University of Pittsburgh

Algorithms and Data Structures 2
CS 1501

Spring 2023

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(Slides are adapted from Dr. Ramirez's and Dr. Farnan's CS1501 slides.)

Announcements

- Upcoming Deadlines
  - Homework 5: this Friday @ 11:59 pm
  - Lab 4: Tuesday 2/21 @ 11:59 pm
  - Assignment 1: Friday 2/17 @ 11:59 pm

Previous lecture

- Digital Search Tree (DST)
- Radix Search Trie (RST)

This Lecture

- multi-way Radix Search Trie
- De La Briandais Trie

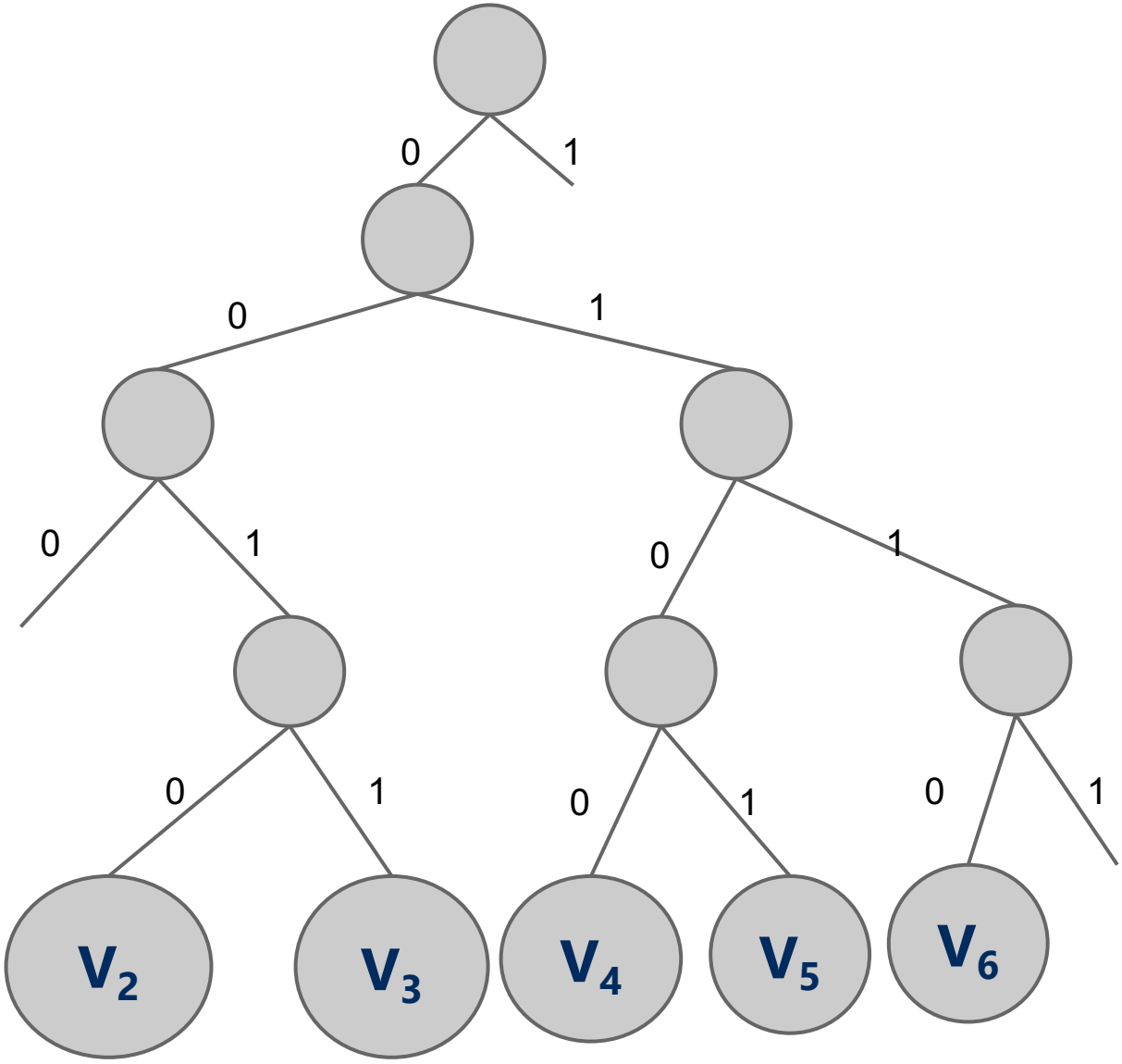
Searching in Radix Search Trie (RST)

- Input: key
- current node ← root
- for each *bit* in the key
  - if current node is null, return *key not found*
  - if bit == 0,
    - current ← current.left
  - if bit == 1,
    - current ← current.right
- if current node is null or the value inside is null
  - return key *not found*
- else return current.value

RST example

Search:

3 0011



RST example

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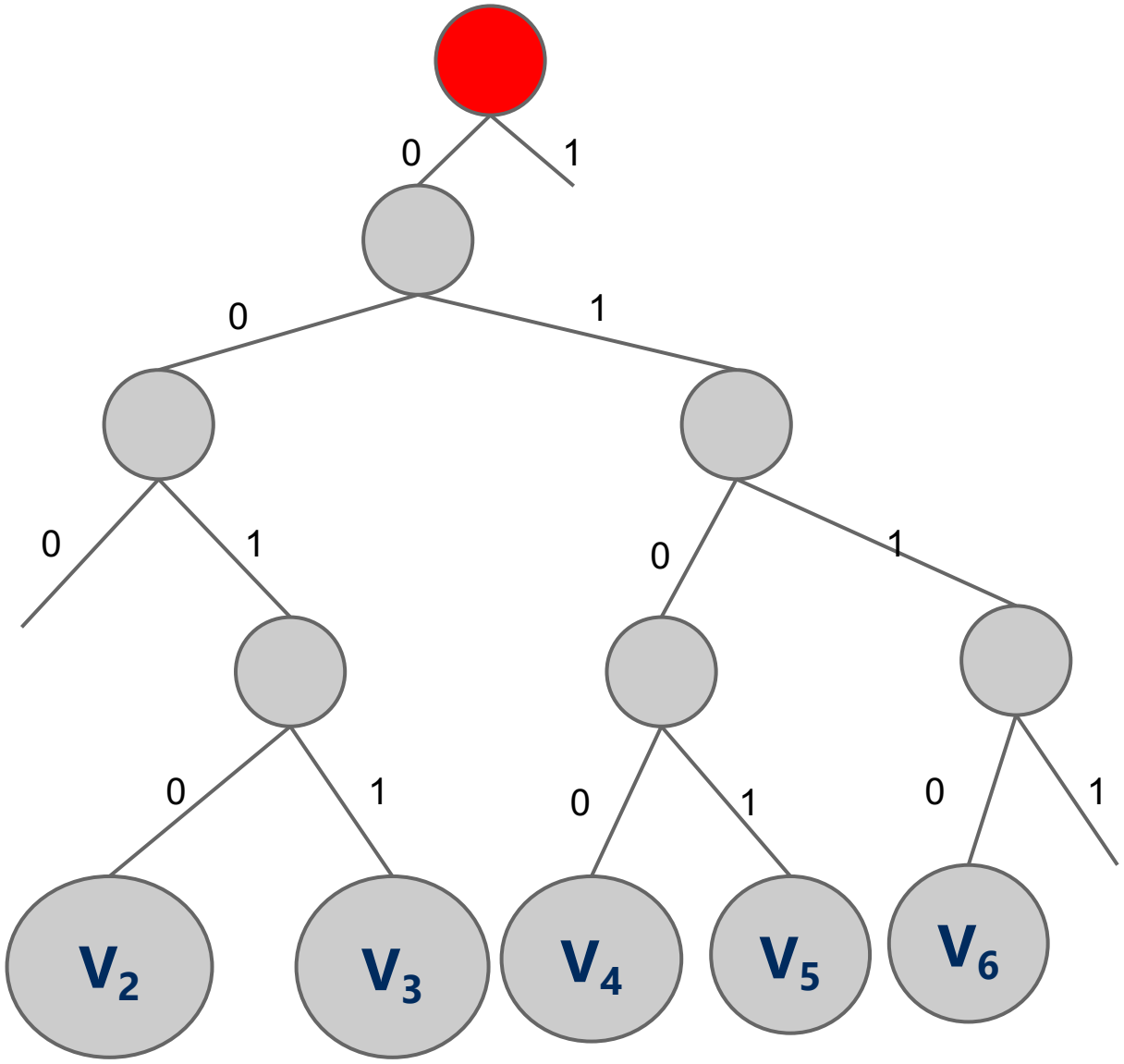
7 0111



RST example

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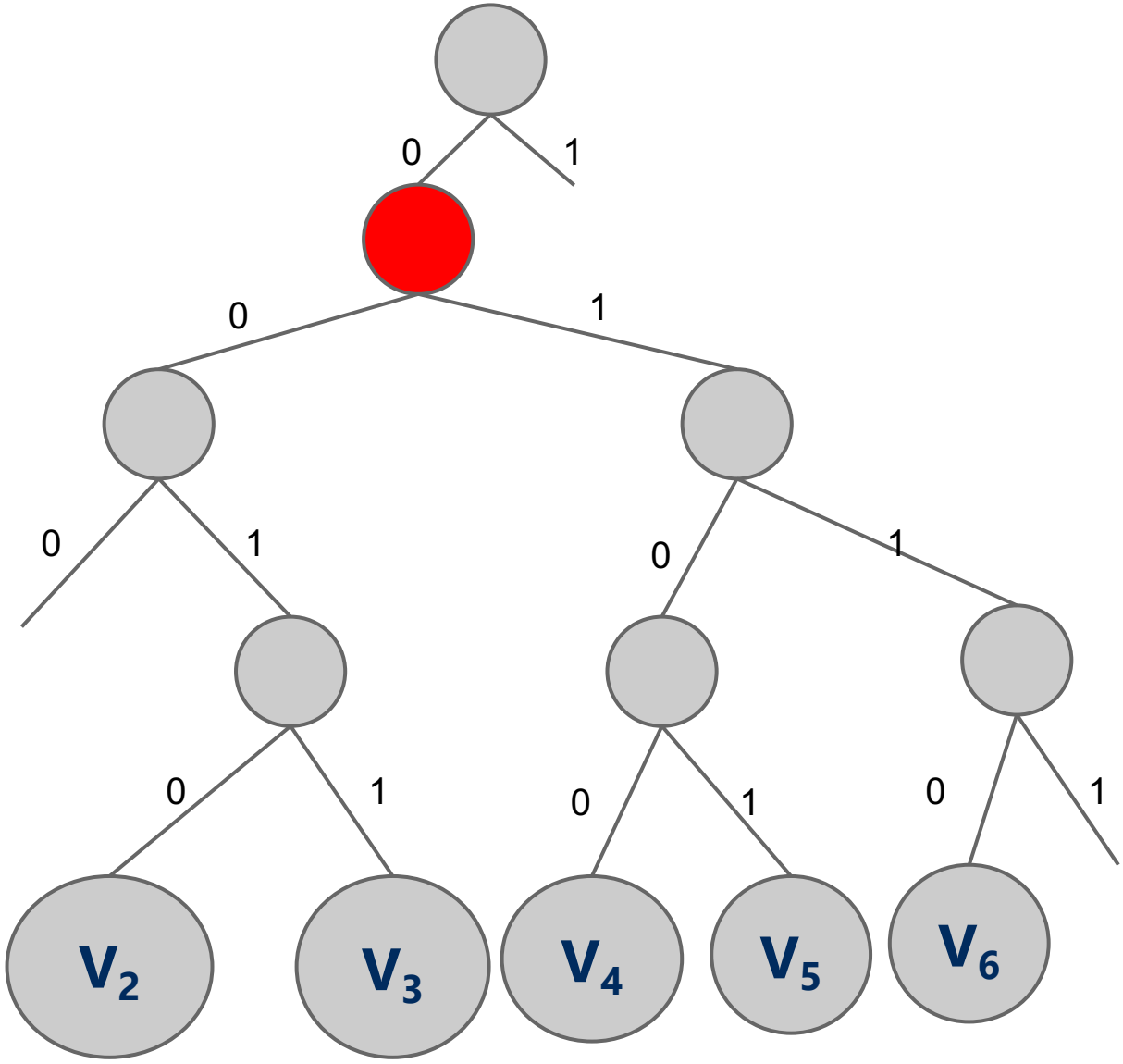

RST example

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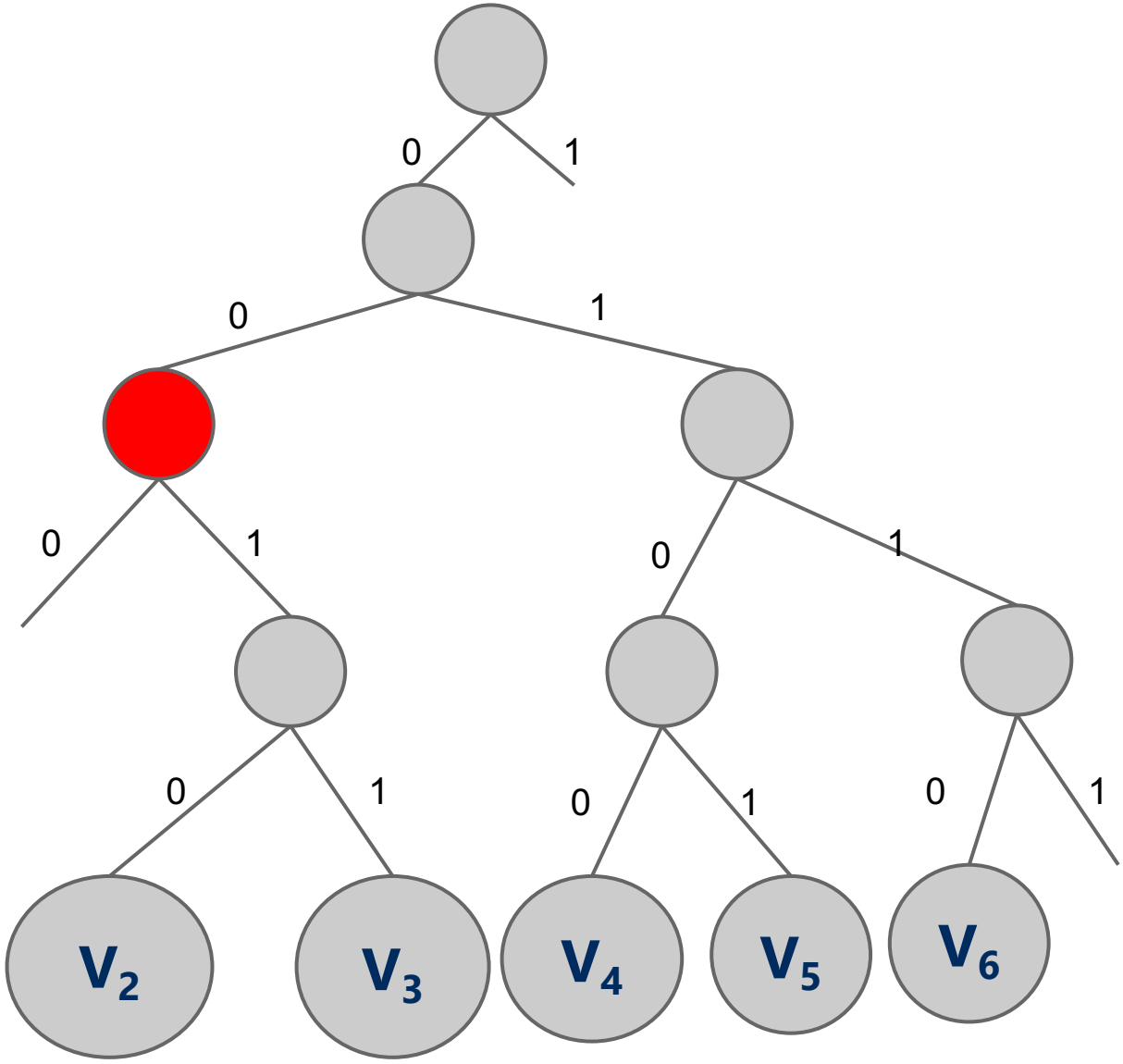

RST example

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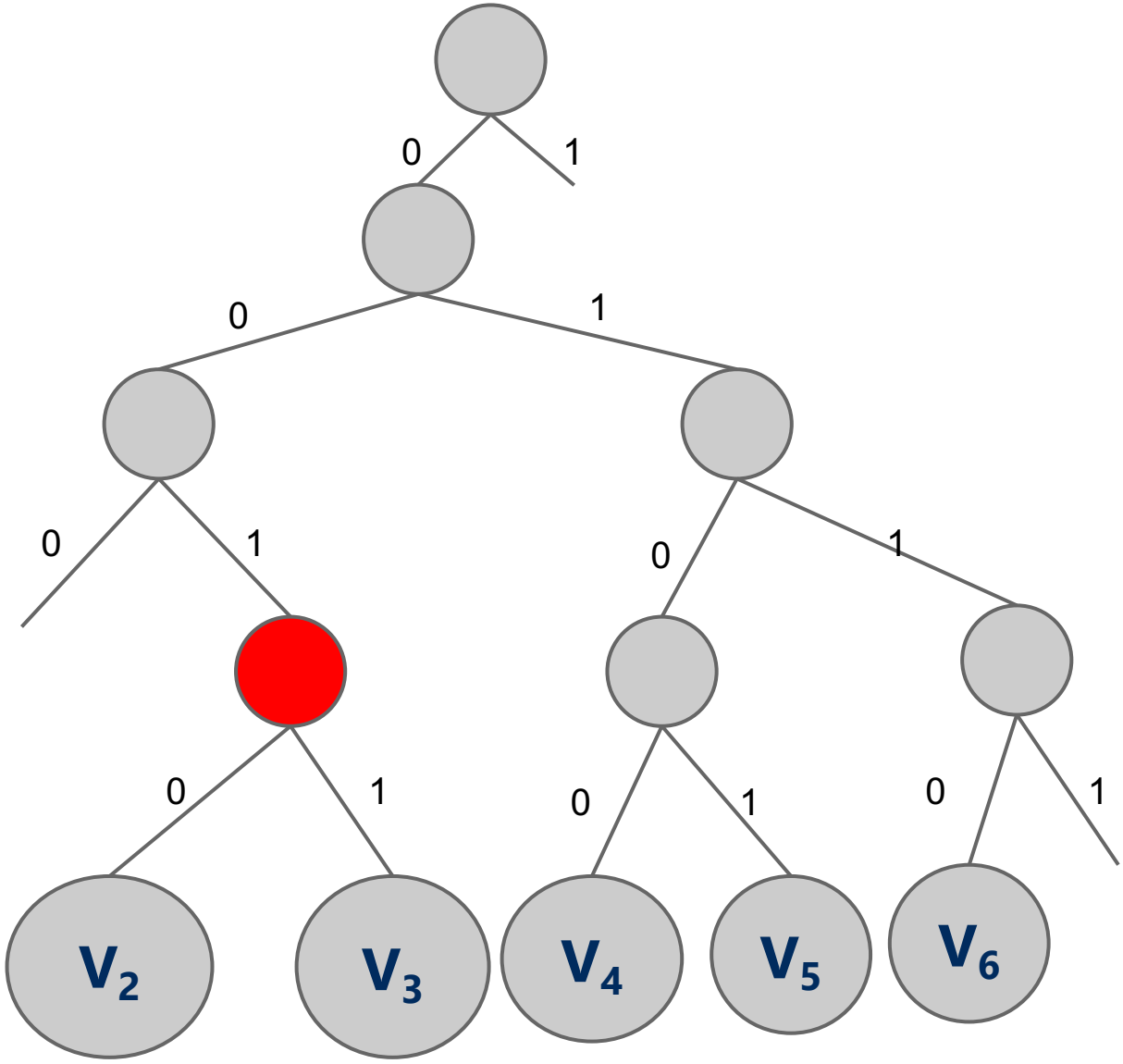

RST example

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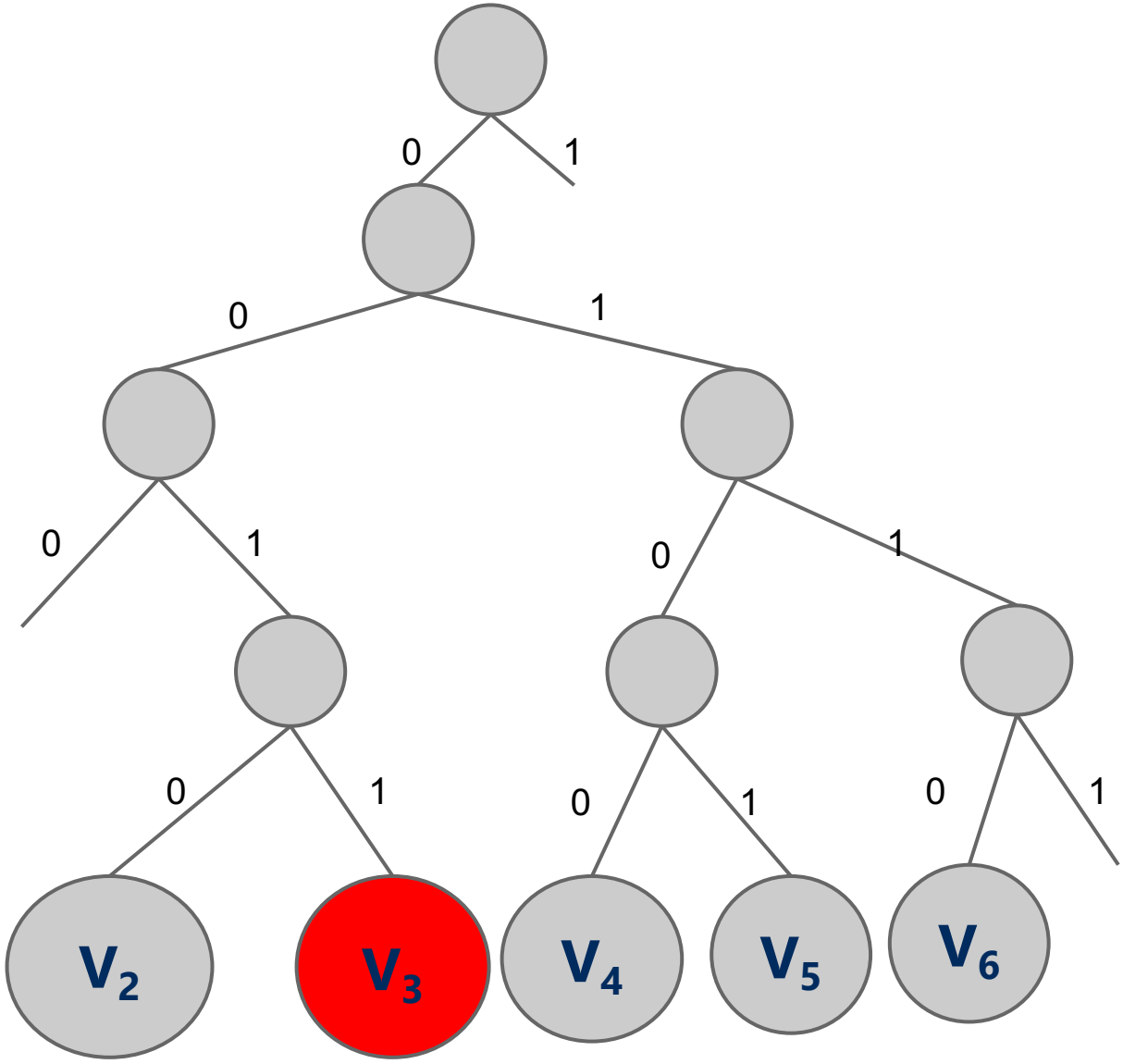

RST example

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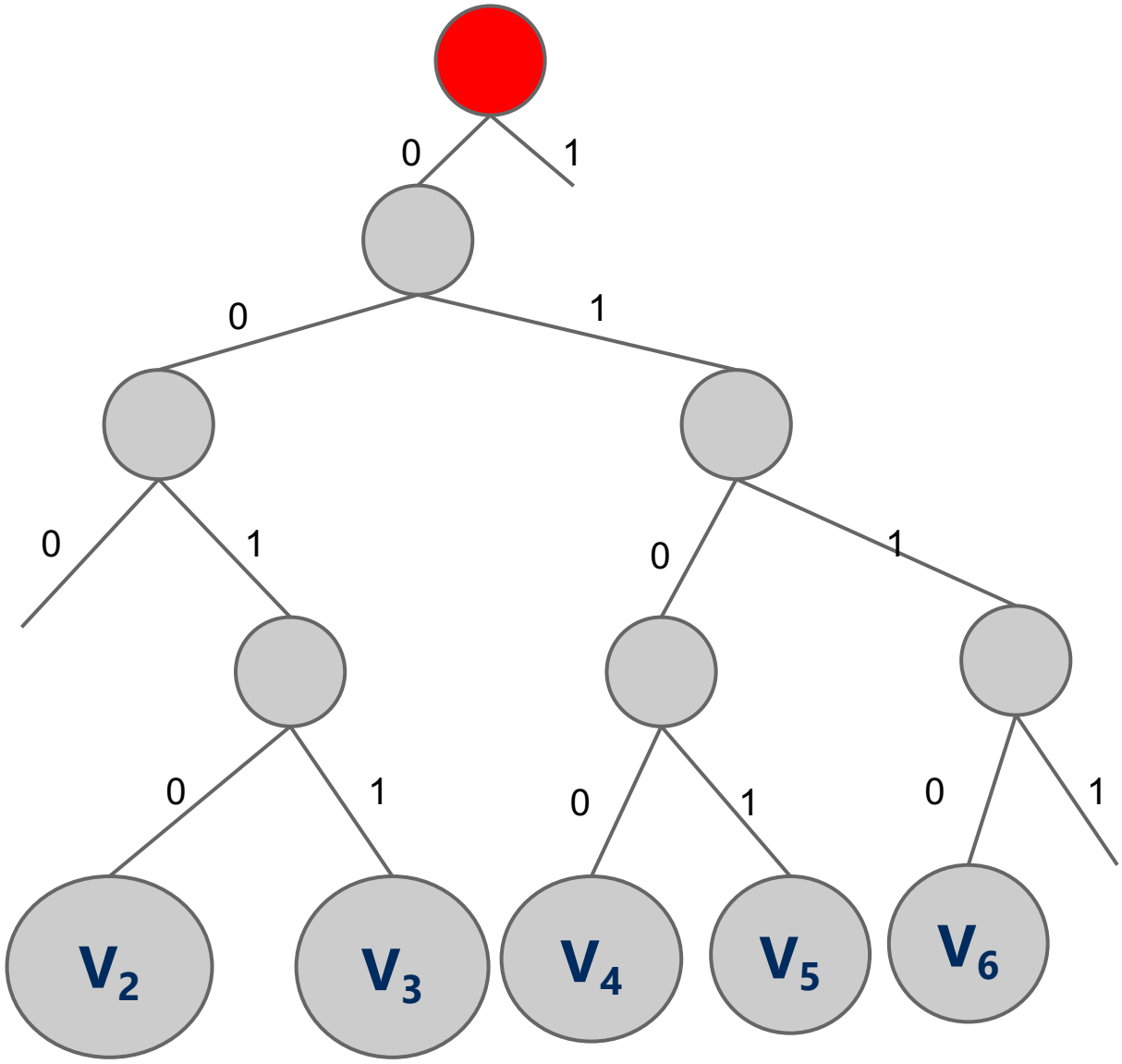
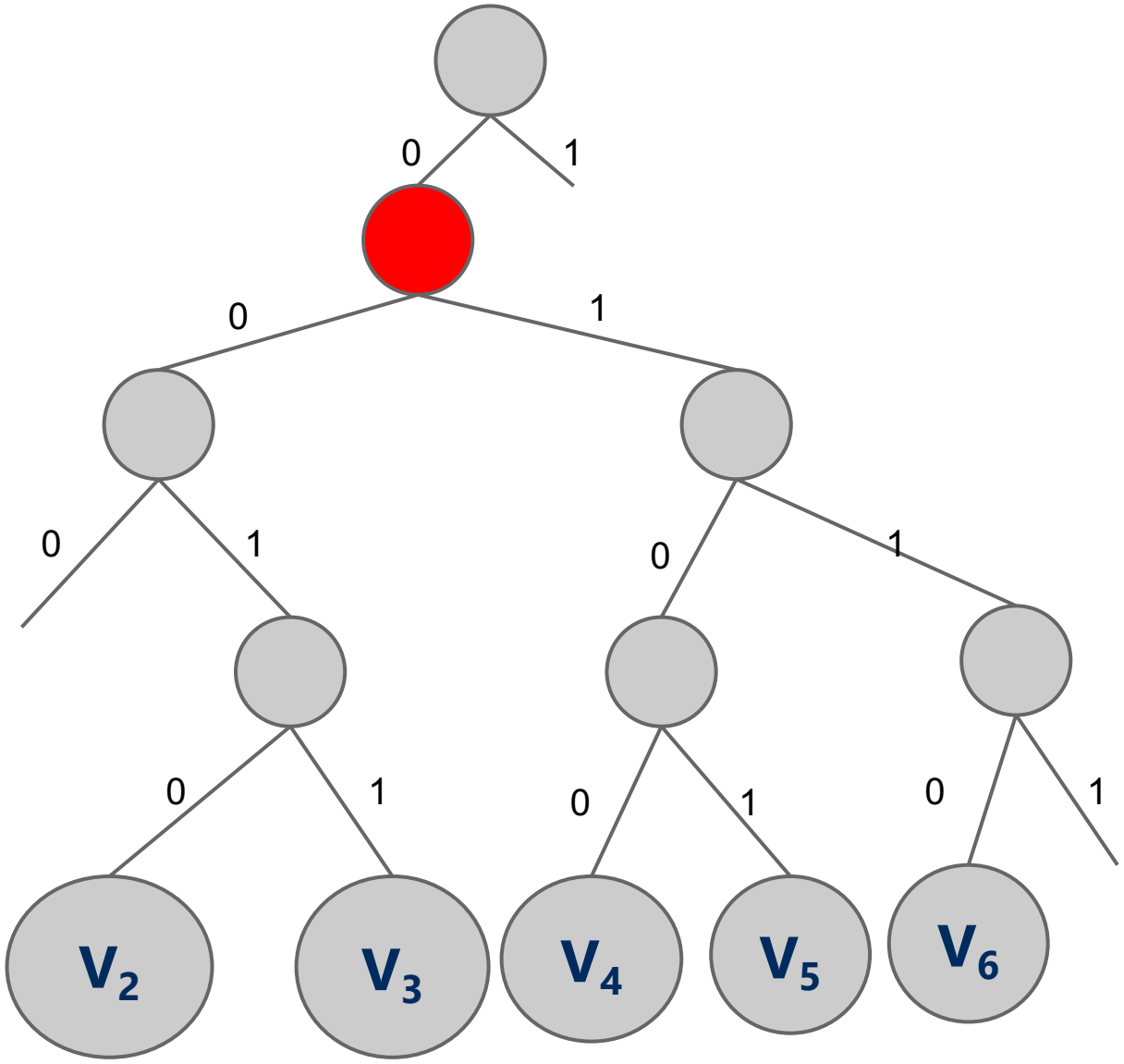


RST example

Search:

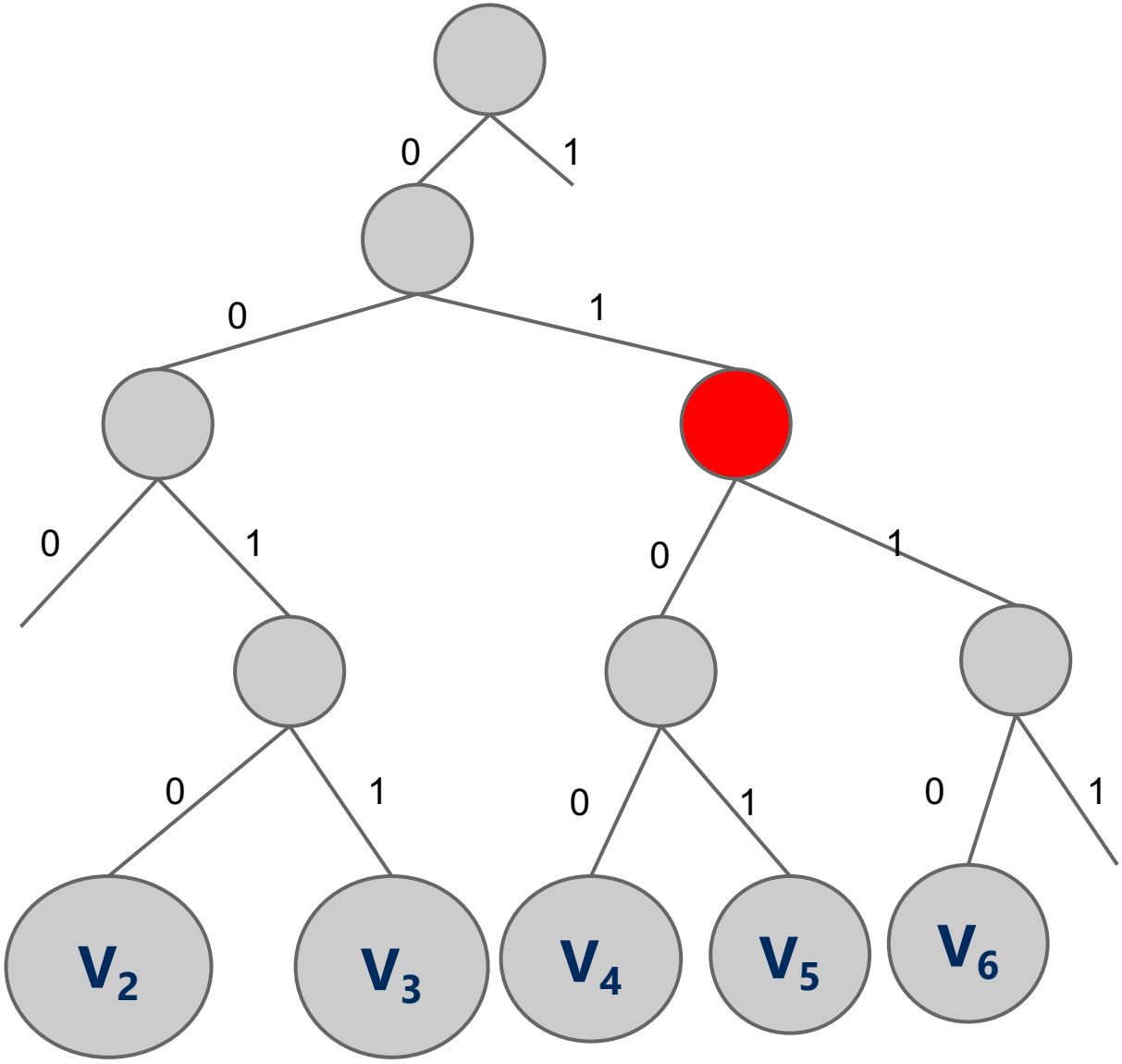
7 0111



RST example

Search:

7 0111



RST example

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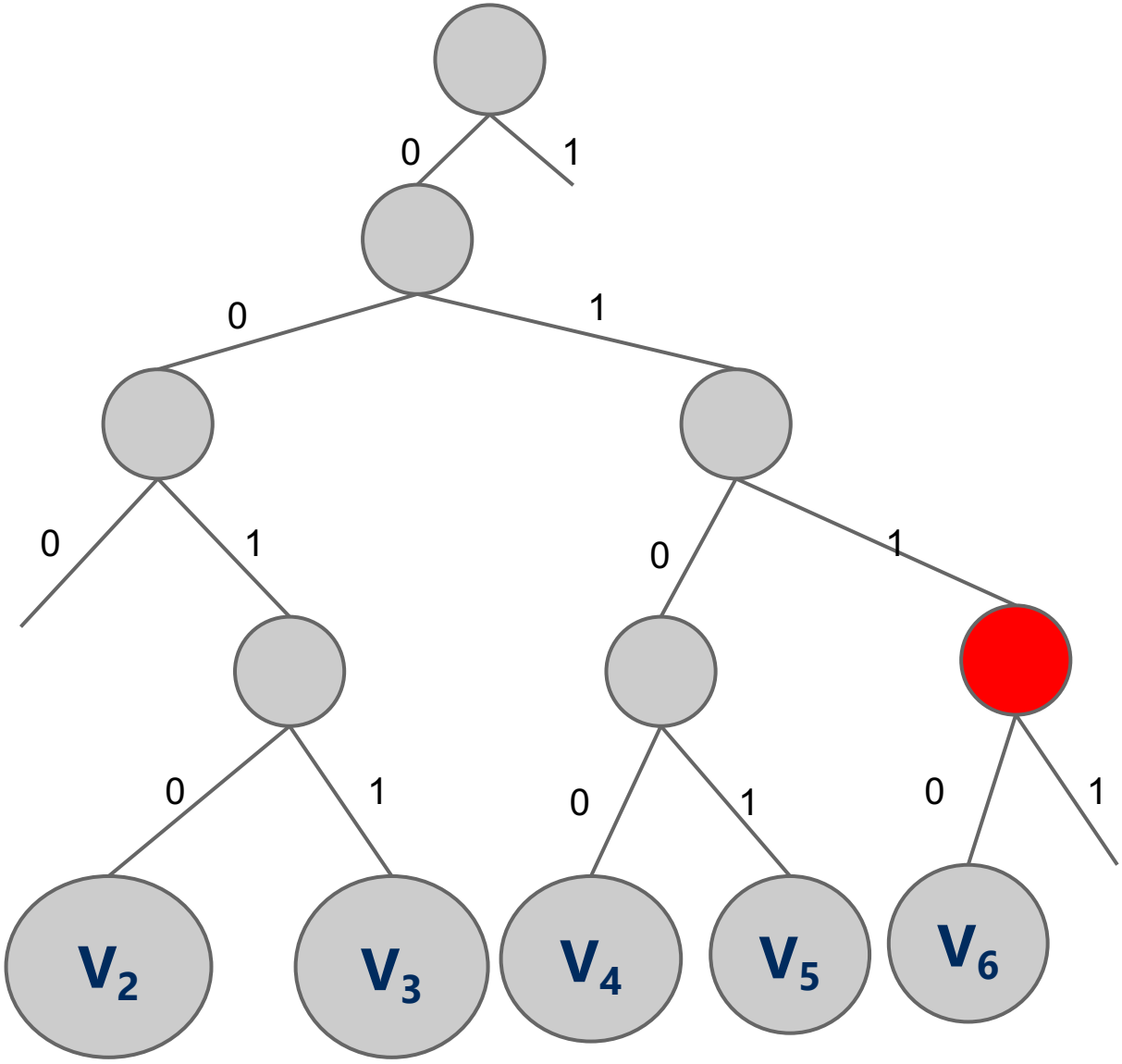

RST example

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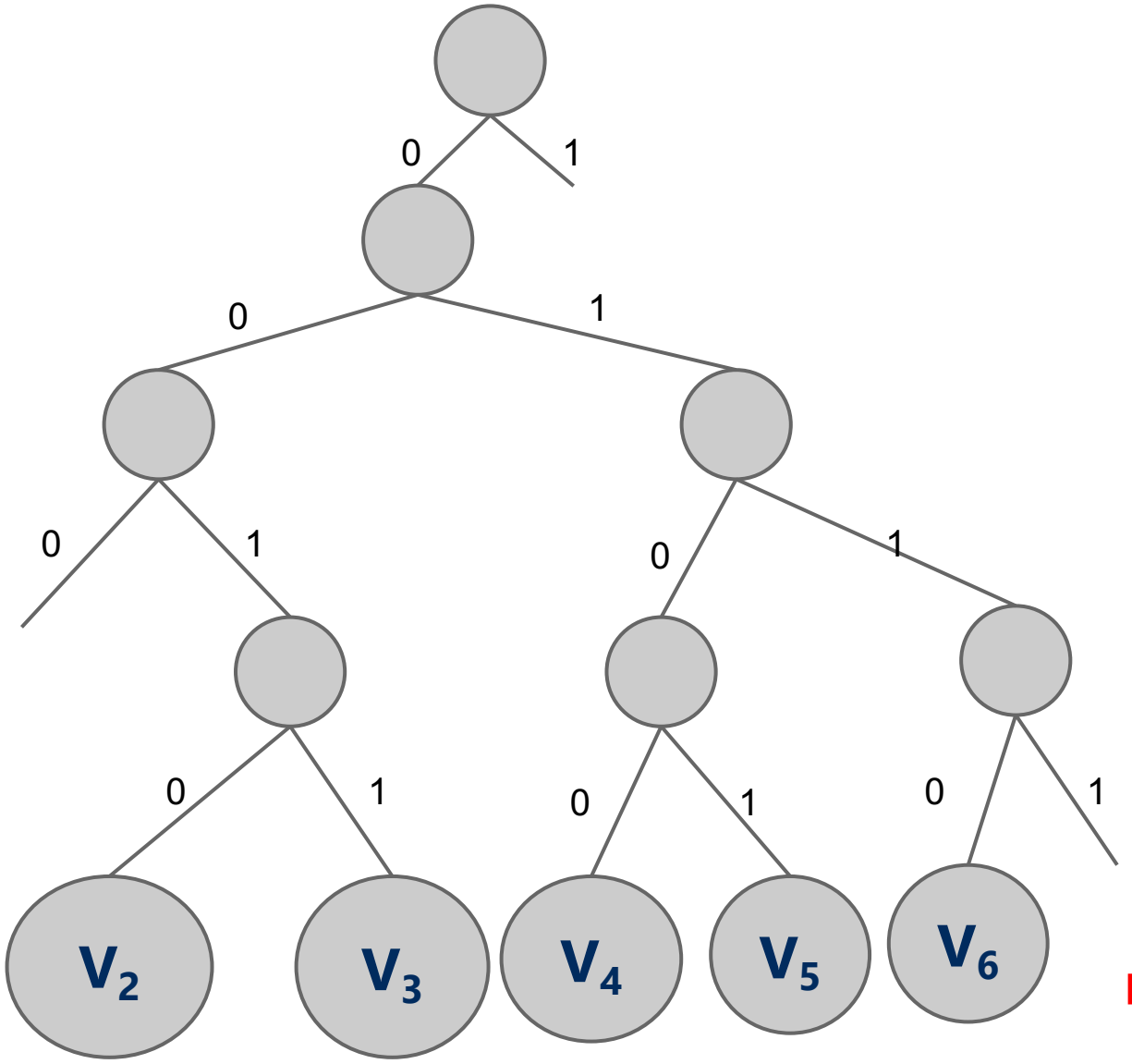


RST Runtime analysis

- **for add:**
  - Theta(b): b is the bit length of the key
  - However, this time we don't have full key comparisons
- for search:
  - **search hit**
    - Theta(b)
  - **search miss**
    - maybe less than Theta(b)

RST Limitations

- Would this structure work as well for other key data types?
- Characters?
  - Characters are the same as 8-bit ints (assuming simple ascii)
- Strings?
- May have huge bit lengths
- How to store Strings?

Larger branching factor tries

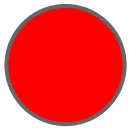
- In our binary-based Radix search trie, we considered one bit at a time
- What if we applied the same method to characters instead of bits in a string?
  - What would this new structure look like?
  - How many children per node?
    - up to R (the alphabet size)
  - Also called R-way radix search tries

Adding to R-way Radix RST

- if root is null, set root ← new node
- current node ← root
- for each *character c* in the key
  - Find the c*th* child of current
    - if child is null, create a new node and attach as the c*th* child
    - move to child
      - current ← child
- insert value into current node

Another trie example

she



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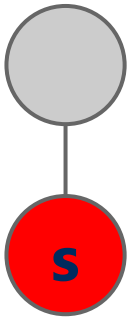

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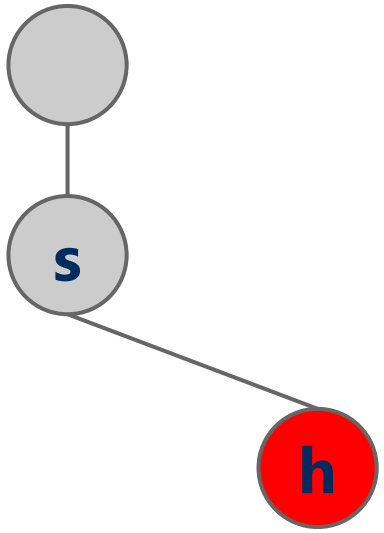

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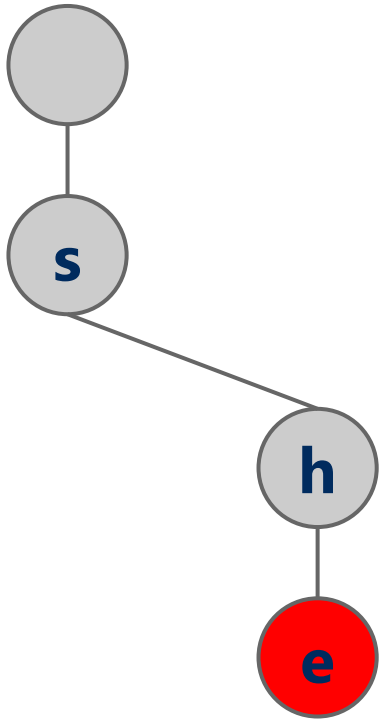

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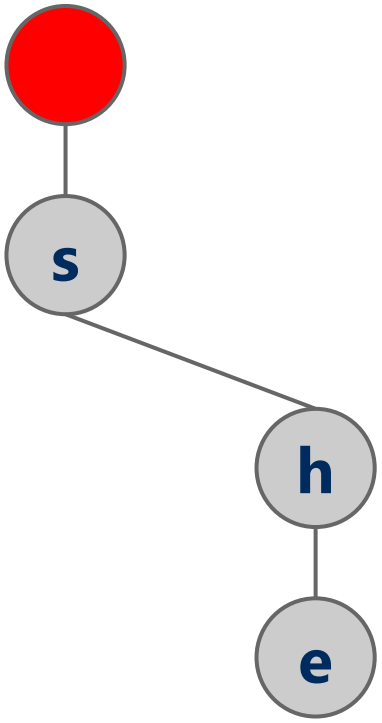
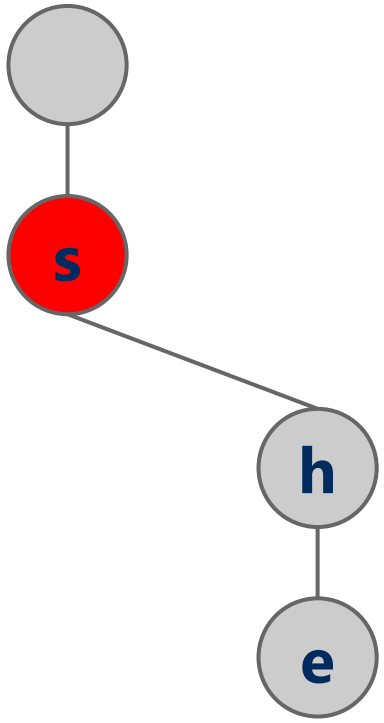


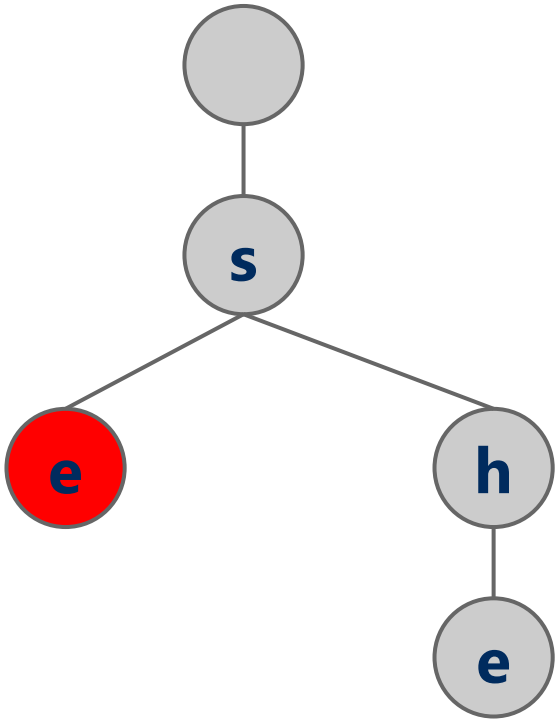
sells

Another trie example



sells

Another trie example



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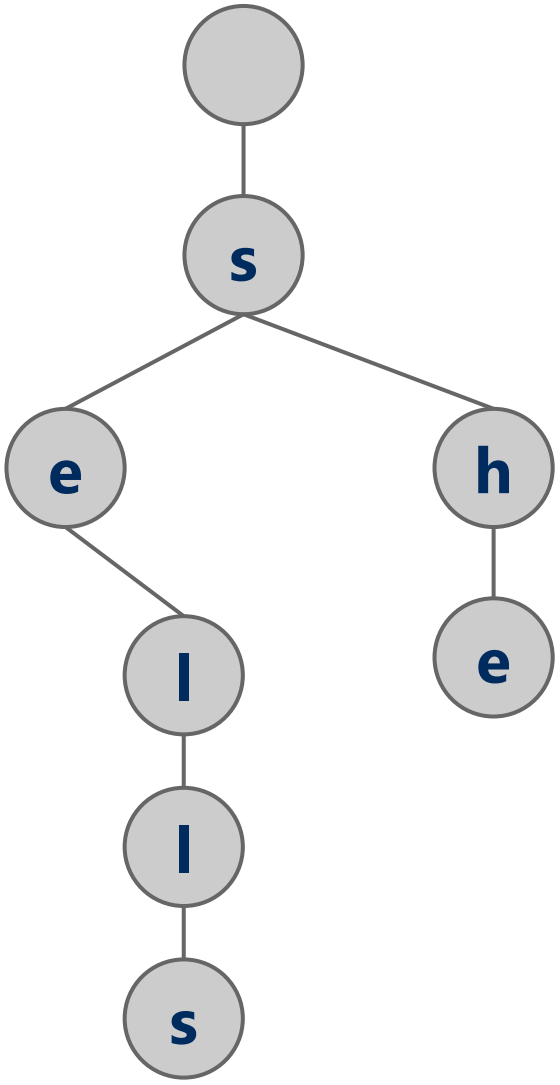
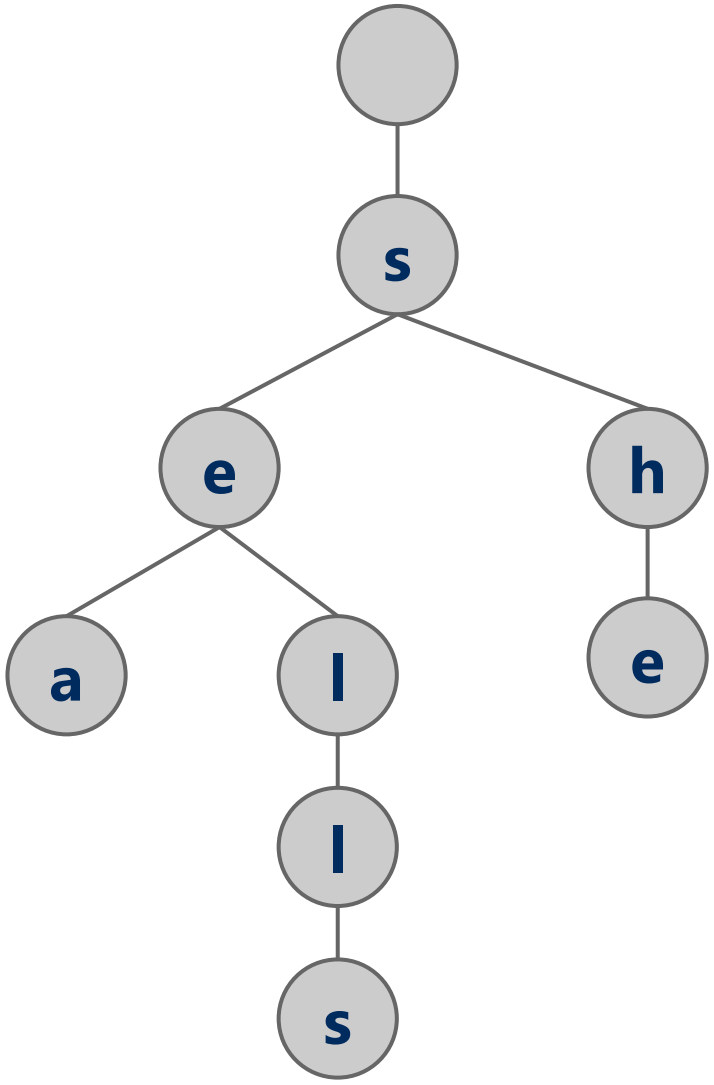


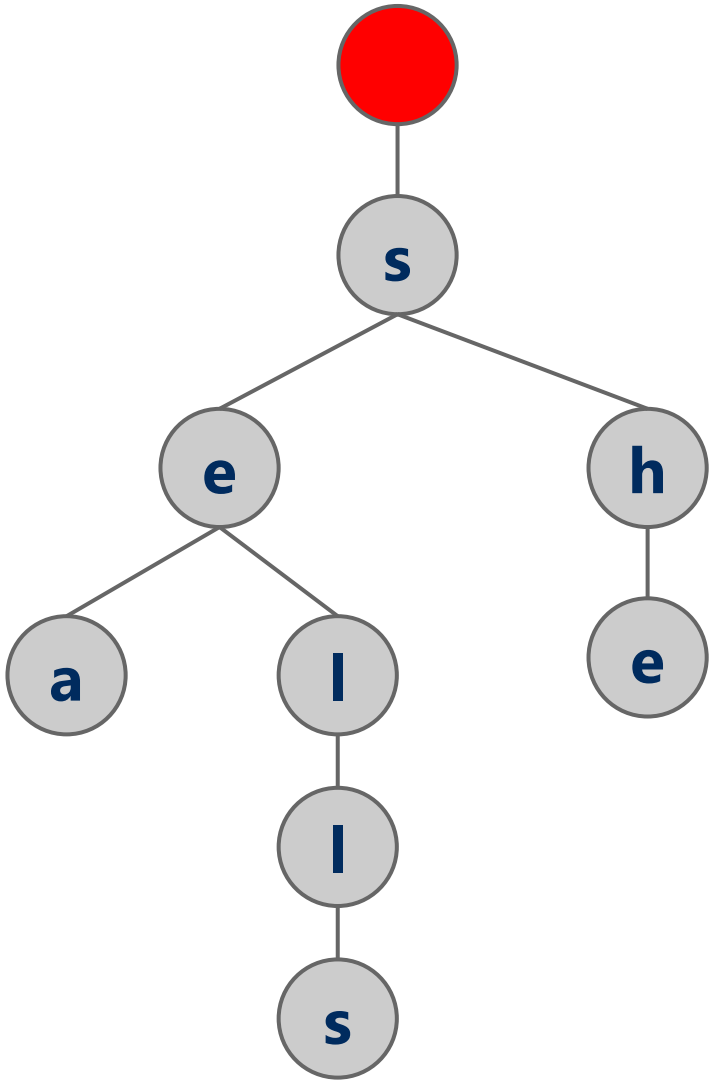
Another trie example

sea



Another trie example

shells



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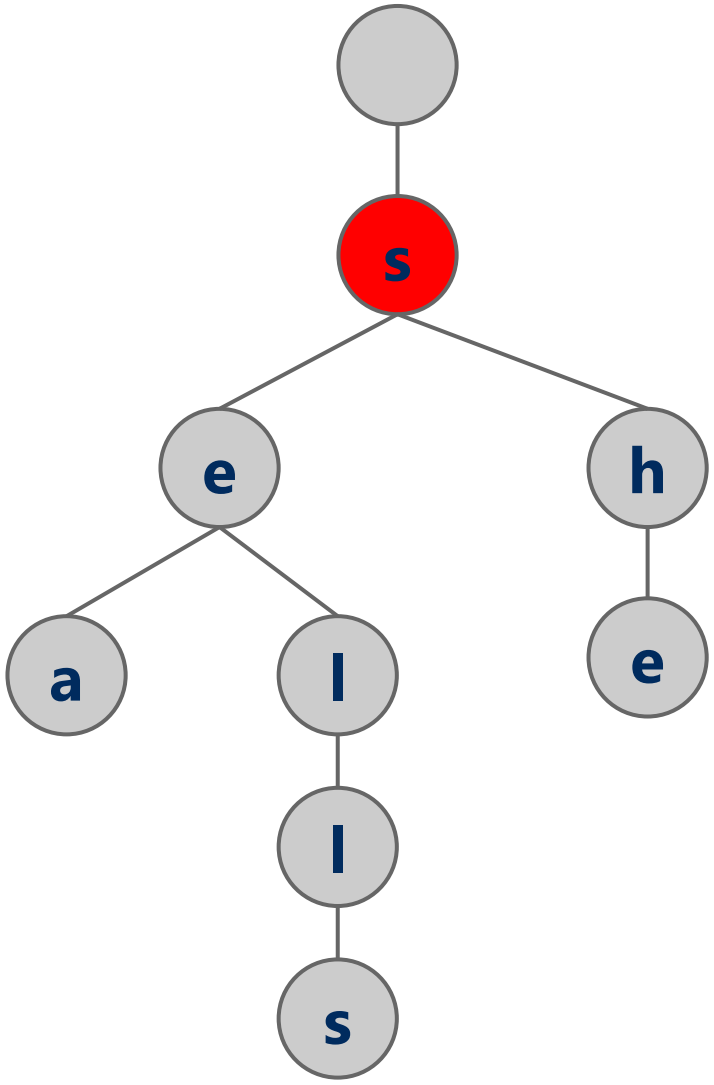

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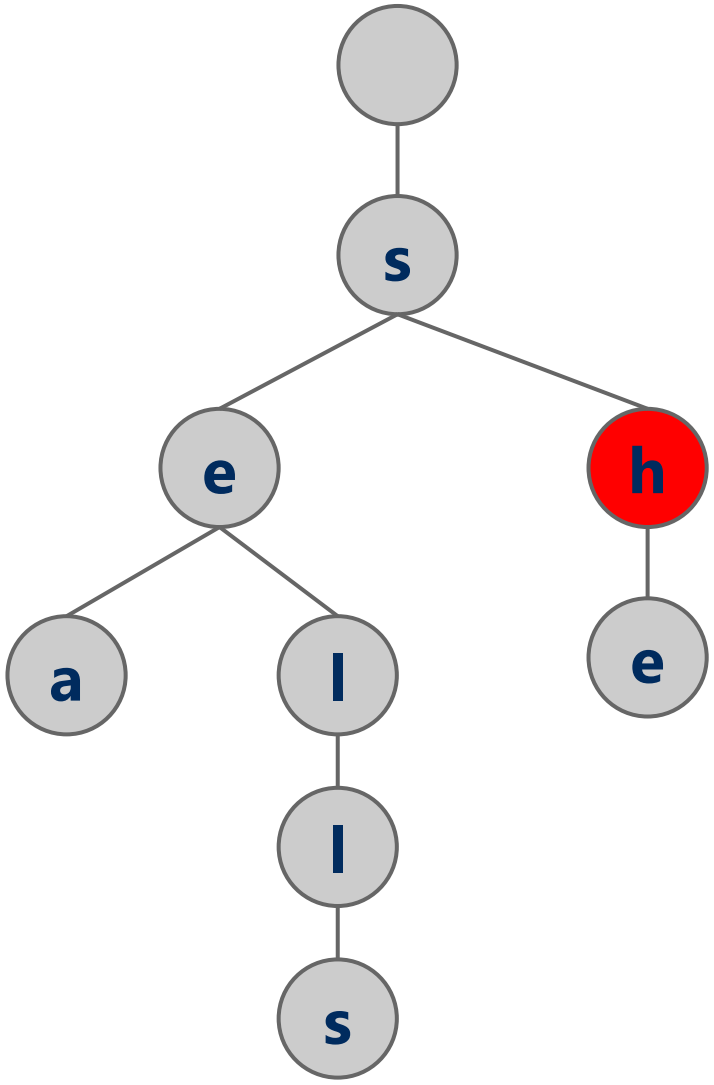

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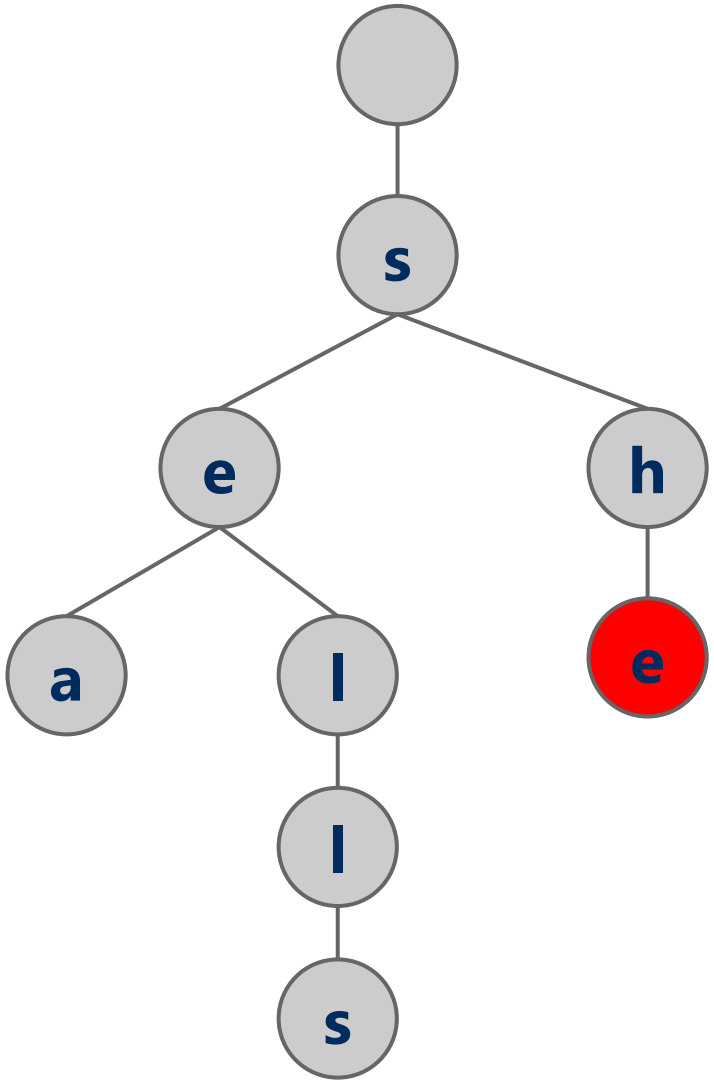

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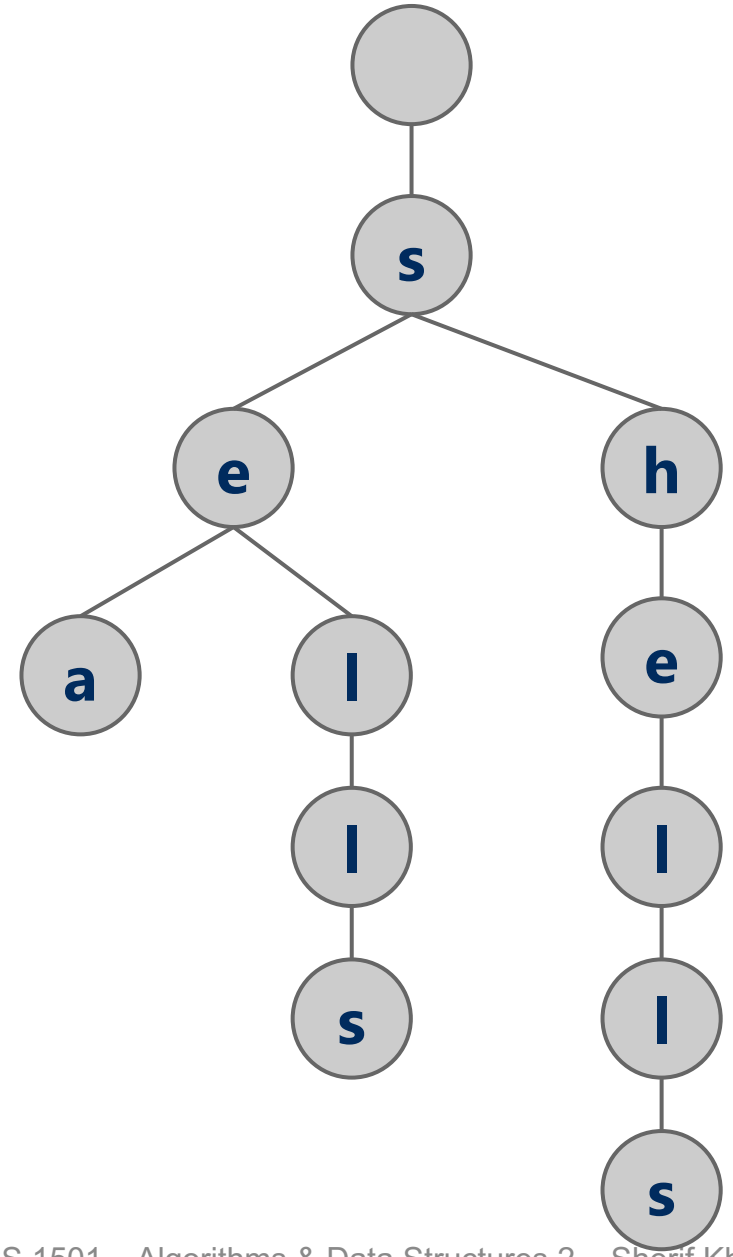

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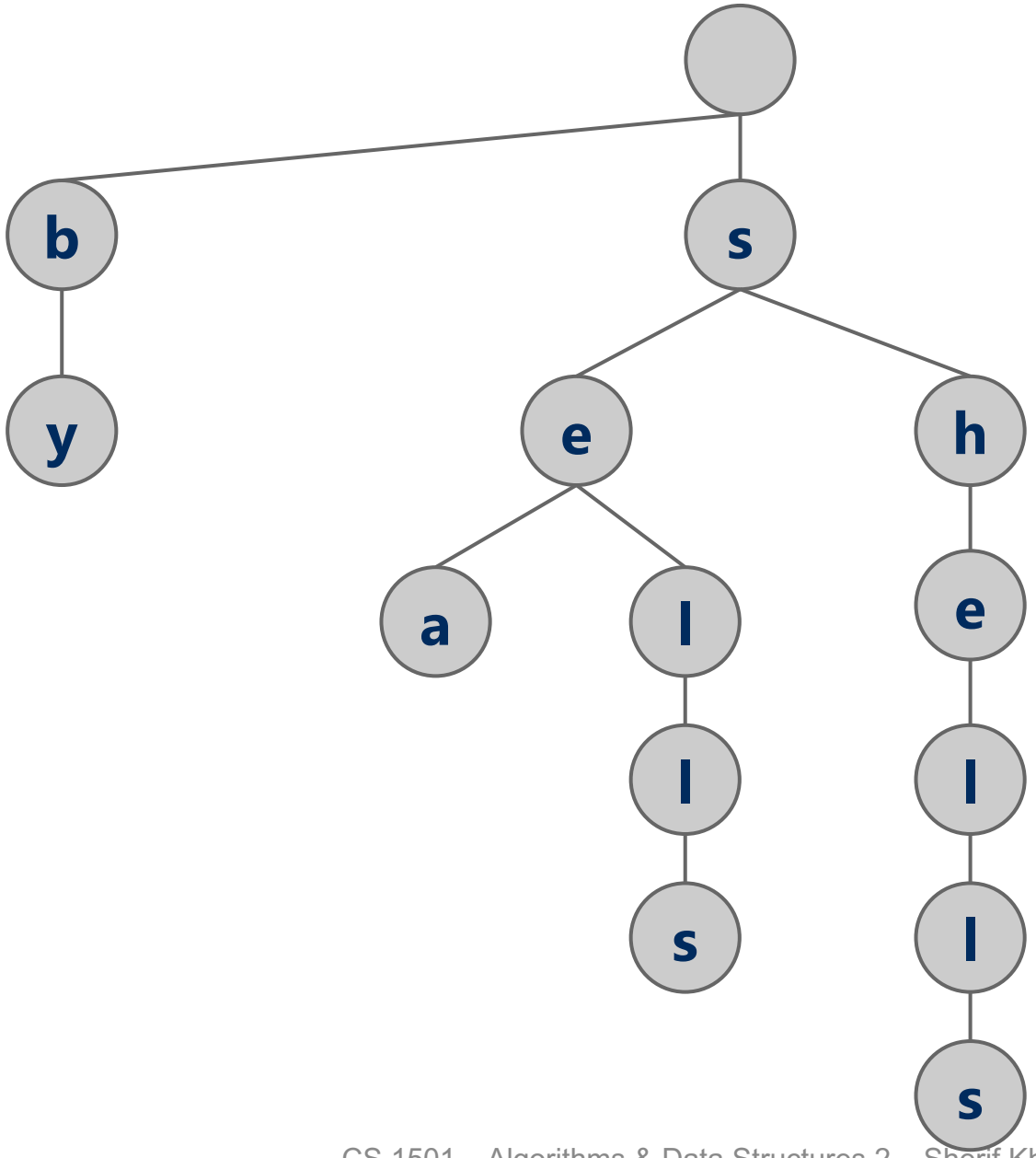
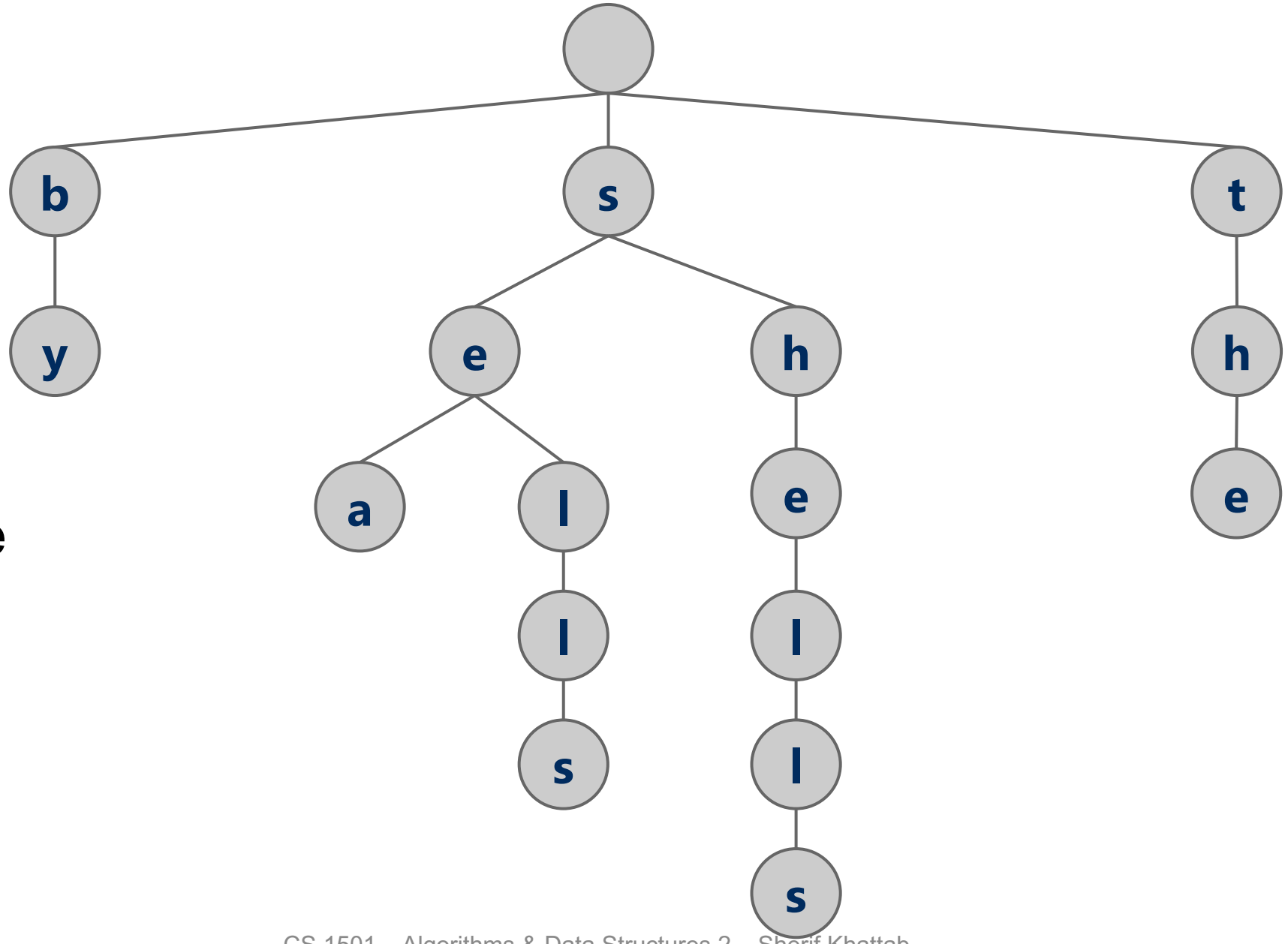


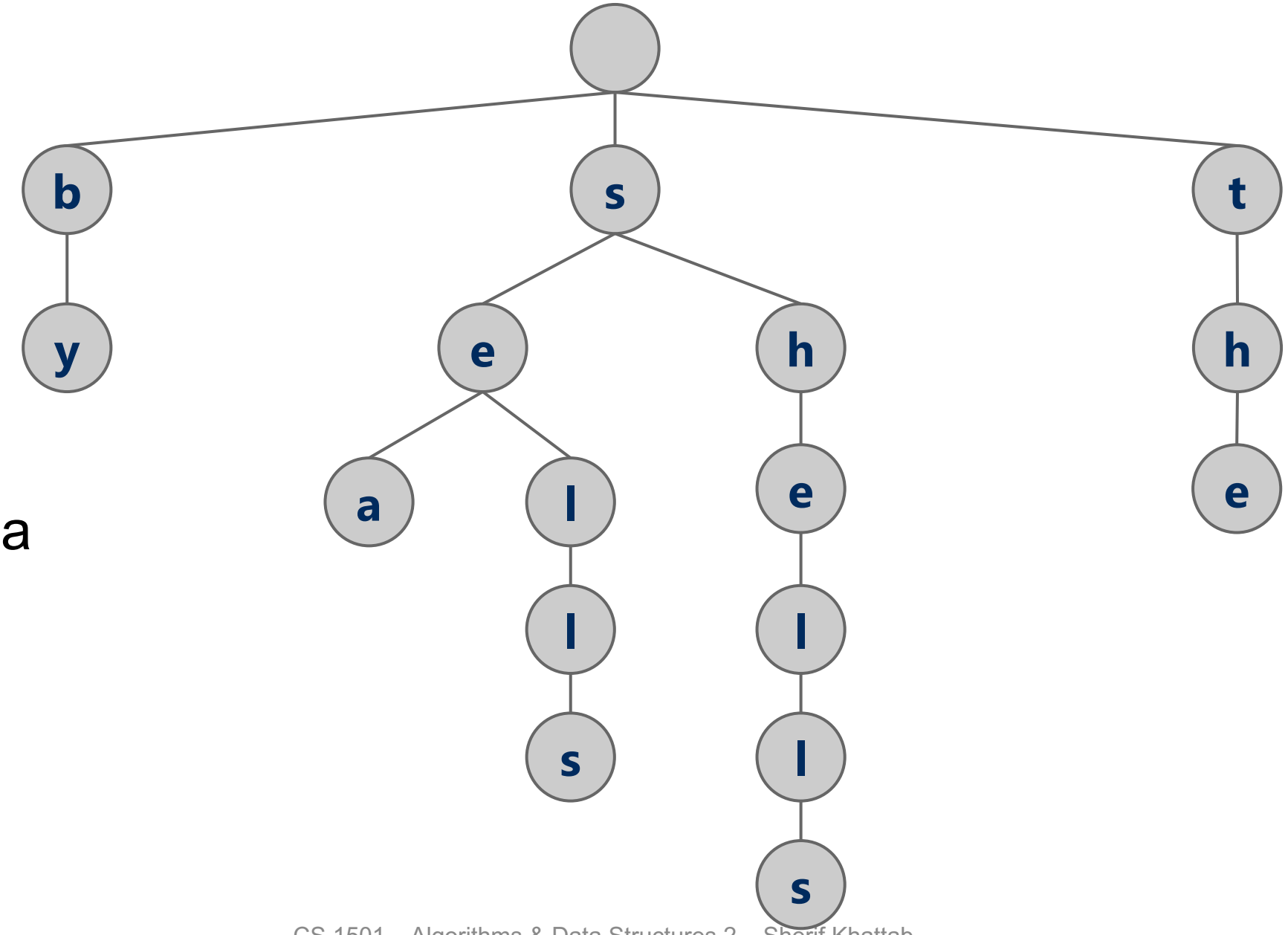
by

Another trie example



the

Another trie example



sea

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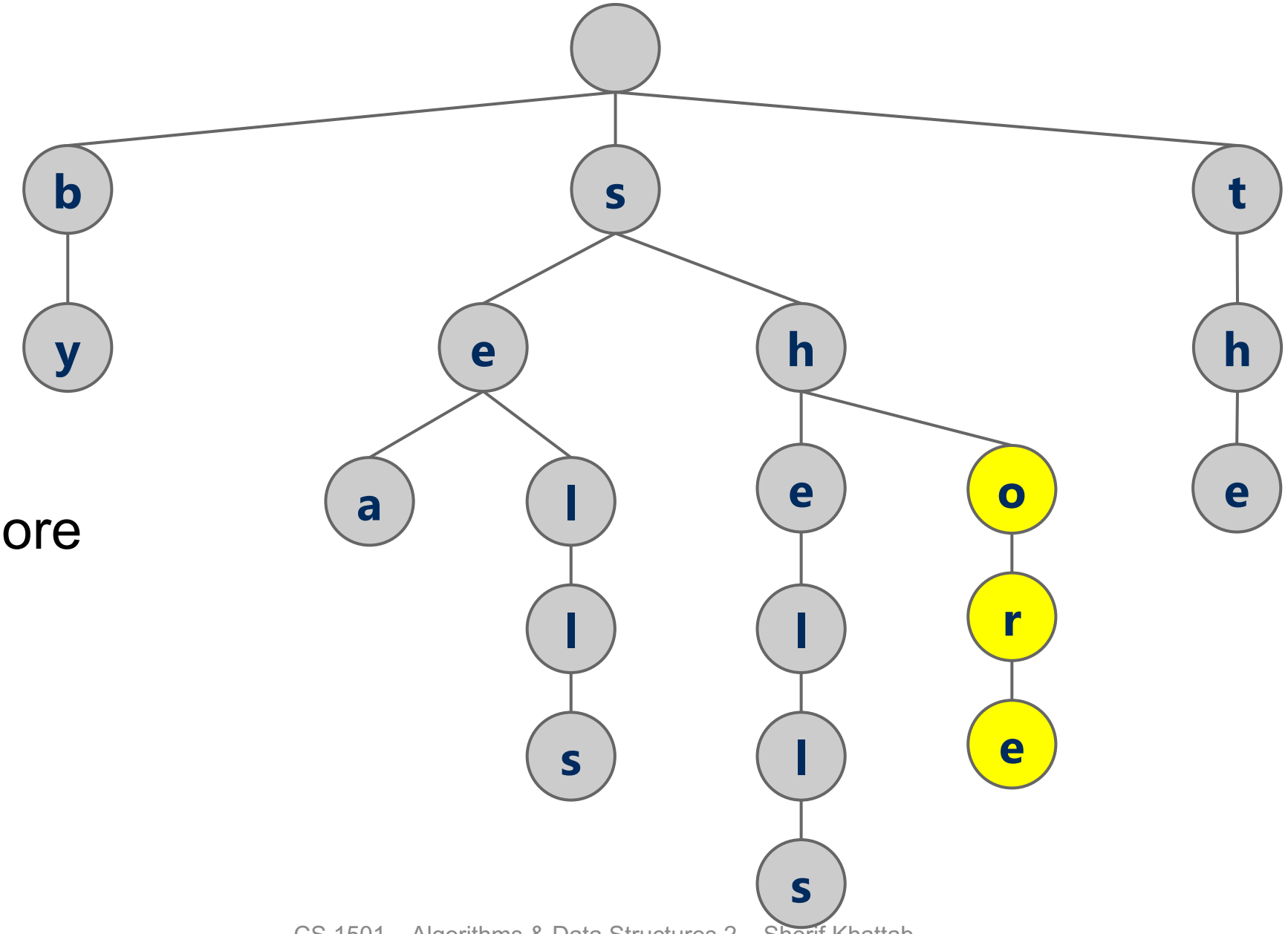


shore

Analysis

- Runtime of add and *search hit*?
- O(w) where w is the character length of the string
  - So, what do we gain over RSTs?
    - $w < b$
    - e.g., assuming fixed-size encoding $w = \frac{b}{\lceil \log R \rceil}$
    - tree height is reduced

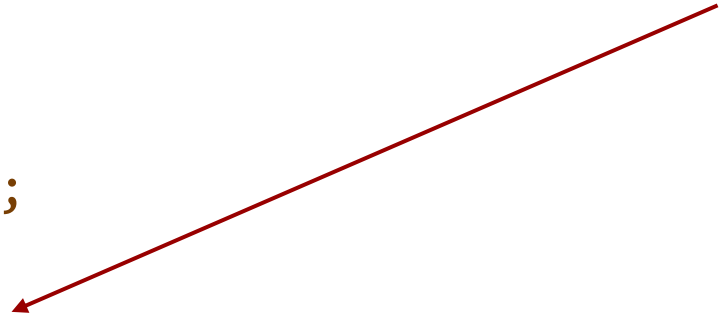
Search Miss

- Search Miss time for R-way RST
  - Require an average of $log_R(n)$ nodes to be examined
    - Proof in Proposition H of Section 5.2 of the text
- Average tree height with 2^{20} keys in an RST?
  - $\log_2 n = \log_2 2^{20} = 20$
- With 2^{20} keys in a large branching factor trie, assuming 8-bits at a time?
  - $\log_R n = \log_{256} 2^{20} = \log_{256} (2^8)^{2.5} = = \log_{256} 256^{2.5} = 2.5$

Implementation Concerns

- See TrieSt.java
  - Implements an R-way trie
- Basic node object:

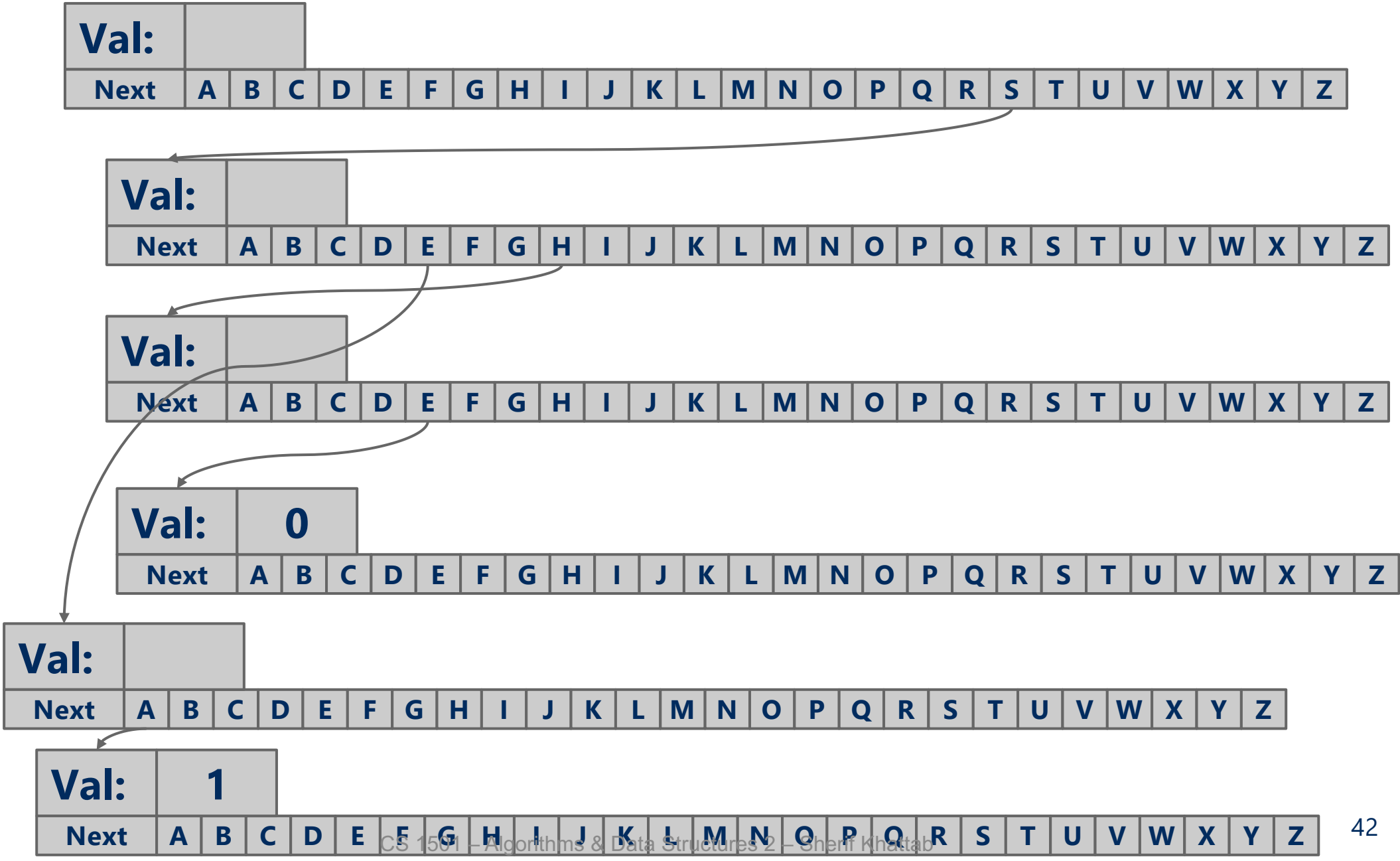
Where R is the branching factor



```
private class Node {
    private Object val;
    private Node[] next;
    private Node(){
      next = new Node[R];
    }
}
```

- Non-null **val** means we have traversed to a valid key
- Again, note that keys are not directly stored in the trie at all

R-way trie example

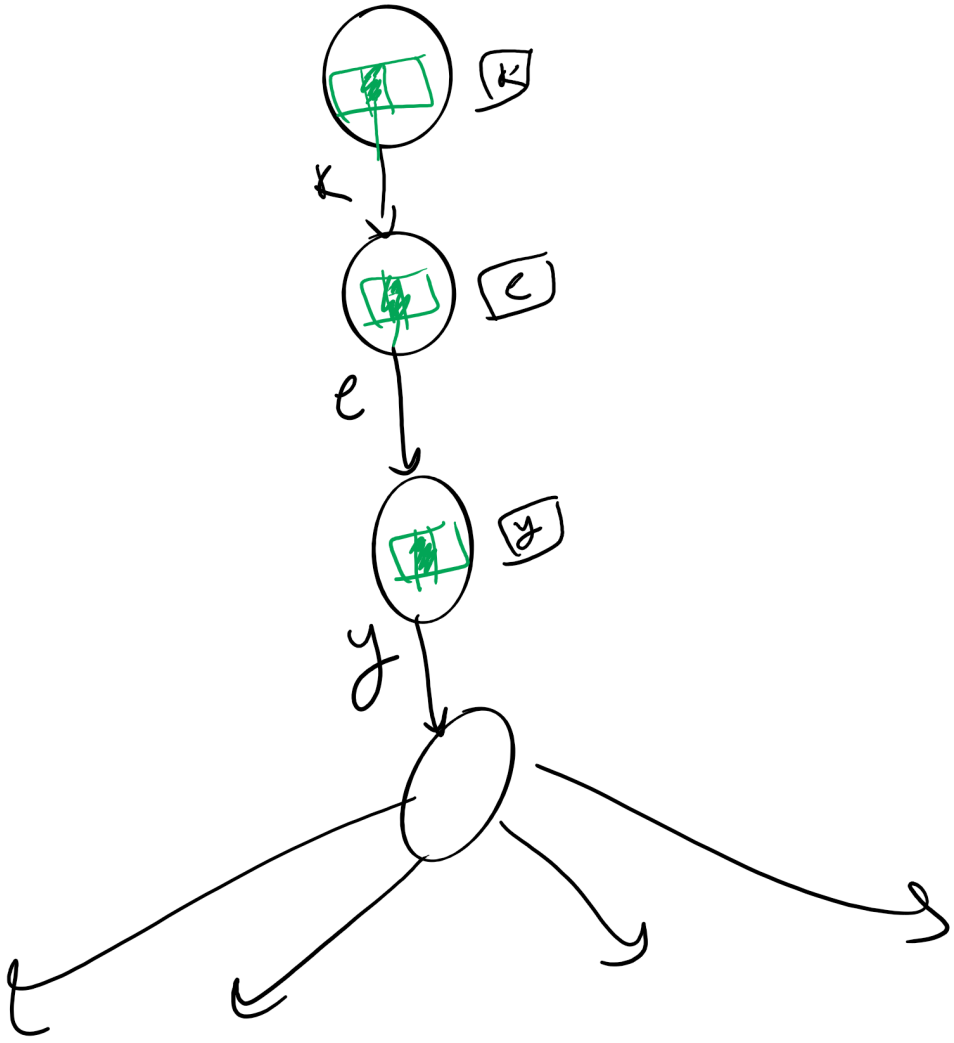


Summary of running time

	insert	search hit	search miss
binary RST	$\Theta(h)$	$\Theta(h)$	$\Theta(\log_2 n)$ on average
multi-way RST	$\Theta(w)$	$\Theta(w)$	$\Theta(\log_R n)$

R-way RST's nodes may waste space!

- Considering 8-bit ASCII, each node contains 2^8 references!
- This is especially problematic as in many cases, a lot of this space is wasted
  - Common paths or prefixes for example, e.g., if all keys begin with "key", thats 255*3 wasted references!
  - At the lower levels of the trie, most keys have probably been separated out and reference lists will be sparse



Solution: De La Briandais tries (DLBs)

Main idea: replace the array inside the node of the R-way trie with a linked-list

De La Briandais (DLB) Trie

- tree-like structure used for searching when keys are sequences of characters
- each nodelet
  - stores one character,
  - points to a sibling (linked list of siblings), and
  - points to a child

Adding to DLB Trie

- if root is null, set root ← new node
- current node ← root
- for each *character c* in the key
  - Search for *c* in the linked list headed at current using sibling links
    - if not found, create a new node and attach as a sibling to the linked list
  - move to child of the found node
    - either recursively or by current ← child
- if at last character of key, insert value into current node and return

DLB Example