Algorithms and Data Structures 2
CS 1501

Spring 2023

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(Slides are adapted from Dr. Ramirez's and Dr. Farnan's CS1501 slides.)

Announcements

- Upcoming Deadlines
  - Homework 5: this Friday @ 11:59 pm
  - Lab 3: Tuesday 2/14 @ 11:59 pm
  - Assignment 1: Friday 2/17 @ 11:59 pm

Previous lecture

- Digital Searching Problem
  - What if we use the fact that data items are represented as bits in computer memory?
- Digital Search Tree (DST)

This Lecture

- Digital Search Tree (DST)
- Radix Search Trie (RST)
- De La Briandais (DLB) Trie

Digital Search Trees (DSTs)

Instead of looking at less than/greater than, lets go left or right based on the bits of the key

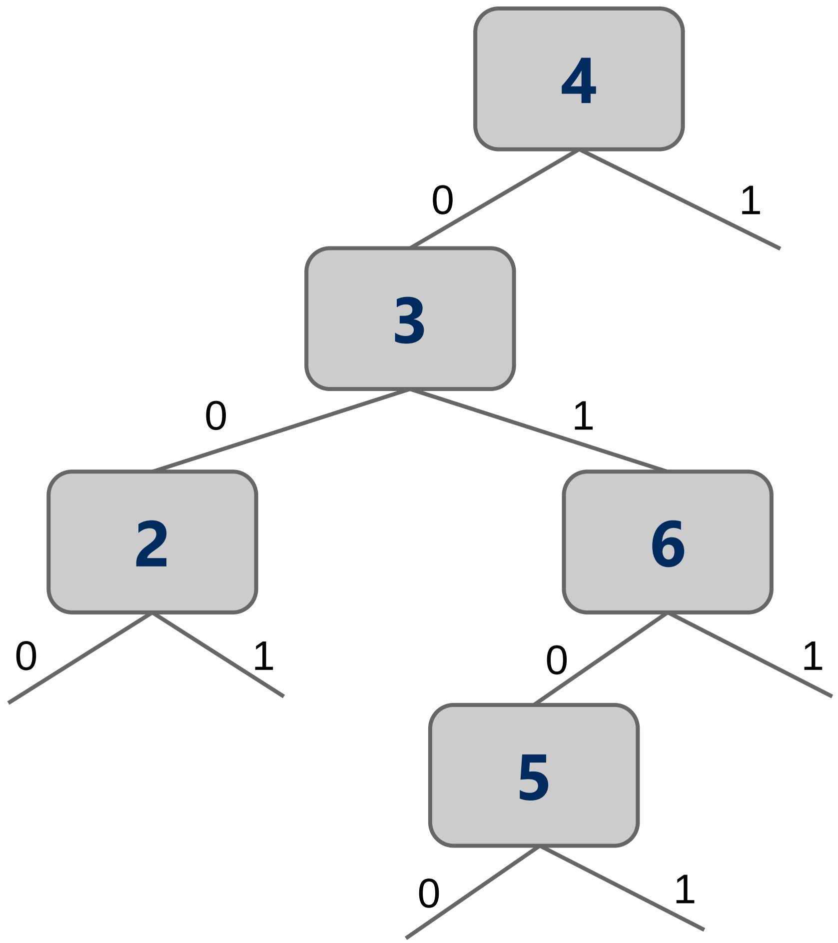
DST example: Insert and Search

Insert:

4	0100
3	0011
2	0010
6	0110
5	0101

Search:

3	0011
7	0111



Inserting into a DST

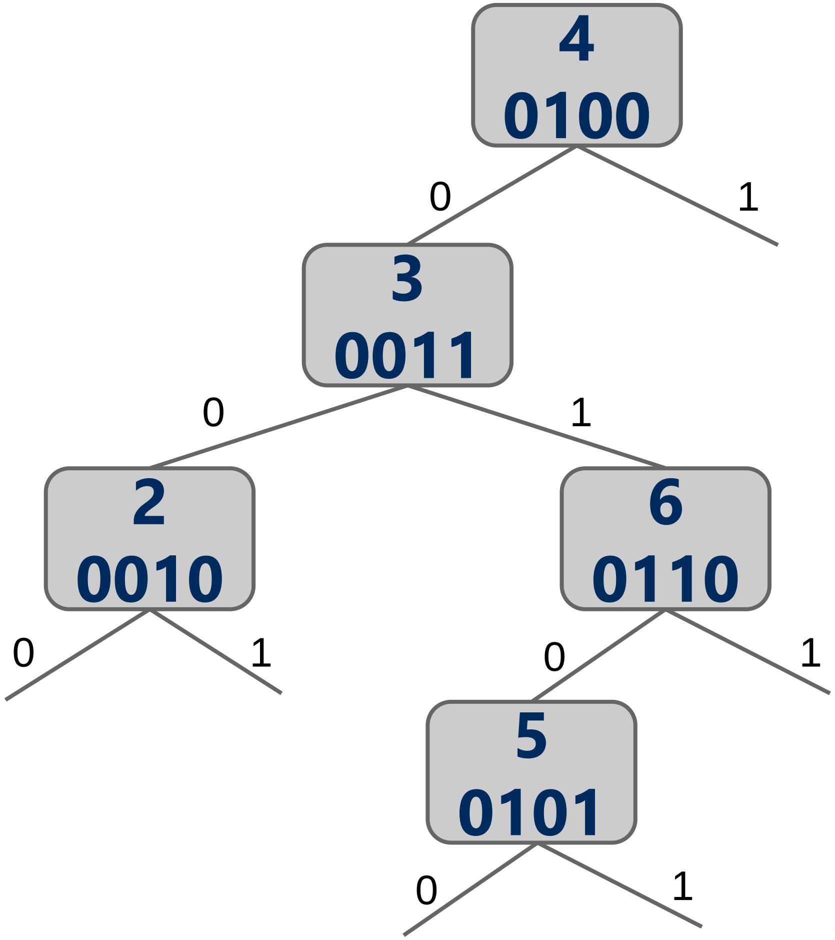
- adding a key *k* and a corresponding value
  - if root is null, add *k* as the root and return
  - current ← root
  - Repeat
    - if k is **equal to** the current.key, replace value and return
    - if current bit of k is 0,
      - if left child is null, add *k* as left child
      - else continue to left child (recursive call or current ← current.left)
    - if current bit of k is 1,
      - if left child is null, add *k* as right child
      - else continue to right child (recursive or current ← current.right)
- When does the algorithm stop?
  - no more bits or
  - hitting a null

Runtime Analysis of Digital Search Trees

- b: the bit length of the target or inserted key
- n: number of nodes in the tree
- Worst-case Runtime?
  - min(b, height of the tree)
- What is the average height of the tree?
  - Assume that having 0 or 1 is equally likely at each bit
  - log(n)
  - In general b >= $\lceil \log n \rceil$

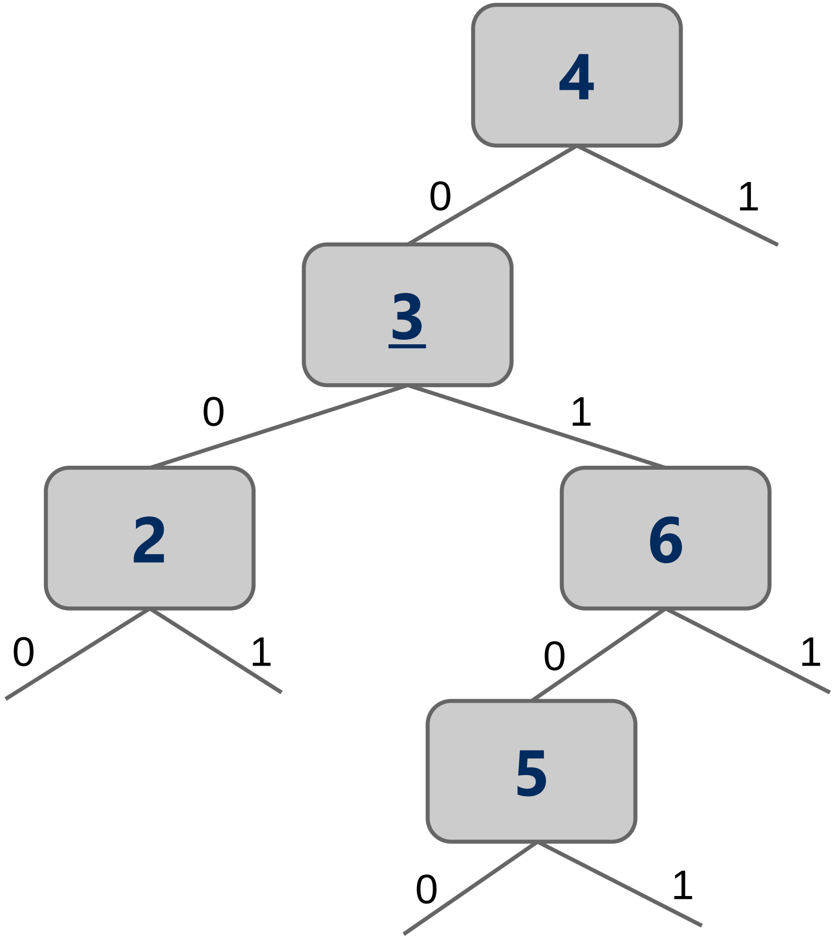
What property does a DST hold?

- In a DST, each node shares a **common prefix of length depth(node)** with all nodes in its subtree
  - e.g., 6 shares the prefix “01” with 5
- In-order traversal doesn’t produce a sorted order of the items
  - Insertion algorithm can be modified to make a DST a BST at the same time



DST example: Delete

- Delete 3
- Can replace it with any leaf in its subtree
- Let's replace it with 2
- OK because 2 shares "0" as a prefix with 3, so it also shares "0" as a prefix with 6 and 5



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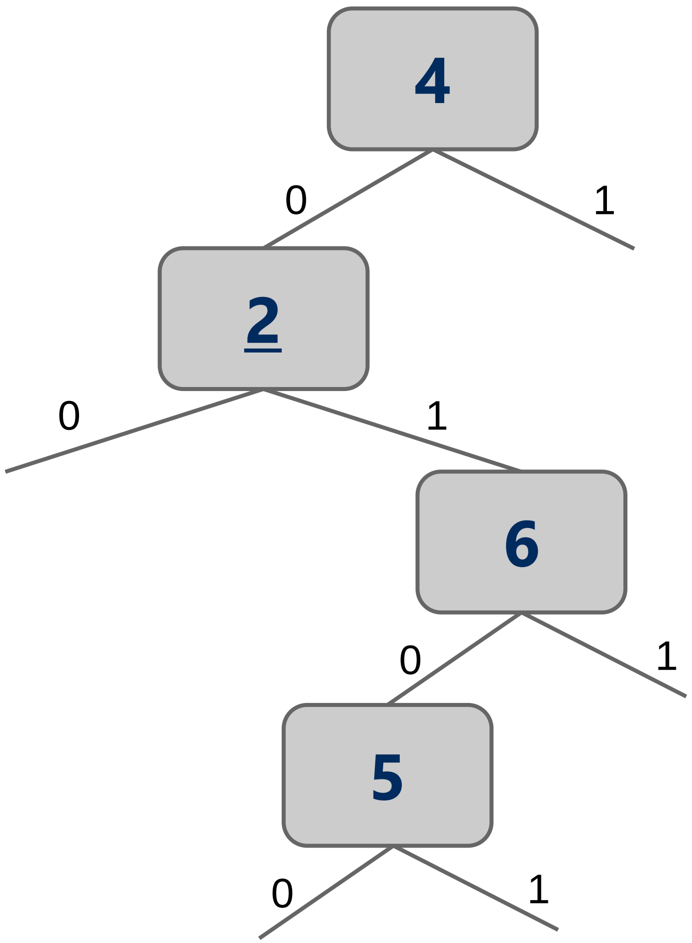


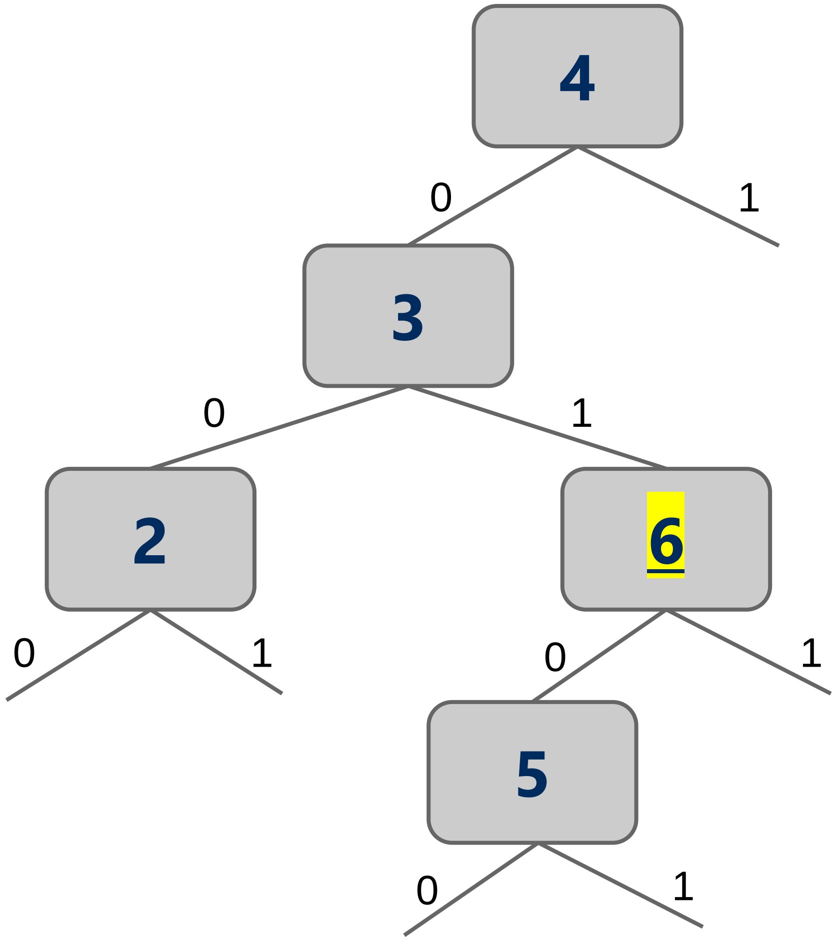
DST example: Variable length keys

- Insert

1 01

- Must be in place of 6

- Replace 6 by 1 and re-insert 6



DST example: Variable length keys

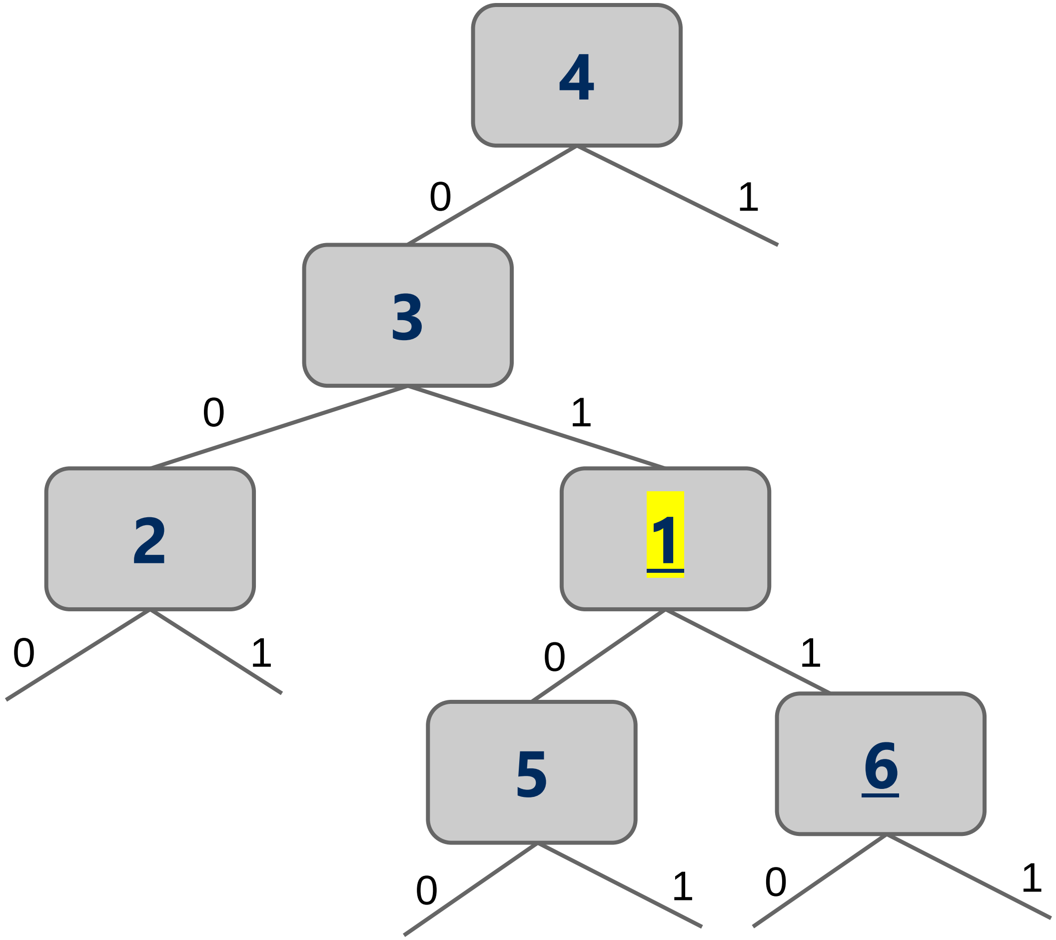
- Insert

1 01

- Must be in place of 6

- Replace 6 by 1 and re-insert

6 0110



Analysis of Digital Search Trees

- We end up doing many **equality** comparisons against the full key
- This is better than less than/greater than comparison in BST
- Can we improve on this?

Radix search tries (RSTs)

- Trie as in retrieve, pronounced the same as “try”
- Instead of storing keys inside nodes in the tree, we store them implicitly as paths down the tree
  - Interior nodes of the tree only serve to direct us according to the bitstring of the key
  - Values can then be stored at the end of key’s bitstring path (i.e., at leaves)
  - RST uses less space than BST and DST

Adding to Radix Search Trie (RST)

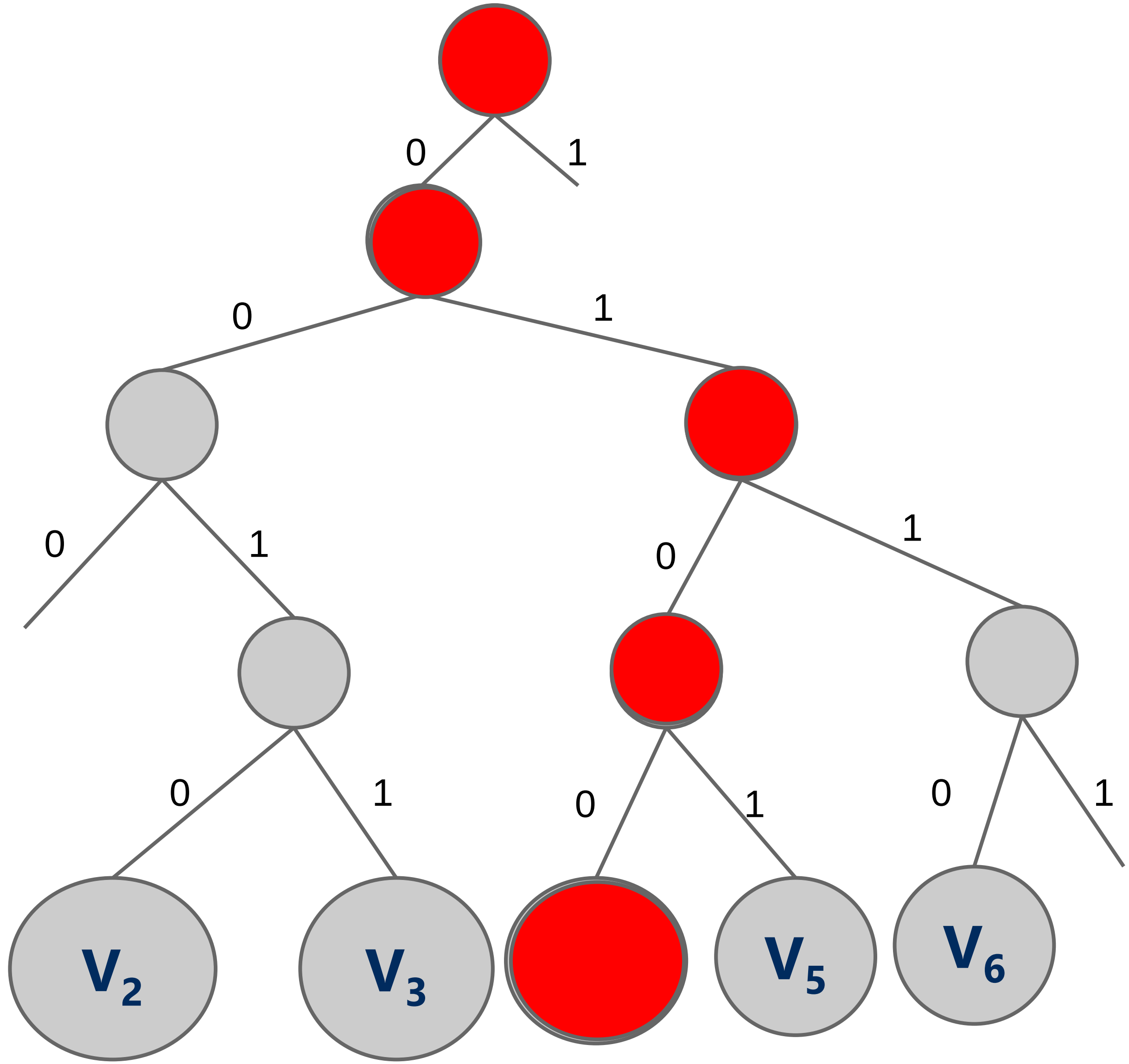
- Input: key and corresponding value
- if root is null, set root ← new node
- current node ← root
- for each *bit* in the key
  - if bit == 0,
    - if current.left is null, set current.left = new node
    - move to left child
      - set current ← current.left
  - if bit == 1,
    - if current.right is null, set current.right = new node

    current ← current.right
- current.value = value

RST example

Insert:

4	0100
3	0011
2	0010
6	0110
5	0101



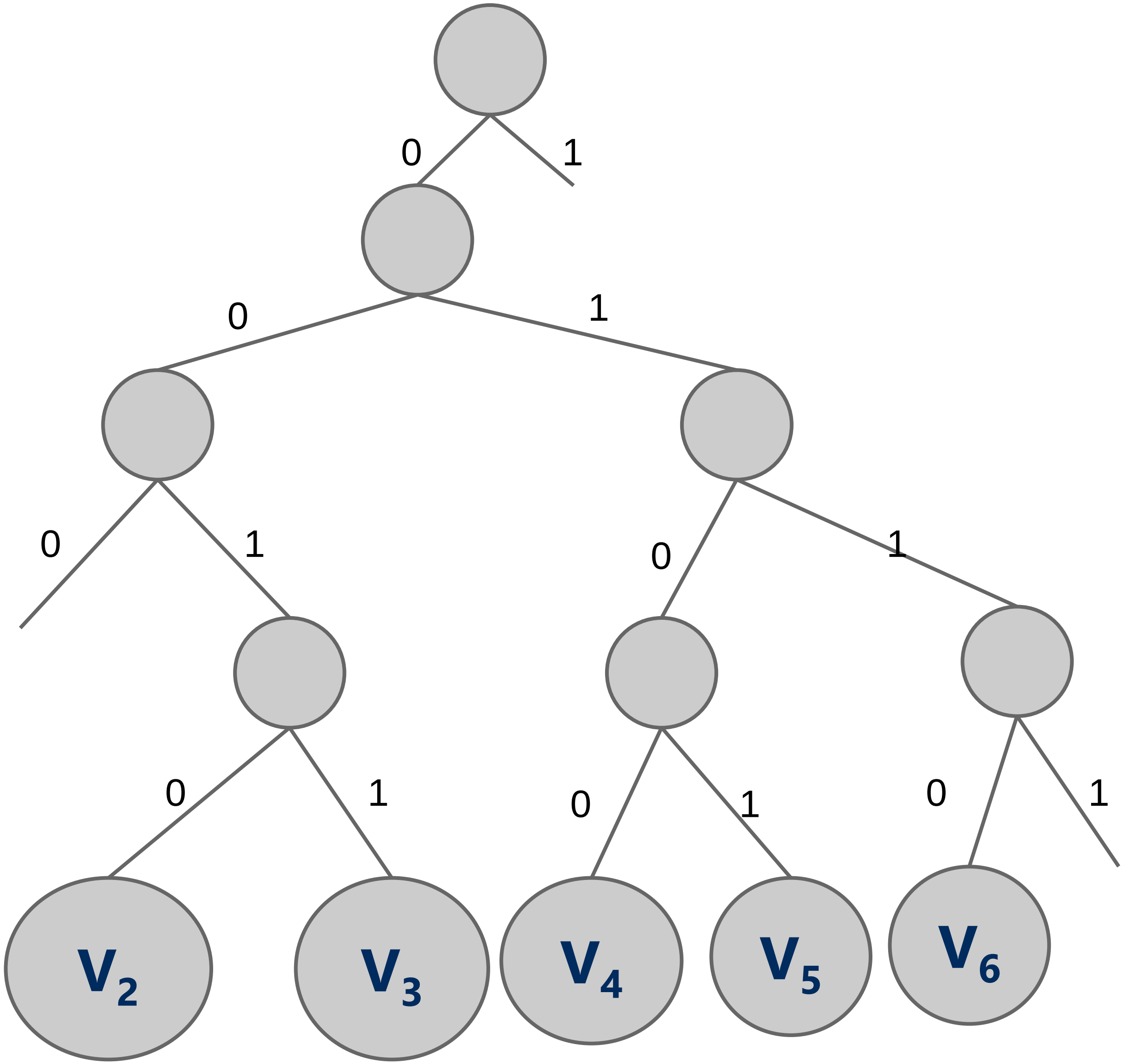
Searching in Radix Search Trie (RST)

- Input: key
- current node ← root
- for each *bit* in the key
  - if current node is null, return *key not found*
  - if bit == 0,
    - current ← current.left
  - if bit == 1,
    - current ← current.right
- if current node is null or the value inside is null
  - return key *not found*
- else return current.value

RST example

Search:

3 0011

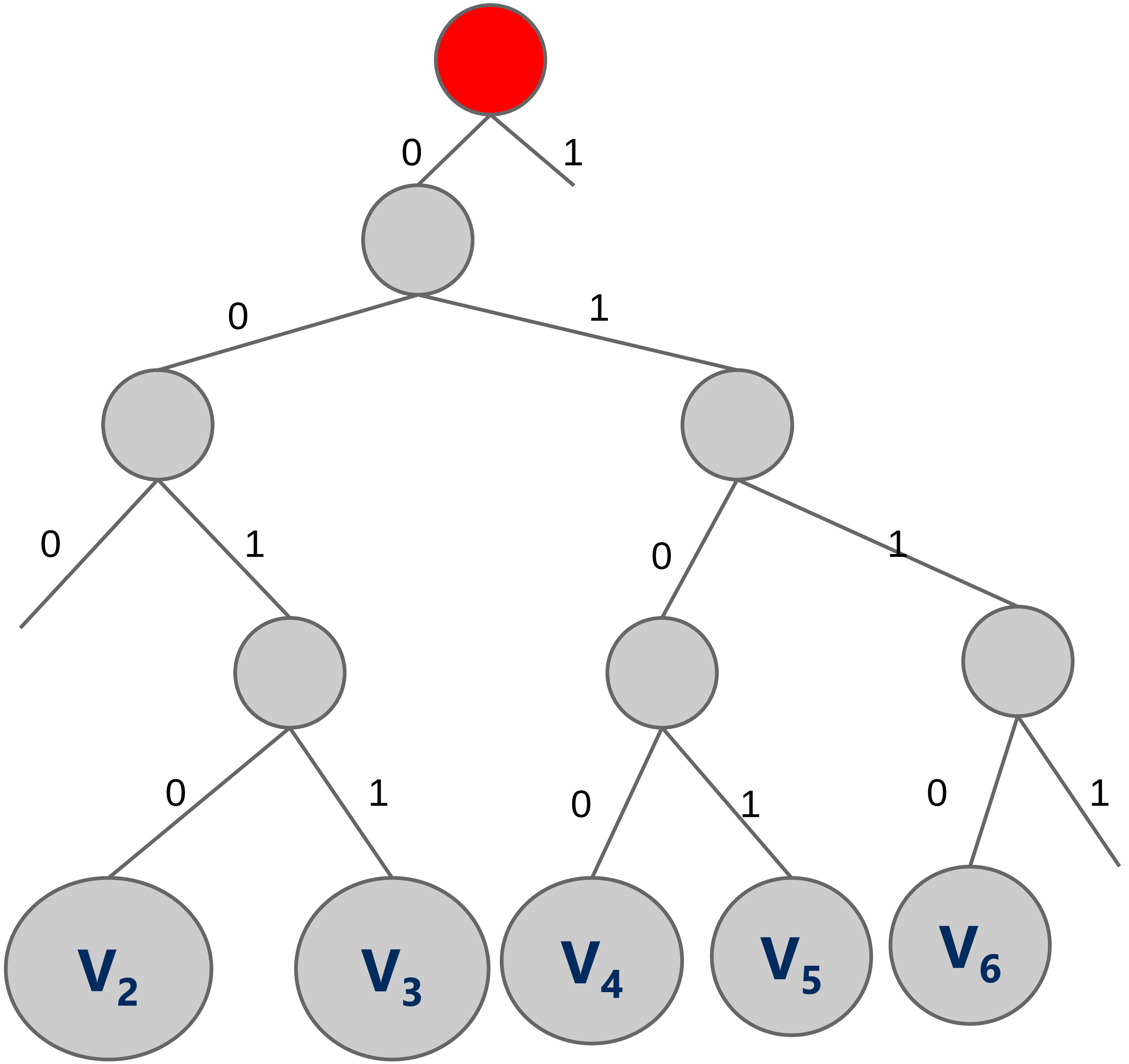


RST example

Search:

3 0011

7 0111



RST example

Search:

3 0011

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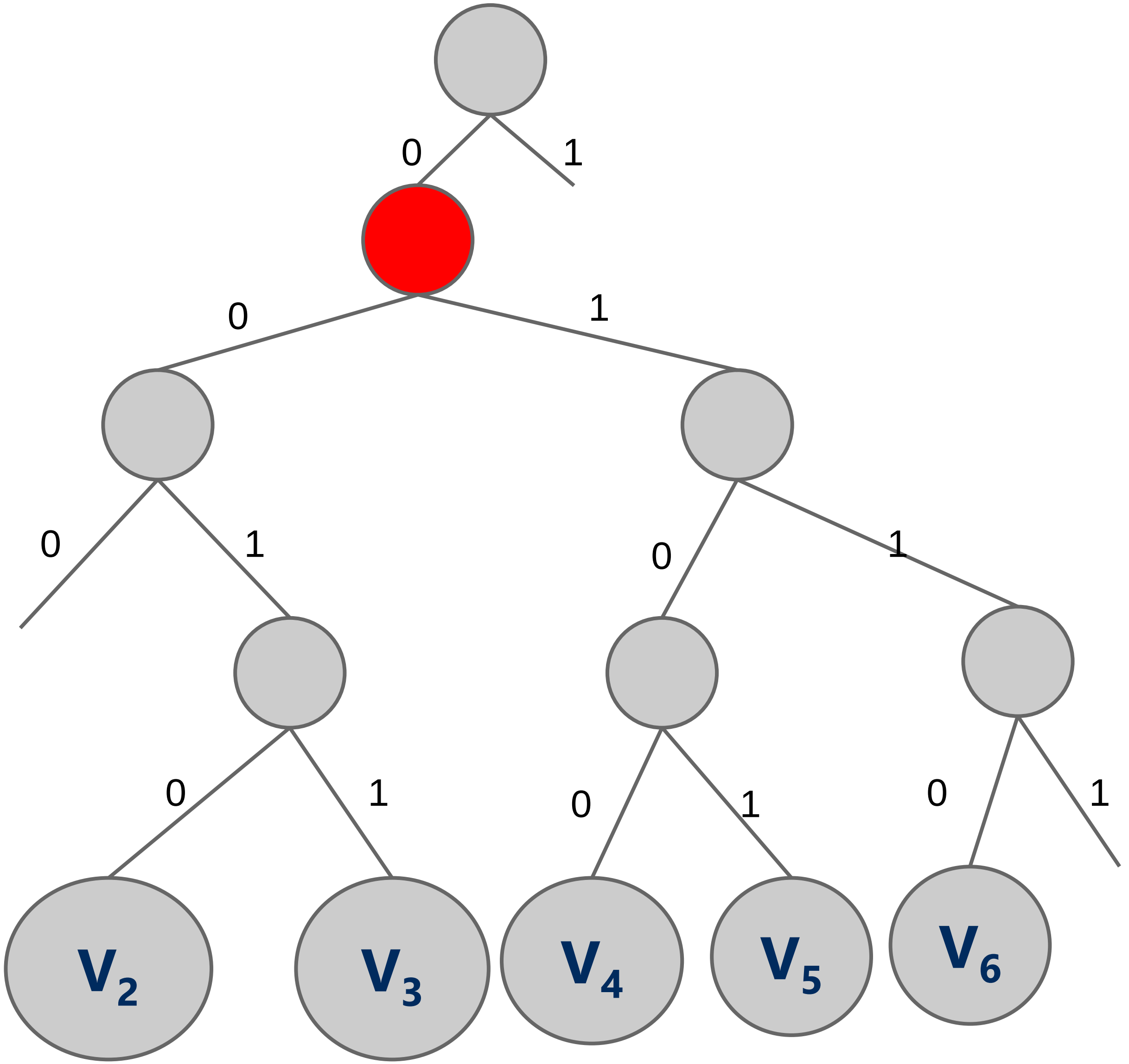

RST example

Search:

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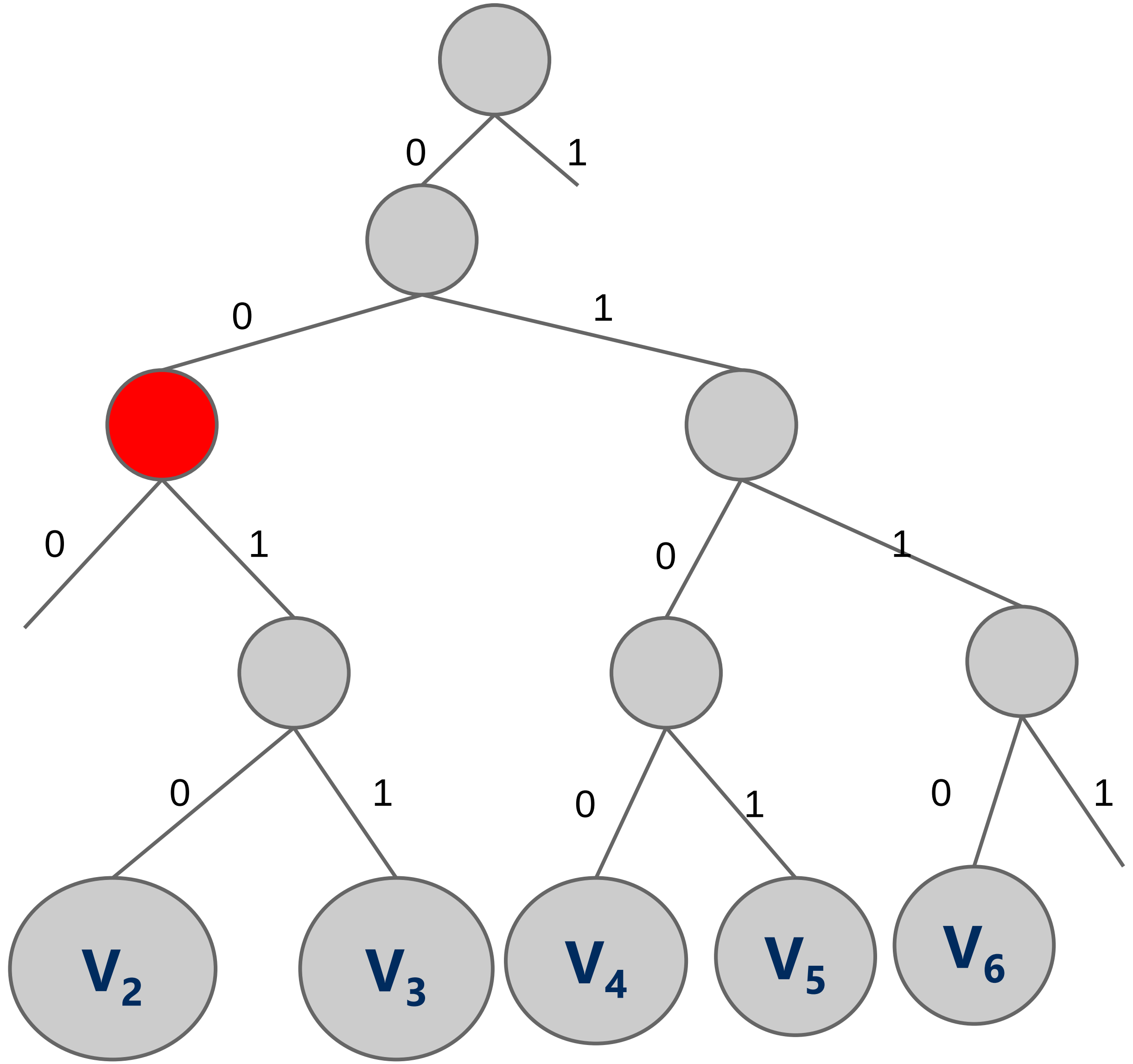

RST example

Search:

3 0011

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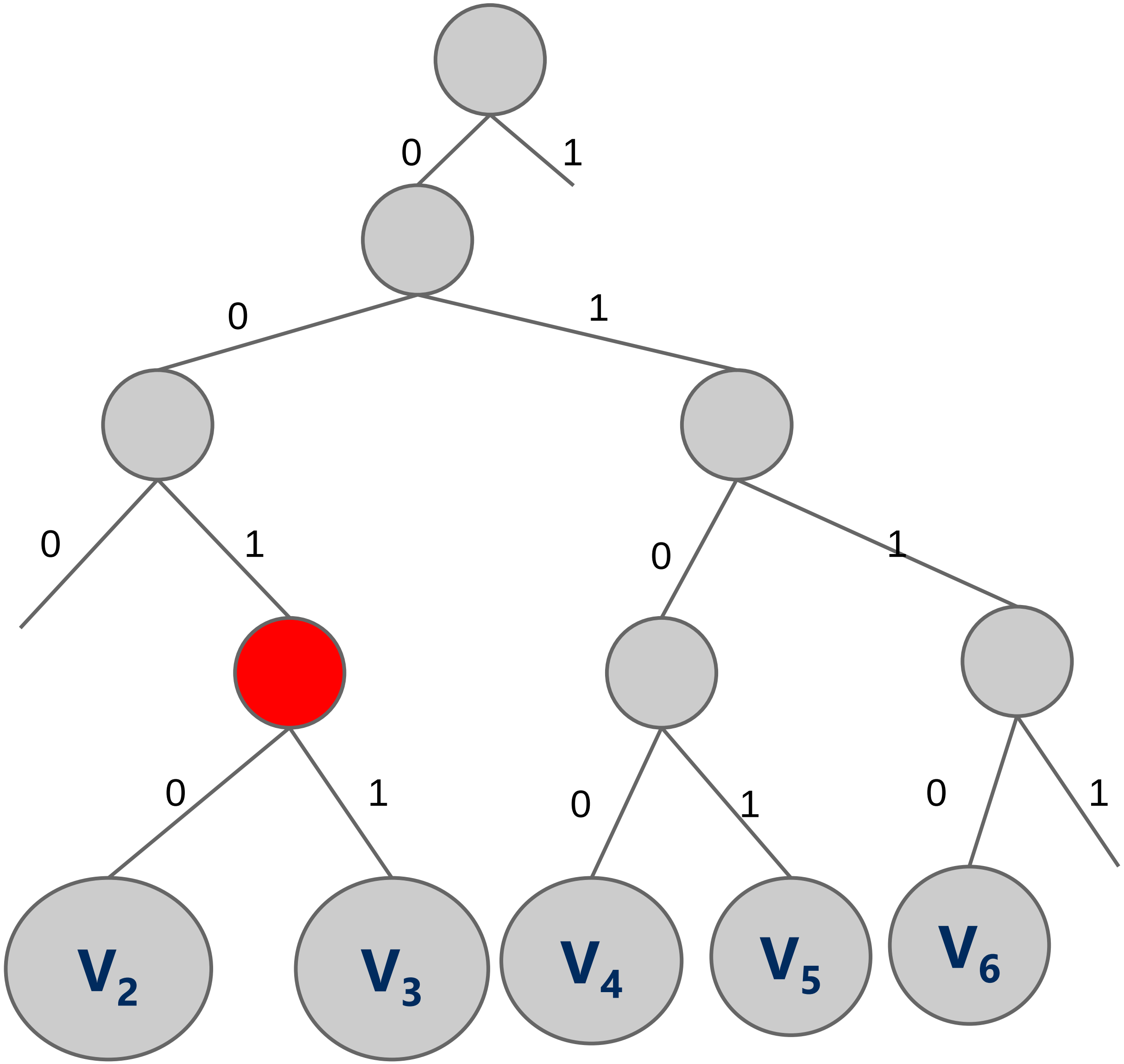
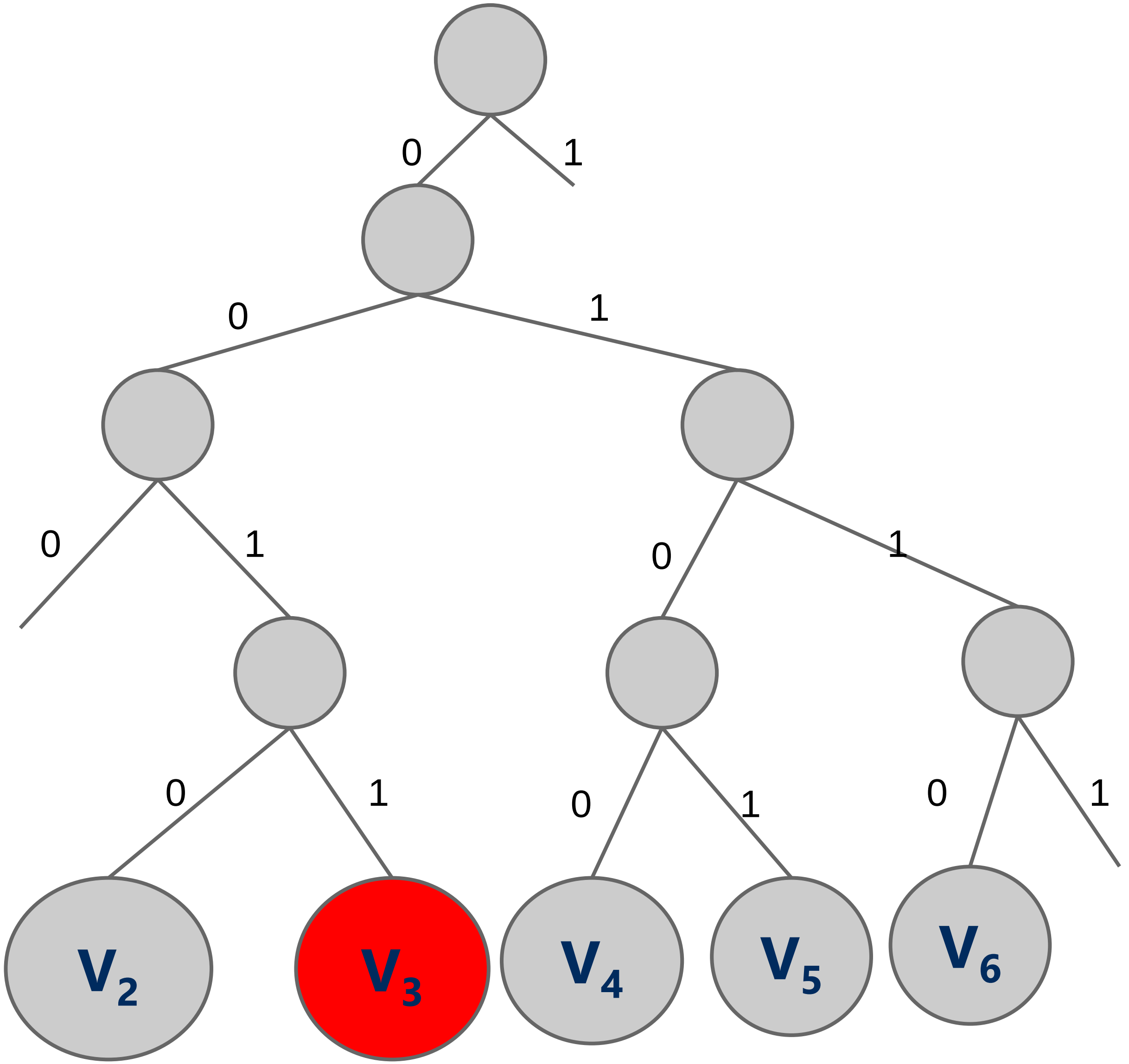


RST example

Search:

3 0011

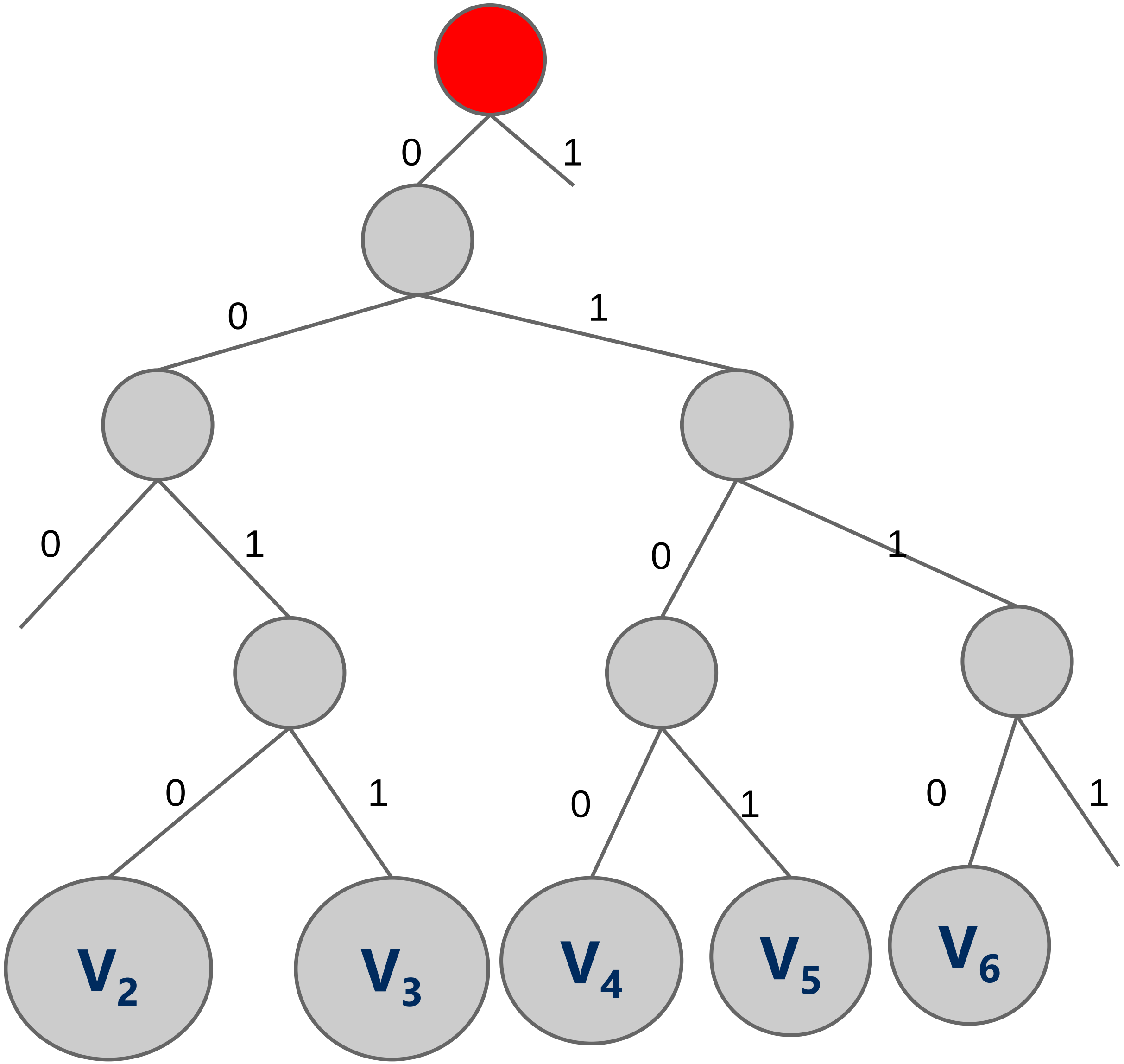
7 0111



RST example

Search:

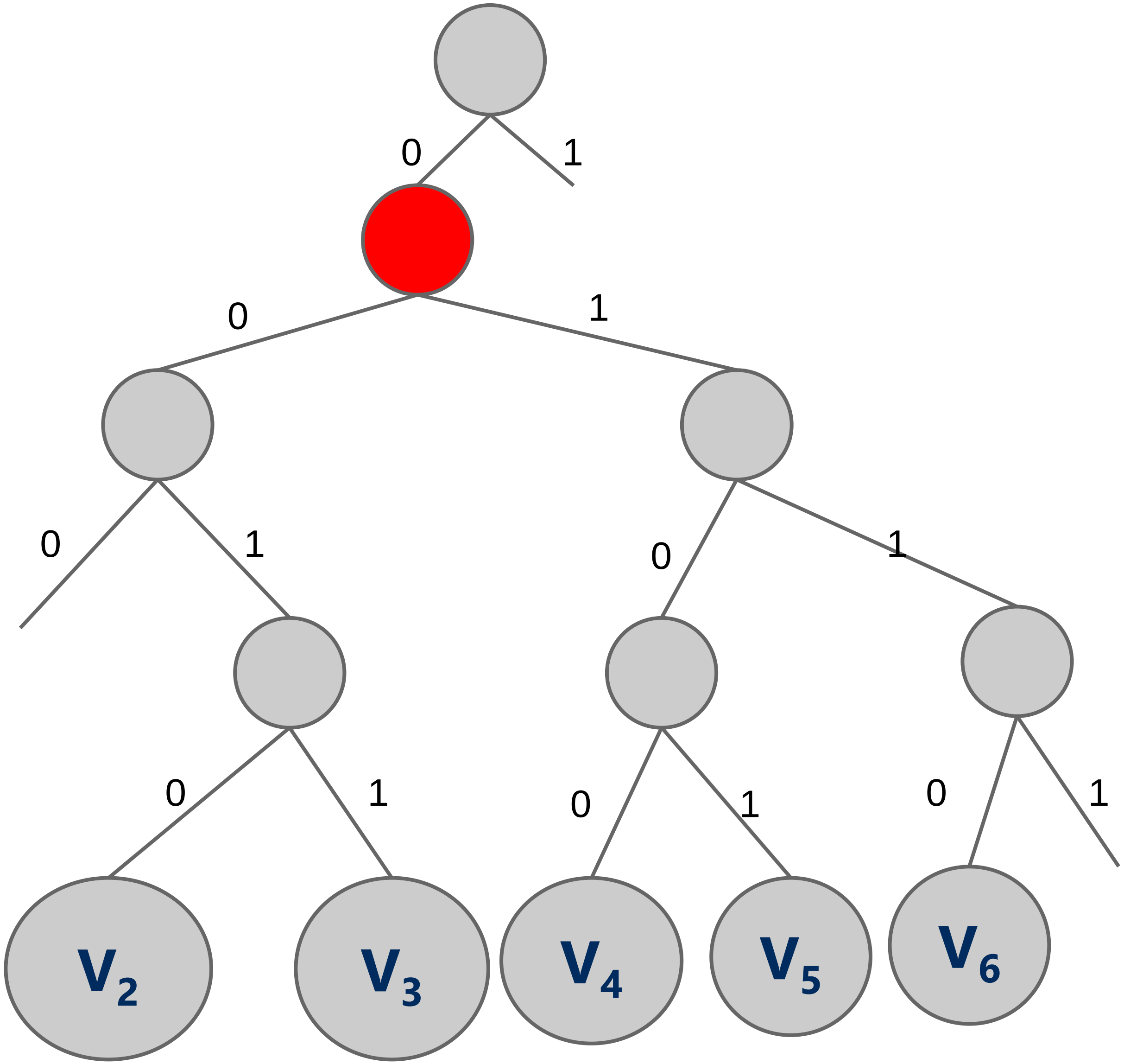
7 0111



RST example

Search:

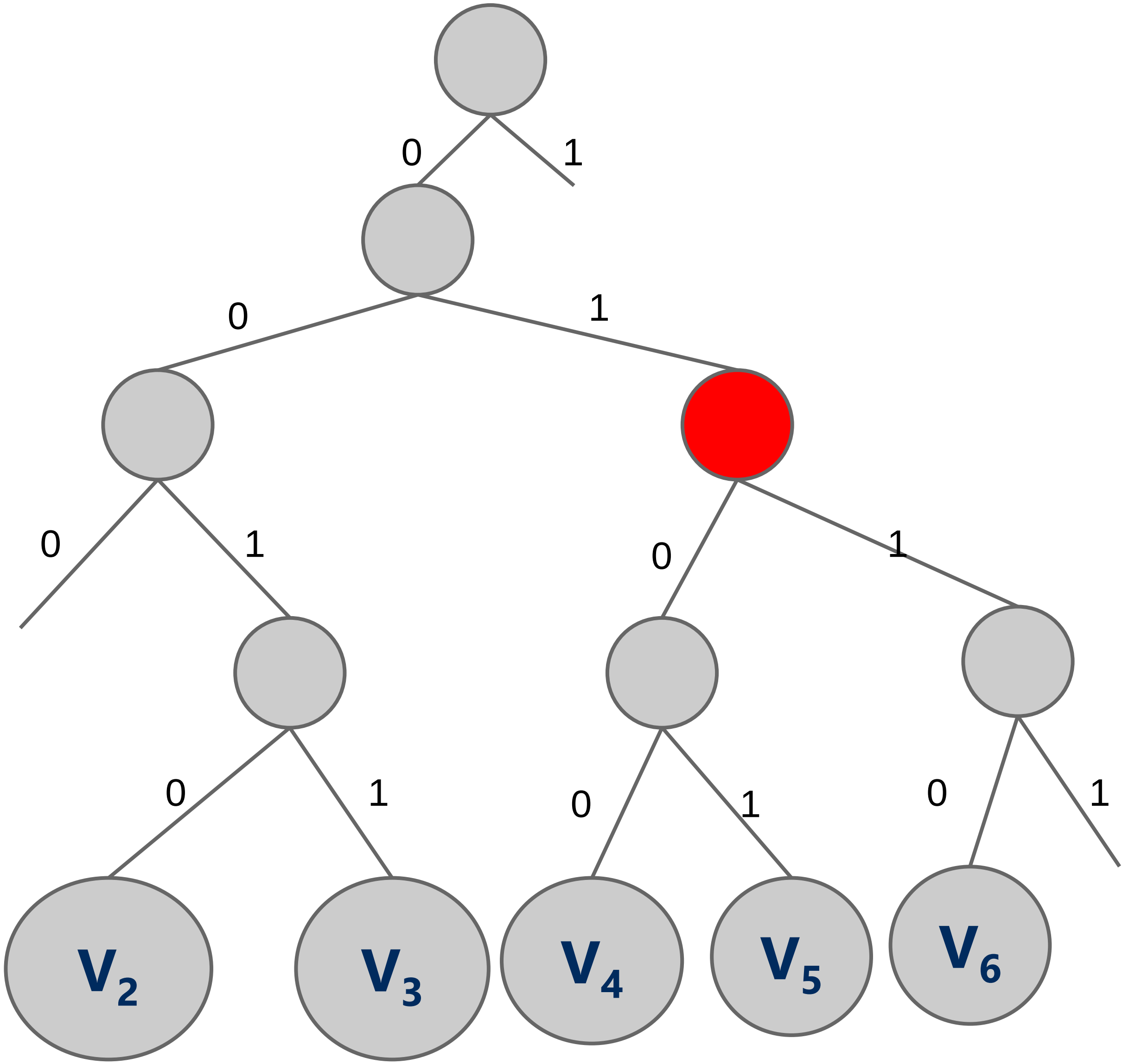
7 0111



RST example

Search:

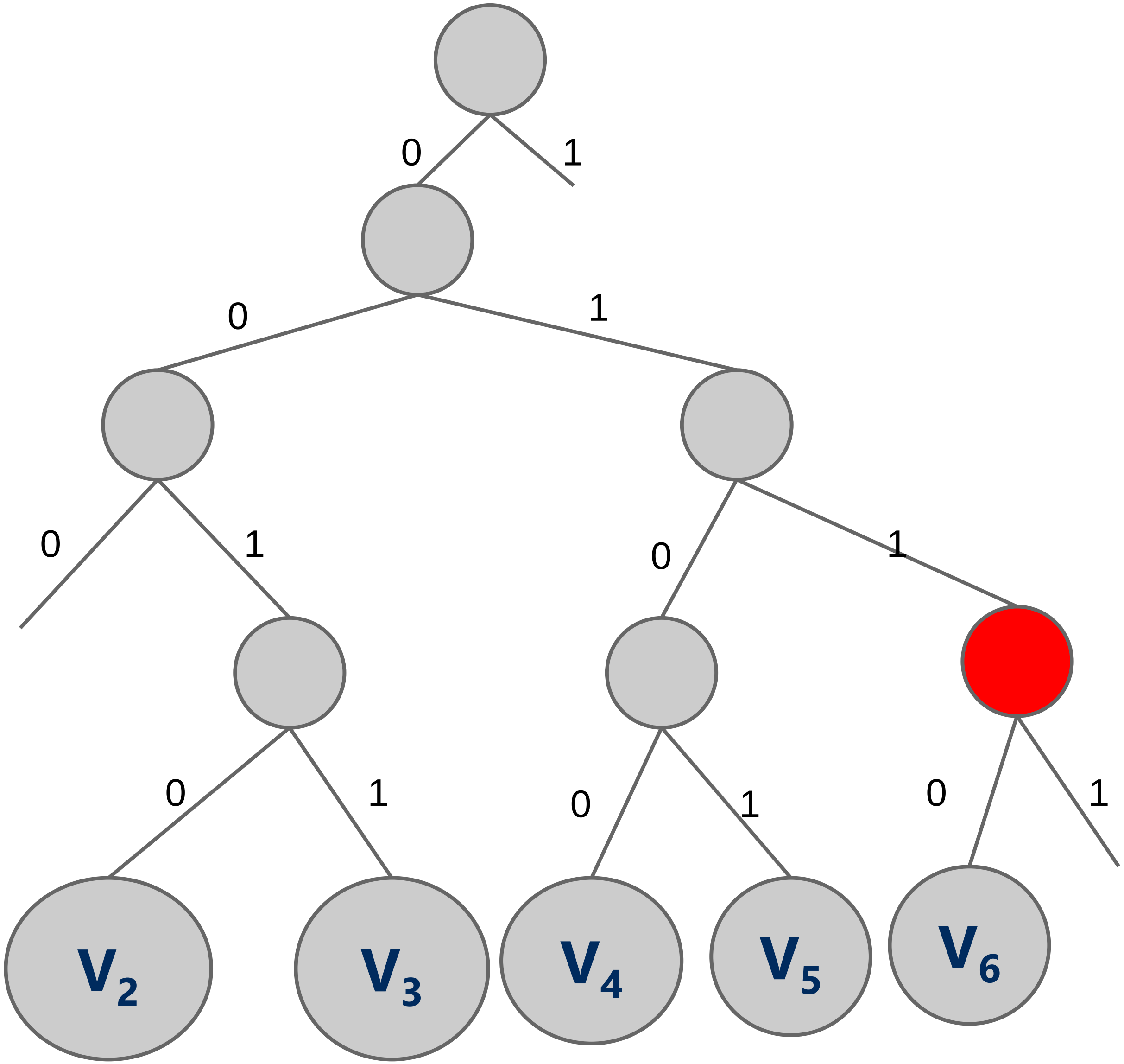
7 0111



RST example

Search:

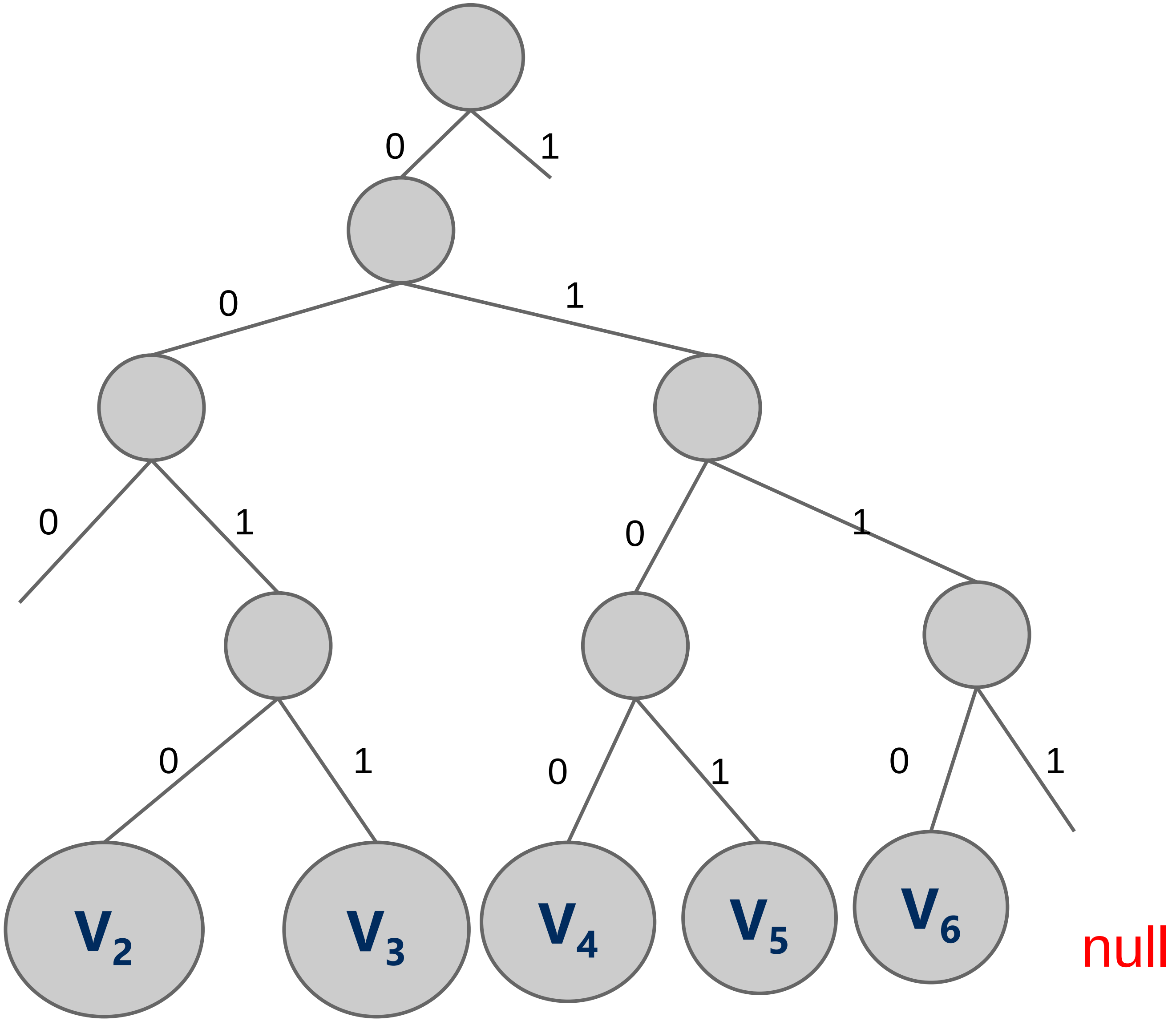
7 0111



RST example

Search:

7 0111



RST Runtime analysis

- **for add:**
  - Theta(b): b is the bit length of the key
  - However, this time we don't have full key comparisons
- for search:
  - **search hit**
    - Theta(b)
  - **search miss**
    - maybe less than Theta(b)

RST Limitations

- Would this structure work as well for other key data types?
- Characters?
    - Characters are the same as 8-bit ints (assuming simple ascii)
- Strings?
- May have huge bit lengths
- How to store Strings?

Larger branching factor tries

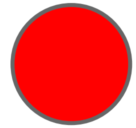
- In our binary-based Radix search trie, we considered one bit at a time
- What if we applied the same method to characters instead of bits in a string?
  - What would this new structure look like?
  - How many children per node?
    - up to R (the alphabet size)
  - Also called R-way radix search tries

Adding to R-way Radix RST

- if root is null, set root ← new node
- current node ← root
- for each *character c* in the key
  - Find the c*th* child of current
    - if child is null, create a new node and attach as the c*th* child
    - move to child
      - current ← child
- insert value into current node

Another trie example

she



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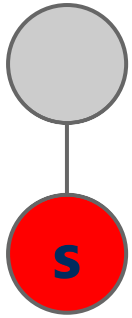

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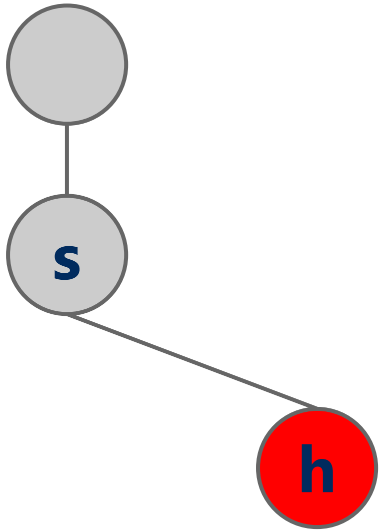

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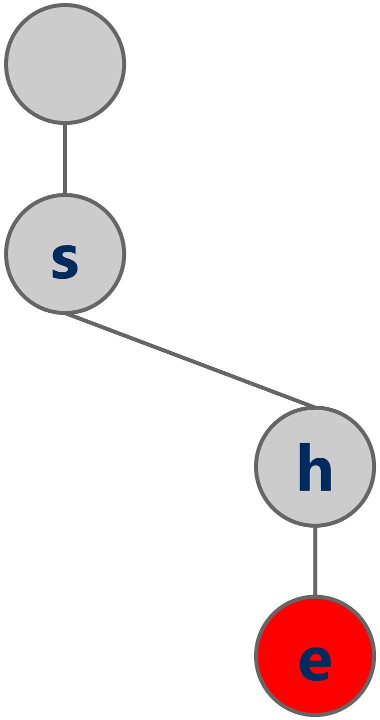

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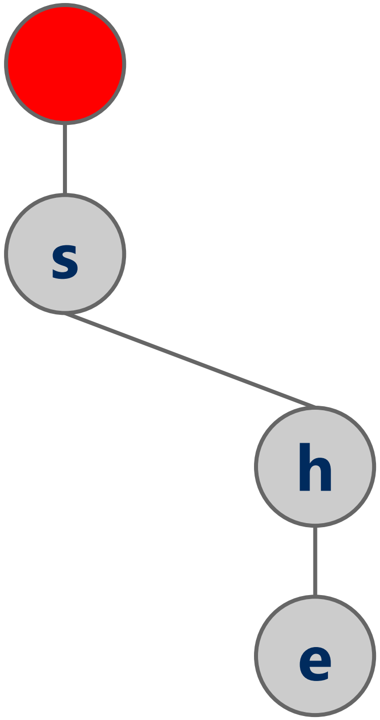
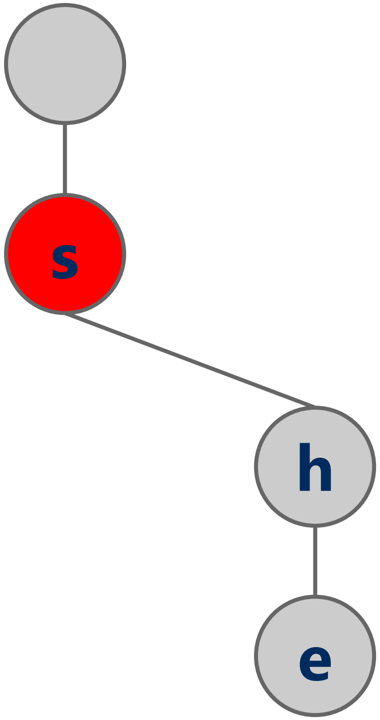


sells

Another trie example



sells

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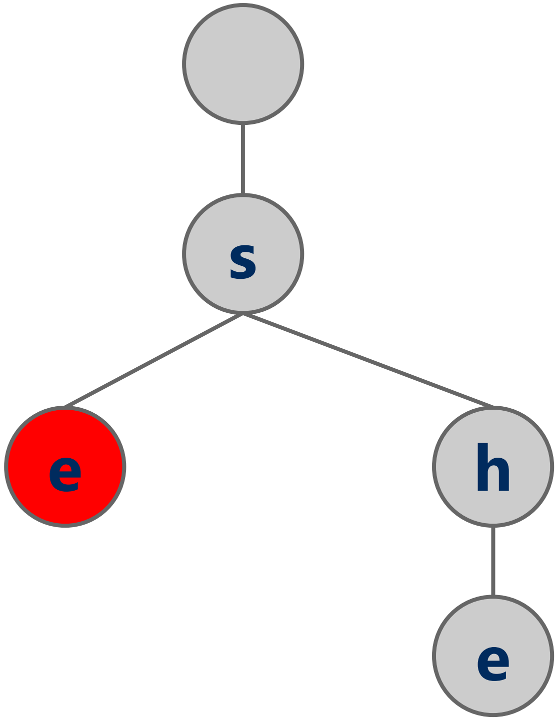

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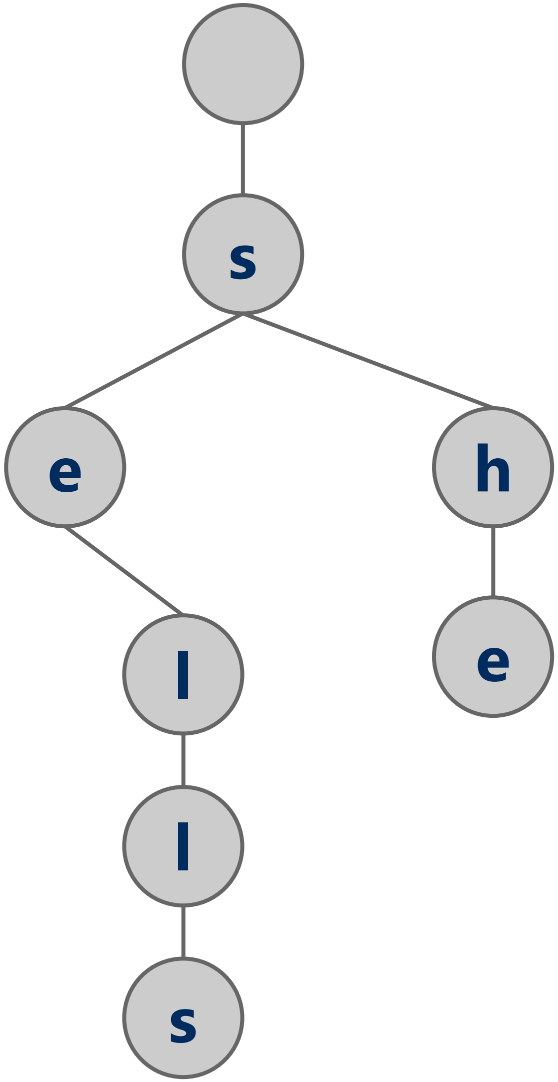

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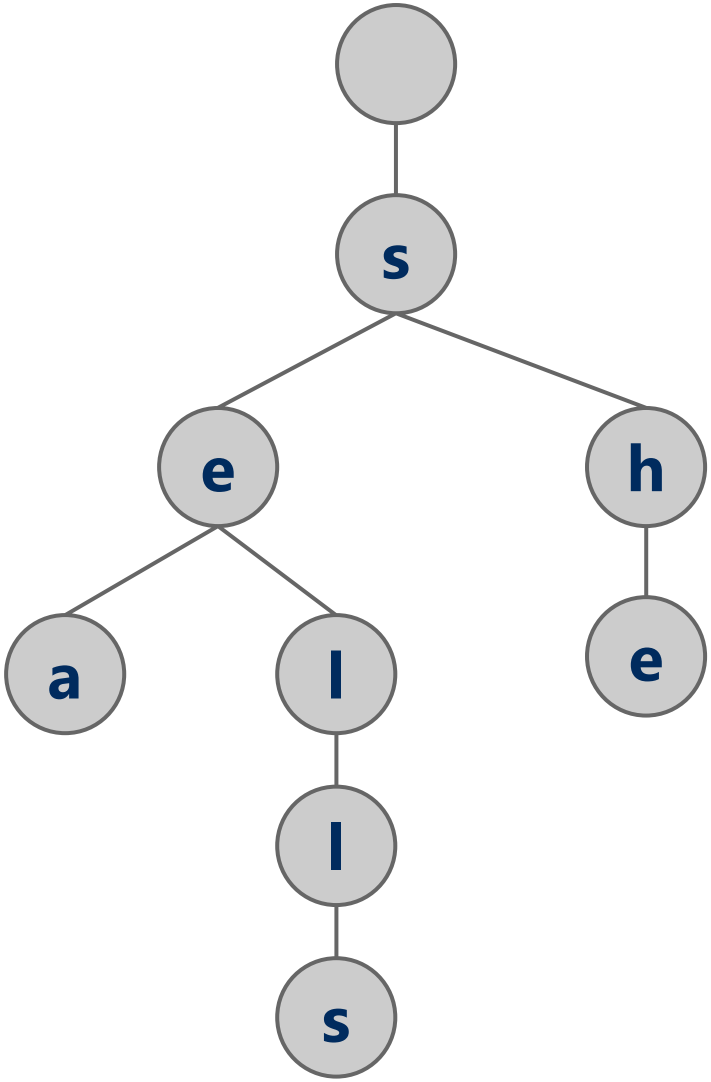
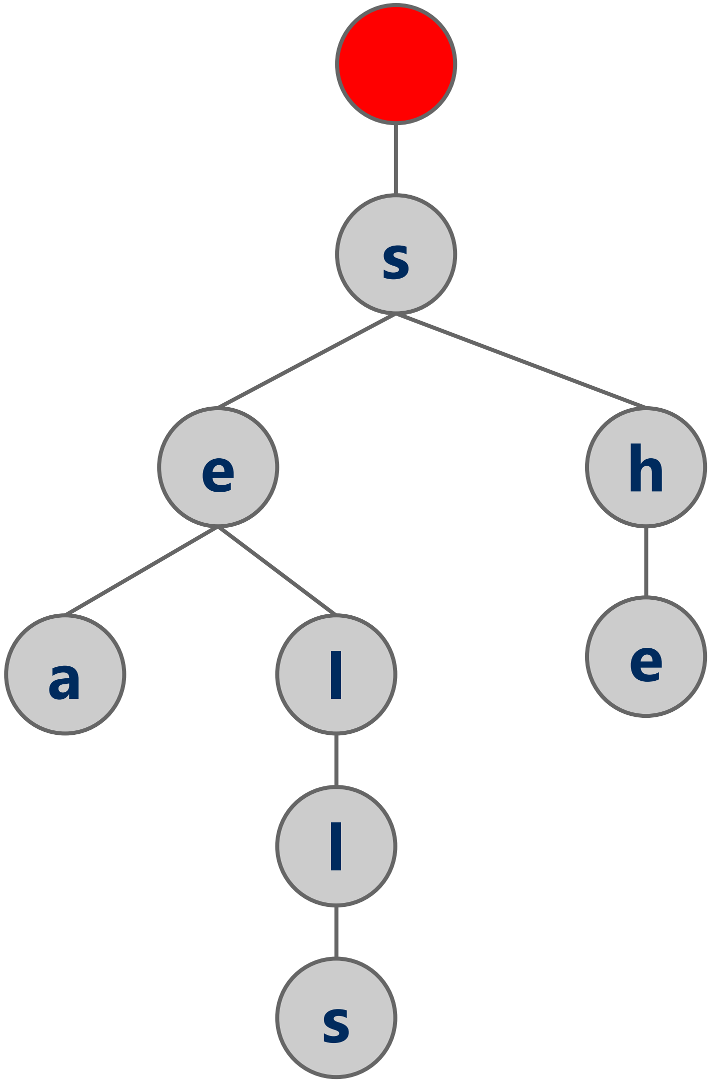


sea

Another trie example

shells



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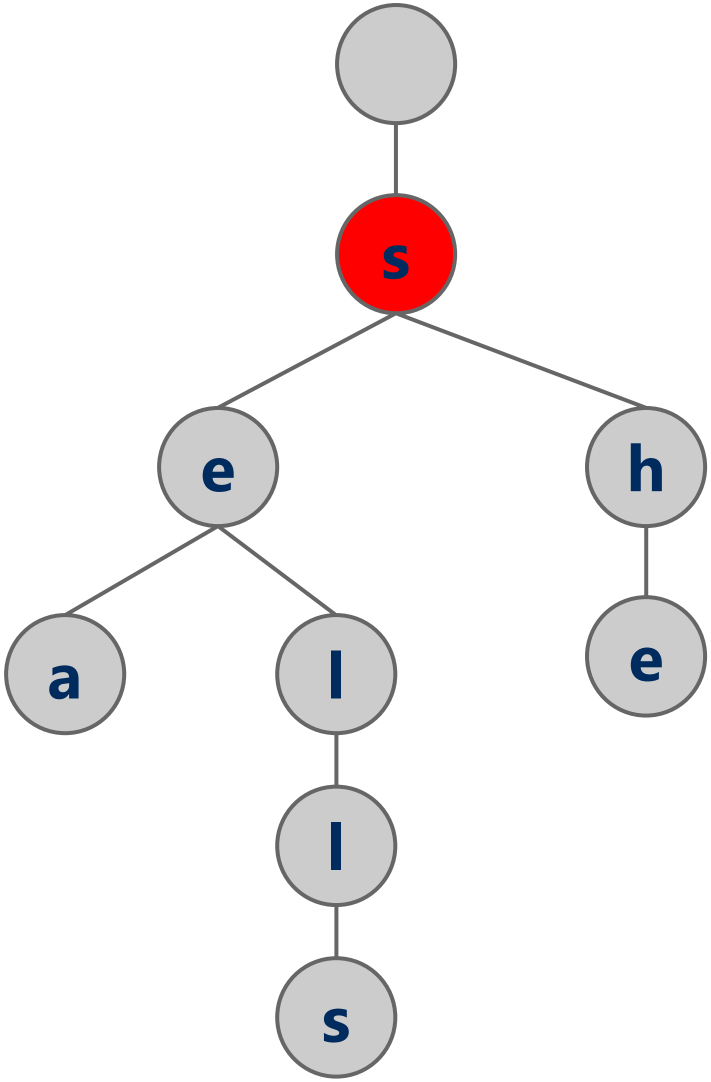

shells

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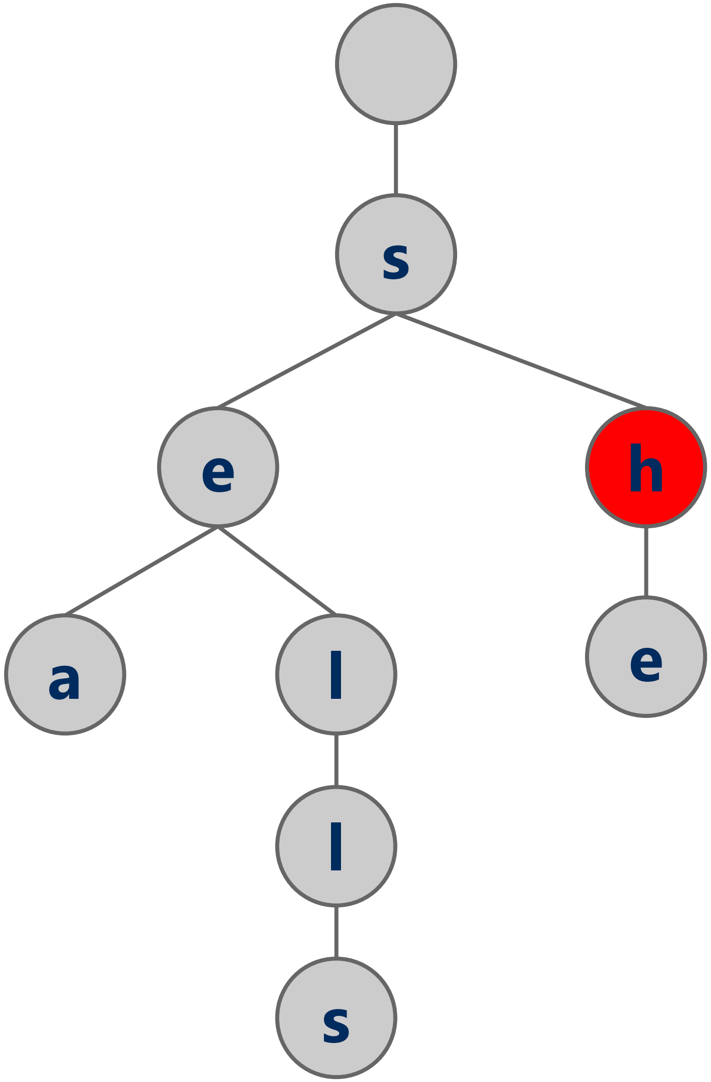

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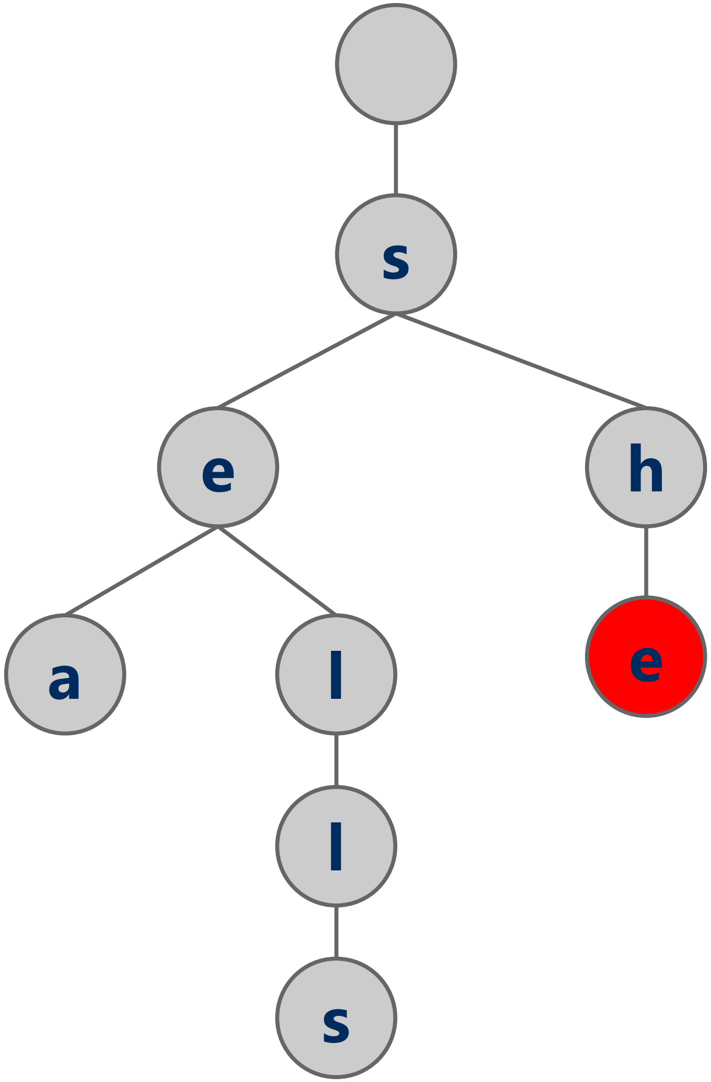

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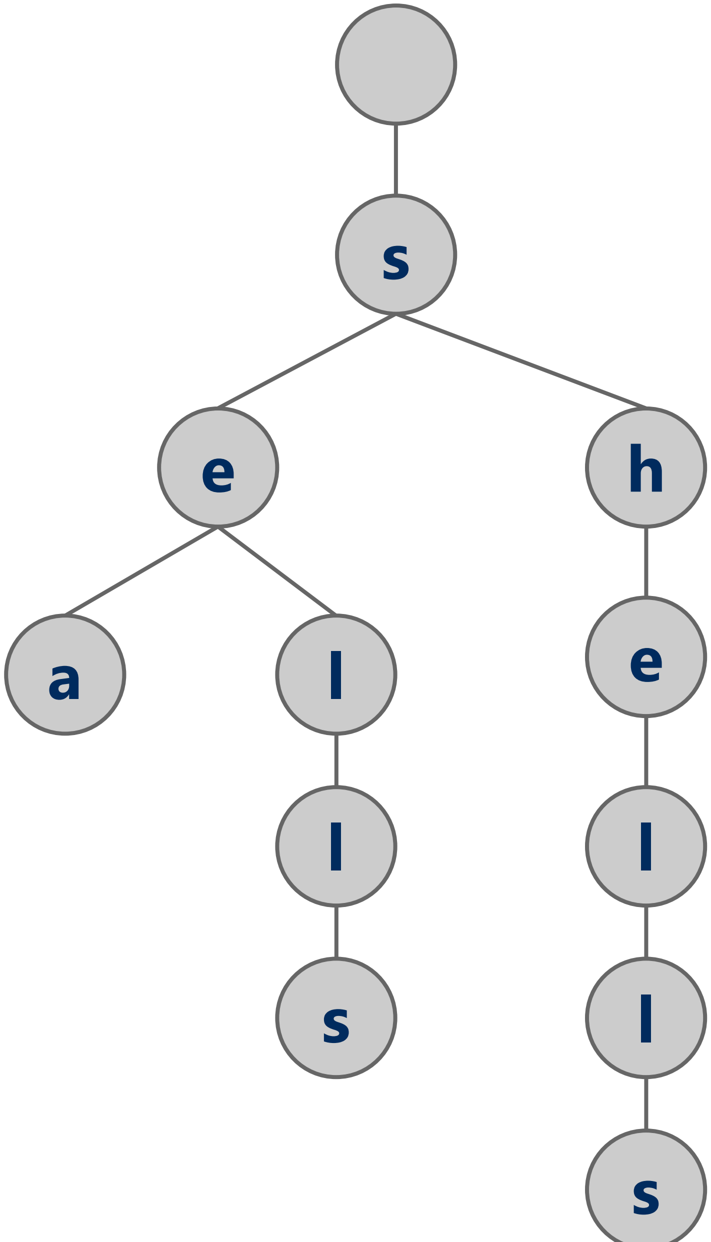

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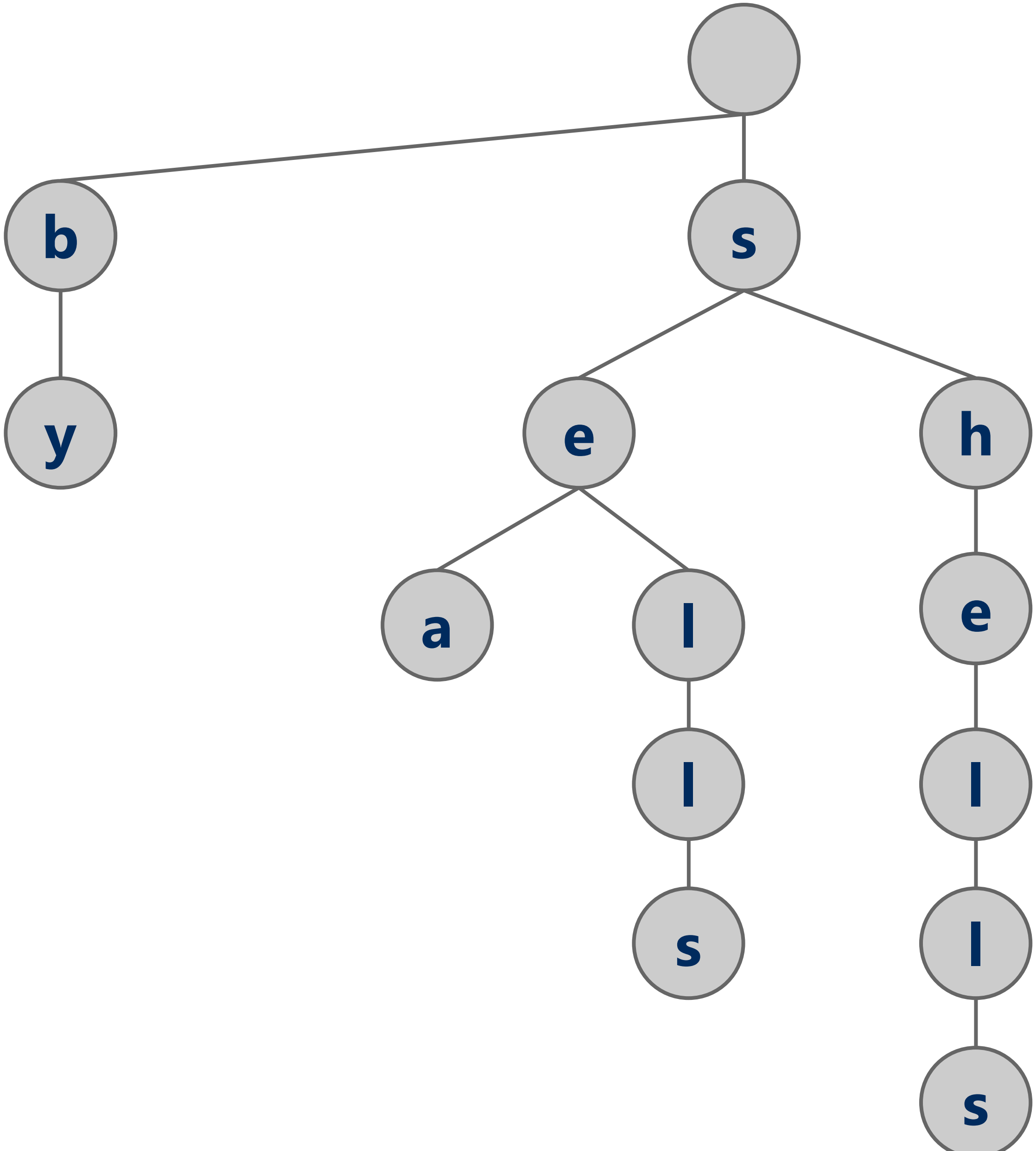
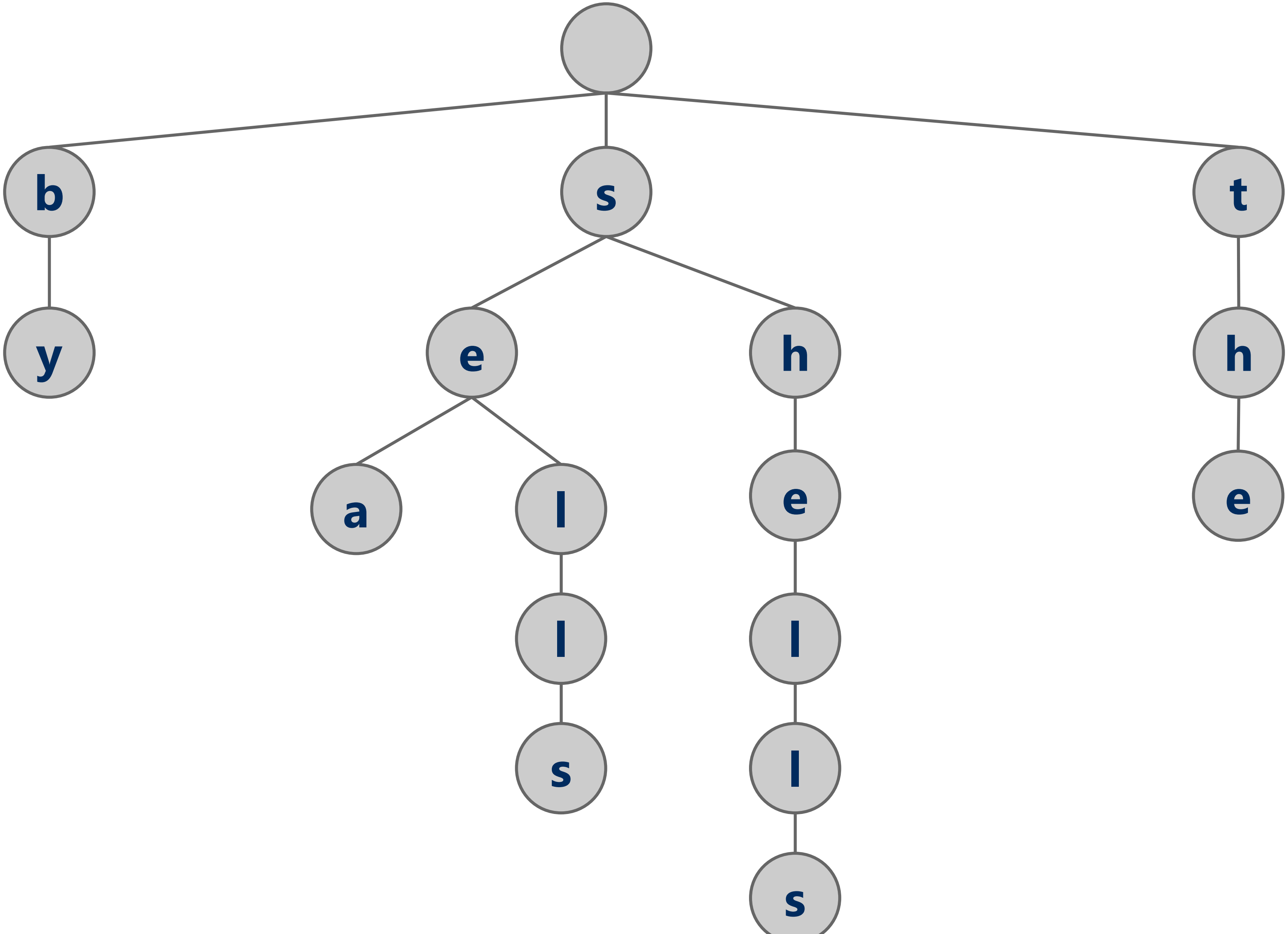


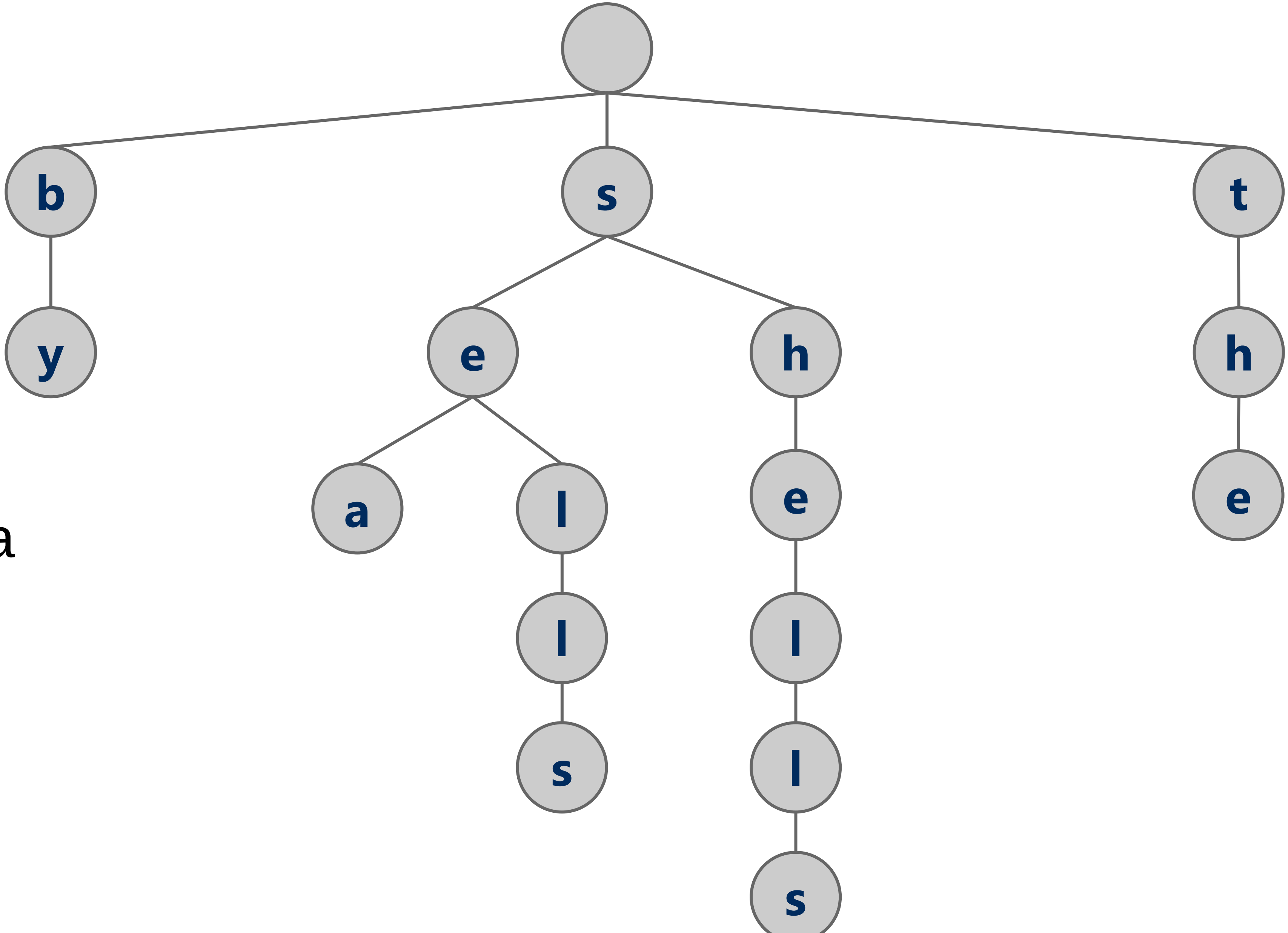
by

Another trie example



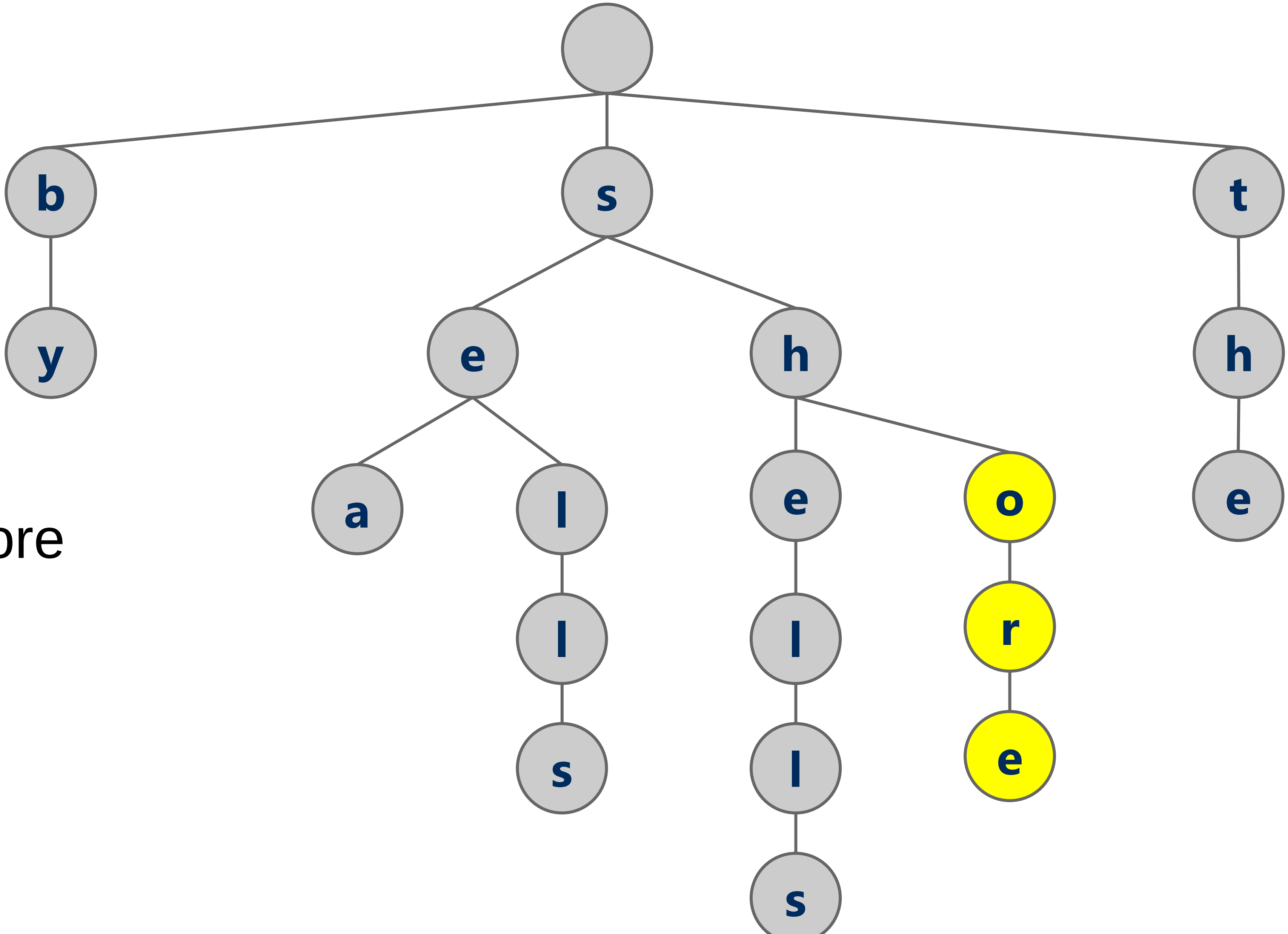
the

Another trie example



sea

Another trie example



shore

Analysis

- Runtime of add and *search hit*?
- O(w) where w is the character length of the string
  - So, what do we gain over RSTs?
    - $w < b$
    - e.g., assuming fixed-size encoding $w = \frac{b}{\lceil \log R \rceil}$
    - tree height is reduced

Search Miss

- Search Miss time for R-way RST
  - Require an average of $log_R(n)$ nodes to be examined
    - Proof in Proposition H of Section 5.2 of the text
- Average tree height with 2^{20} keys in an RST?
  - $\log_2 n = \log_2 2^{20} = 20$
- With 2^{20} keys in a large branching factor trie, assuming 8-bits at a time?
  - $\log_R n = \log_{256} 2^{20} = \log_{256} (2^8)^{2.5} = = \log_{256} 256^{2.5} = 2.5$

Implementation Concerns

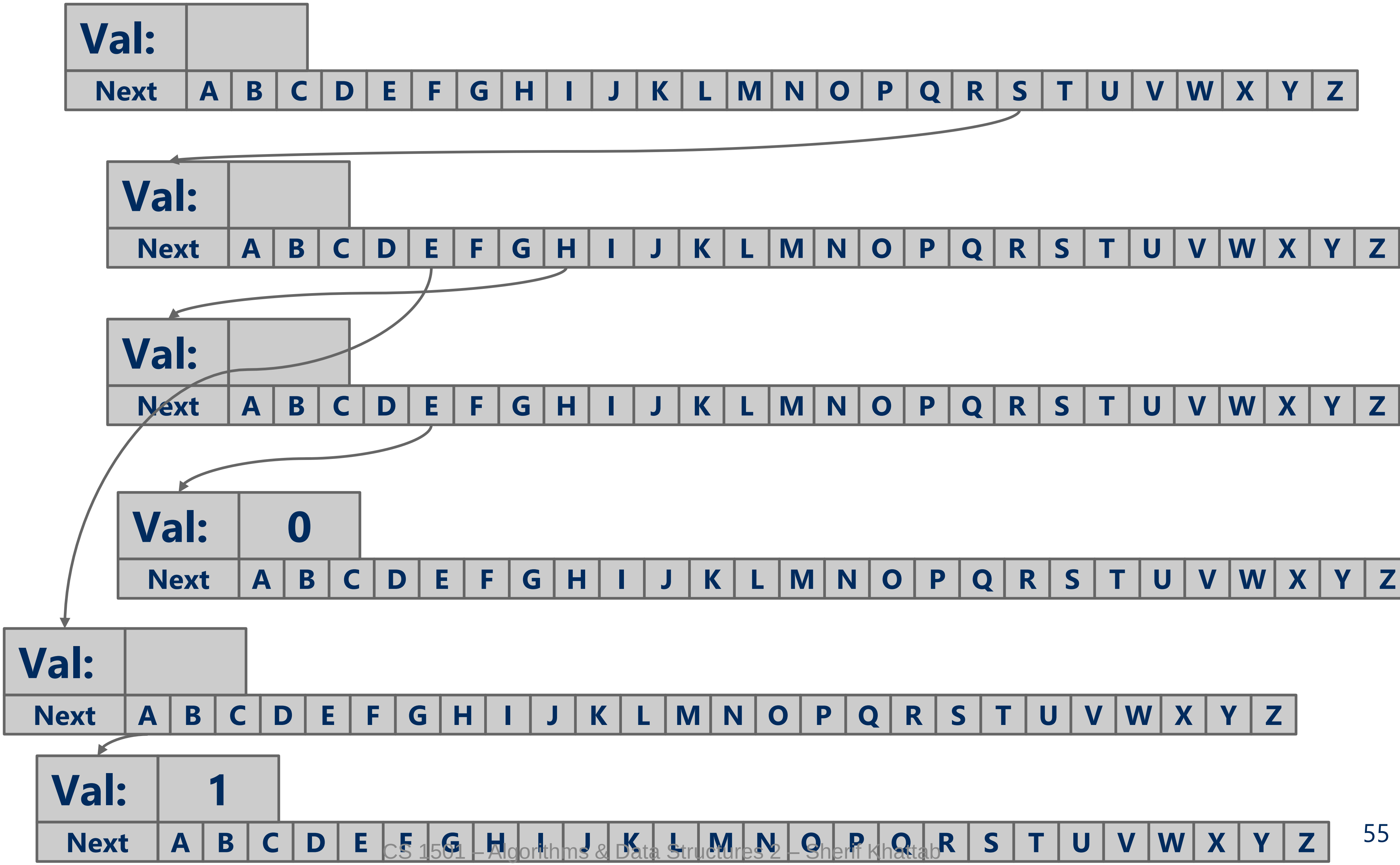
- See TrieSt.java
  - Implements an R-way trie
- Basic node object:

Where R is the branching factor

```
private class Node {
    private Object val;
    private Node[] next;
    private Node(){
      next = new Node[R];
    }
}
```

- Non-null **val** means we have traversed to a valid key
- Again, note that keys are not directly stored in the trie at all

R-way trie example

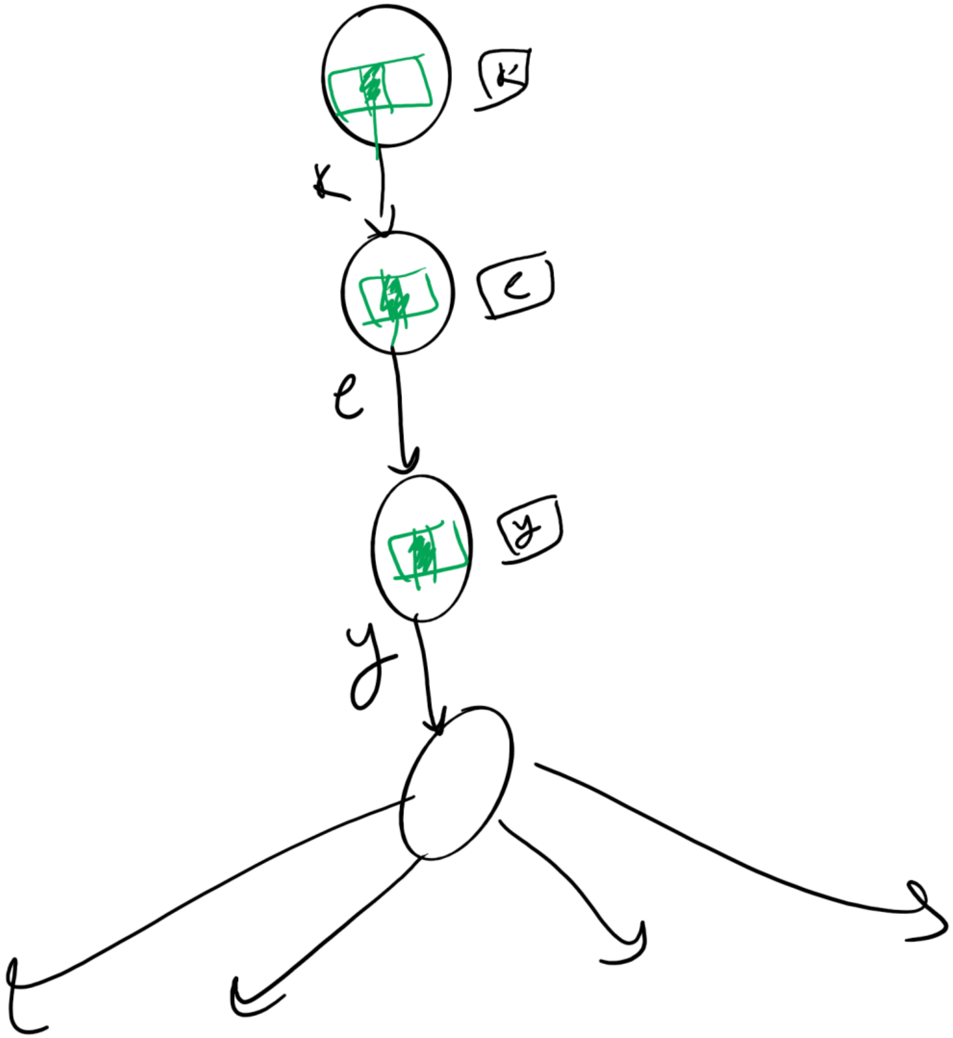


Summary of running time

	insert	search hit	search miss
binary RST	$\Theta(b)$	$\Theta(b)$	$\Theta(\log_2 n)$ on average
multi-way RST	$\Theta(w)$	$\Theta(w)$	$\Theta(\log_R n)$

R-way RST's nodes may waste space!

- Considering 8-bit ASCII, each node contains 2^8 references!
- This is especially problematic as in many cases, a lot of this space is wasted
  - Common paths or prefixes for example, e.g., if all keys begin with "key", thats 255*3 wasted references!
  - At the lower levels of the trie, most keys have probably been separated out and reference lists will be sparse



Solution: De La Briandais tries (DLBs)

Main idea: replace the array inside the node of the R-way trie with a linked-list

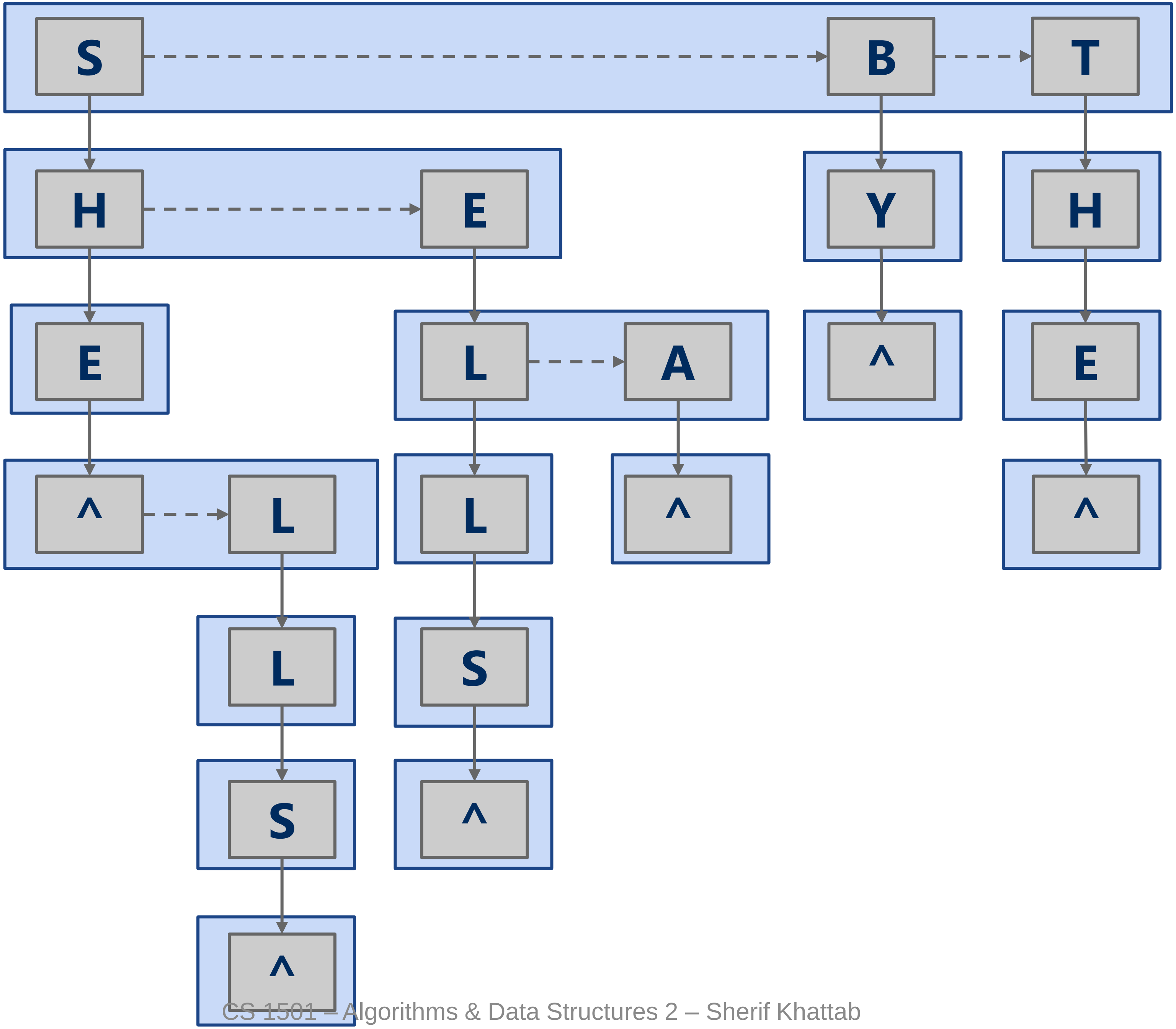
De La Briandais (DLB) Trie

- tree-like structure used for searching when keys are sequences of characters
- each nodelet
  - stores one character,
  - points to a sibling (linked list of siblings), and
  - points to a child

Adding to DLB Trie

- if root is null, set root ← new node
- current node ← root
- for each *character c* in the key
  - Search for *c* in the linked list headed at current using sibling links
    - if not found, create a new node and attach as a sibling to the linked list
  - move to child of the found node
    - either recursively or by current ← child
- if at last character of key, insert value into current node and return

DLB Example



DLB analysis

- How does DLB performance differ from R-way tries?
- Which should you use?

	search hit insert	
R-way RST	$\theta(w)$	
DLB	$\theta(wR)$	

De La Briandais (DLB) Trie

- worst-case running time is O(wR)
  - w: number of characters in the key
  - R: alphabet size
- worst-case can be avoided by using DLB only when the sibling lists are short

DLB vs. R-way RST: Space comparison example

- **Q: How does DLB save space over R-way RST?** Assume the following set of keys:
  - ksm1 … ksm9
- How big does an 256-way RST take vs. a DLB trie?

R-way RST

```
private class Node {
    private Object val;
    private Node[] next;
     private Node(){
        next = new Node[R];
     }
}
```

Each node takes 4*(R+1) = 4*257 = 1028 bytes, assuming 4 bytes per reference variable

R-way RST

Val:
null
Next
k
Val:
null
Next
s
Val:
null
Next
m
Val:
null
Next
1
2
3
4
5
6
7
8
9
Val:
V_{ksm1}
Next
Val:
V_{ksm2}
Next

R-way RST

We will end up with 4 + 9 = 13 nodes

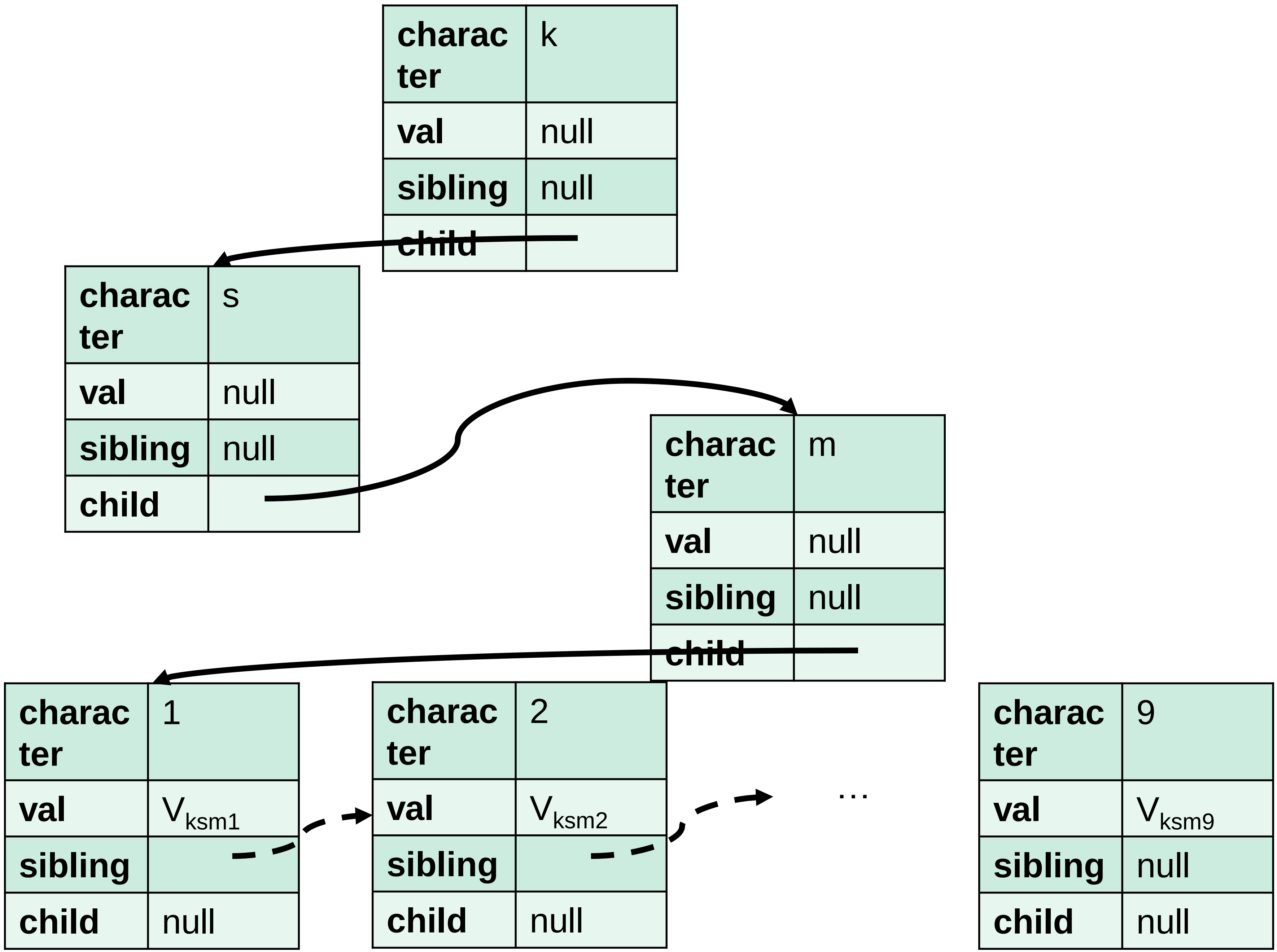
Total space is 13*1028 = 13,364 bytes

DLB Trie

```
private class DLBNode<T> {
    private Character character;
    private Object val;
    private Node sibling;
    private Node child;
}
```

Each node takes 4*4 = 16 bytes, assuming 4 bytes per reference variable

DLB Trie



character k
val null
sibling null
child
character s
val null
sibling null
child
character m
val null
sibling null
child
character 1
val V_{ksm1}
sibling
child null
character 2
val V_{ksm2}
sibling
child null
…
character 9
val V_{ksm9}
sibling null
child null

DLB Trie

We will end up with 12 nodes

Total space is 12*16 = 192 bytes

Compare to 13,364 bytes with an R-way RST

Runtime Comparison for Search Trees/Tries

	Search hit	Search miss (average)	insert
BST	$\Theta(n)$	$\Theta(\log n)$	$\Theta(n)$
RB-BST	$\Theta(\log n)$	$\Theta(\log n)$	$\Theta(\log n)$
DST	$\Theta(b)$	$\Theta(\log n)$	$\Theta(b)$
RST	$\Theta(b)$	$\Theta(\log n)$	$\Theta(b)$
R-way RST	$\Theta(w)$	$\Theta(\log_R n)$	$\Theta(w)$
DLB	$\Theta(wR)$	$\Theta(\log_R n \cdot R)$	$\Theta(w \cdot R)$

Final notes on Search Tree/Tries

- We did not present an exhaustive look at search trees/tries, just the sampling that we're going to focus on
- Many variations on these techniques exist and perform quite well in different circumstances
  - Ternary search Tries
  - R-way tries without 1-way branching
- See the table at the end of Section 5.2 of the text