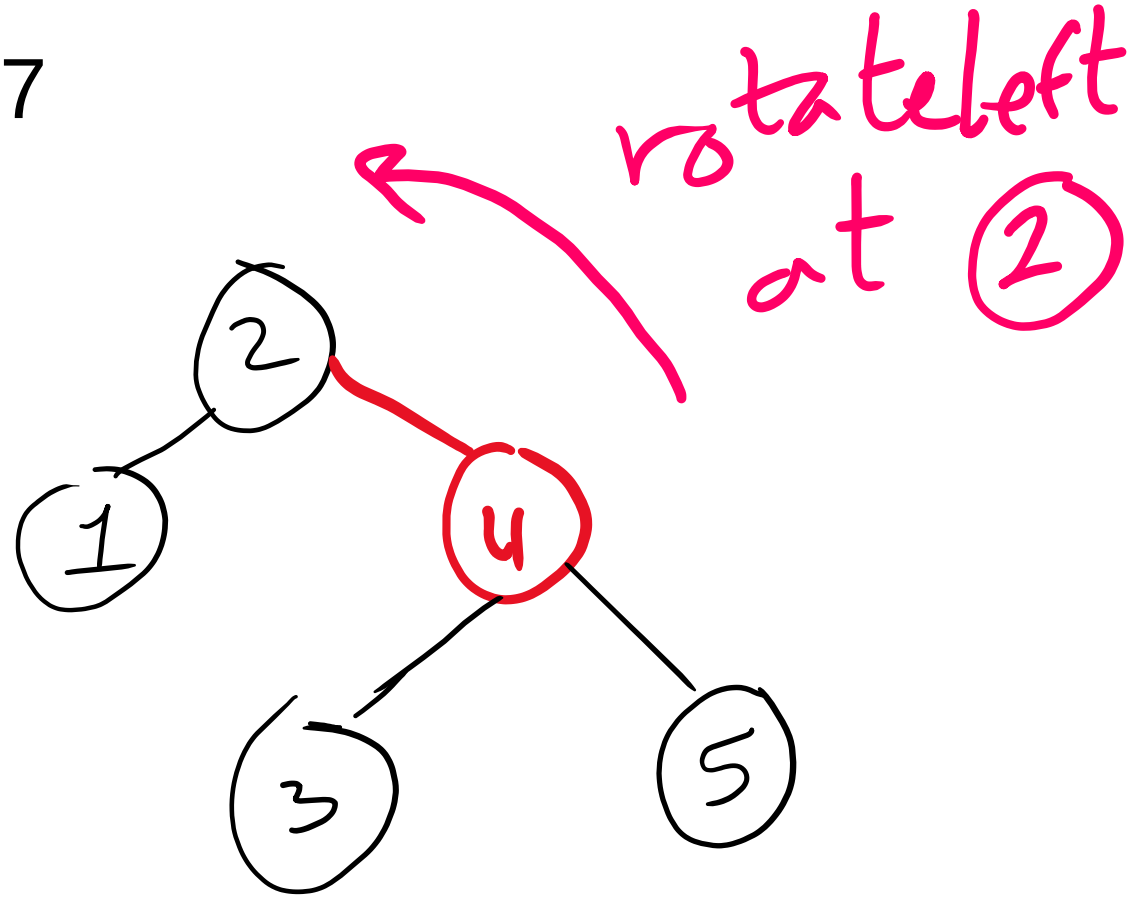

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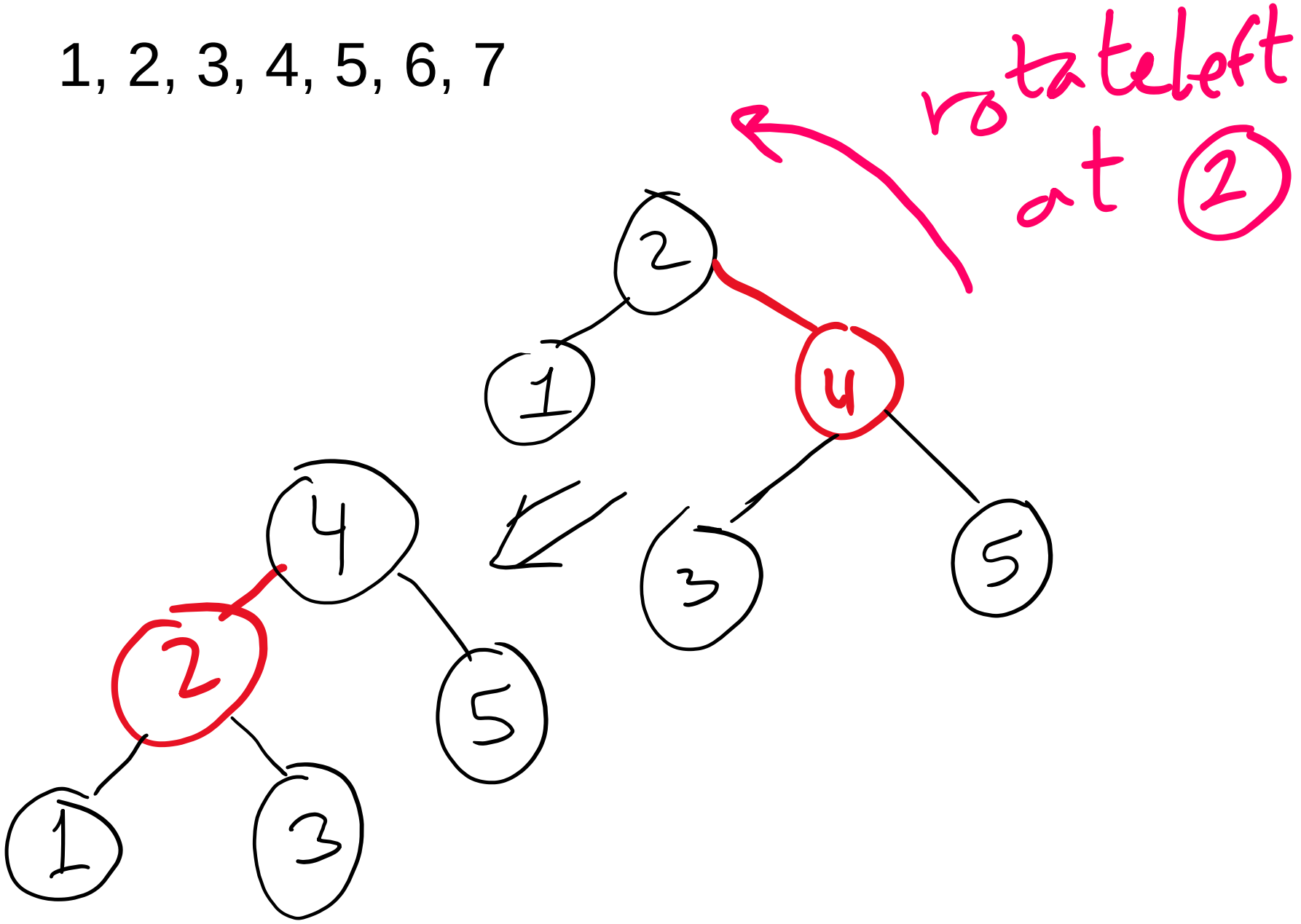

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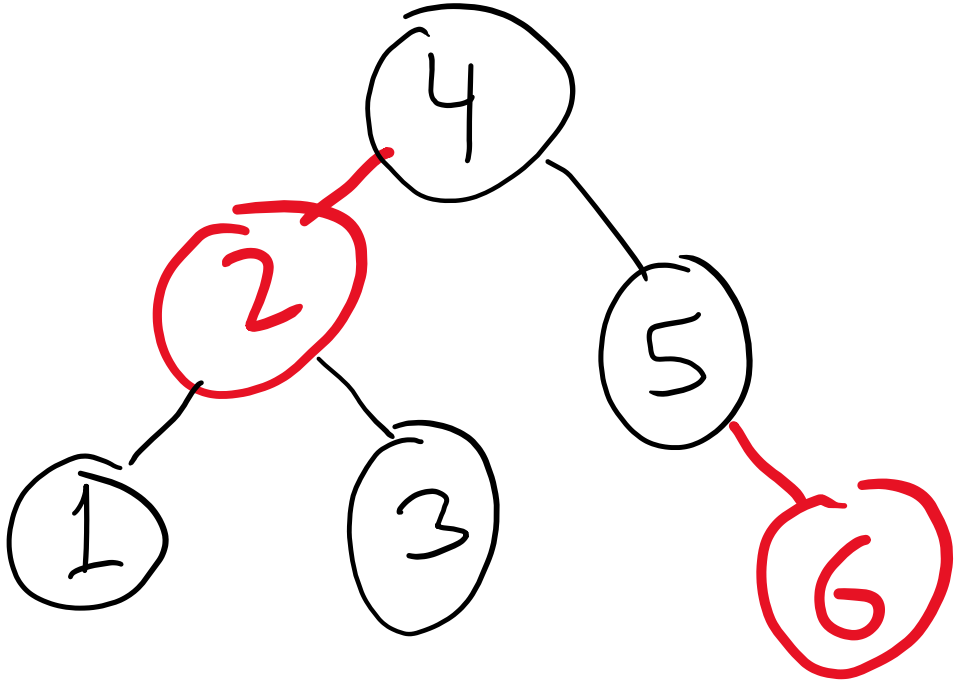

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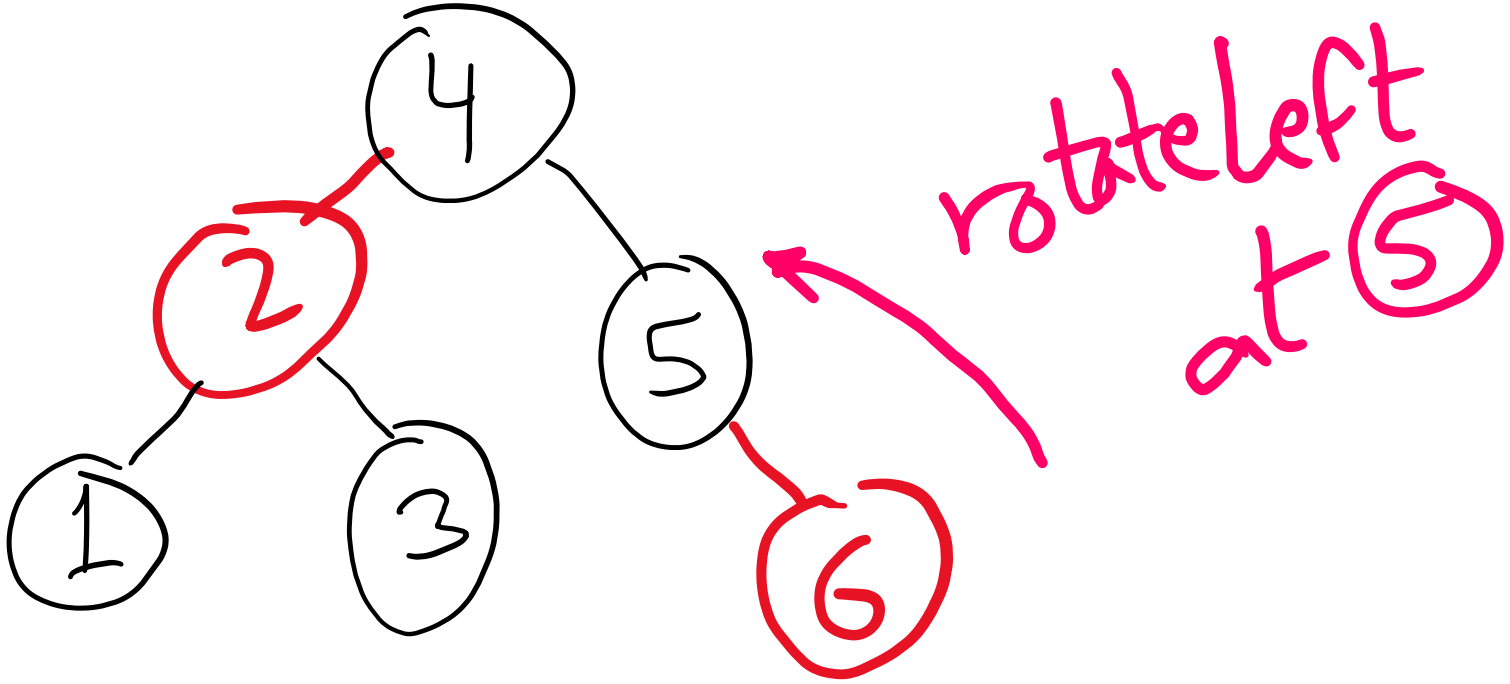

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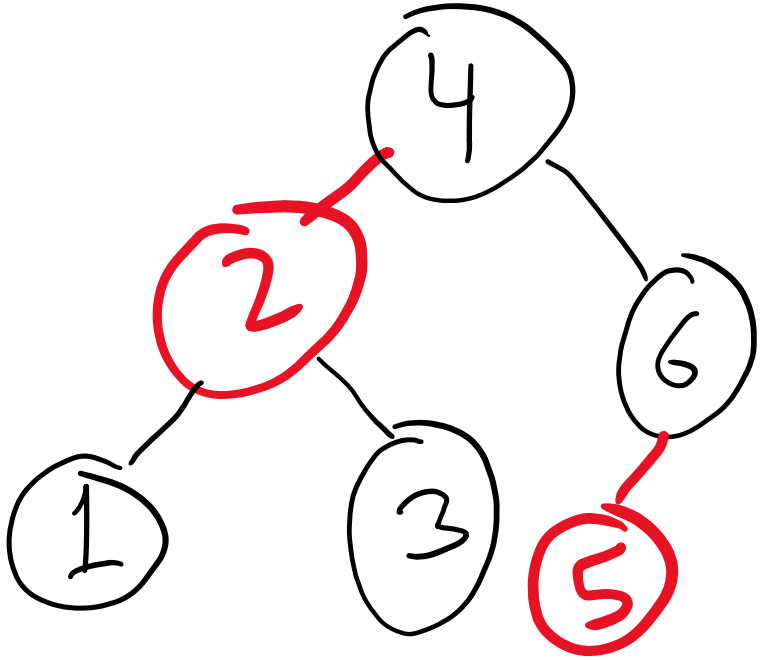

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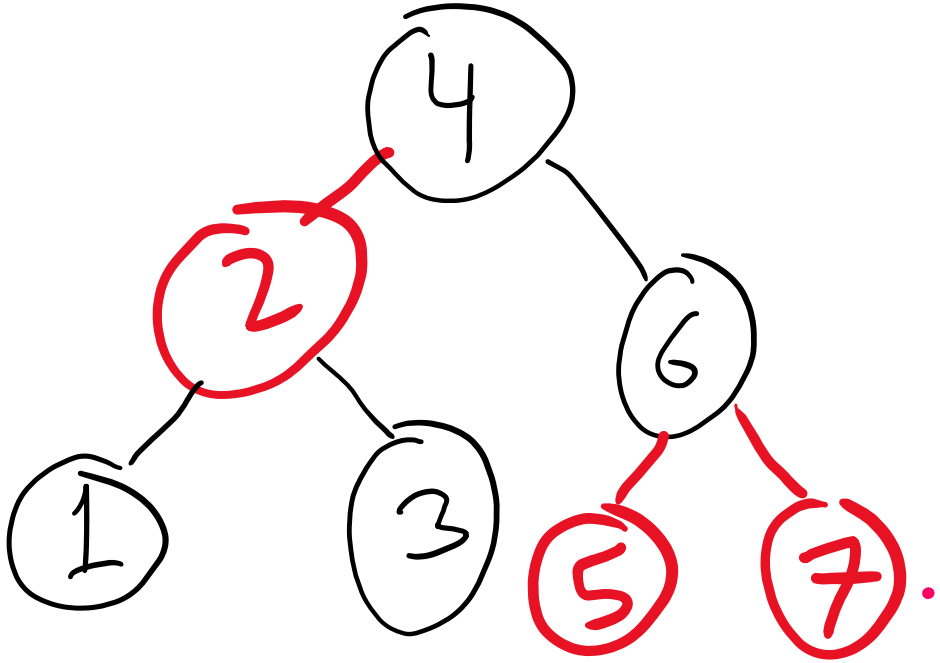

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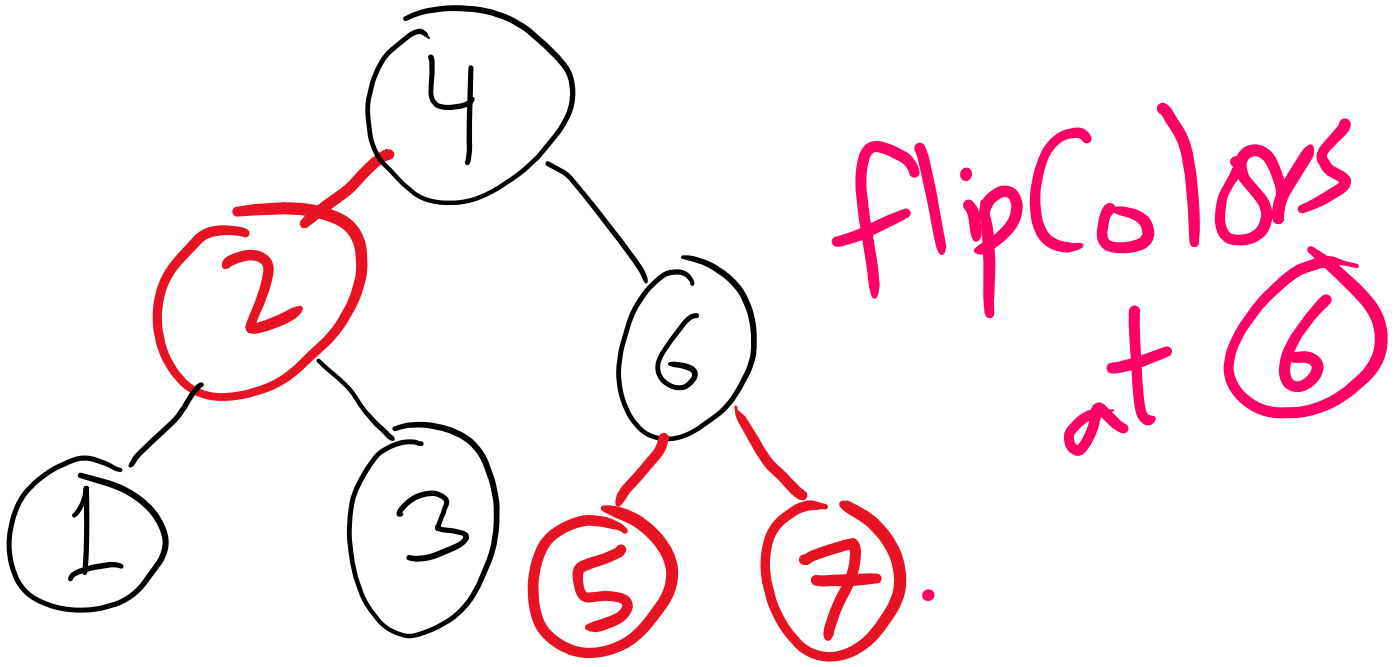

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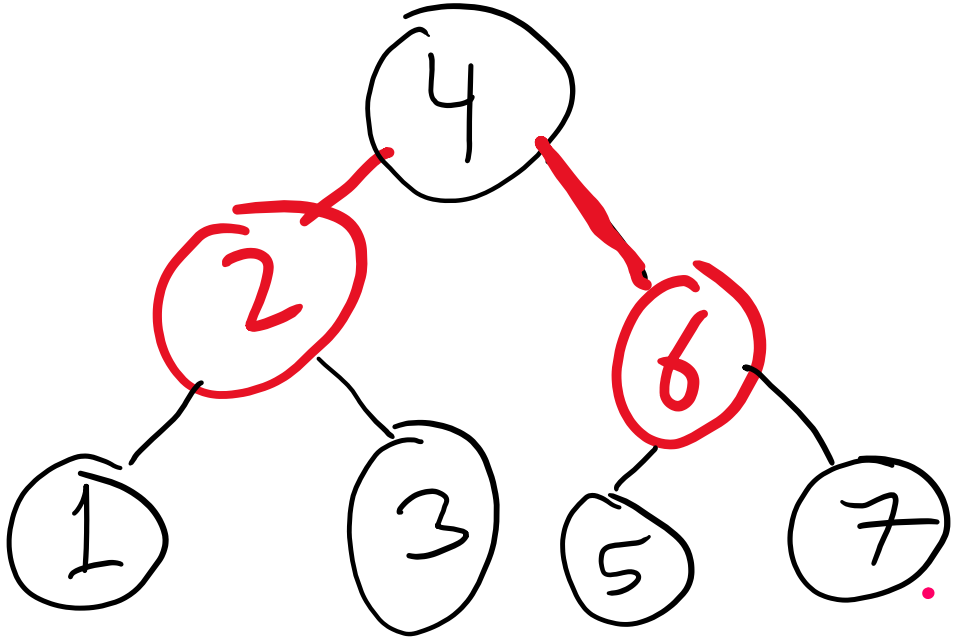

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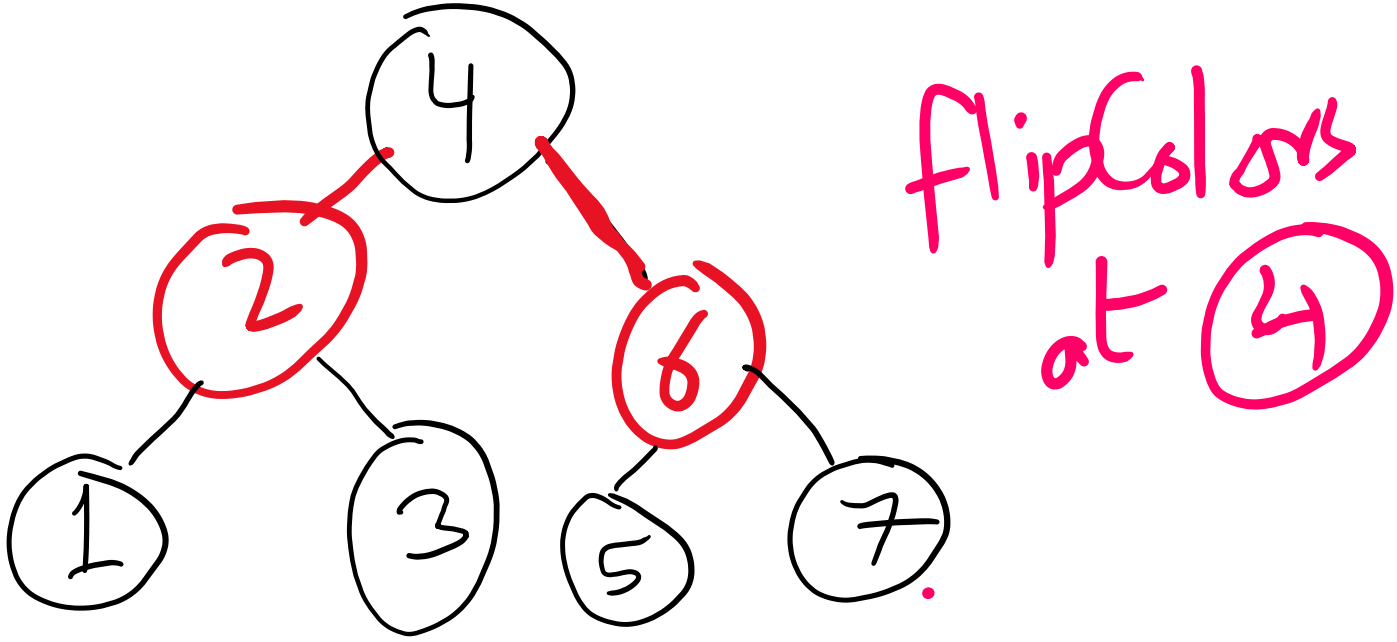

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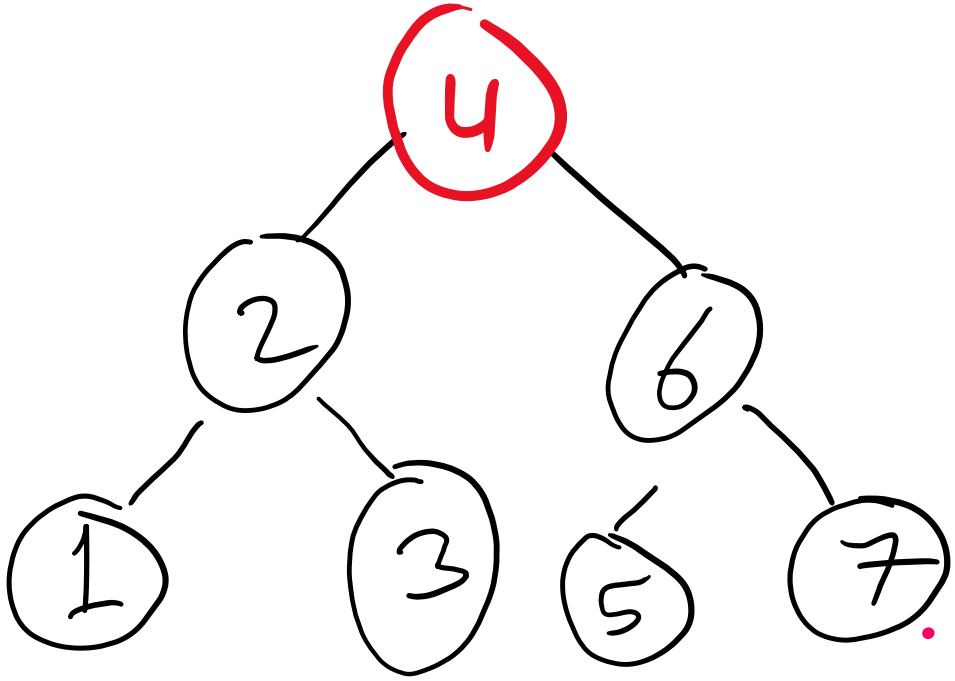

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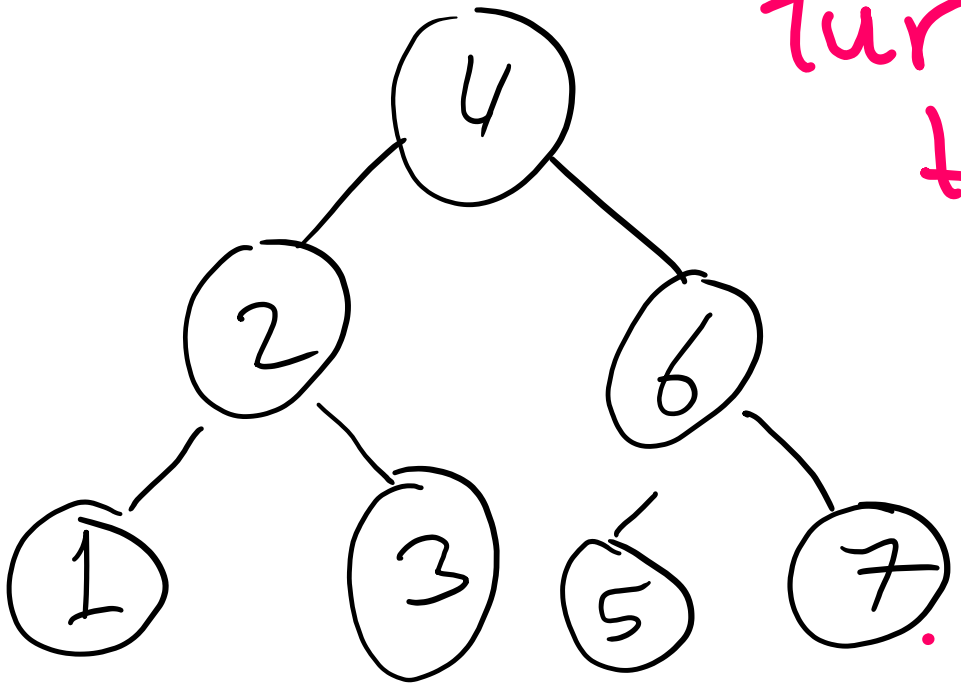
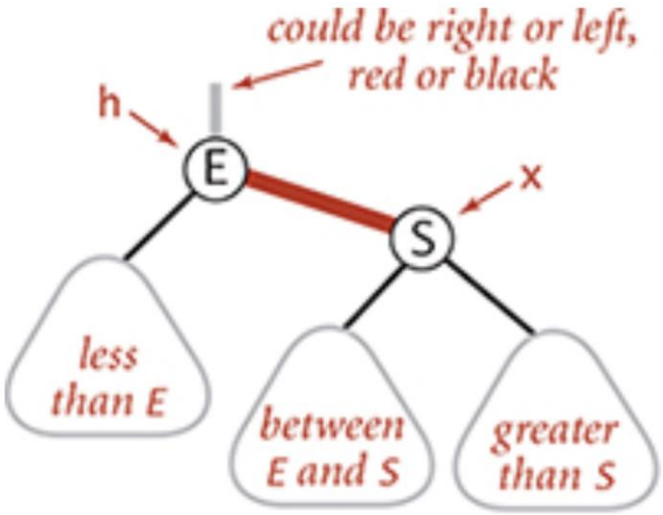
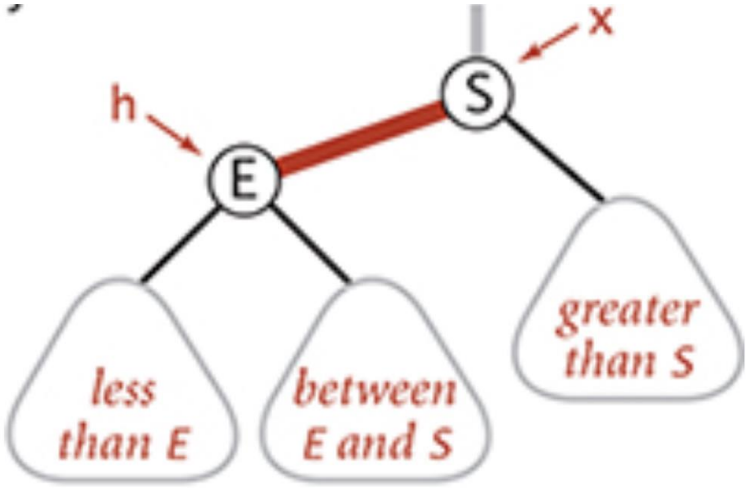


turn root to black

Left Rotate

- **BinaryNode<T> rotateLeft(BinaryNode<T> h)** performs left rotation at node *h*. The code emerges from contrasting the before and after pictures



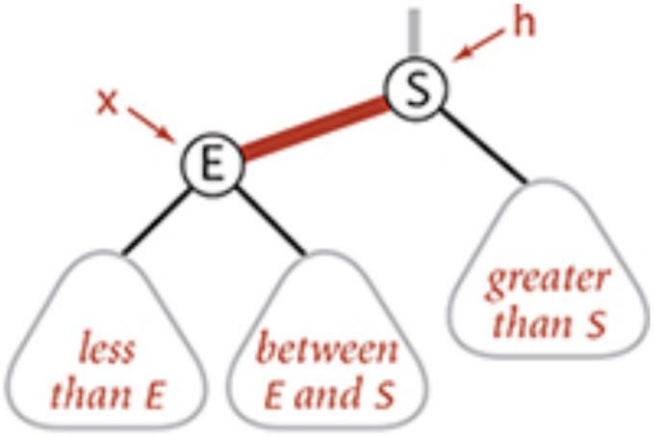


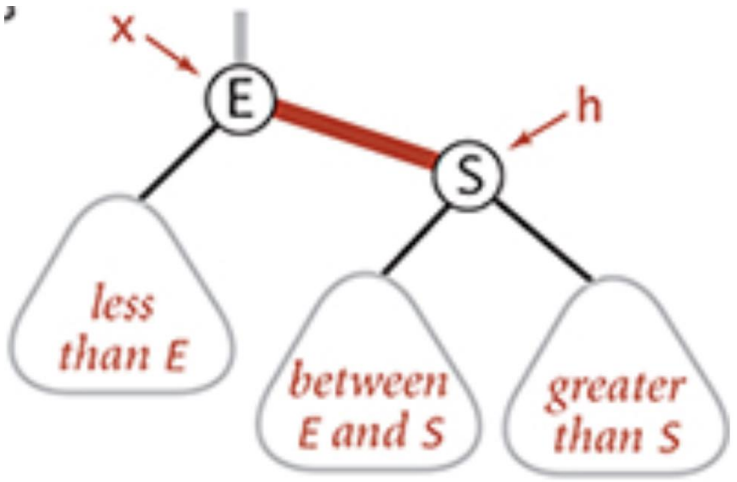

```
x
h.right = x.left;
x.left = h;
x.color = h.color;
h.color = RED;
return x;
```

- rotateLeft maintains the search tree property
  - why?
- It also maintains the perfect balance property
  - why?

Right Rotate

- **BinaryNode<T> rotatRight(BinaryNode<T> h)** performs right rotation at node *h*. The code emerges from contrasting the before and after pictures



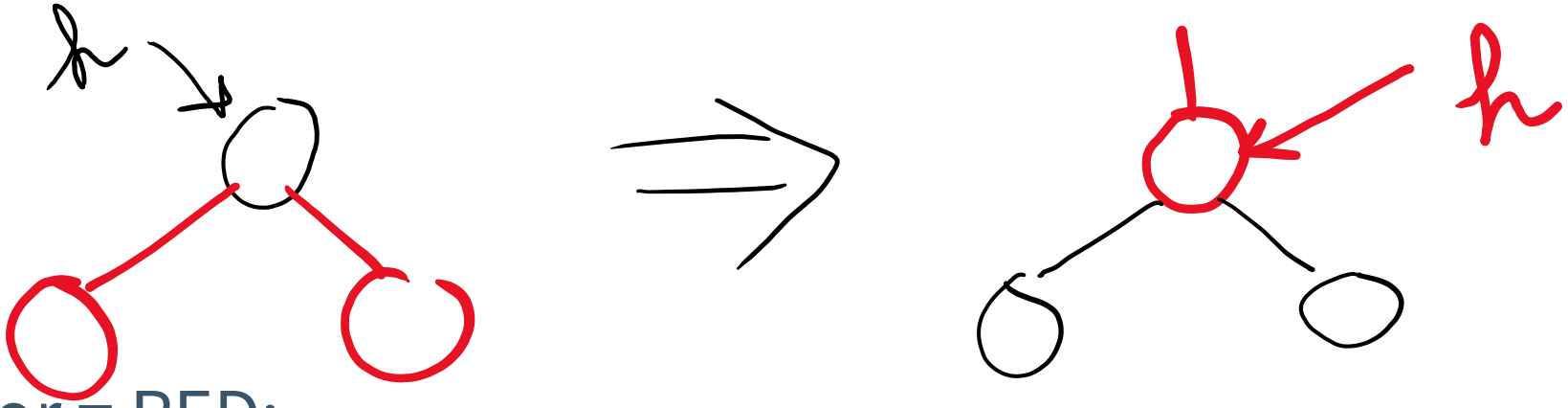



```
x = h.left;
h.left = x.right;
x.right = h;
x.color = h.color;
h.color = RED;
return x;
```

- rotateRight maintains the search tree property
  - why?
- It also maintains the perfect balance property
  - why?

Color Flip

- **BinaryNode<T> flipColors(BinaryNode<T> h)** performs color flip at node *h*. The code emerges from contrasting the before and after pictures




```
h.color = RED;
h.right.color = BLACK;
h.left.color = BLACK;
return h;
```

- flipColors maintains the search tree property
  - why?
- It also maintains the perfect balance property
  - why?

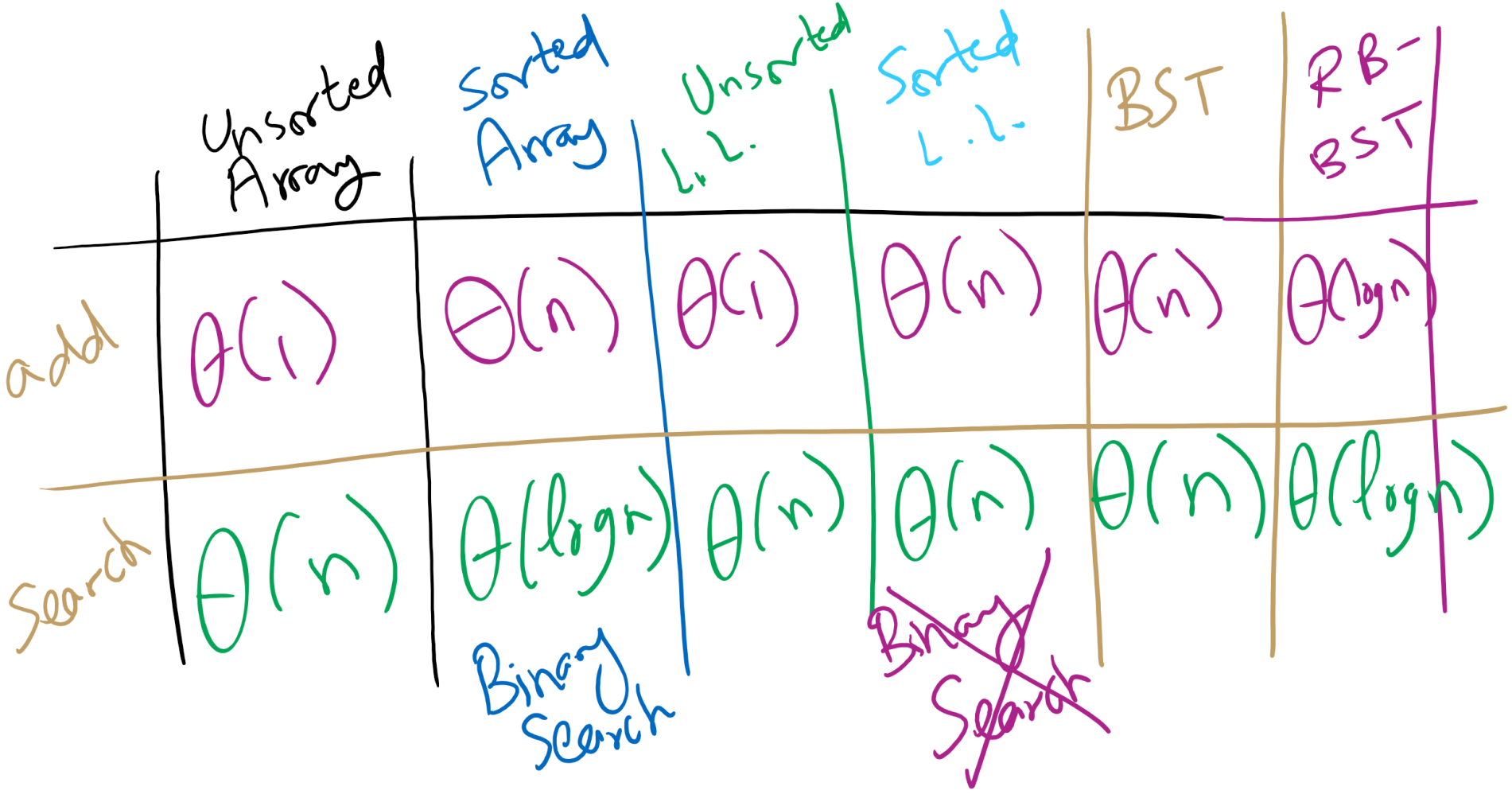
Deleting a node

- Make sure that we are not deleting a black node
  - as we go down the tree, make sure that the next node down is red
    - using a different set of operations
  - as we go back up the tree, correct any violations
    - same as we did while adding
  - if deleting a node with 2 children
    - replace with minimum of right subtree
    - delete minimum of right subtree
    - similar trick to delete in regular BST

Other BST operations

- Find successor and predecessor of an item
  - Lab 3
- Find all items within a specific range
  - Please check the keys methods in RedBlackBST.java inside the TreeADT folder in the code handouts
- Same code as regular BST!
- **worst-case runtime = Theta(log n)**

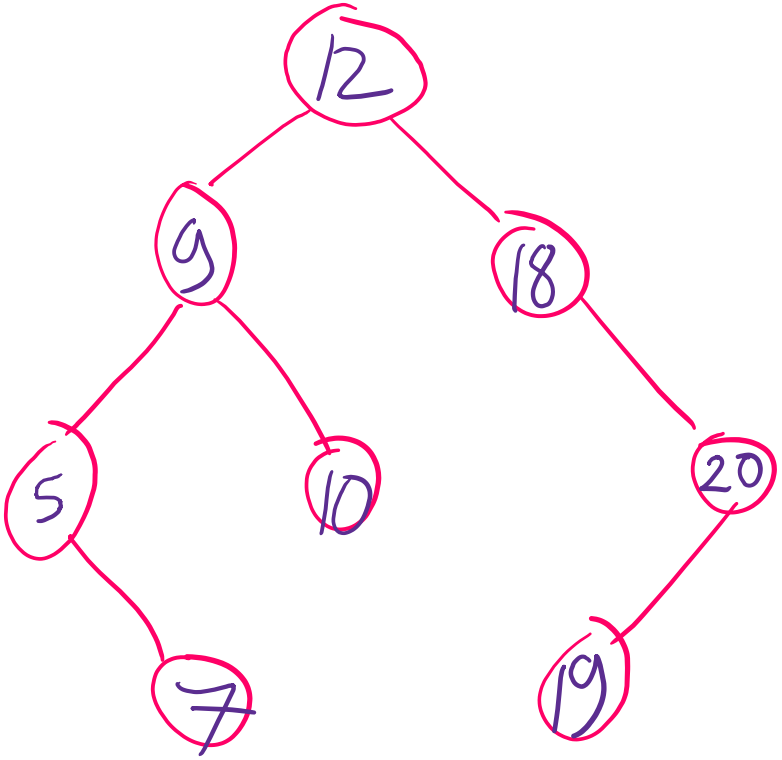
Symbol Table Implementations

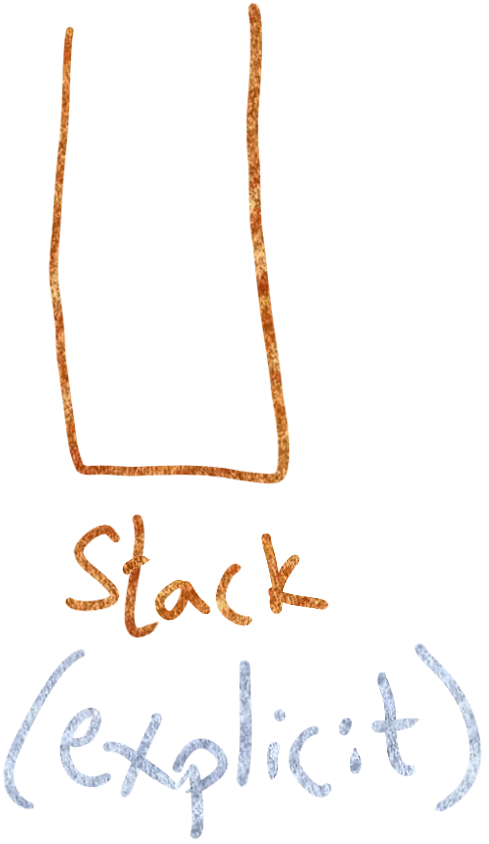


Why use iteration instead of recursion?

- Iteration is faster than recursion
  - No function call overhead (stack allocation)
- Especially useful for frequently used operations
  - Search is a good example of these operations

Iterative Preorder traversal





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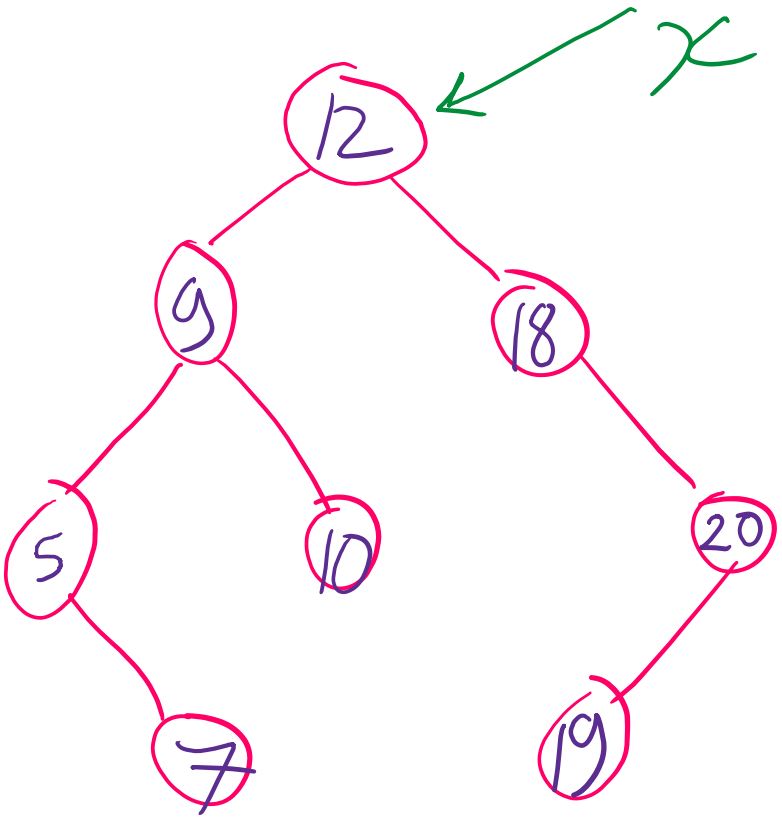
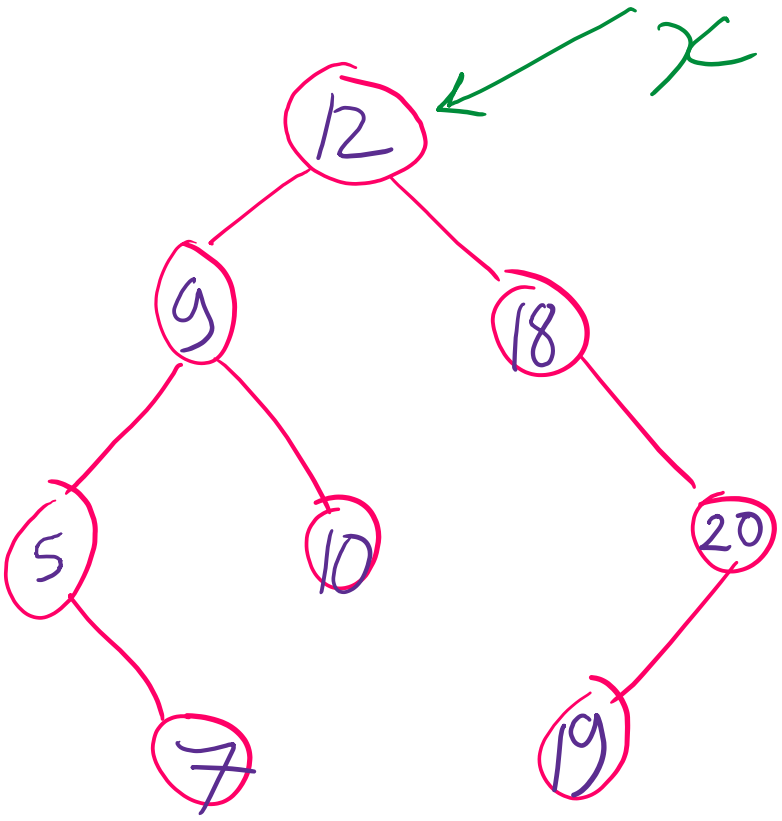


Stack
(explicit)

while (stack not empty)
- pop
- visit
- Push right
- Push left

Visit order: 12

Iterative Preorder traversal



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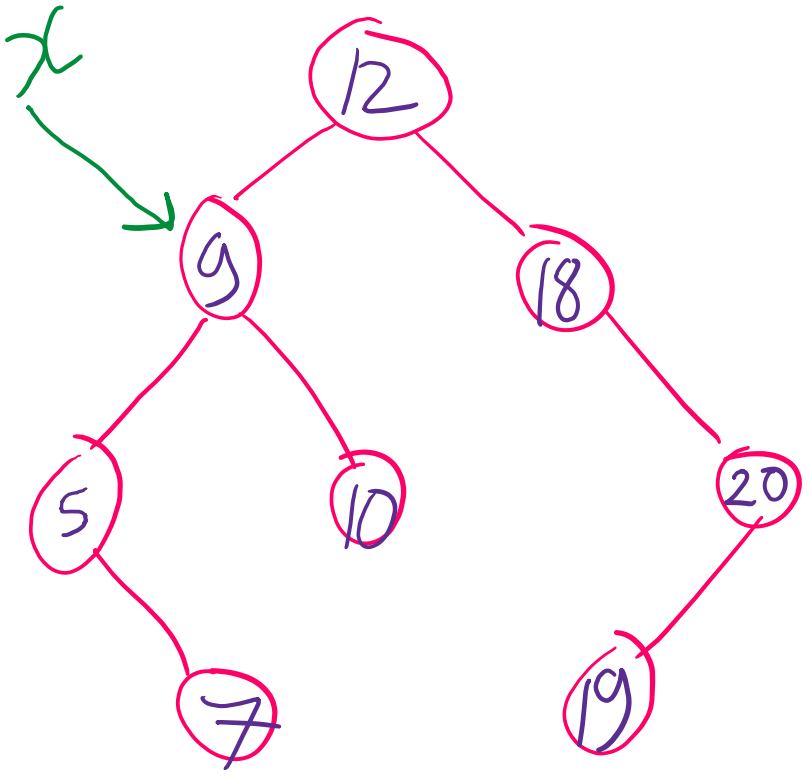

Stack (explicit)

while (stack not empty)

pop

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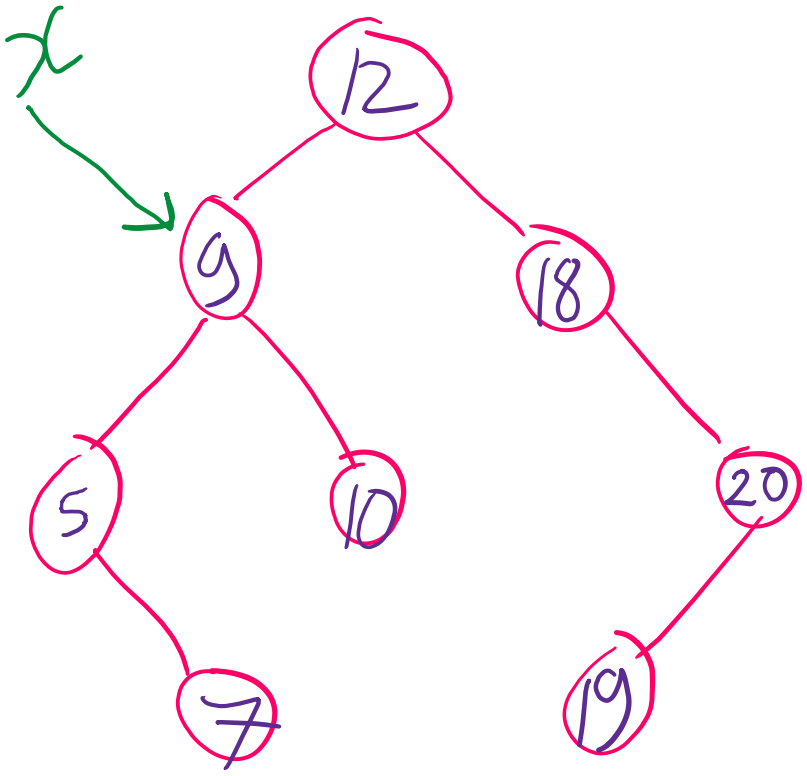
Push right

Push left

Visit order: 12, 9

Iterative Preorder traversal



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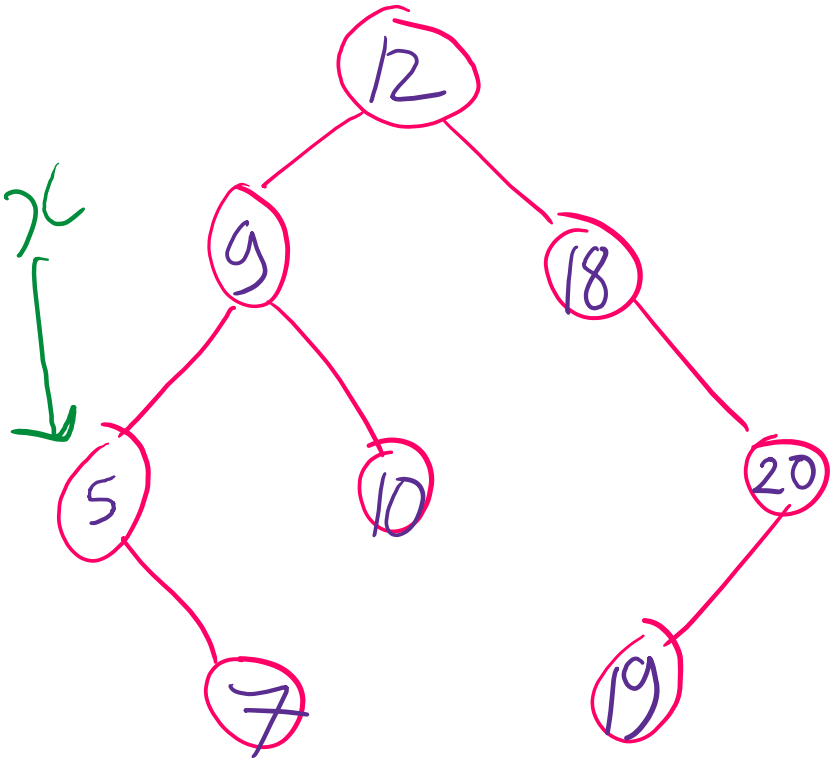
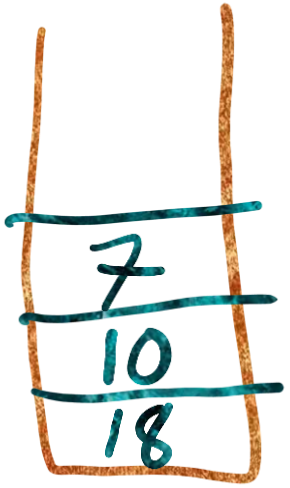
Iterative Preorder traversal





Stack (explicit)

while (stack not empty)

pop

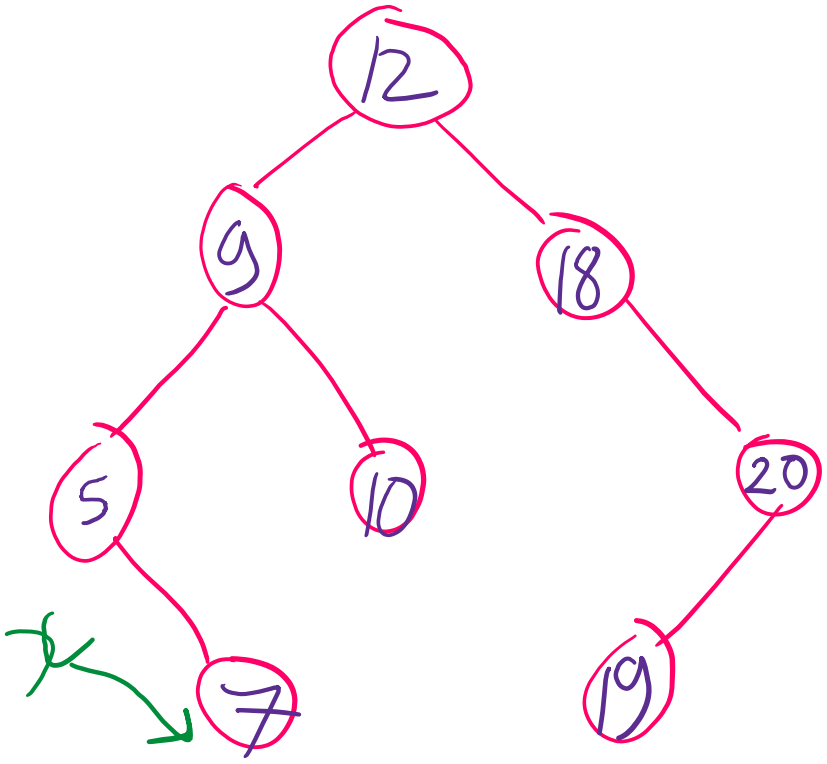
visit

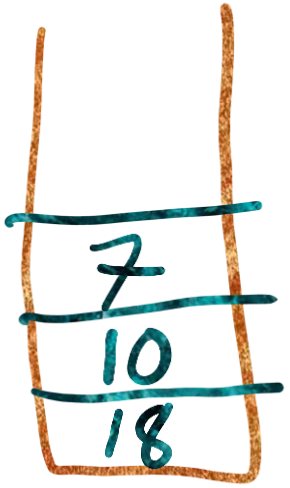
Push right

Push left

Visit order: 12, 9, 5

Iterative Preorder traversal





Stack (explicit)

while (stack not empty)

pop

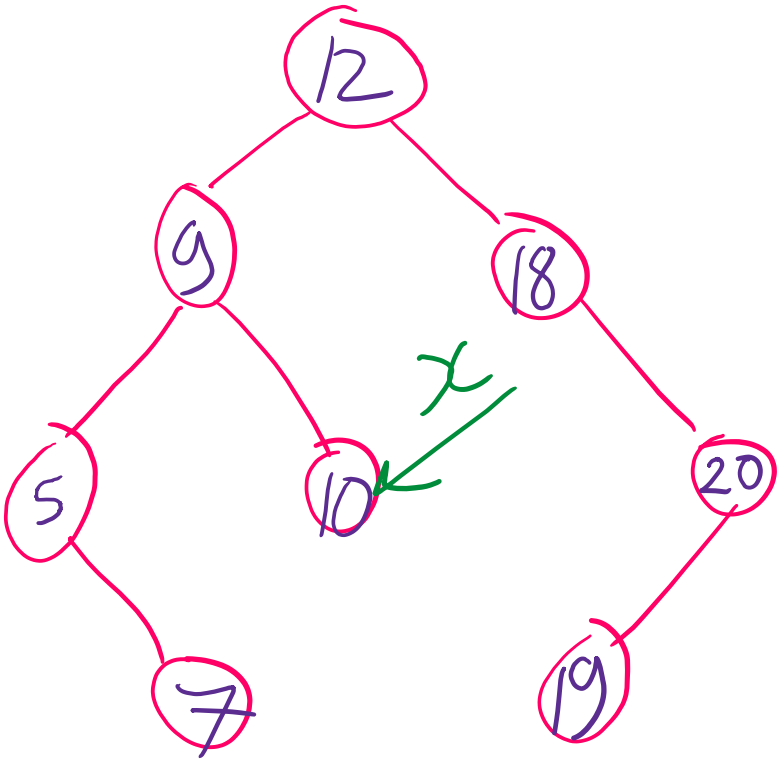
visit

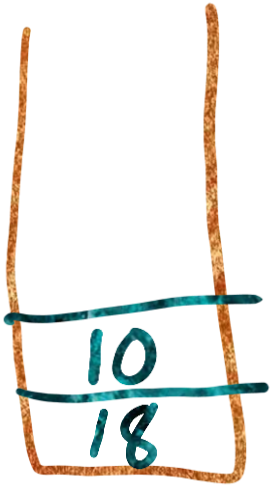
Push right

Push left

Visit order: 12, 9, 5, 7

Iterative Preorder traversal





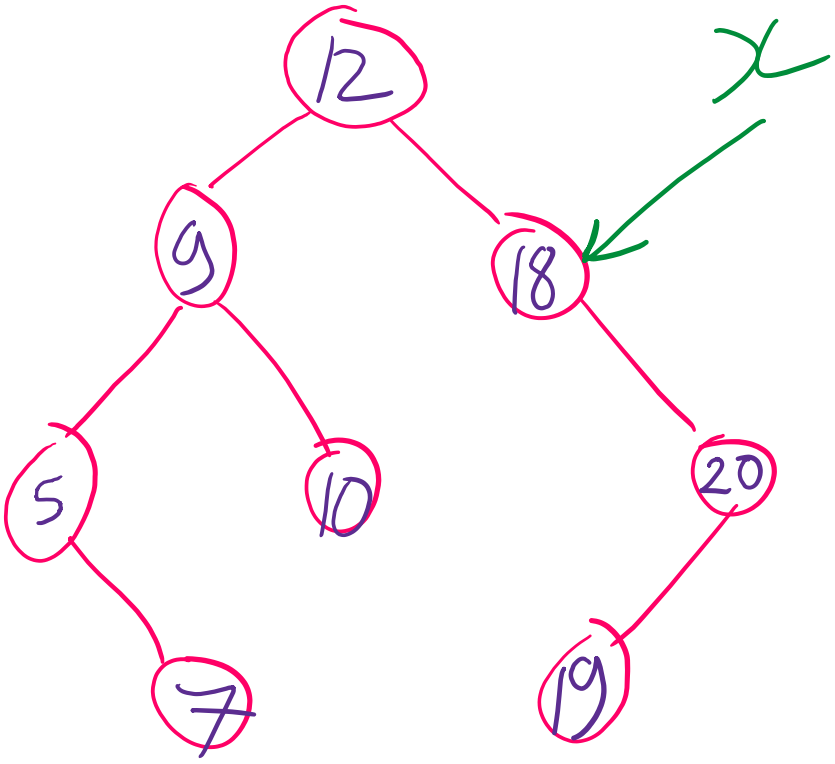
Stack
(explicit)

while (stack not empty)

pop
visit
Push right
Push left

Visit order: 12, 9, 5, 7, 10

Iterative Preorder traversal



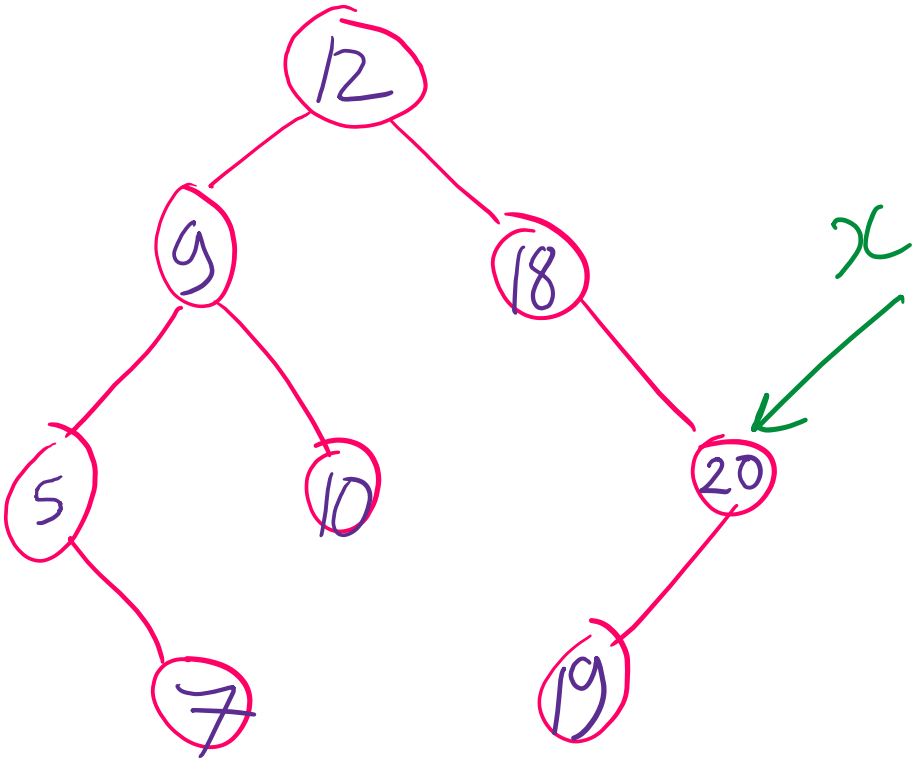
Stack (explicit)

while (stack not empty)

pop
visit
Push right
Push left

Visit order: 12, 9, 5, 7, 10, 18

Iterative Preorder traversal



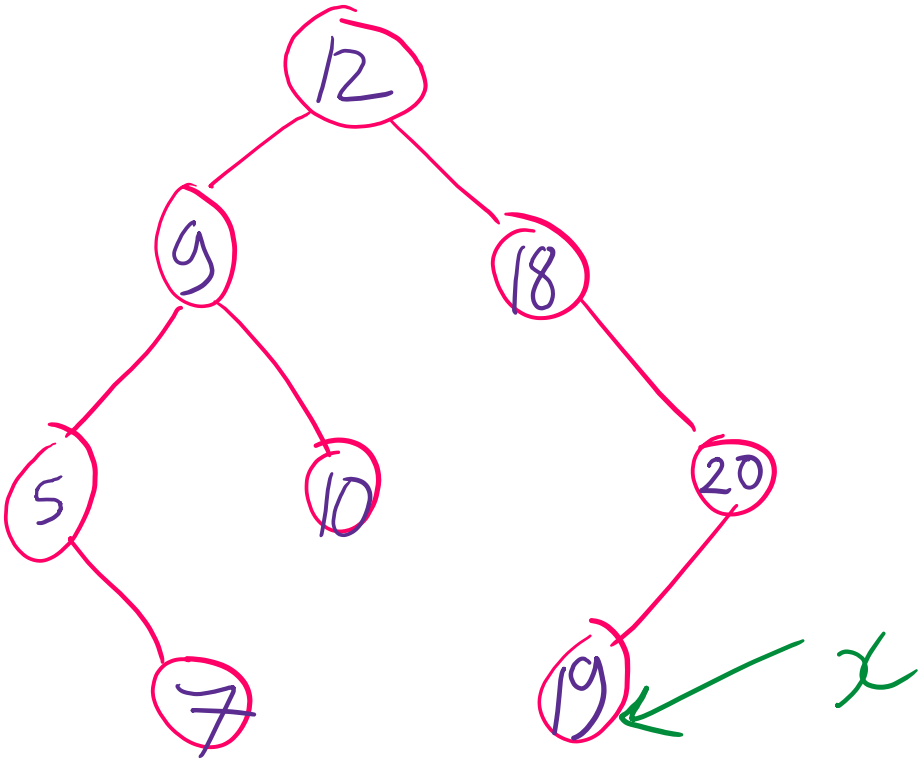
Stack (explicit)

while (stack not empty)

pop
visit
Push right
Push left

Visit order: 12, 9, 5, 7, 10, 18, 20

Iterative Preorder traversal



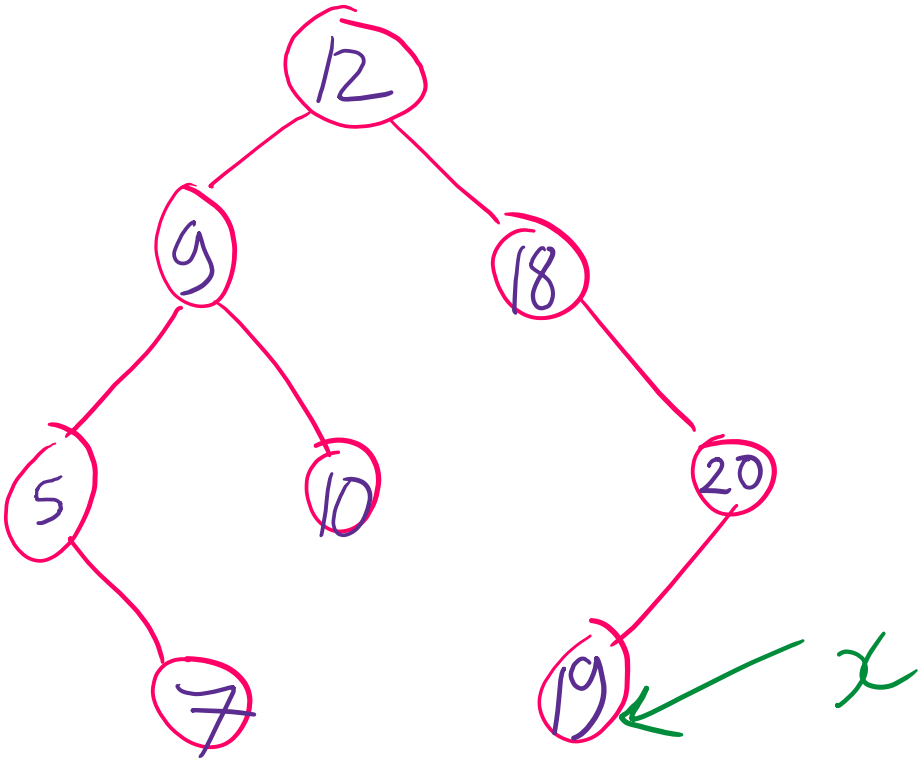
Stack
(explicit)

while (stack not empty)

pop
visit
Push right
Push left

Visit order: 12, 9, 5, 7, 10, 18, 20, 19

Iterative Preorder traversal



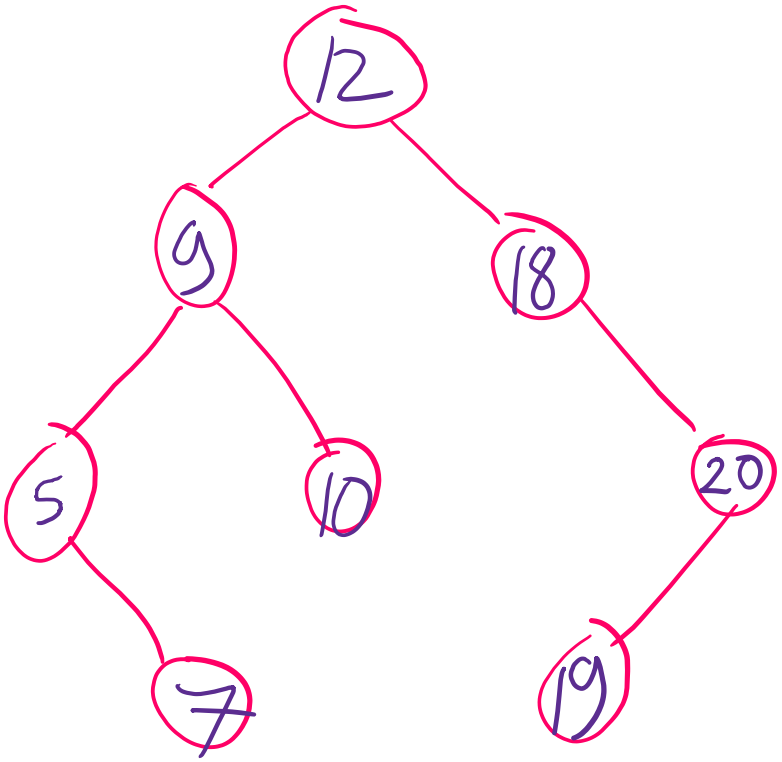
Stack
(explicit)

while (stack not empty)

pop
visit
Push right
Push left

Visit order: 12, 9, 5, 7, 10, 18, 20, 19

Iterative Inorder traversal



repeat until stack empty & x == null

x = leftmost node

Push to stack while moving down

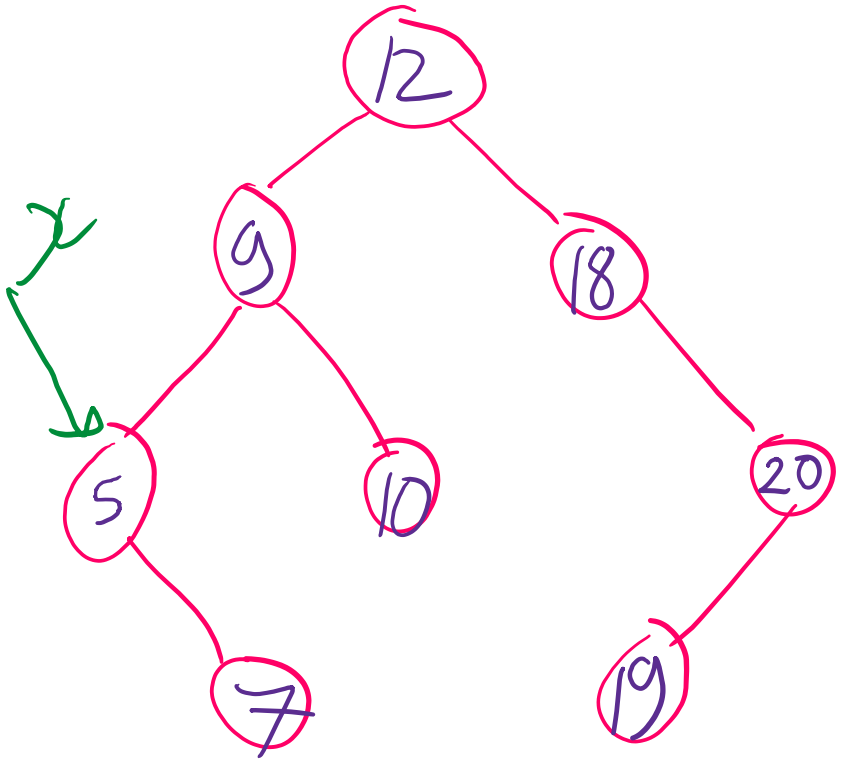
Pop & visit

x = x.right

Stack (explicit)

Visit order:

Iterative Inorder traversal



repeat until stack empty & x == null

x = leftmost node

Push to stack while moving down

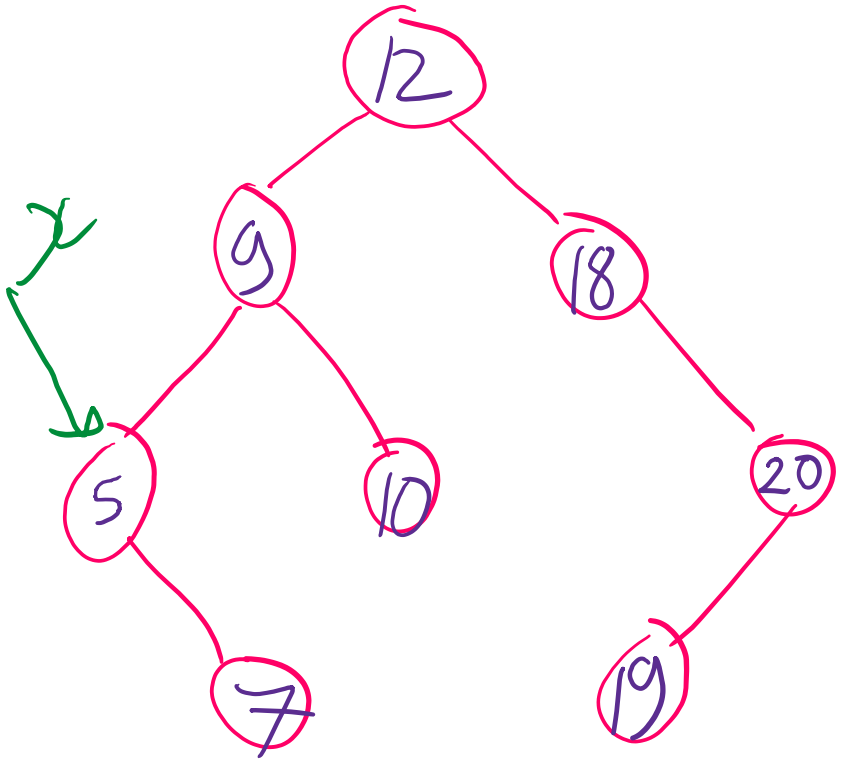
Pop & visit

x = x.right

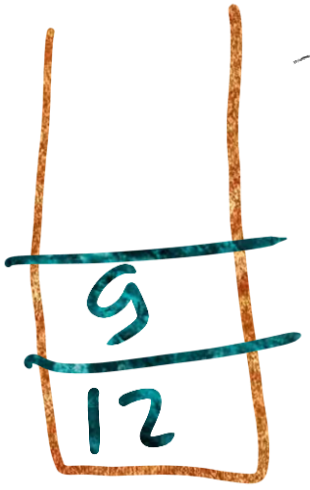
Stack (explicit)

Visit order:

Iterative Inorder traversal



repeat until stack empty & x == null



x = leftmost node

Push to stack while moving down

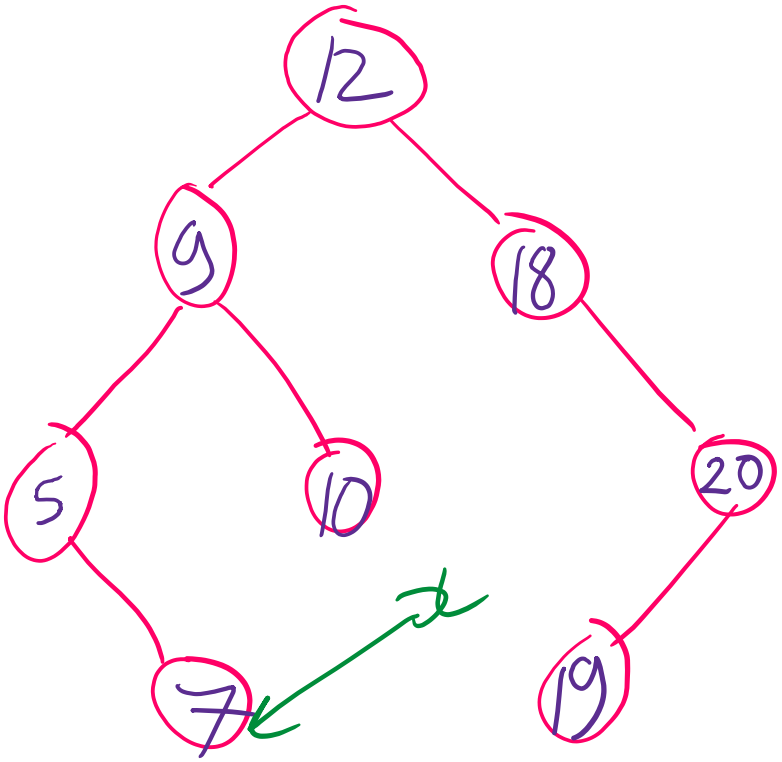
Pop & visit

x = x.right

Stack (explicit)

Visit order: 5

Iterative Inorder traversal



repeat until stack empty & x == null

Stack (explicit)

x = leftmost node

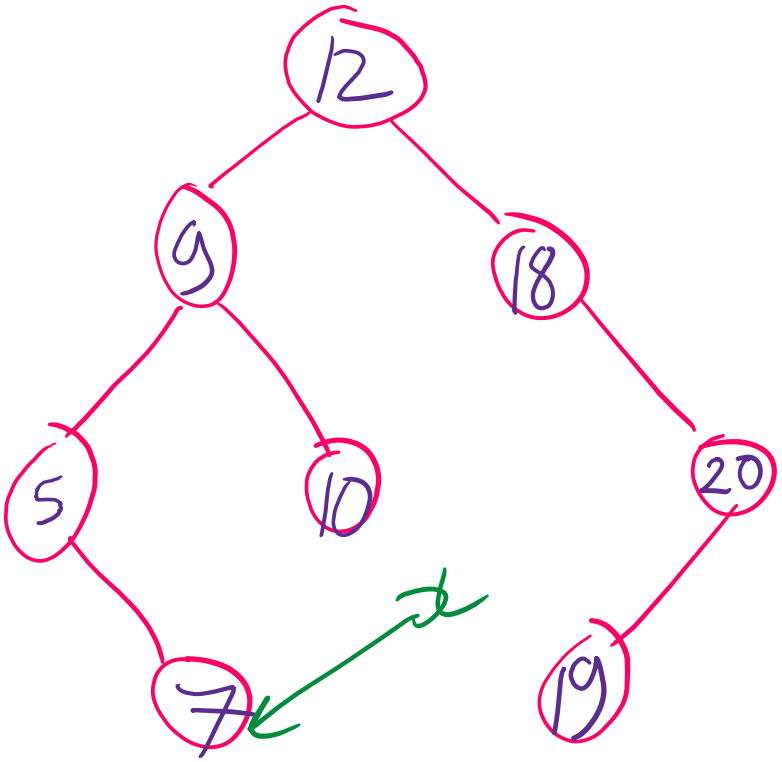
Push to stack while moving down

Pop & visit

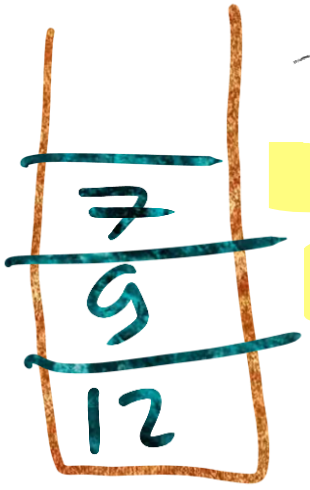
x = x.right

Visit order: 5

Iterative Inorder traversal



repeat until stack empty & x == null



x = leftmost node

Push to stack while moving down

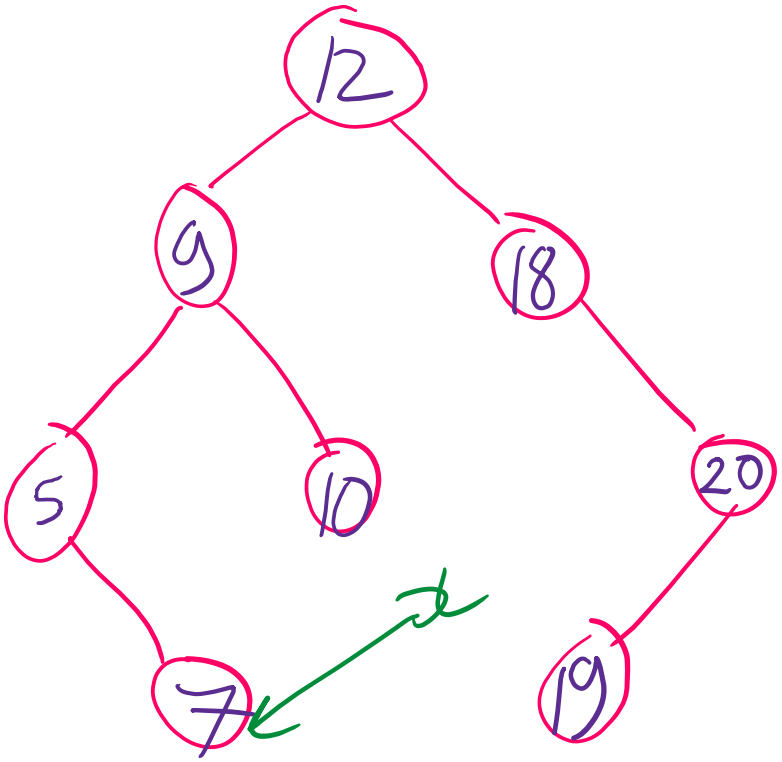
Pop & visit

x = x.right

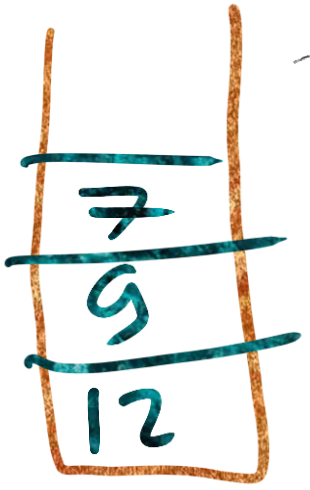
Stack (explicit)

Visit order: 5

Iterative Inorder traversal



repeat until stack empty & x == null



x = leftmost node

Push to stack while moving down

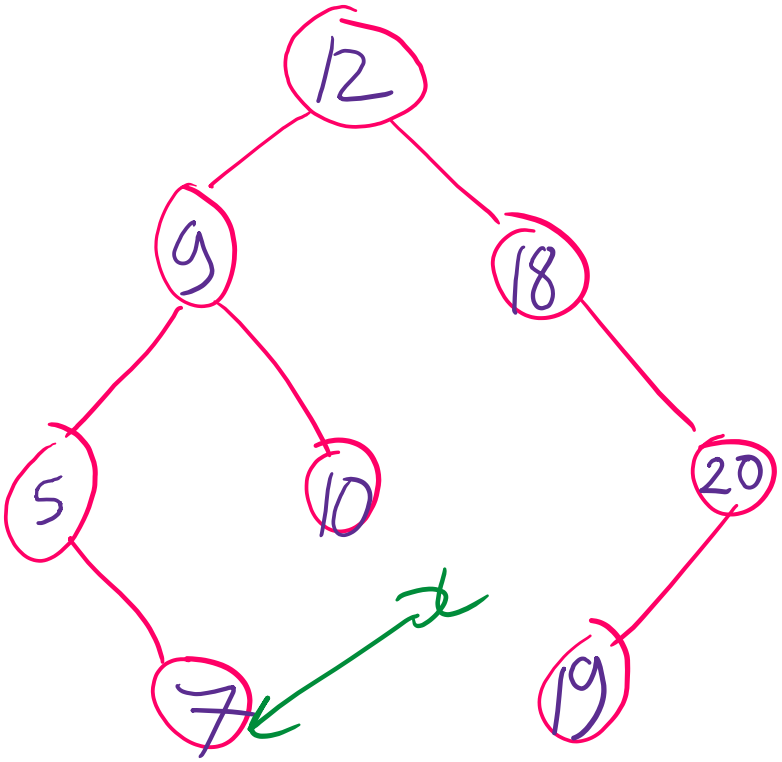
Pop & visit

x = x.right

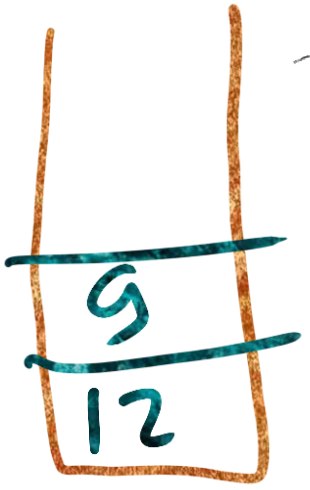
Stack (explicit)

Visit order: 5, 7

Iterative Inorder traversal



repeat until stack empty & x == null



x = leftmost node

Push to stack while moving down

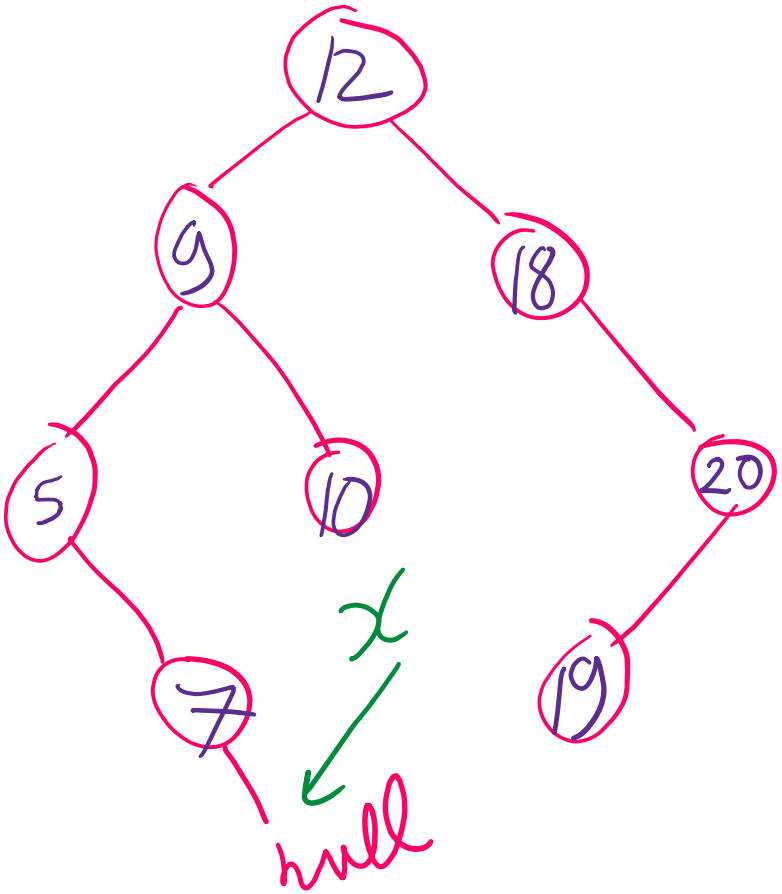
Pop & visit

x = x.right

Stack (explicit)

Visit order: 5, 7

Iterative Inorder traversal



Visit order: 5, 7

repeat until stack empty & x == null

x = leftmost node

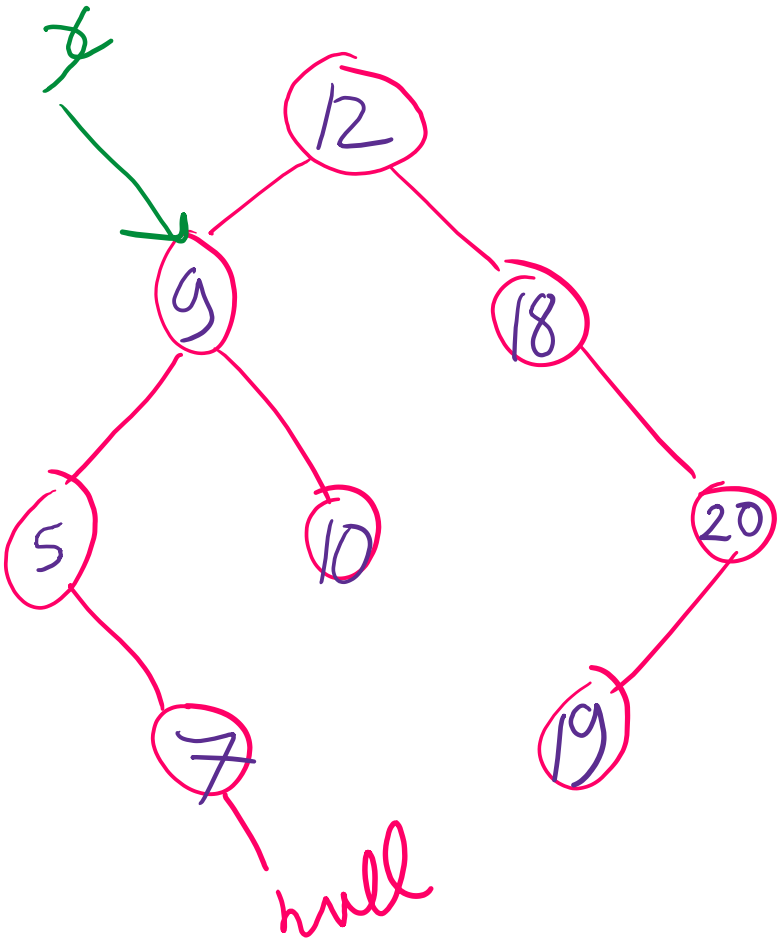
Push to stack while moving down

Pop & visit

x = x.right

Stack (explicit)

Iterative Inorder traversal



repeat until stack empty & x == null

x = leftmost node

Push to stack while moving down

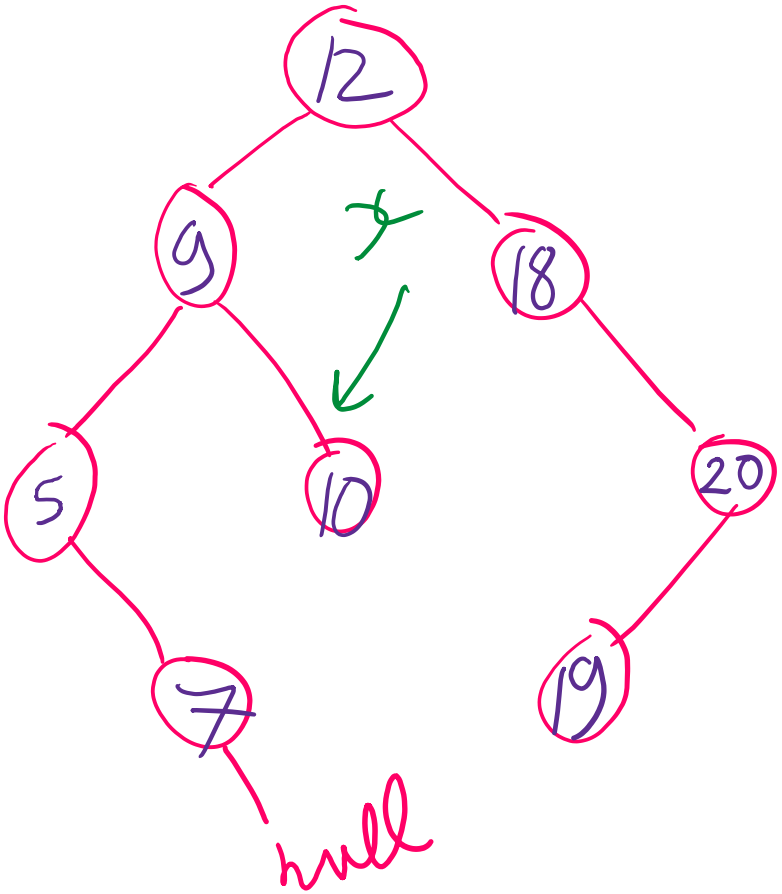
Pop & visit

x = x.right

Stack (explicit)

Visit order: 5, 7, 9

Iterative Inorder traversal



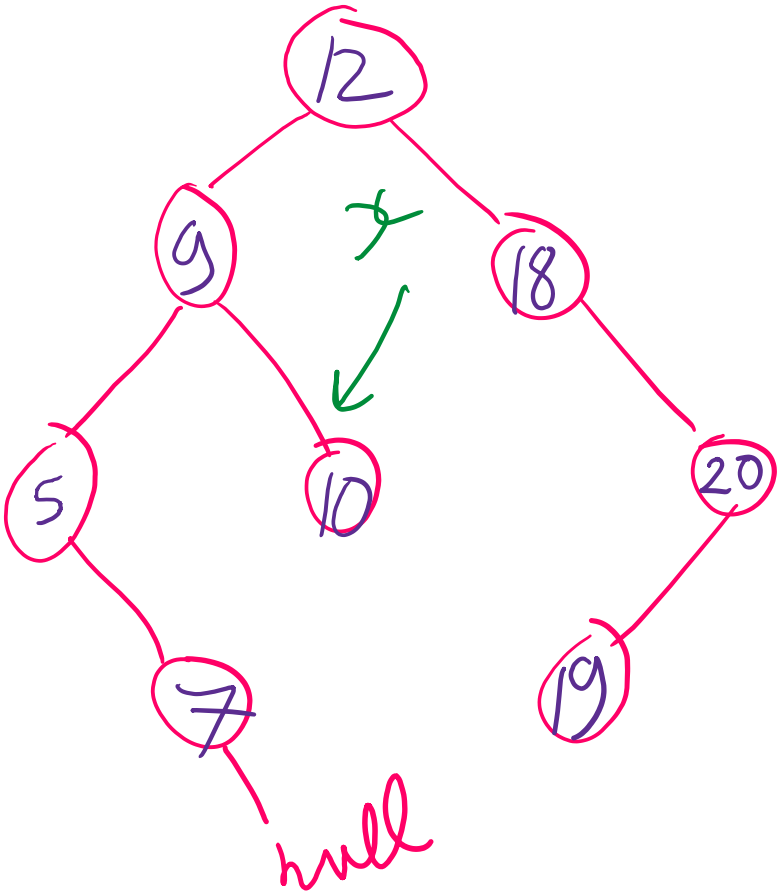
repeat until stack empty & x == null

x = leftmost node
Push to stack while moving down
Pop & visit
x = x.right

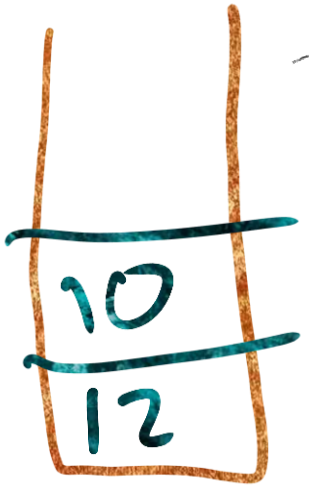
Stack (explicit)

Visit order: 5, 7, 9

Iterative Inorder traversal



repeat until stack empty & x == null



x = leftmost node

Push to stack while moving down

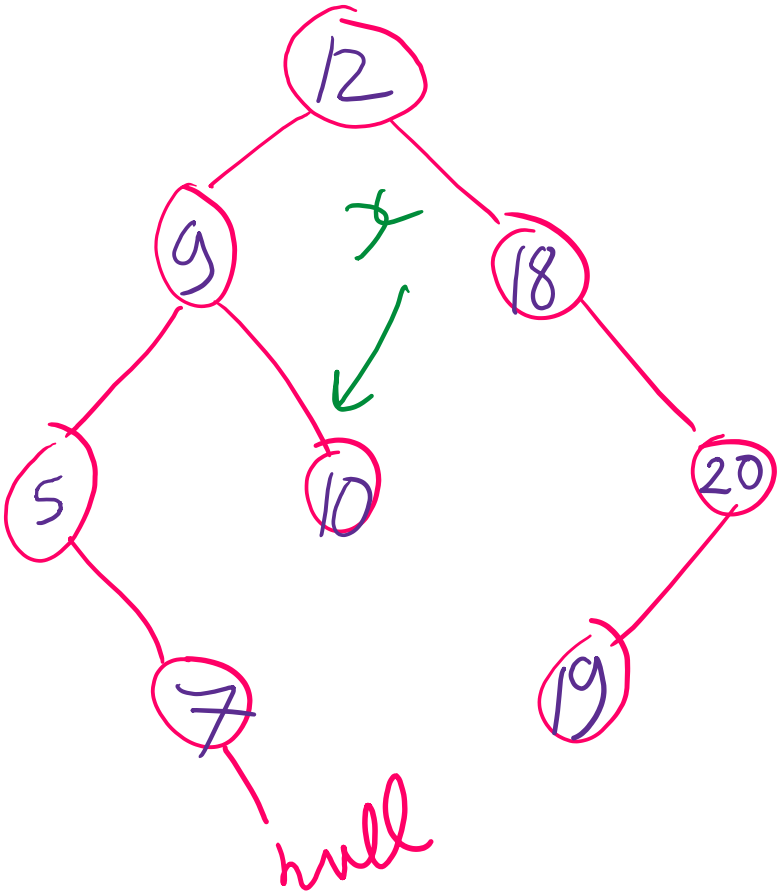
Pop & visit

x = x.right

Stack (explicit)

Visit order: 5, 7, 9

Iterative Inorder traversal



repeat until stack empty & x == null

x = leftmost node

Push to stack while moving down

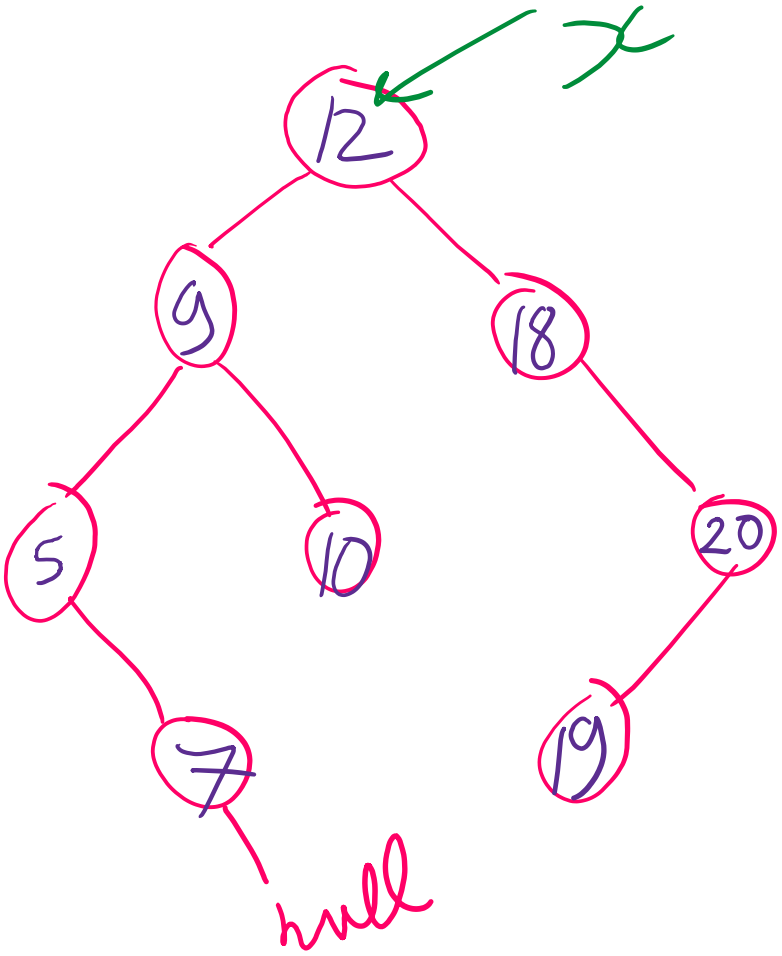
Pop & visit

x = x.right

Stack (explicit)

Visit order: 5, 7, 9, 10

Iterative Inorder traversal



repeat until stack empty & x == null

x = leftmost node

Push to stack while moving down

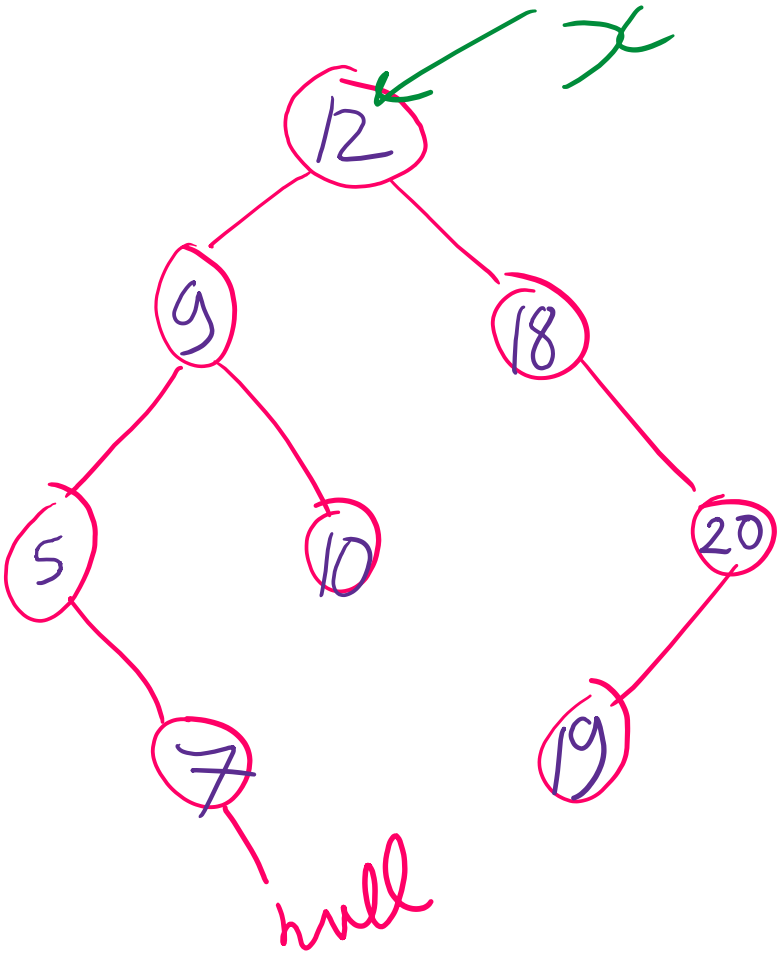
Pop & visit

x = x.right

Stack (explicit)

Visit order: 5, 7, 9, 10

Iterative Inorder traversal



repeat until stack empty & x == null

x = leftmost node

Push to stack while moving down

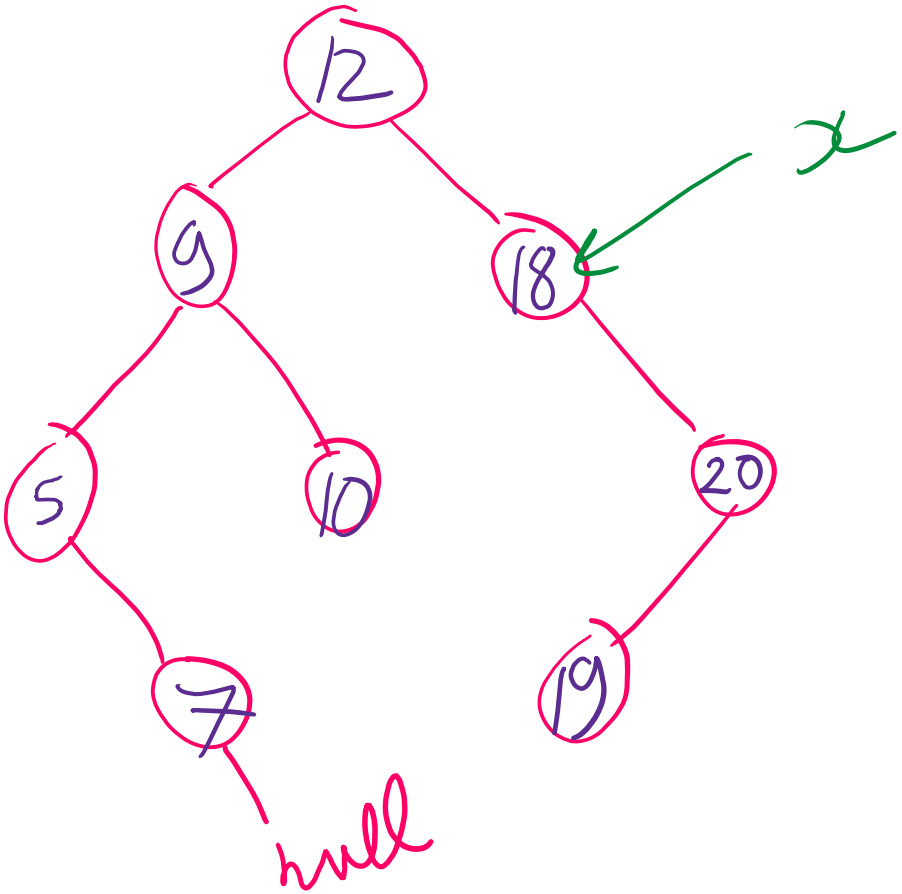
Pop & visit

x = x.right

Stack (explicit)

Visit order: 5, 7, 9, 10, 12

Iterative Inorder traversal



repeat until stack empty & x == null

x = leftmost node

Push to stack while moving down

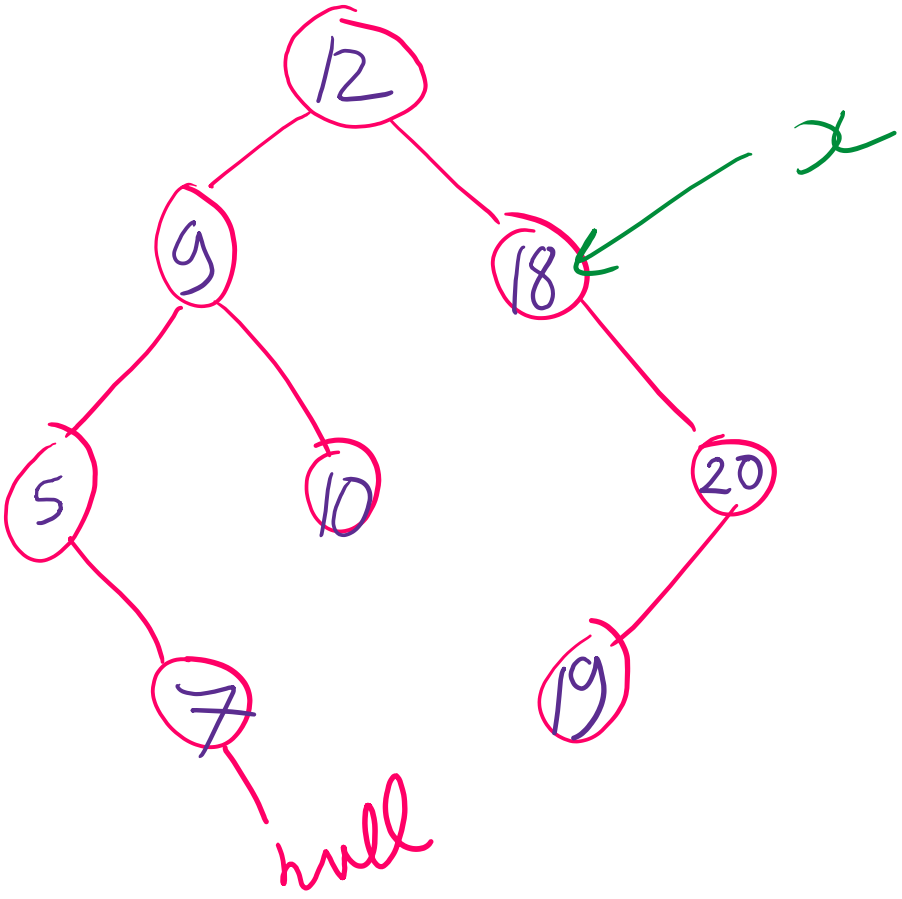
Pop & visit

x = x.right

Stack (explicit)

Visit order: 5, 7, 9, 10, 12

Iterative Inorder traversal



repeat until stack empty & x == null

x = leftmost node

Push to stack while moving down

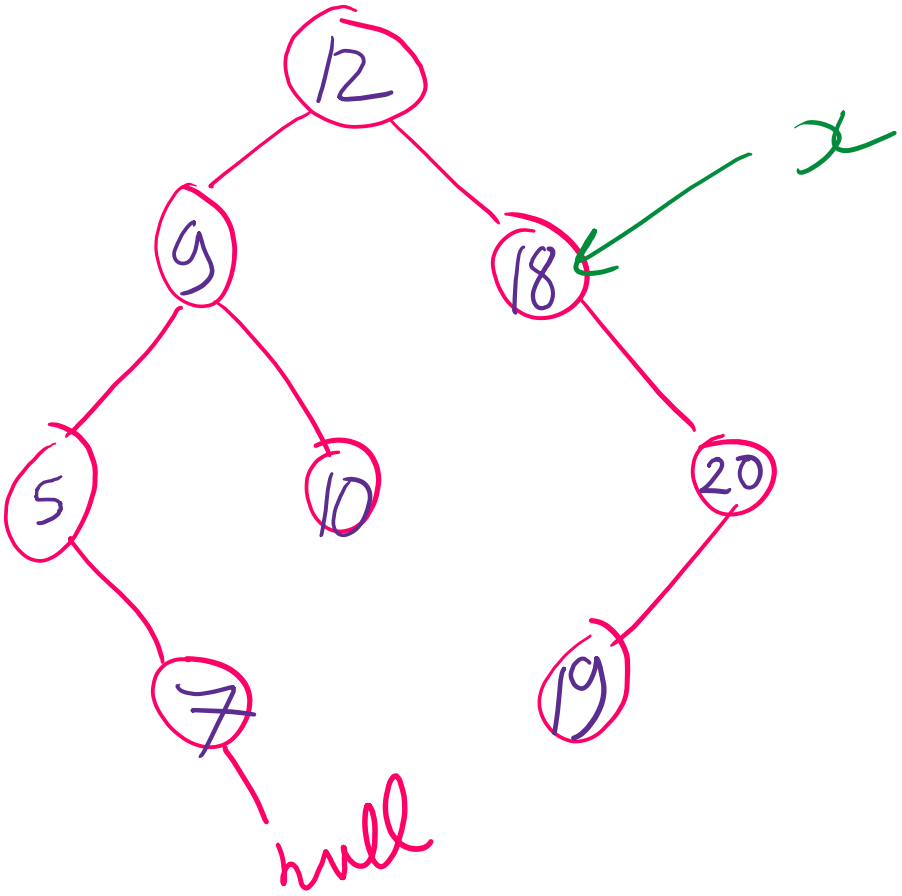
Stack (explicit)

Pop & visit

x = x.right

Visit order: 5, 7, 9, 10, 12

Iterative Inorder traversal



repeat until stack empty & x == null

x = leftmost node

Push to stack while moving down

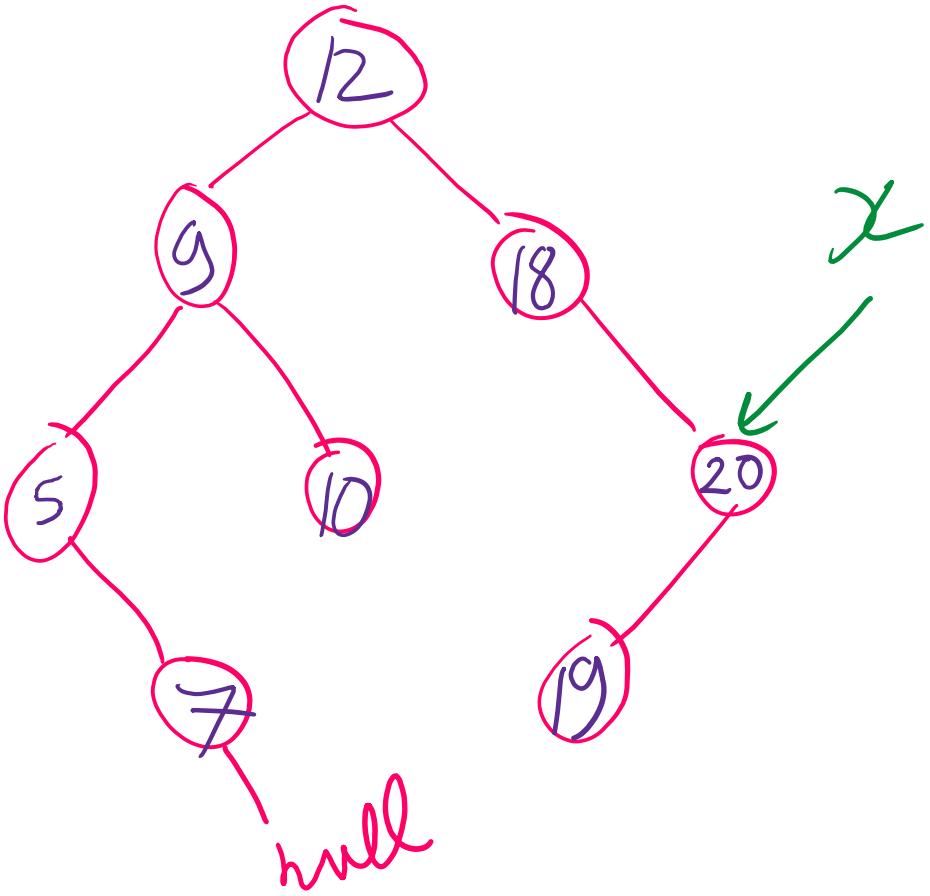
Pop & visit

x = x.right

Stack (explicit)

Visit order: 5, 7, 9, 10, 12, 18

Iterative Inorder traversal



repeat until stack empty & x == null

x = leftmost node

Push to stack while moving down

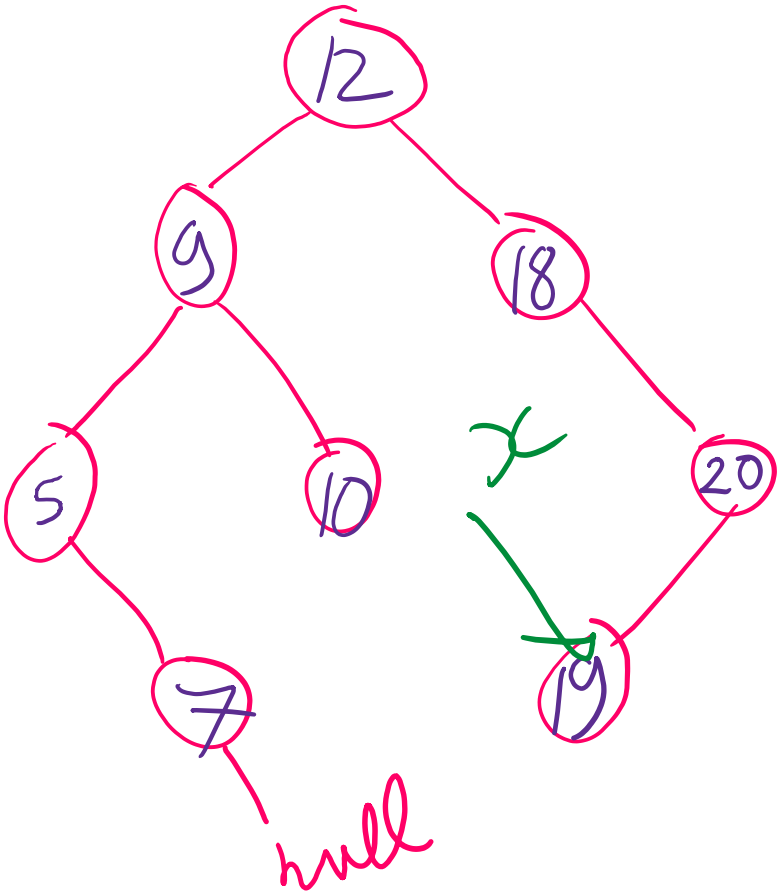
Pop & visit

x = x.right

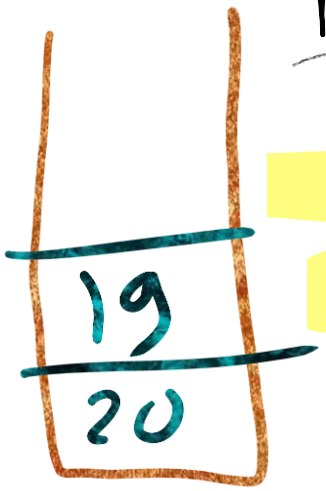
Stack (explicit)

Visit order: 5, 7, 9, 10, 12, 18

Iterative Inorder traversal



repeat until stack empty & x == null



x = leftmost node

Push to stack while moving down

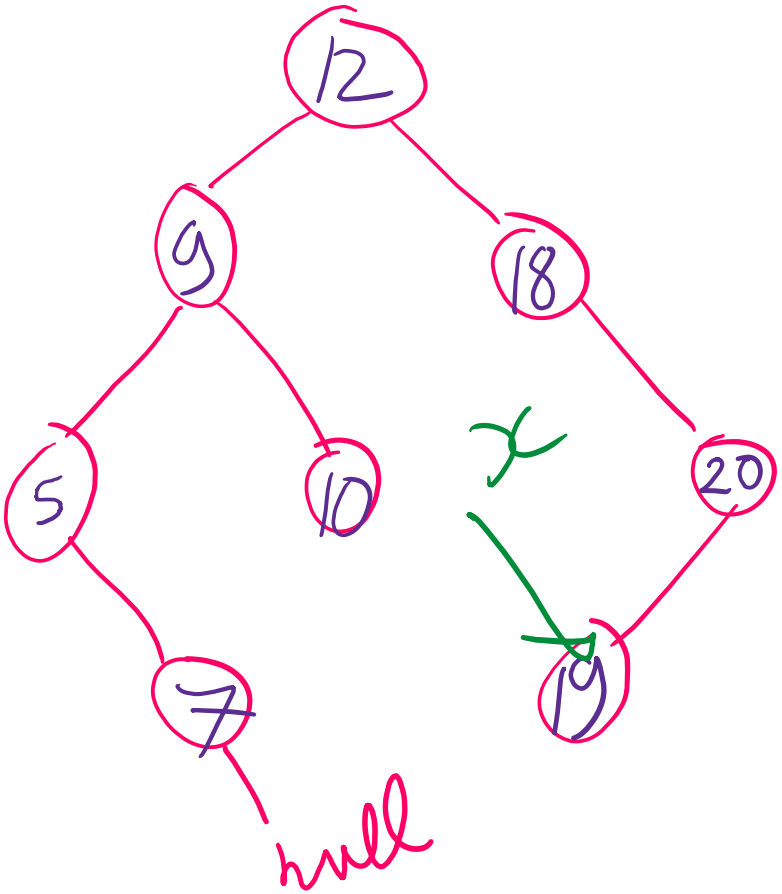
Pop & visit

x = x.right

Stack (explicit)

Visit order: 5, 7, 9, 10, 12, 18

Iterative Inorder traversal



repeat until stack empty & x == null

x = leftmost node

Push to stack while moving down

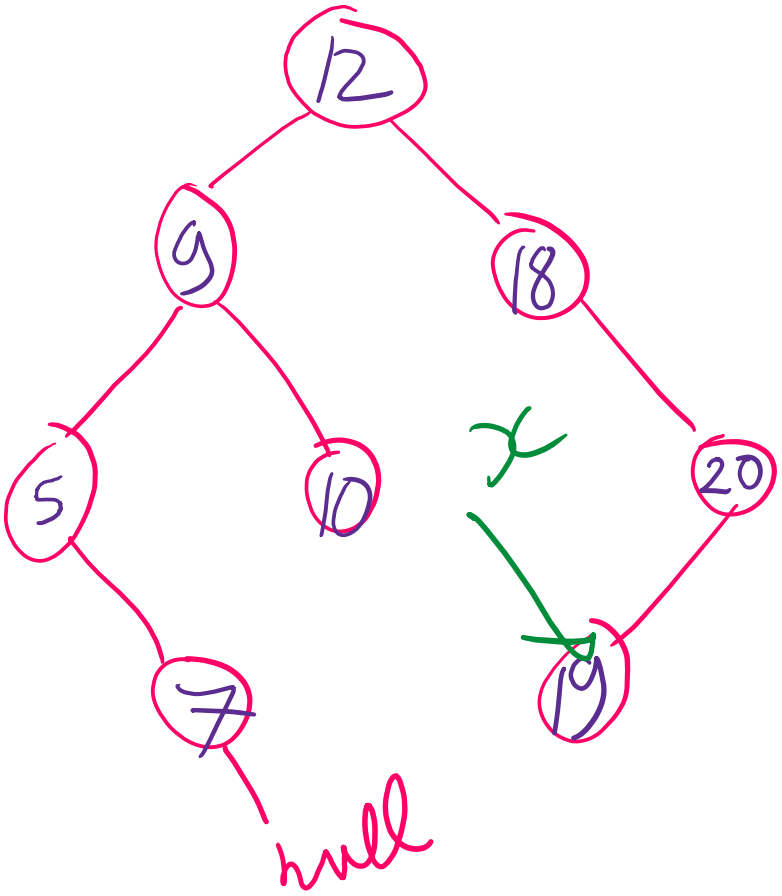
Pop & visit

x = x.right

Stack (explicit)

Visit order: 5, 7, 9, 10, 12, 18, 19

Iterative Inorder traversal



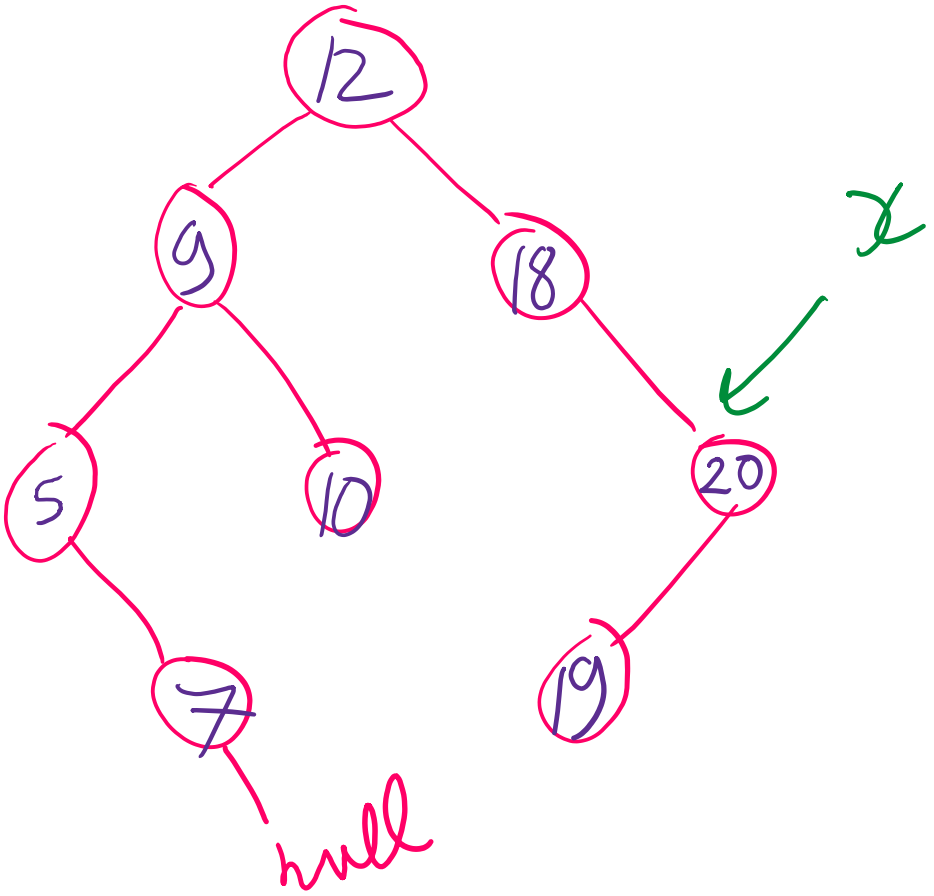
repeat until stack empty & x == null

Stack (explicit)

x = leftmost node
Push to stack while moving down
Pop & visit
x = x.right

Visit order: 5, 7, 9, 10, 12, 18, 19

Iterative Inorder traversal



repeat until stack empty & x == null

x = leftmost node

Push to stack while moving down

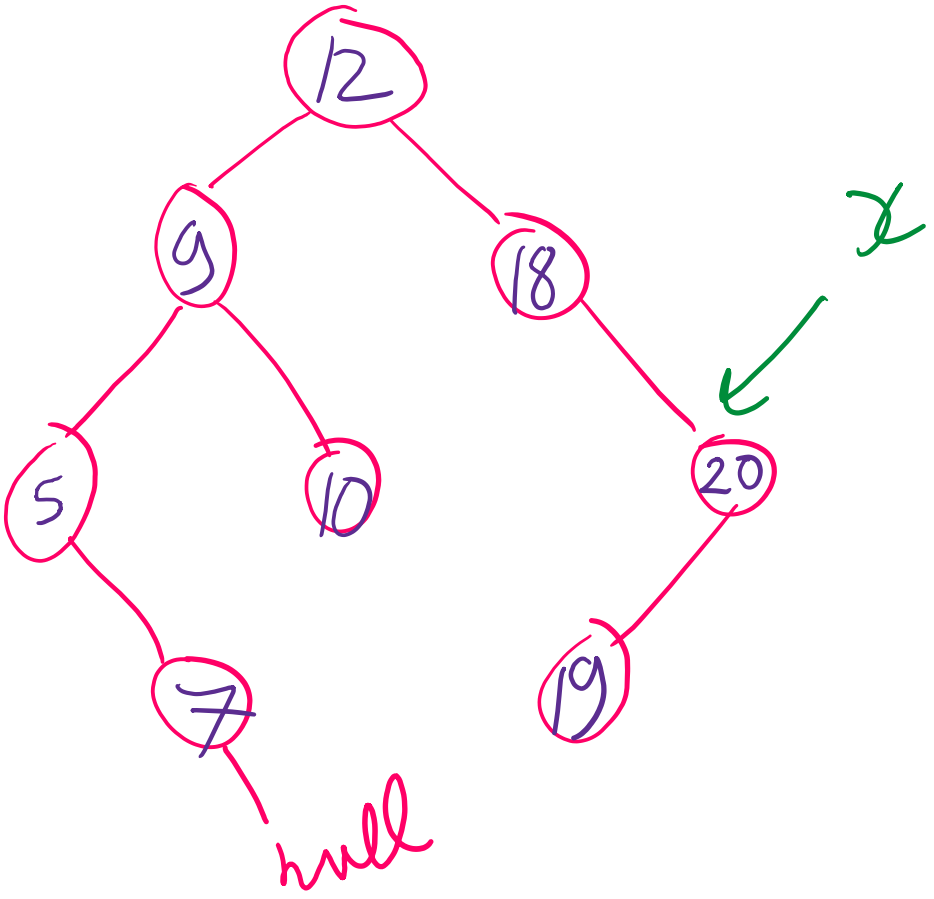
Pop & visit

x = x.right

Stack (explicit)

Visit order: 5, 7, 9, 10, 12, 18, 19, 20

Iterative Inorder traversal



repeat until stack empty & x == null

x = leftmost node

Push to stack while moving down

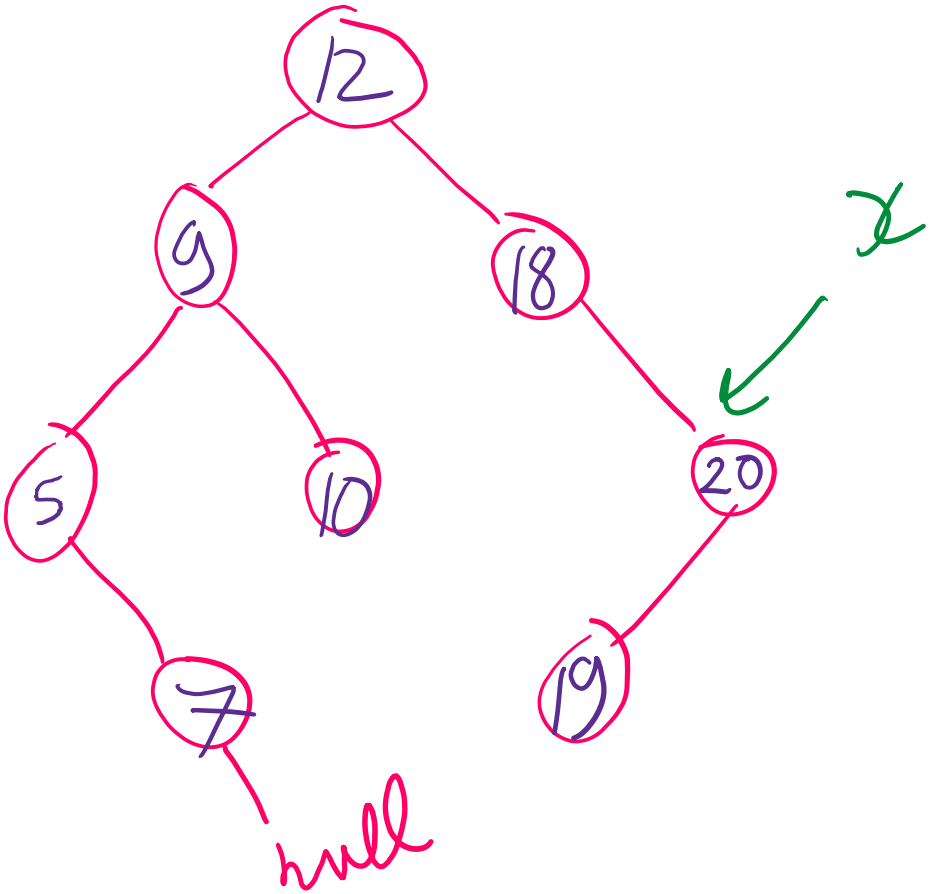
Pop & visit

x = x.right

Stack (explicit)

Visit order: 5, 7, 9, 10, 12, 18, 19, 20

Iterative Inorder traversal



repeat until stack empty & x == null

Stack (explicit)

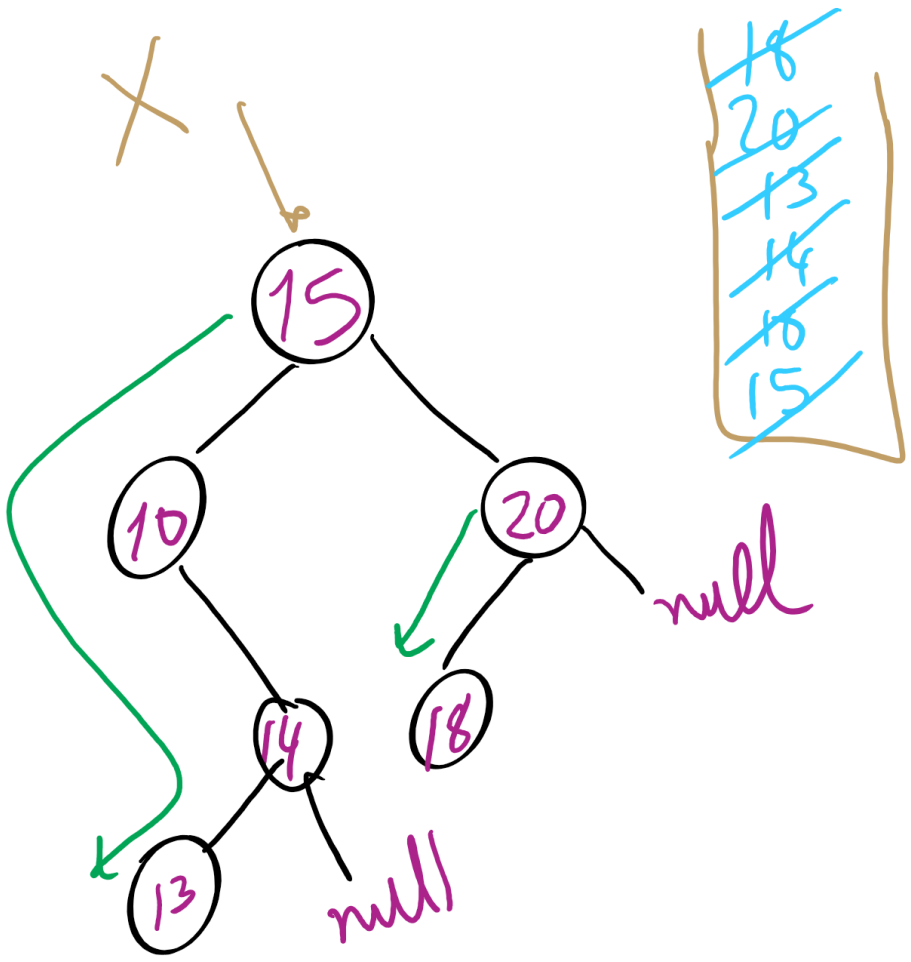
x = leftmost node
Push to stack while moving down
Pop & visit
x = x.right

Visit order: 5, 7, 9, 10, 12, 18, 19, 20

Traversals of a Binary Tree

- Preorder traversal
  - Visit root before we visit root's subtrees
- Inorder traversal
  - Visit root of a binary tree between visiting nodes in root's subtrees.
- Postorder traversal
  - Visit root of a binary tree after visiting nodes in root's subtrees
  - left then right then root

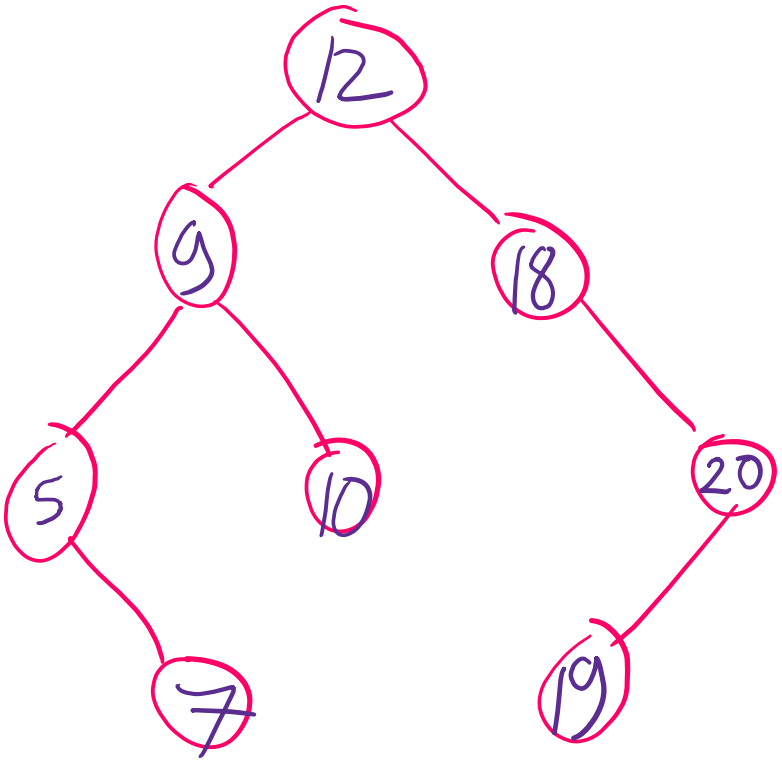
Iterative Post-order Traversal

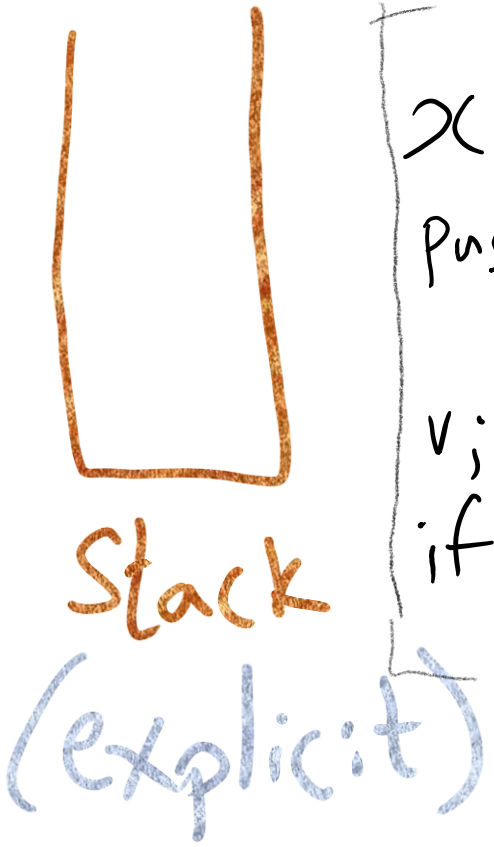


13, 14, 10, 18, 20 15

```
stack = empty stack
x = root
While( x != null ||
       stack is not empty){
  -find leftmost leaf
  -push nodes as we
     go down the tree
  - x = pop() & visit x
  - if x is a left child
       x = right sibling
  else
       x = null
}
```

Iterative Postorder traversal





repeat until stack empty & x == null

x = leftmost leaf

Push to stack while moving down

visit x

if x is right child

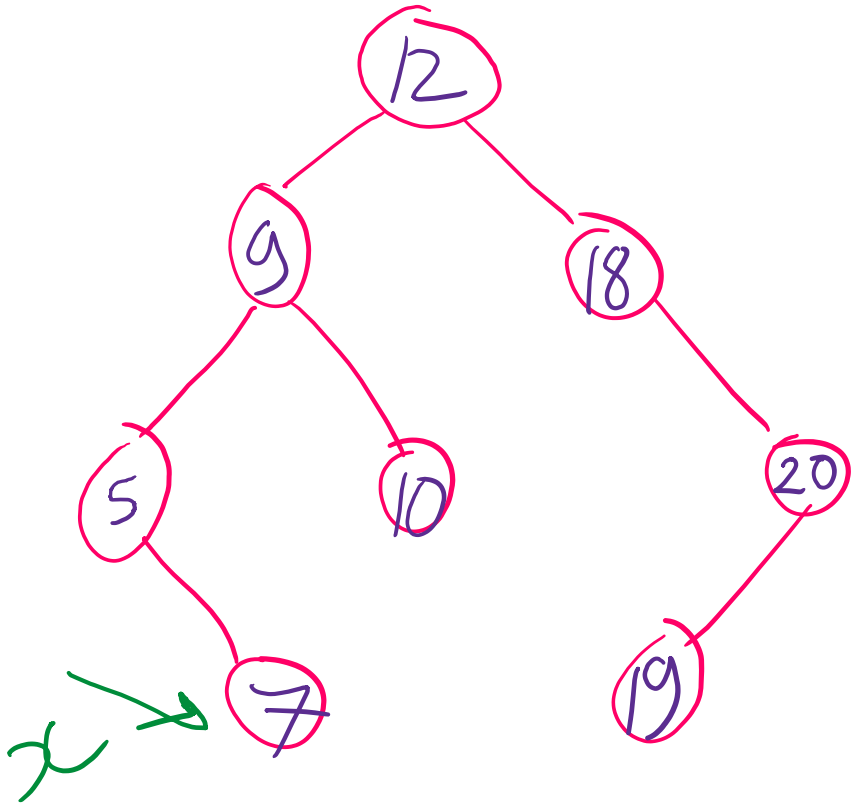
pop & visit

skip leftmost leaf

else x = parent.right

Visit order:

Iterative Postorder traversal



Stack (explicit)

repeat until stack empty & x == null

x = leftmost leaf
Push to stack while moving down

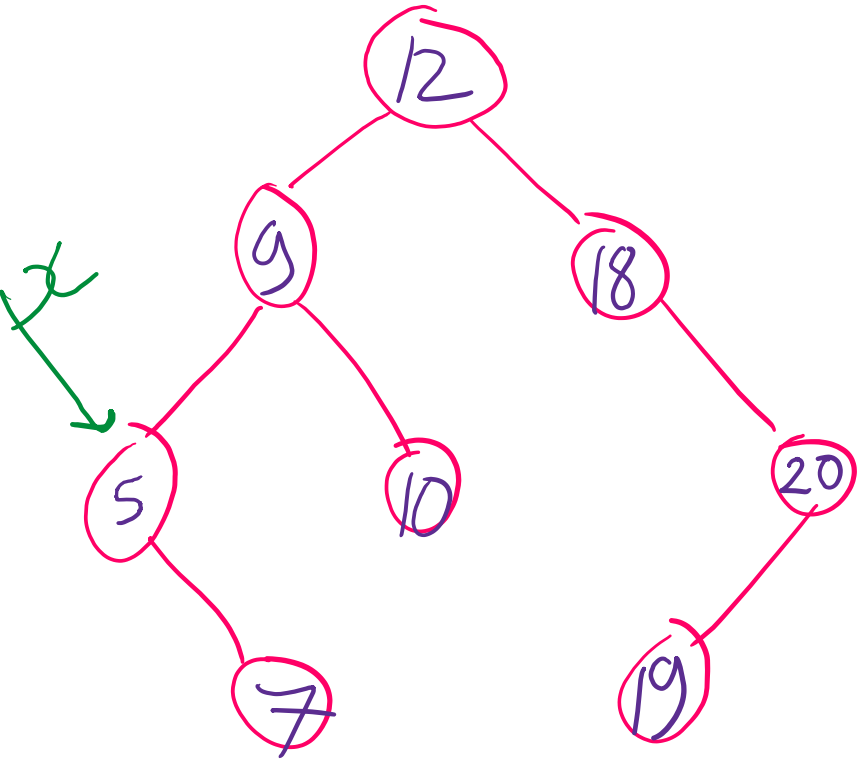
visit x

if x is rightchild
pop & visit
skip leftmost leaf

else x = parent.right

Visit order: 7

Iterative Postorder traversal



Stack (explicit)

repeat until stack empty & x == null

x = leftmost leaf

Push to stack while moving down

visit x

if x is rightchild

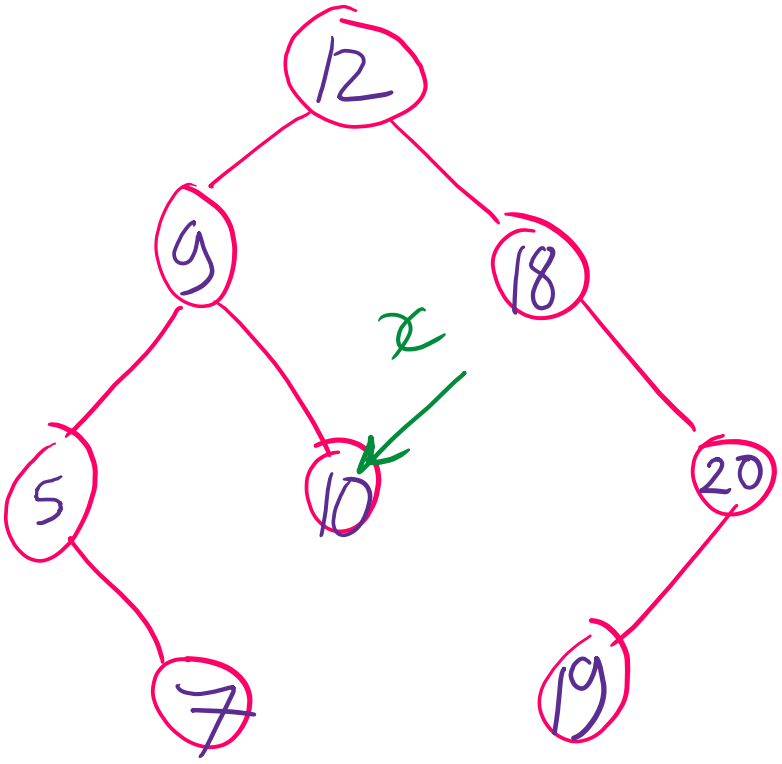
pop & visit

skip leftmost leaf

else x = parent.right

Visit order: 7, 5

Iterative Postorder traversal



Stack (explicit)

repeat until stack empty & x == null

x = leftmost leaf

Push to stack while moving down

visit x

if x is right child

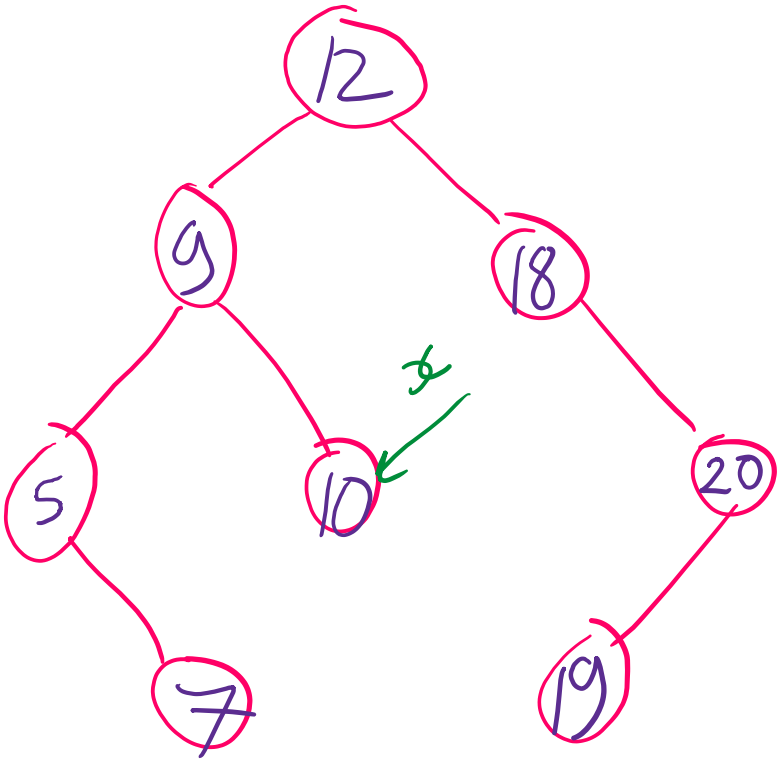
pop & visit

skip leftmost leaf

else x = parent.right

Visit order: 7, 5

Iterative Postorder traversal



repeat until stack empty & x == null

Stack (explicit)

x = leftmost leaf

Push to stack while moving down

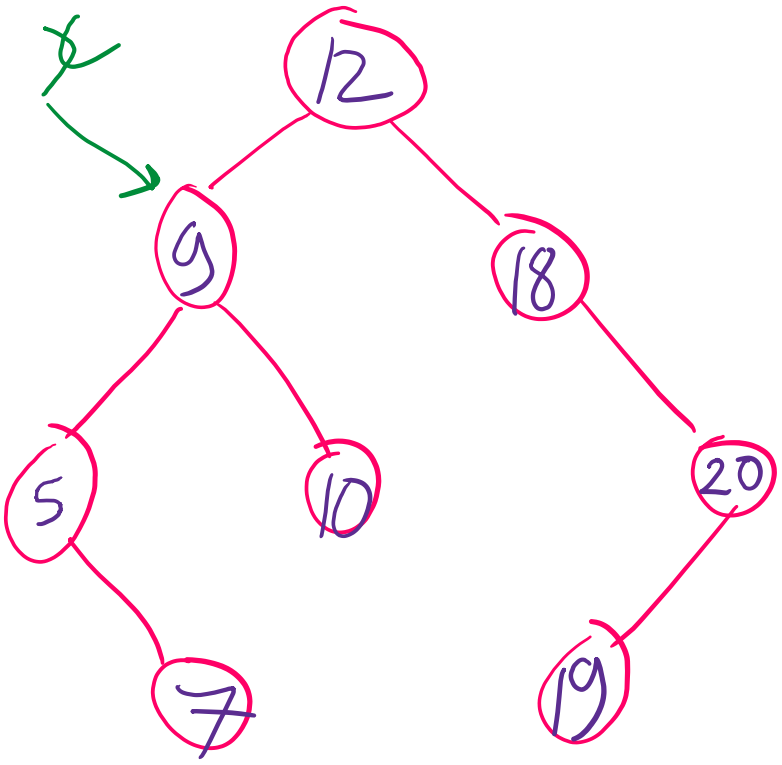
visit x

if x is right child
pop & visit
skip leftmost leaf

else x = parent.right

Visit order: 7, 5, 10

Iterative Postorder traversal



Stack (explicit)

repeat until stack empty & x == null

x = leftmost leaf

Push to stack while moving down

visit x

if x is right child

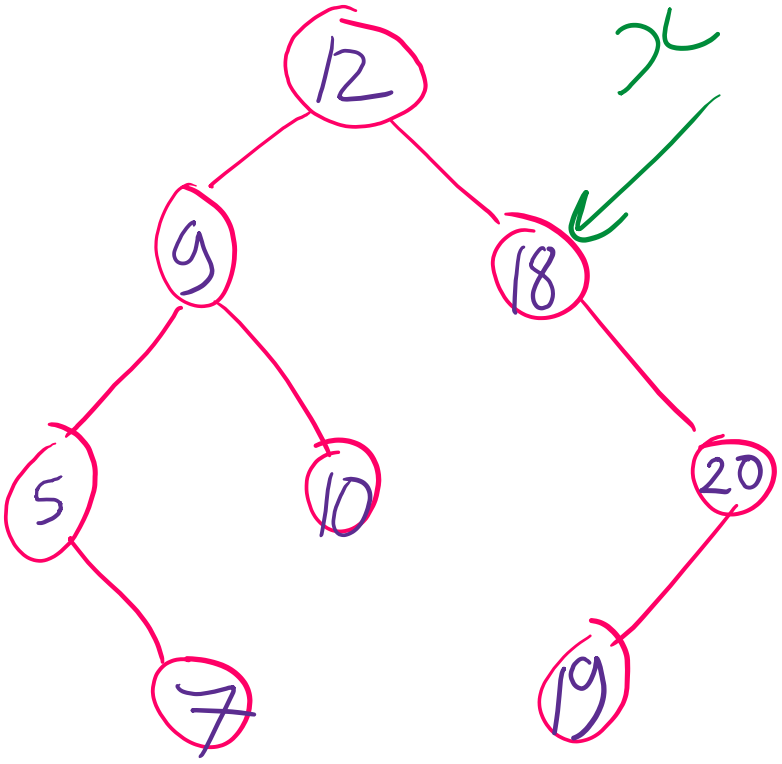
pop & visit

skip leftmost leaf

else x = parent.right

Visit order: 7, 5, 10, 9

Iterative Postorder traversal



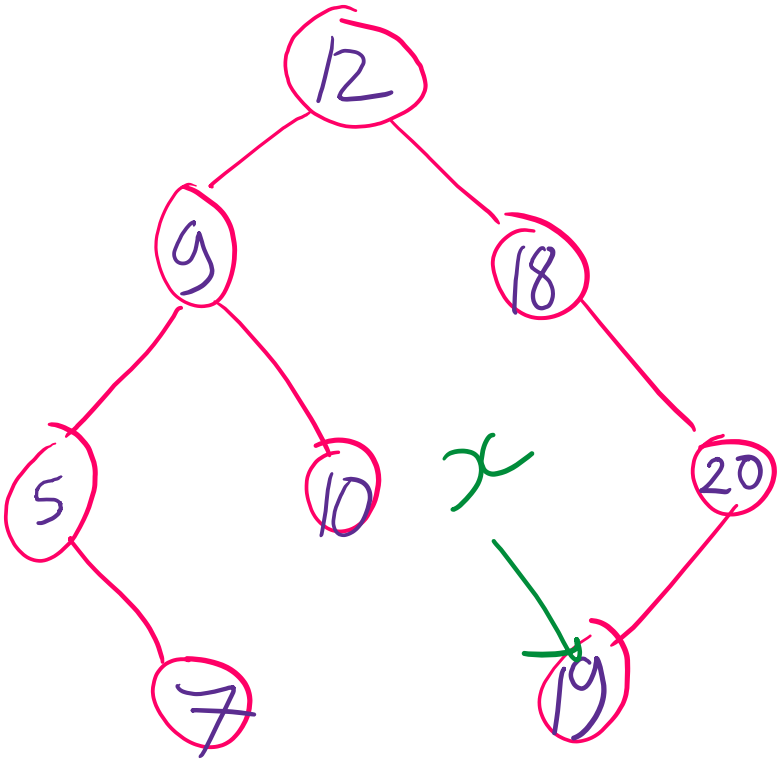
Stack (explicit)

repeat until stack empty & x == null

x = leftmost leaf
Push to stack while moving down
visit x
if x is rightchild
pop & visit
skip leftmost leaf
else x = parent.right

Visit order: 7, 5, 10, 9

Iterative Postorder traversal



repeat until stack empty & x == null

Stack (explicit)

x = leftmost leaf

Push to stack while moving down

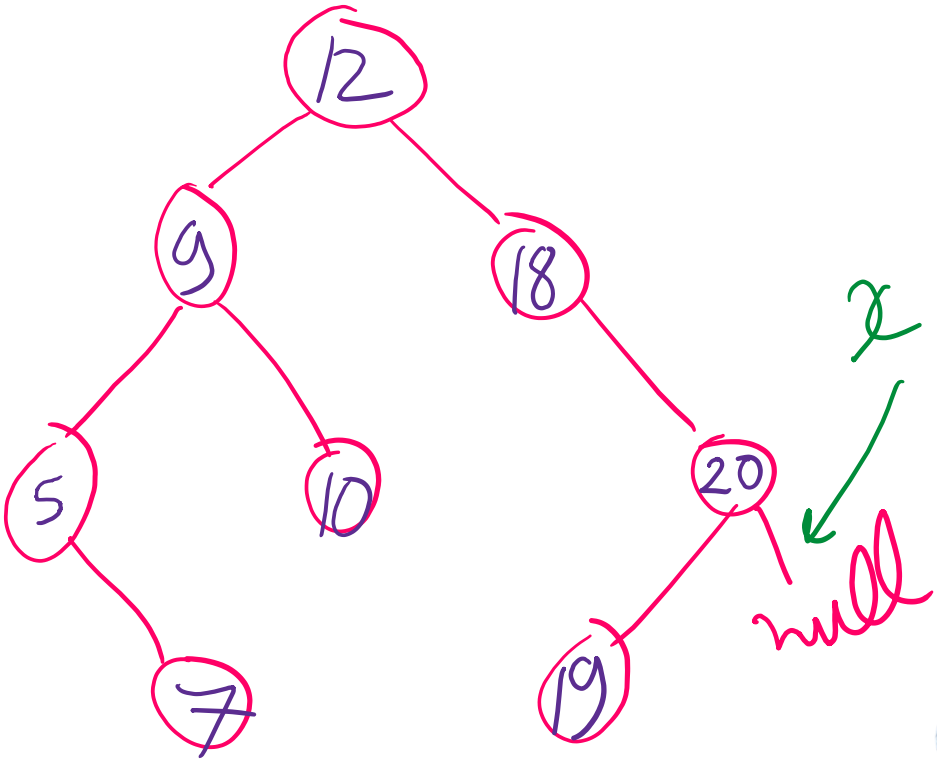
visit x

if x is right child
pop & visit
skip leftmost leaf

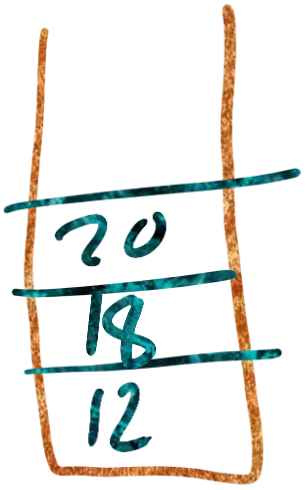
else x = parent.right

Visit order: 7, 5, 10, 9, 19

Iterative Postorder traversal



x



Stack (explicit)

repeat until stack empty & x == null

x = leftmost leaf

Push to stack while moving down

visit x

if x is right child

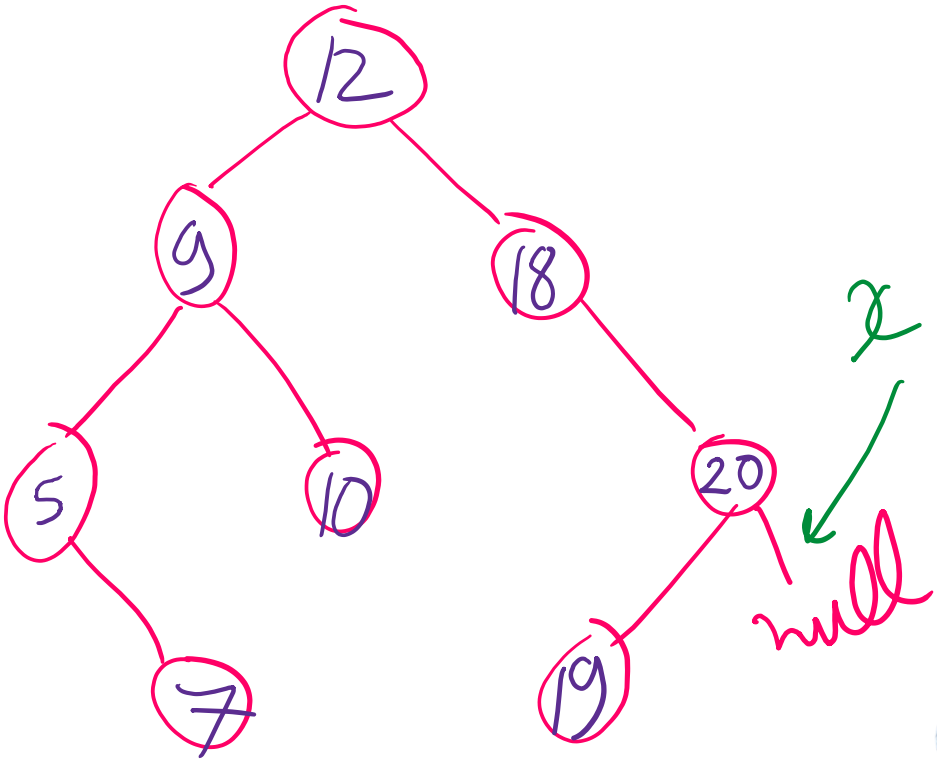
pop & visit

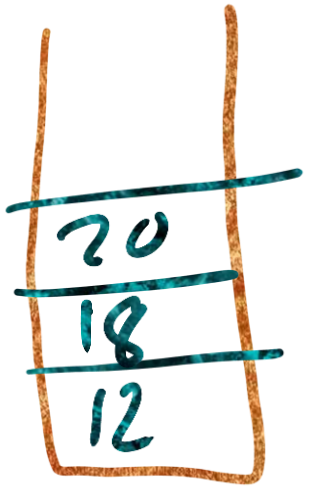
skip leftmost leaf

else x = parent.right

Visit order: 7, 5, 10, 9, 19

Iterative Postorder traversal





Stack (explicit)

repeat until stack empty & x == null

x = leftmost leaf

Push to stack while moving down

visit x

if x is right child

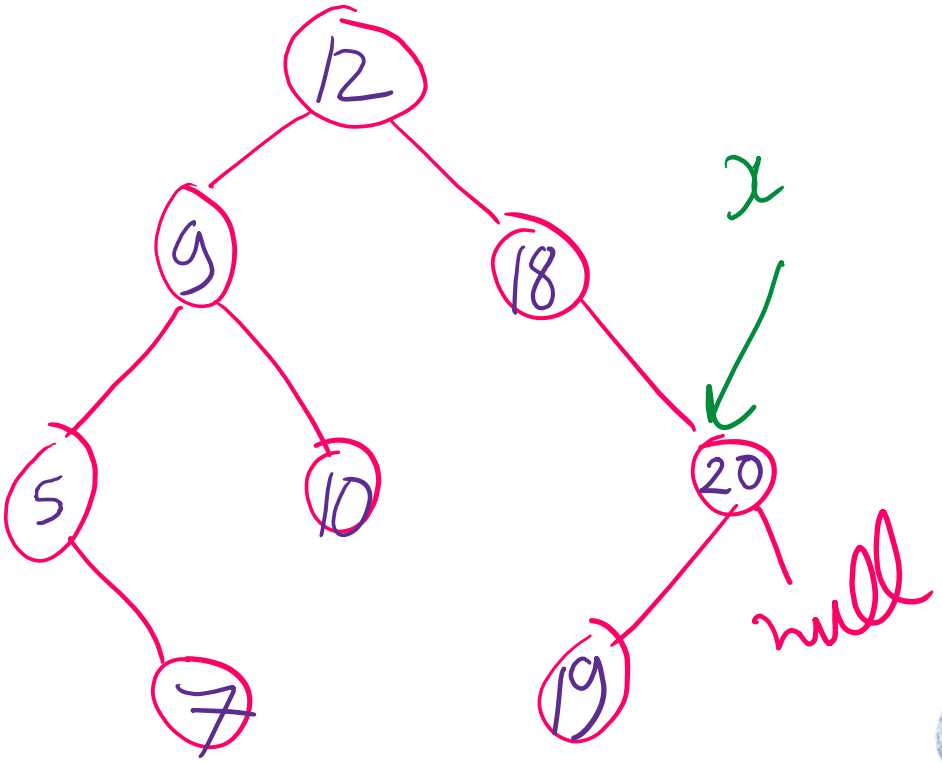
pop & visit

skip leftmost leaf

else x = parent.right

Visit order: 7, 5, 10, 9, 19

Iterative Postorder traversal



Stack (explicit)

repeat until stack empty & x == null

x = leftmost leaf

Push to stack while moving down

visit x

if x is right child

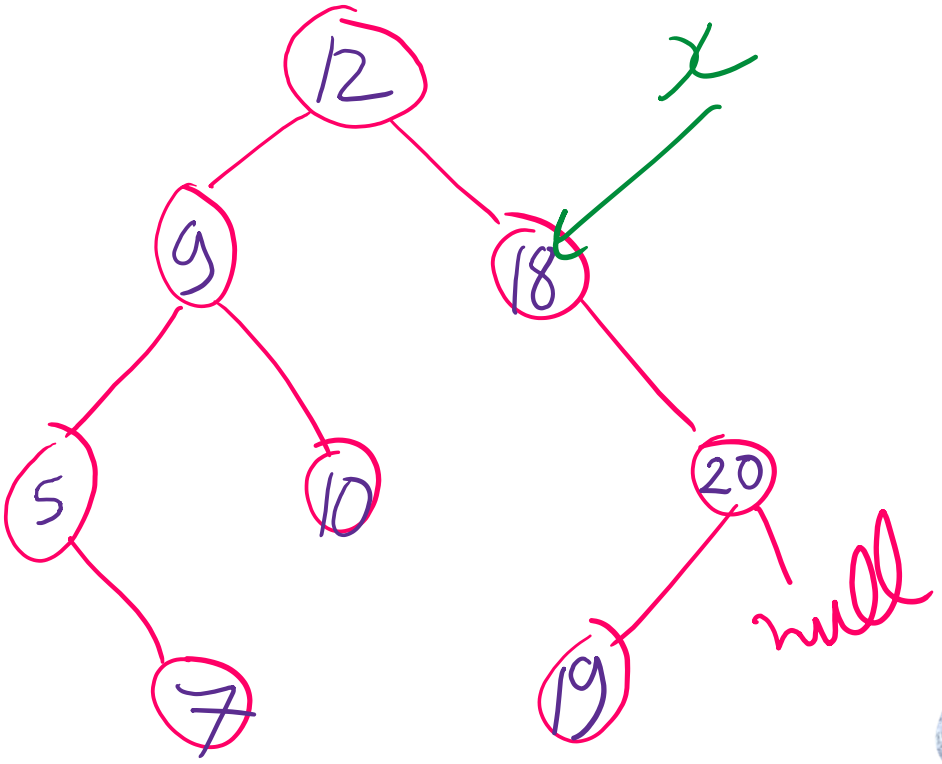
pop & visit

skip leftmost leaf

else x = parent.right

Visit order: 7, 5, 10, 9, 19, 20

Iterative Postorder traversal



Stack (explicit)

repeat until stack empty & x == null

x = leftmost leaf

Push to stack while moving down

visit x

if x is right child

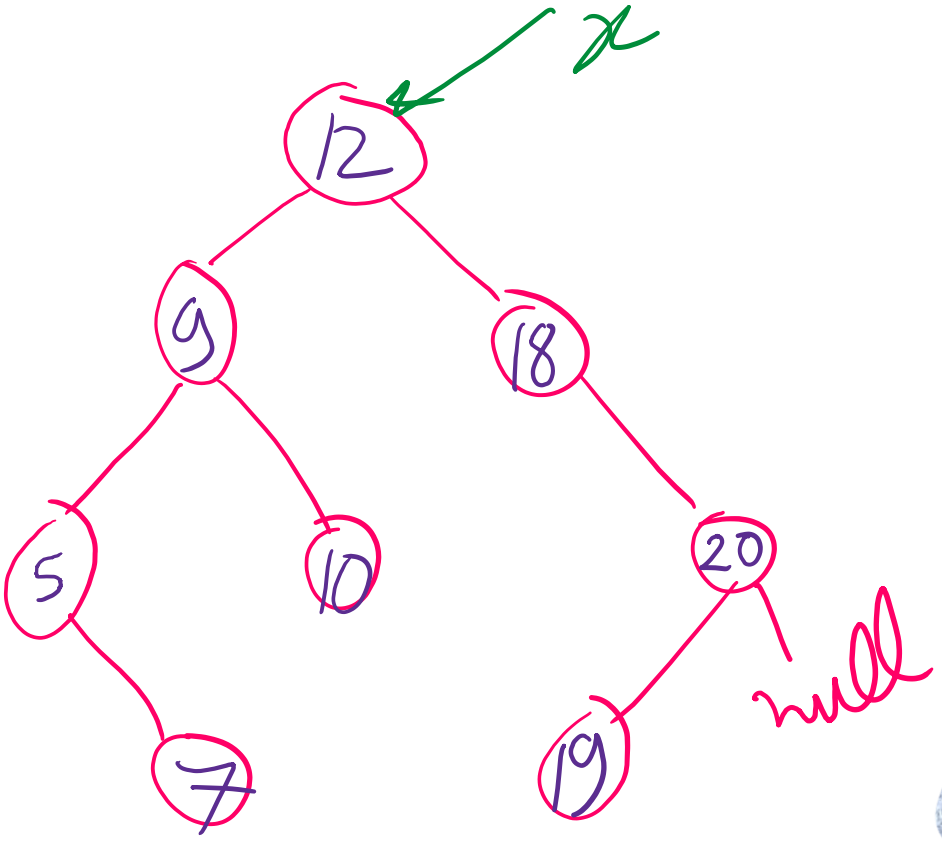
pop & visit

skip leftmost leaf

else x = parent.right

Visit order: 7, 5, 10, 9, 19, 20, 18

Iterative Postorder traversal



Stack (explicit)

repeat until stack empty & x == null

x = leftmost leaf
push to stack while moving down

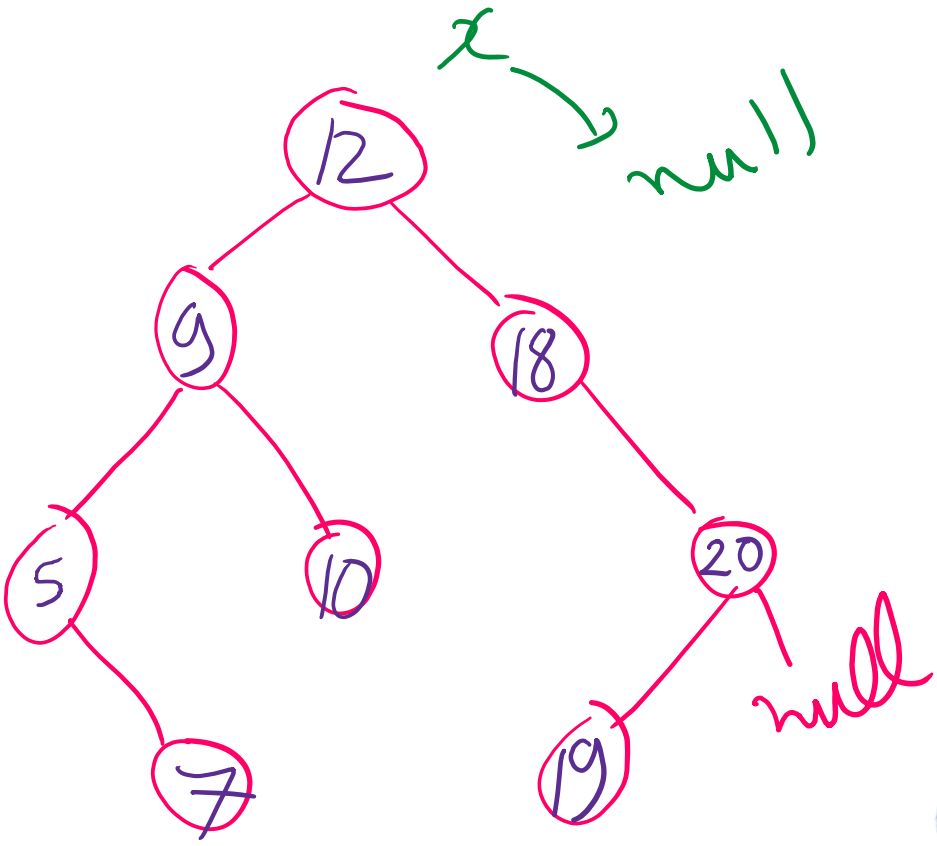
visit x

if x is right child
pop & visit
skip leftmost leaf

else x = parent.right

Visit order: 7, 5, 10, 9, 19, 20, 18, 12

Iterative Postorder traversal



Stack (explicit)

repeat until stack empty & x == null

x = leftmost leaf
Push to stack while moving down

visit x

if x is right child
pop & visit
skip leftmost leaf

else x = parent.right

Visit order: 7, 5, 10, 9, 19, 20, 18, 12

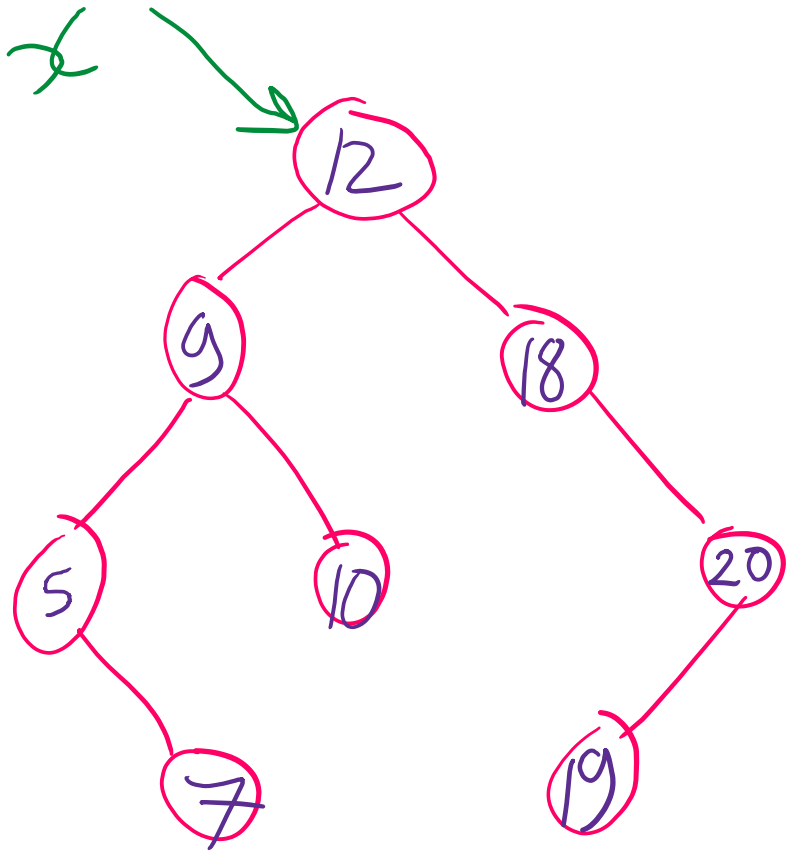
Traversals of a Binary Tree

- Preorder traversal
  - Visit root before we visit root's subtrees
- Inorder traversal
  - Visit root of a binary tree between visiting nodes in root's subtrees.
- Postorder traversal
  - Visit root of a binary tree after visiting nodes in root's subtrees
- Level-order traversal
  - Begin at root and visit nodes one level at a time
  - We will see the implementation when we learn Breadth-First Search of Graphs

Traversals of Binary Search Trees

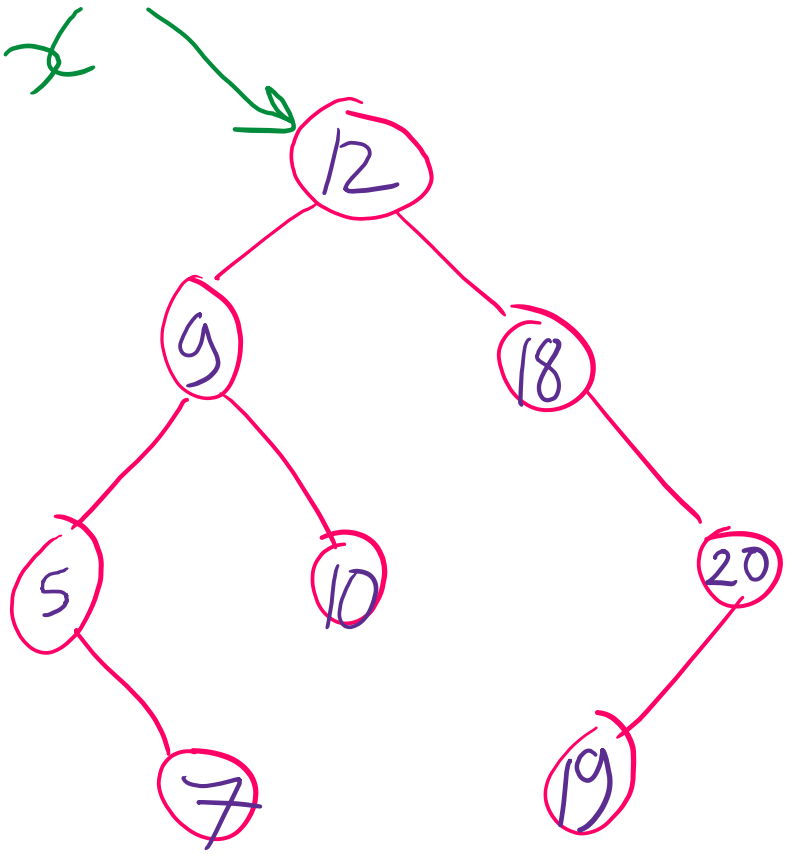
- Traversal for enumerating all items
- Traversal for finding an item

BST search using iteration




```
x = root
while(x != null){
    if equal break;
    if < x = x.left
    if > x = x.right
}
```

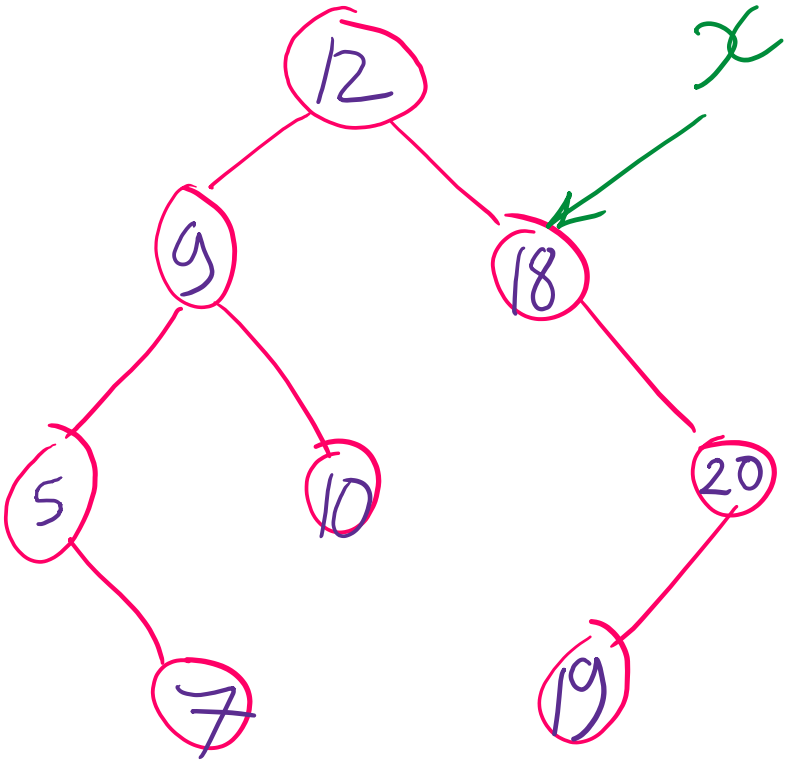
BST search using iteration




```
x = root
while(x != null){
    if equal break;
    if < x = x.left
    if > x = x.right
}
```

Search for 19

BST search using iteration




```
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    if equal break;
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    if > x = x.right
}
```
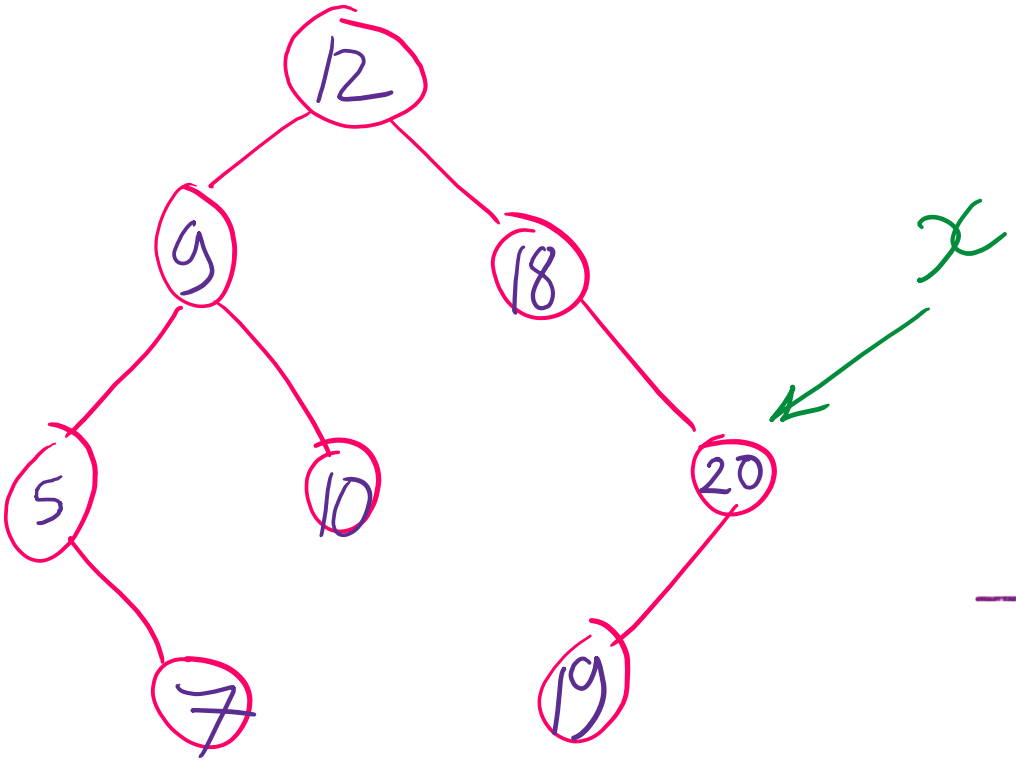

Search for 19

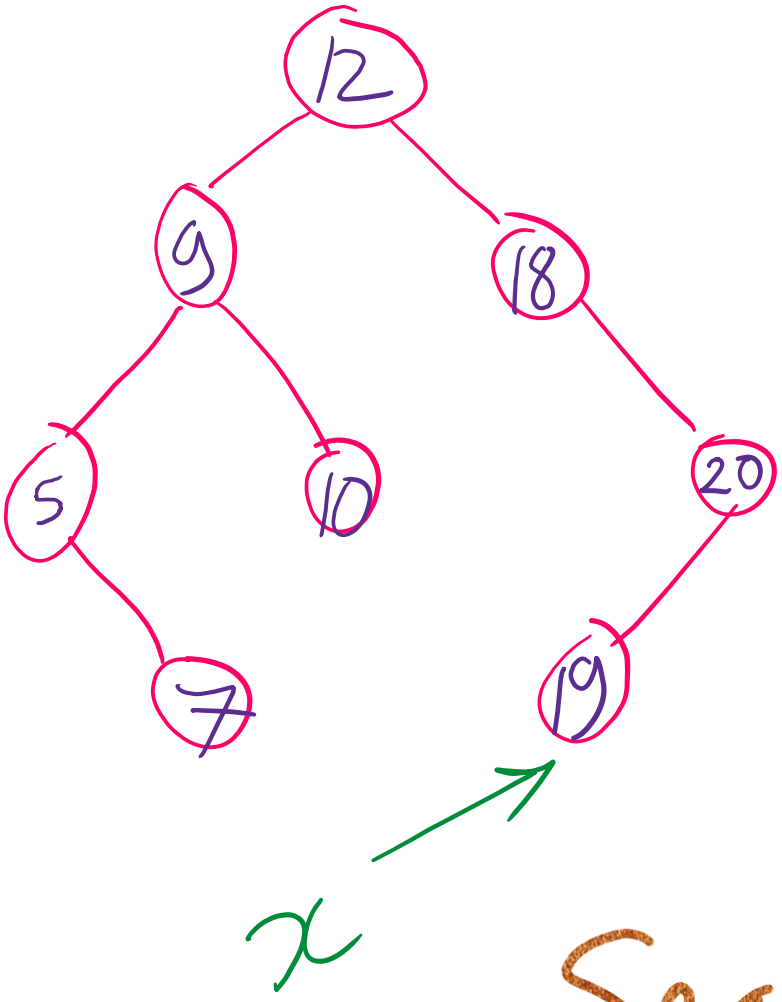
BST search using iteration




```
x = root
while(x != null){
    if equal break;
    if < x = x.left
    if > x = x.right
}
```

Search for 19

BST search using iteration




```
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    if > x = x.right
}
```
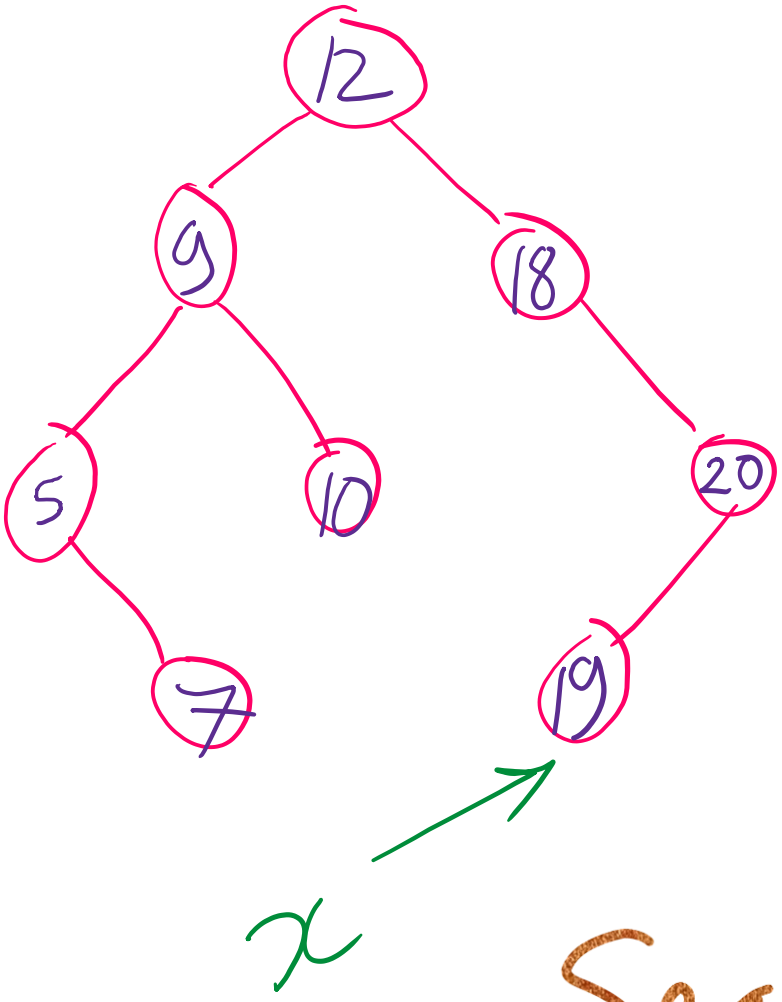

Search for 19

BST search using iteration

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Search for 19