Algorithms and Data Structures 2
CS 1501

Fall 2022

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(Slides are adapted from Dr. Ramirez's and Dr. Farnan's CS1501 slides.)

Announcements

- Upcoming Deadlines
  - Homework 4: this Friday @ 11:59 pm
  - Lab 3: next Monday @ 11:59 pm
  - Assignment 1: Monday Oct 10th @ 11:59 pm
- **Live support session** for Assignment 1
  - Over Zoom this Friday @ 5:00 pm
- **Student Support Hours** of the teaching team are posted on the Syllabus page

Previous lecture

- Digital Searching Problem
  - Searching when keys are represented as a sequence of digits (e.g., bits) or alphabetic characters
  - Digital Search Trees
  - Radix Search Tries

This Lecture

- R-way Radix Search Tries
- De La Briandais (DLB) Tries

Adding to Radix Search Trie (RST)

- Input: key and corresponding value
- if root is null, set root ← new node
- current node ← root
- for each *bit* in the key
  - if bit == 0,
    - if left child of current node is null, create a new node and attach as the left child
    - move to left child
      - either recursively or by setting current ← current.left
  - if bit == 1,
    - if right child of current node is null, create a new node and attach as the right child
    - move to right child
      - either recursively or by setting current ← current.right
- insert corresponding value into current node

RST example

Insert:

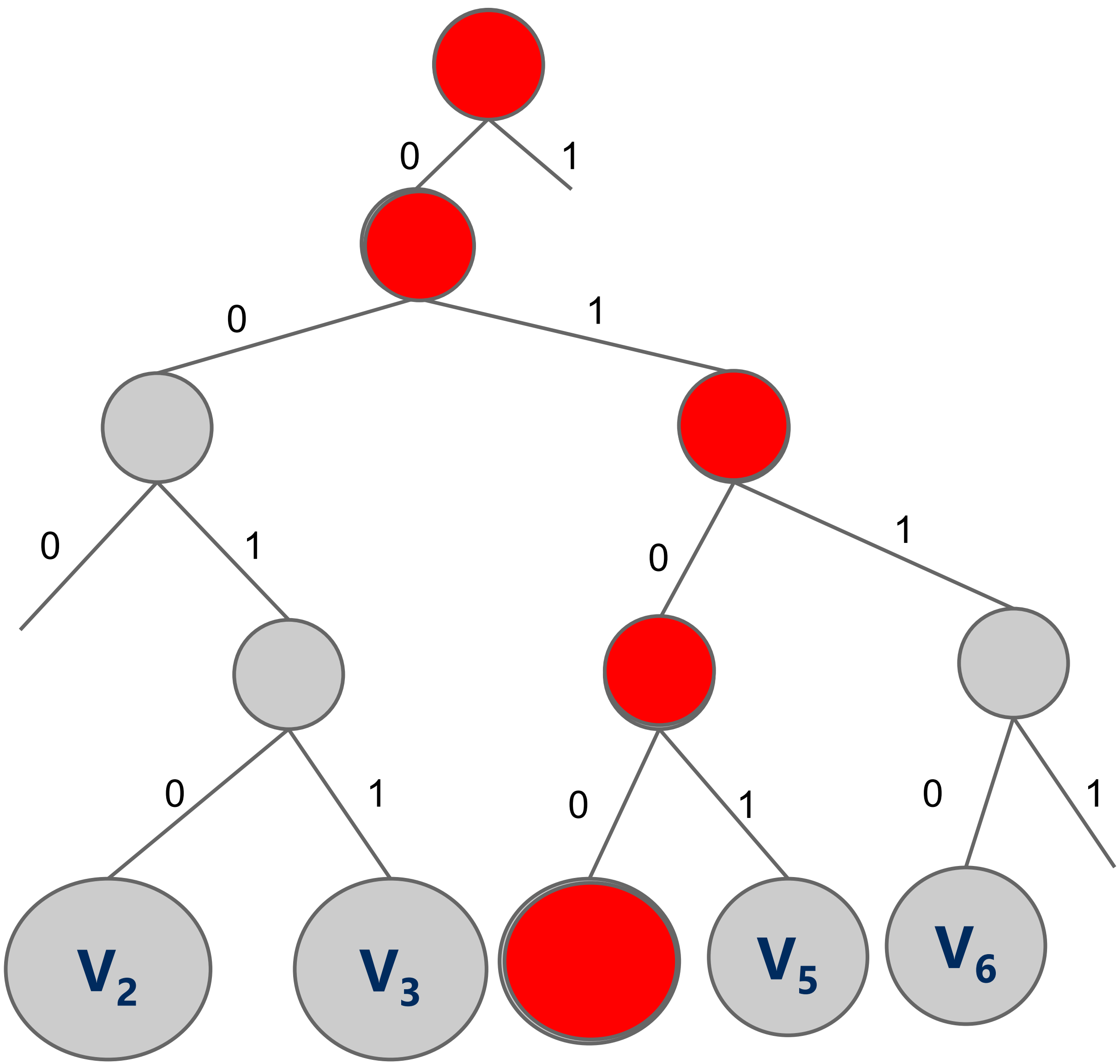
4 0100

3 0011

2 0010

6 0110

5 0101



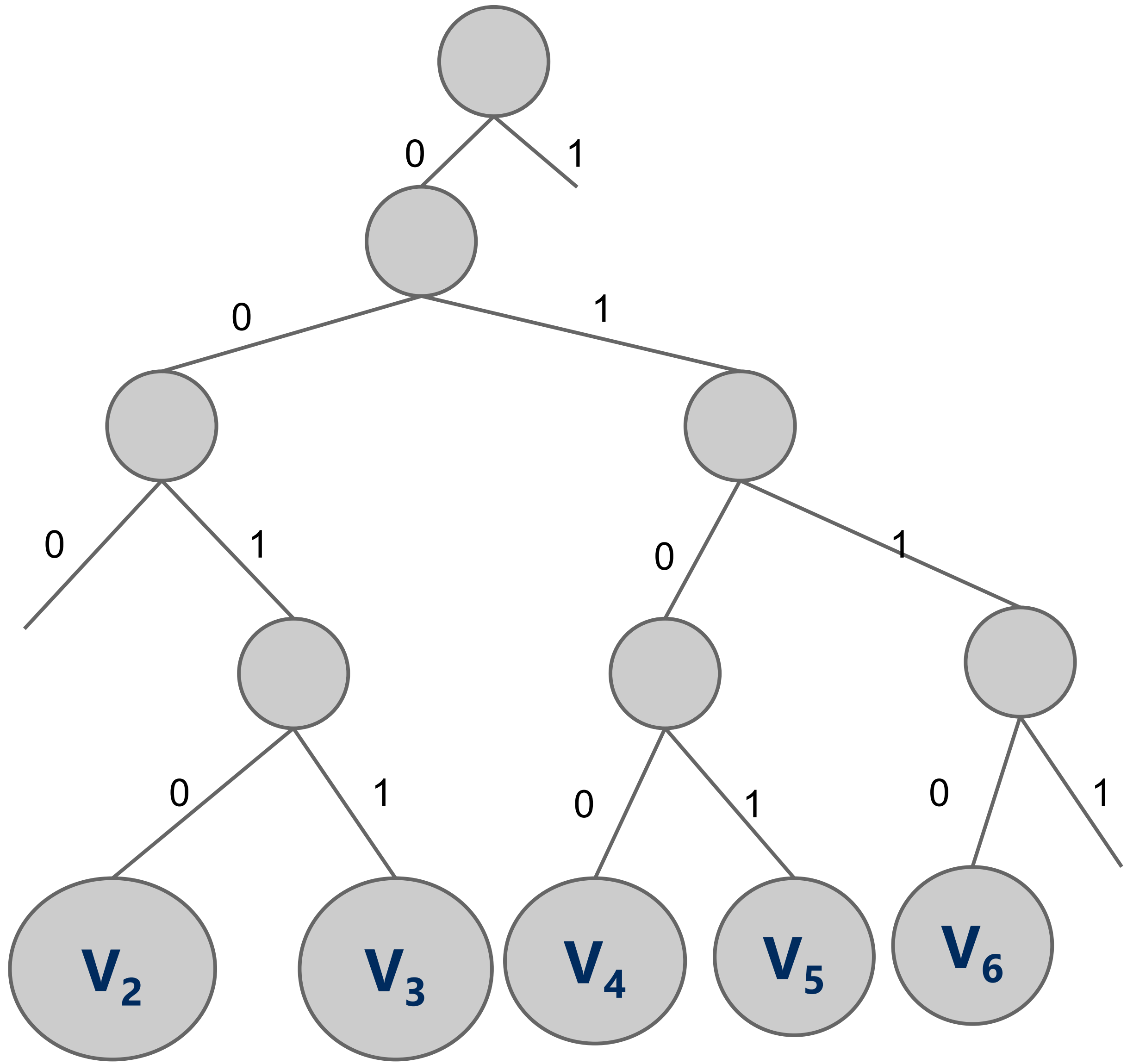
Searching in Radix Search Trie (RST)

- Input: key
- current node ← root
- for each *bit* in the key
  - if current node is null, return *key not found*
  - if bit == 0,
    - move to left child
      - either recursively or by setting current ← current.left
  - if bit == 1,
    - move to right child
      - either recursively or by setting current ← current.right
- if current node is null or the value inside is null
  - return key *not found*
- else return the value stored in current node

RST example

Search:

3 0011

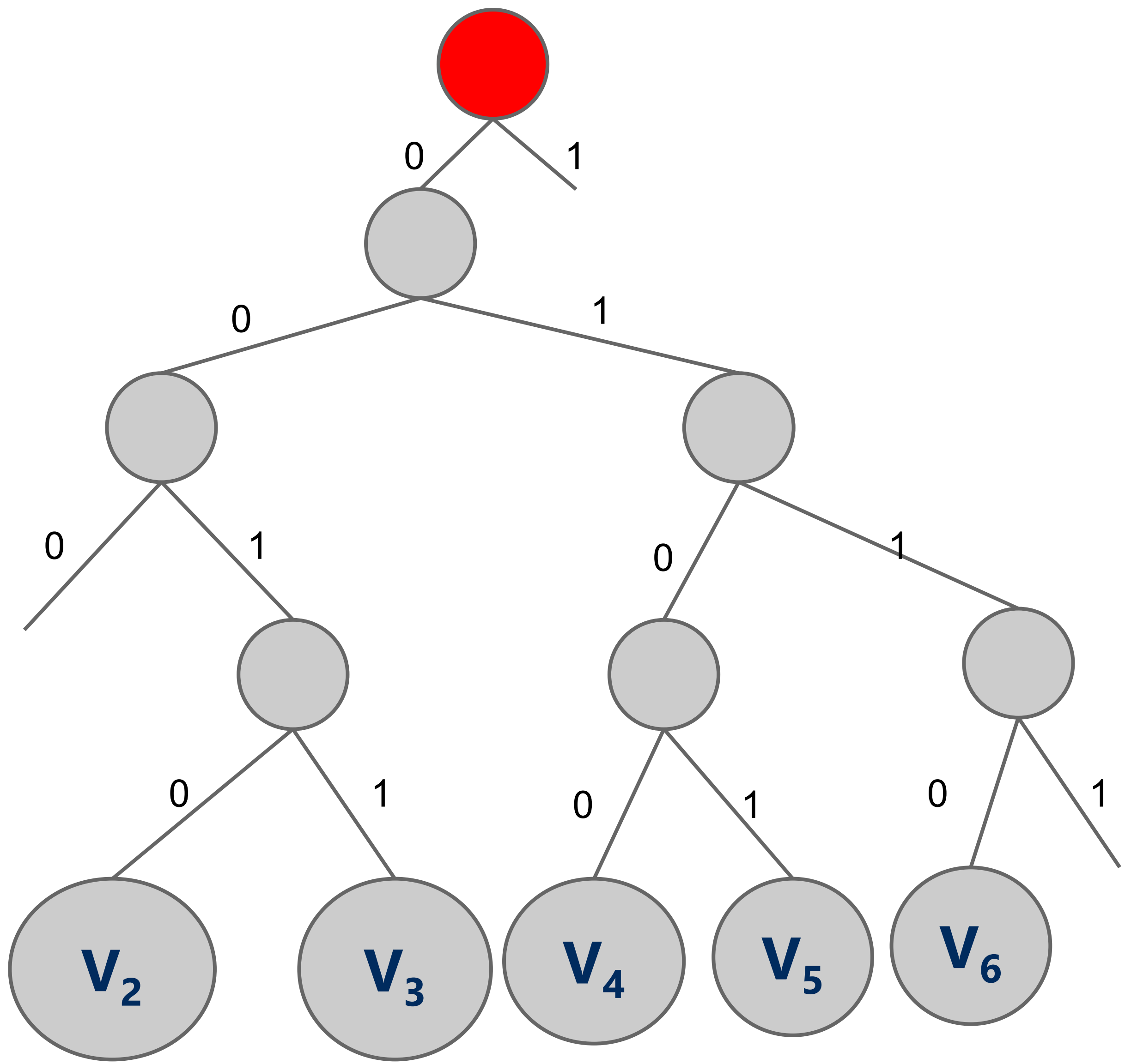


RST example

Search:

3 0011

7 0111

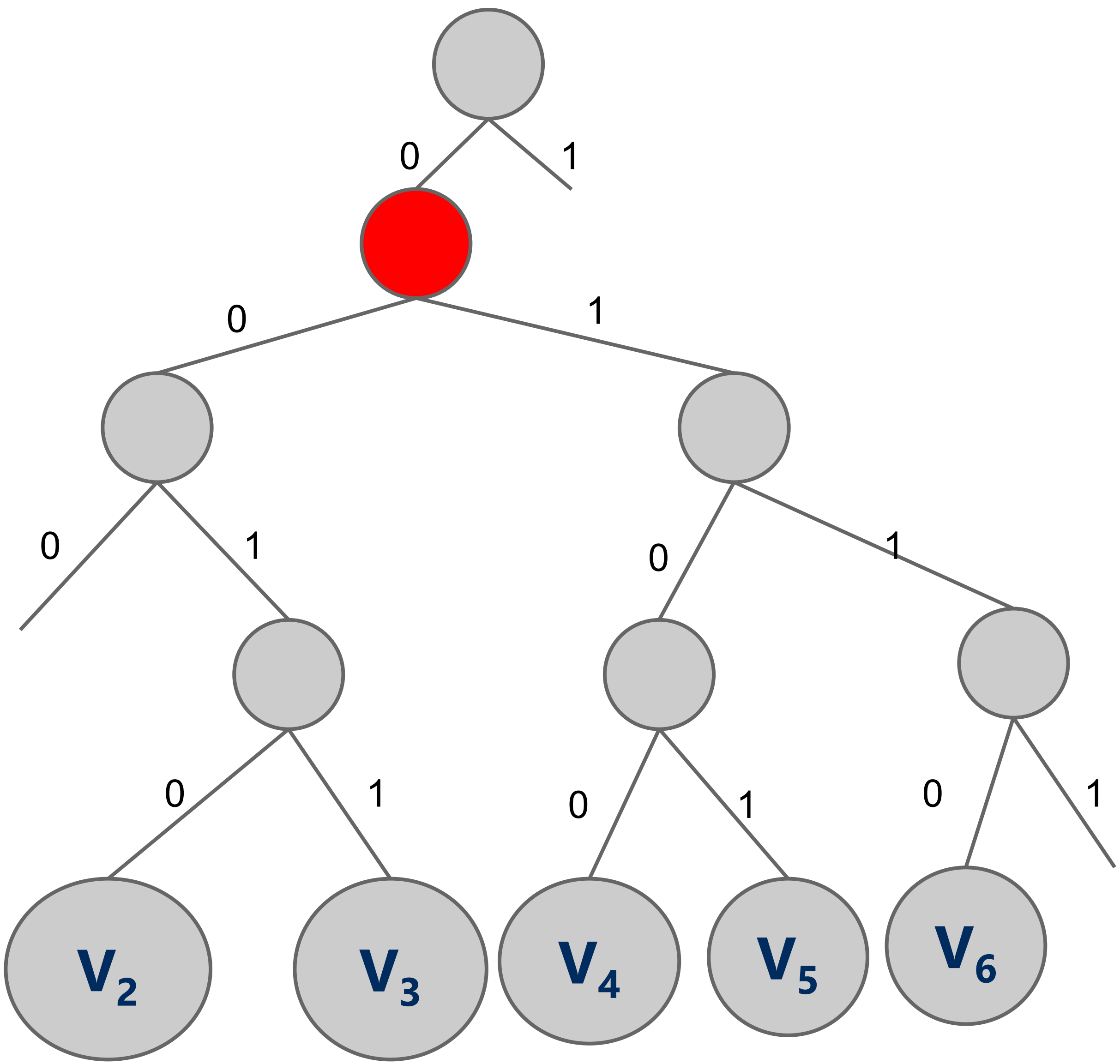


RST example

Search:

3 0011

7 0111

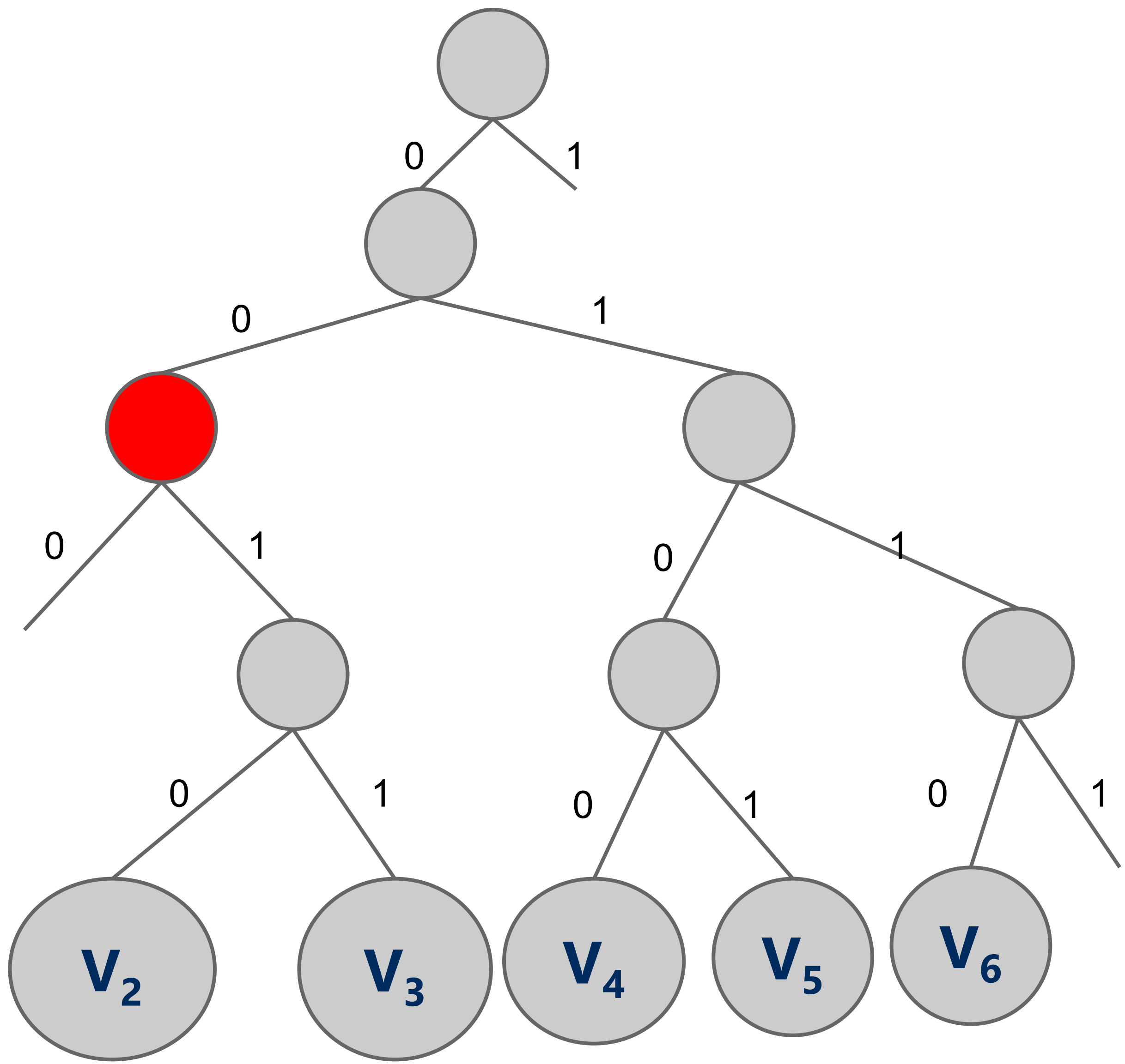


RST example

Search:

3 0011

7 0111

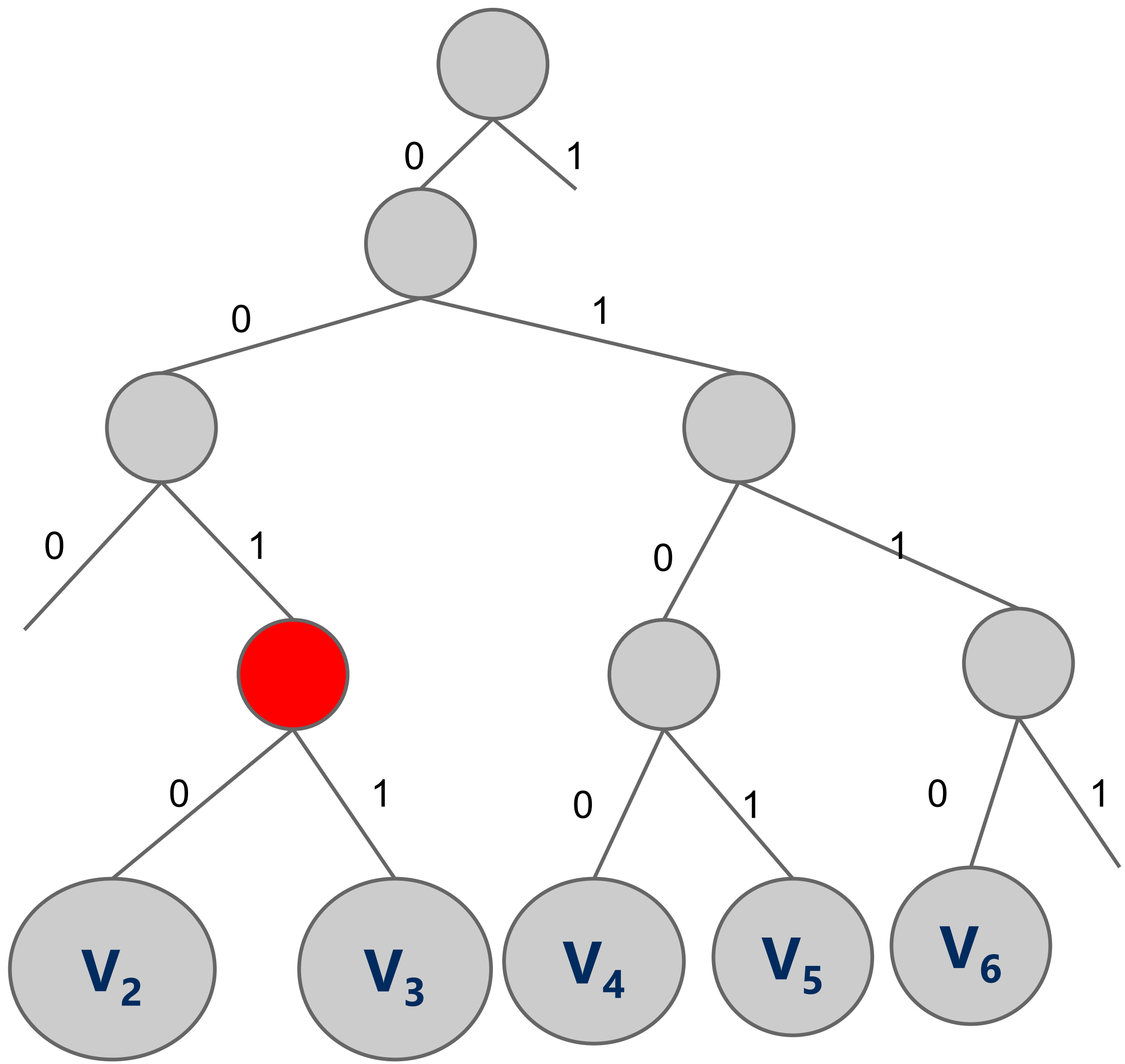


RST example

Search:

3 0011

7 0111



RST example

Search:

3 0011

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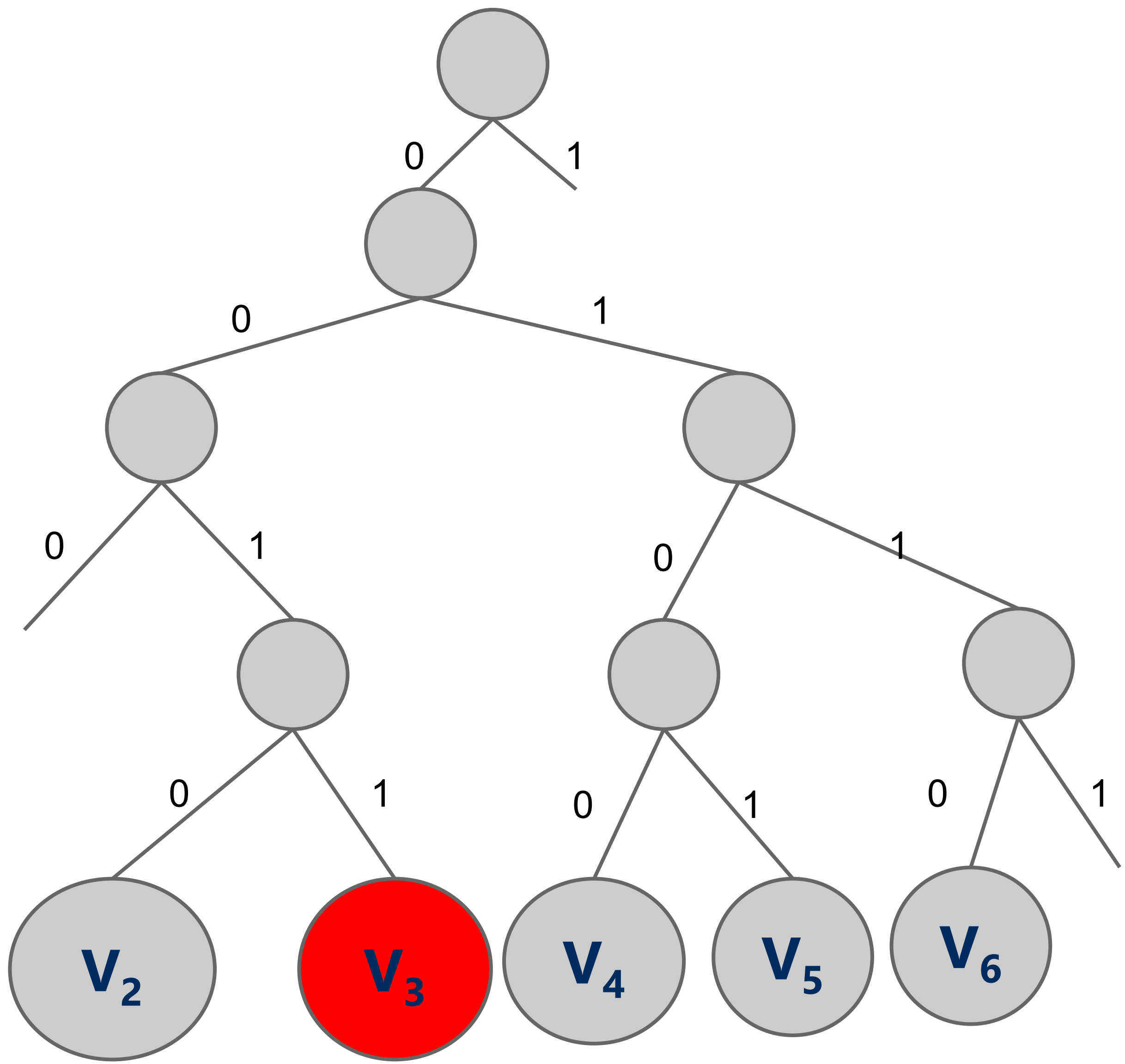
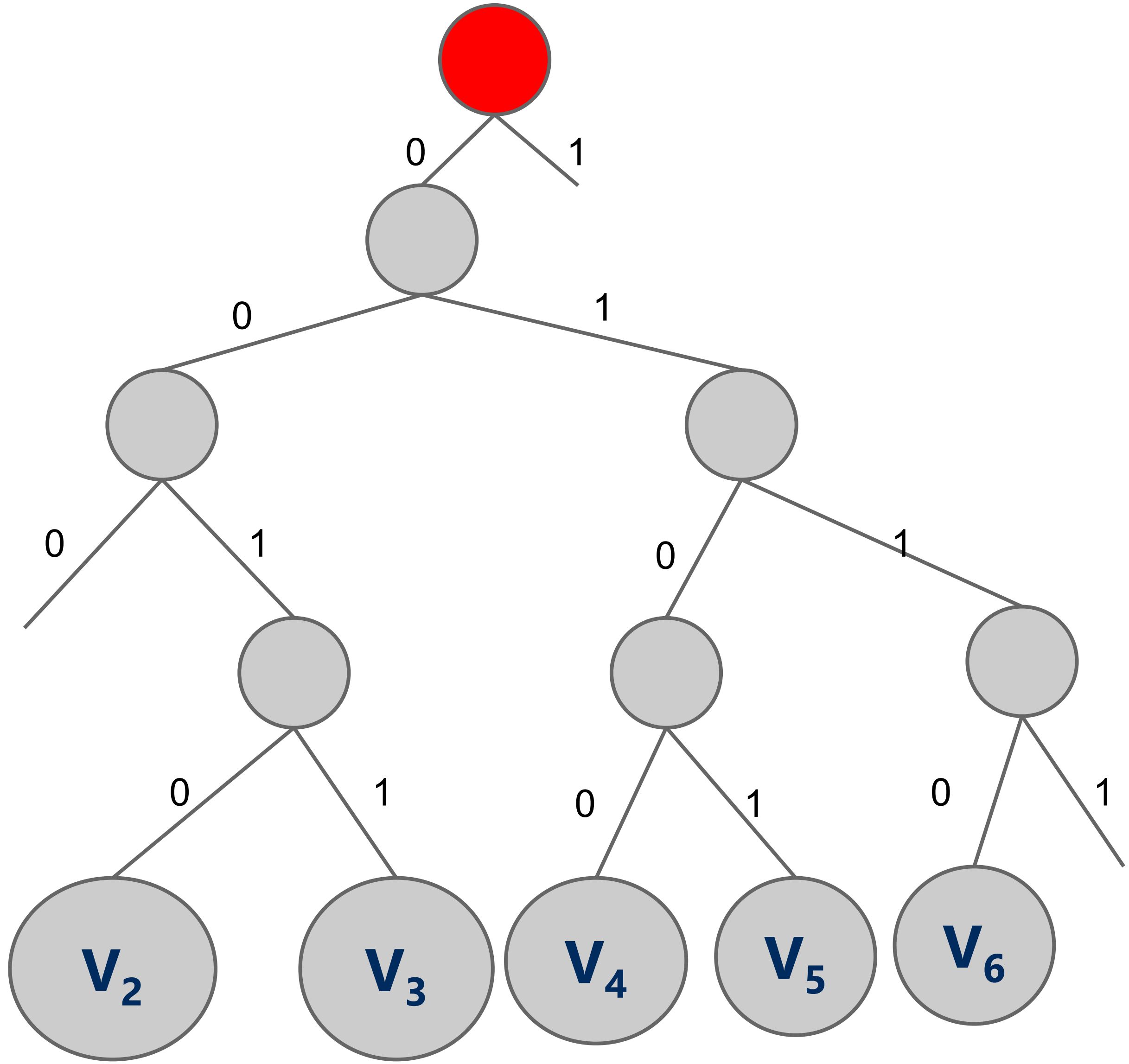


RST example

Search:

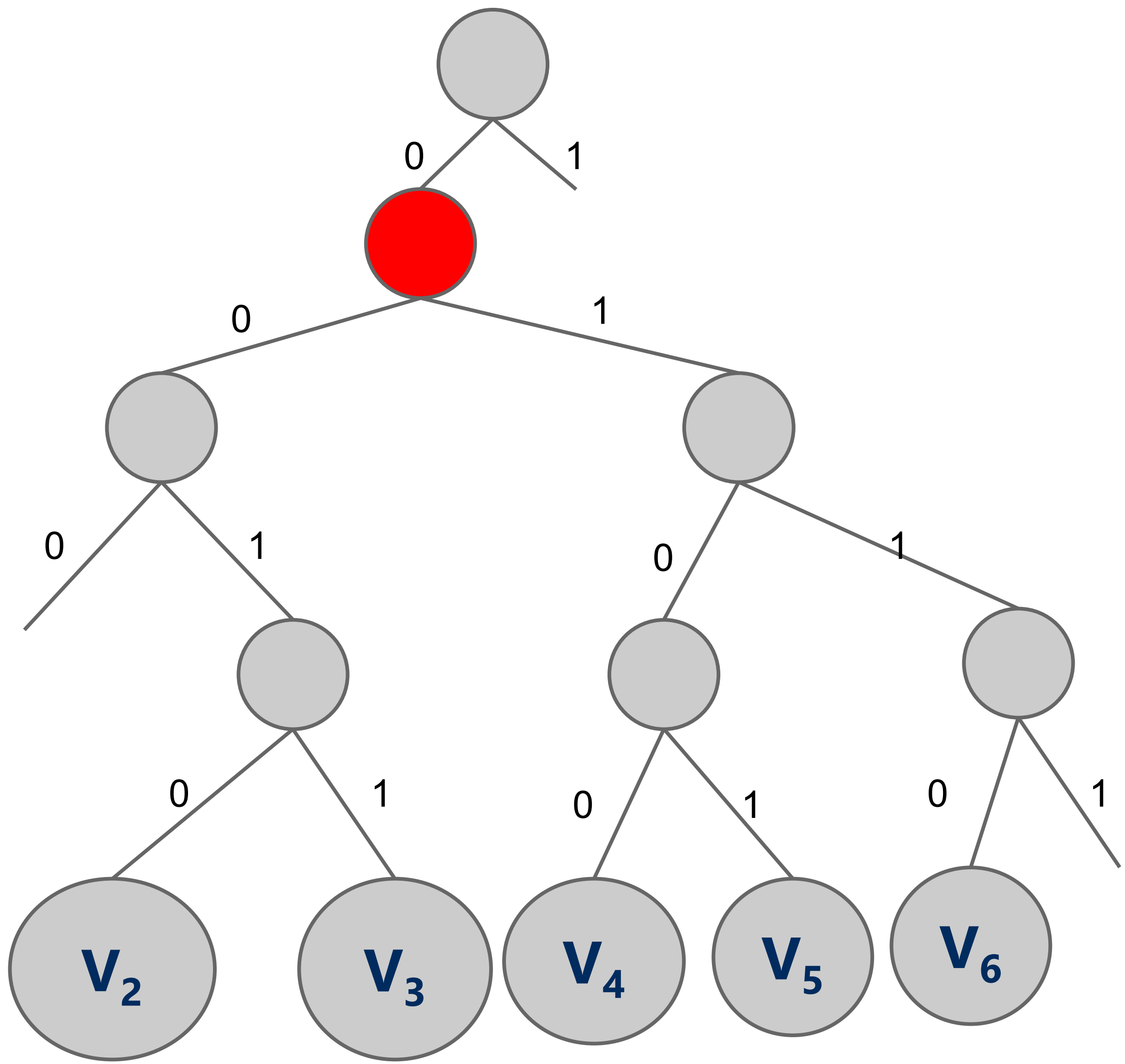
7 0111



RST example

Search:

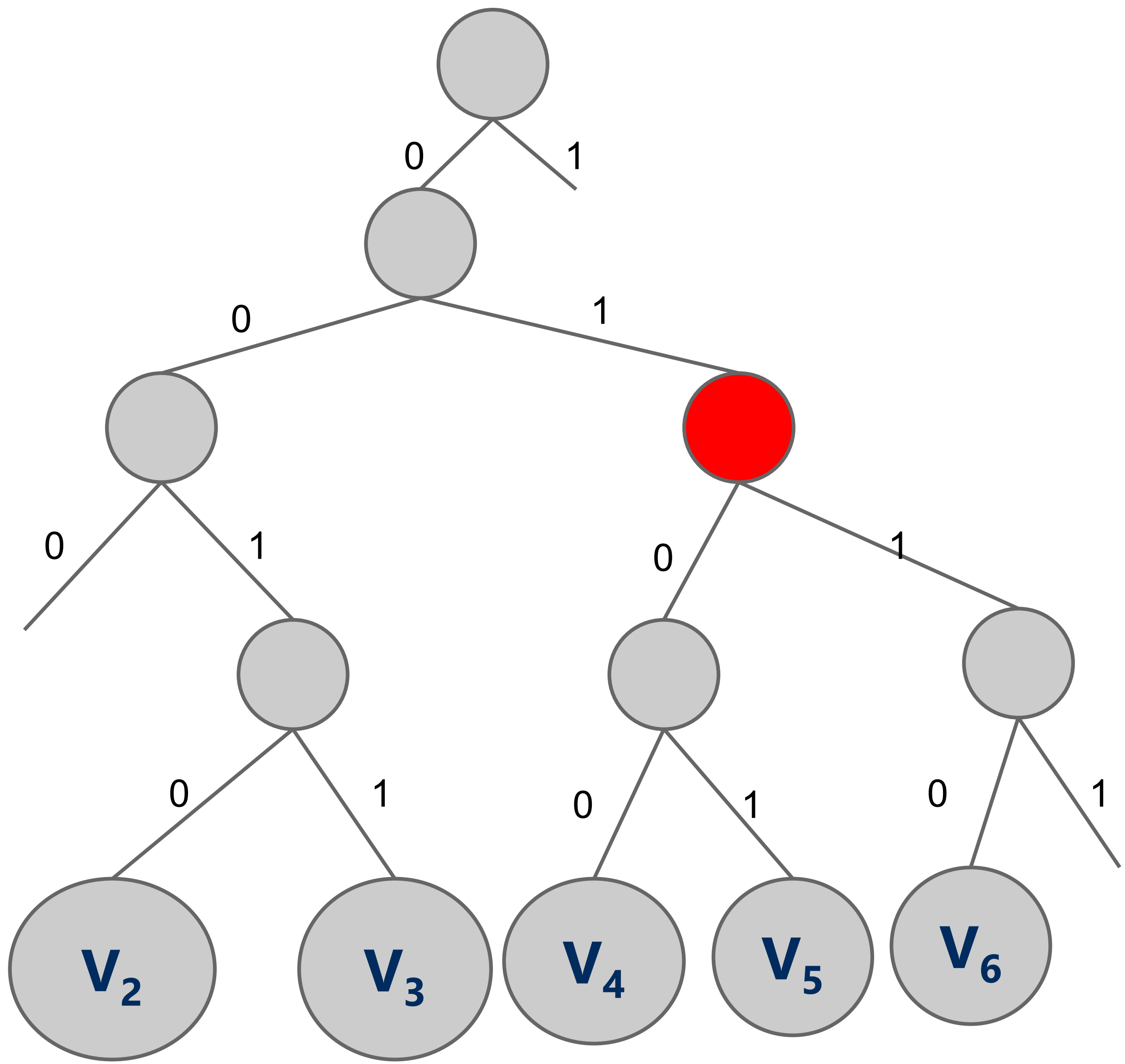
7 0111



RST example

Search:

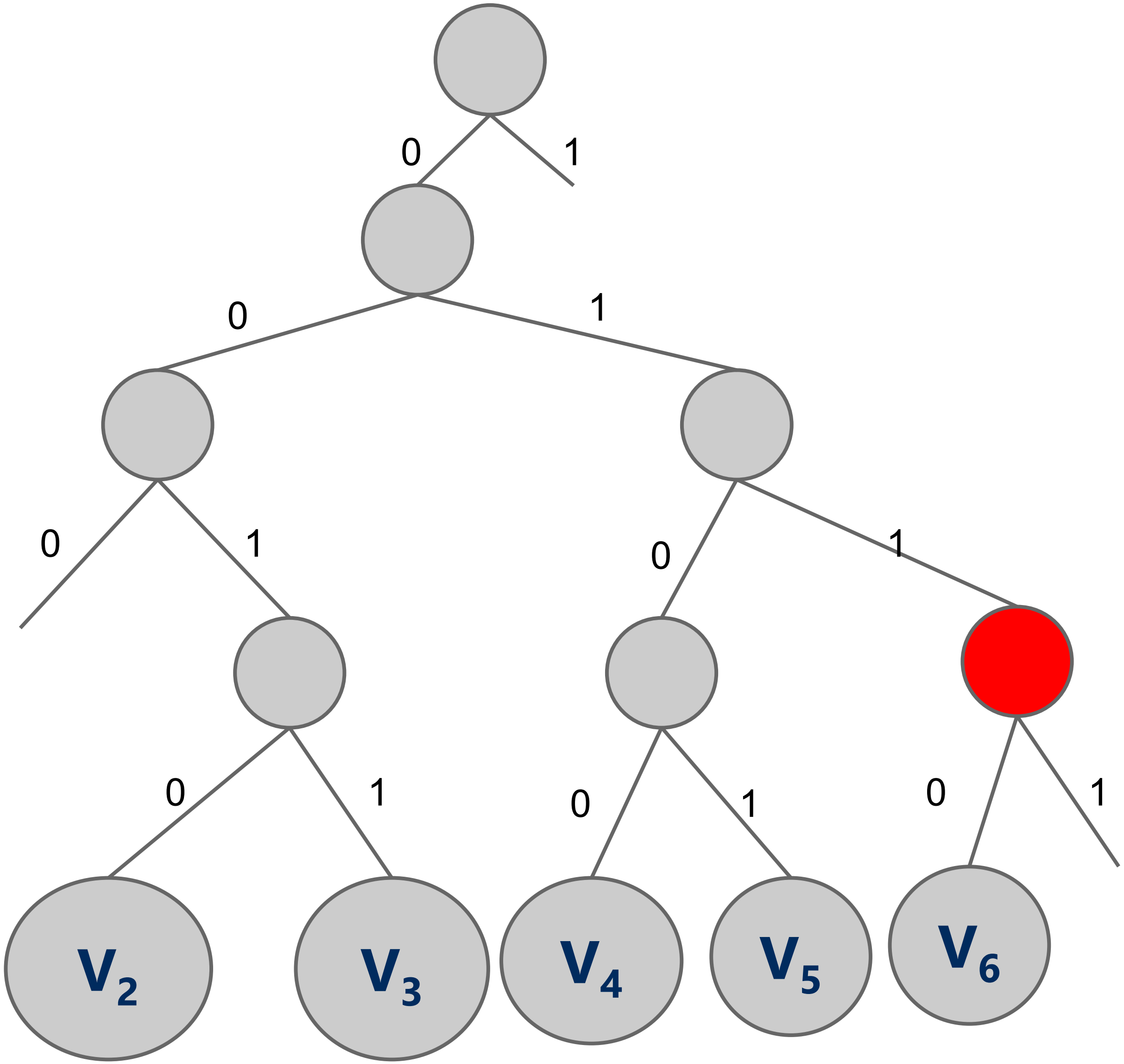
7 0111



RST example

Search:

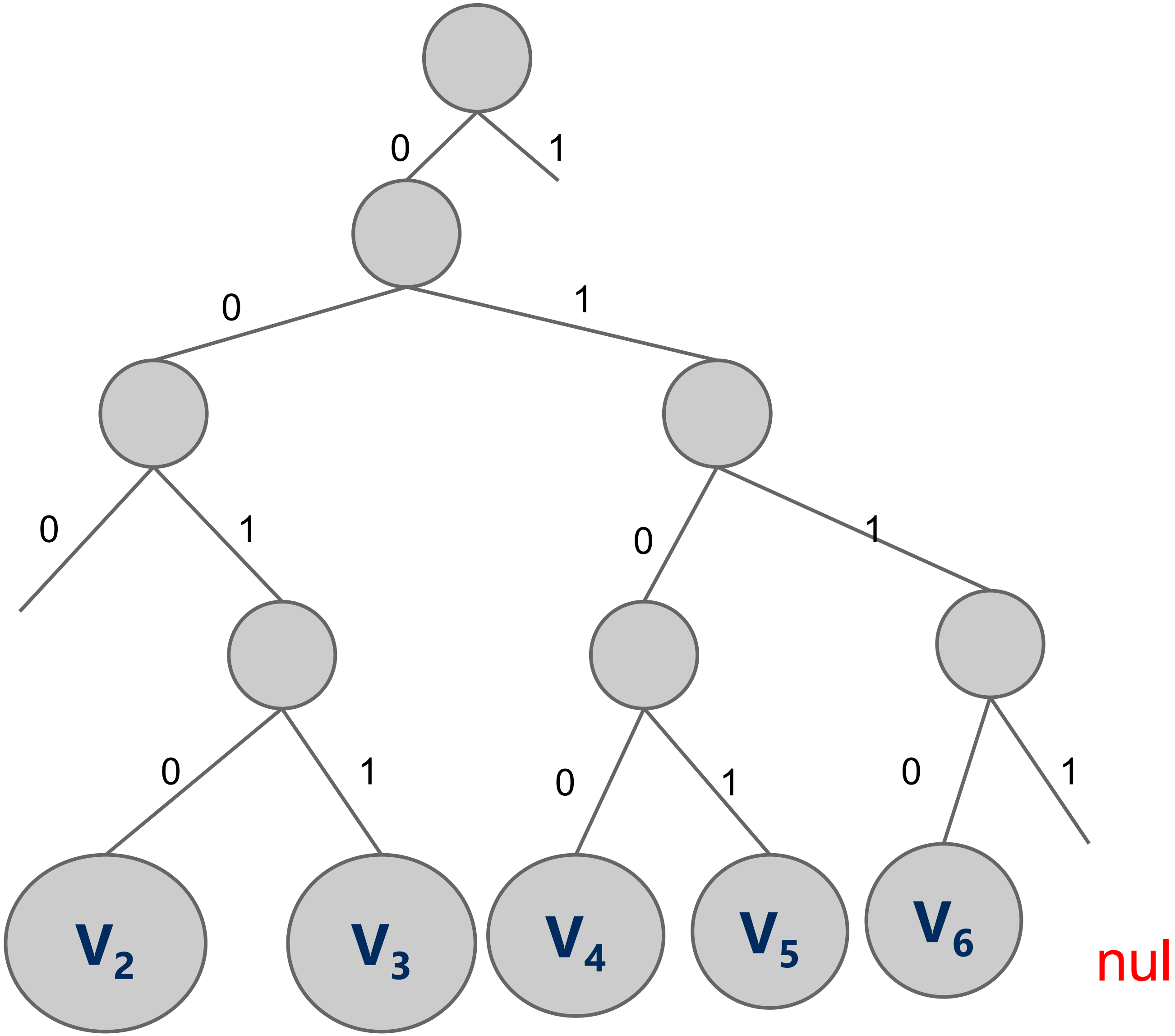
7 0111



RST example

Search:

7 0111



RST analysis

- Runtime?
- O(b), the bit length of the key
  - However, this time we don't have full key comparisons
- Would this structure work as well for other key data types?
- Characters?
  - Characters are the same as 8-bit ints (assuming simple ascii)
- Strings?
- May have huge bit lengths
- How to store Strings?

Larger branching factor tries

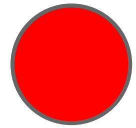
- In our binary-based Radix search trie, we considered one bit at a time
- What if we applied the same method to characters instead of bits in a string?
  - What would this new structure look like?
  - How many children per node?
    - up to R (the alphabet size)
  - Also called R-way radix search tries

Adding to R-way Radix RST

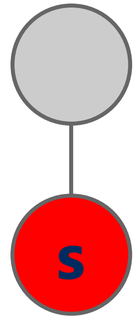
- if root is null, set root ← new node
- current node ← root
- for each *character c* in the key
    - Find the *cth* child
        - if child is null, create a new node and attach as the *cth* child
        - move to child
            - either recursively or by current ← child
- if at last character of key, insert value into current node

Another trie example

she

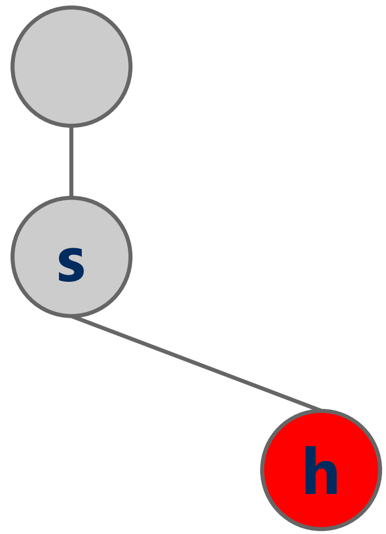


Another trie example



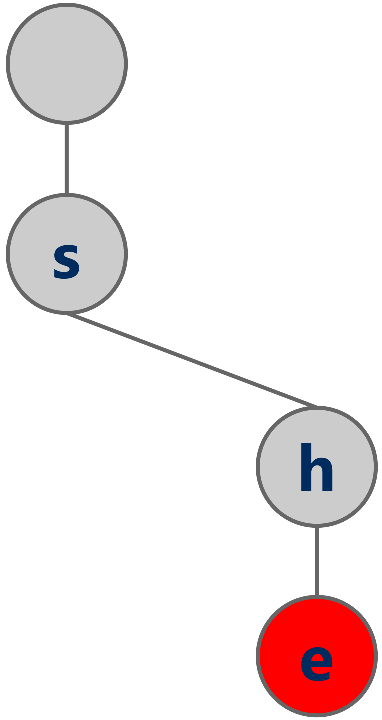
she

Another trie example



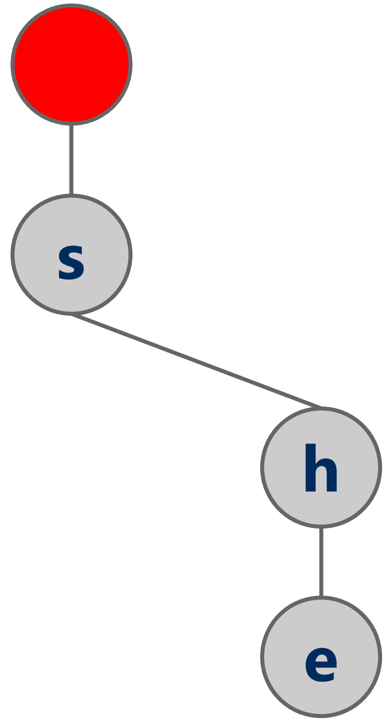
she

Another trie example



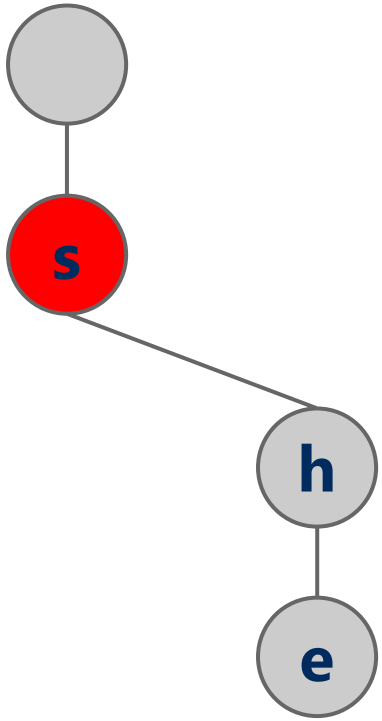
she

Another trie example



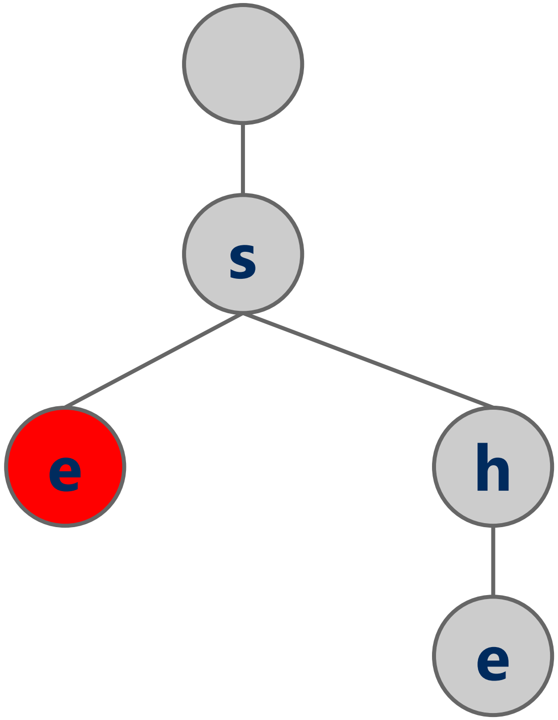
sells

Another trie example



sells

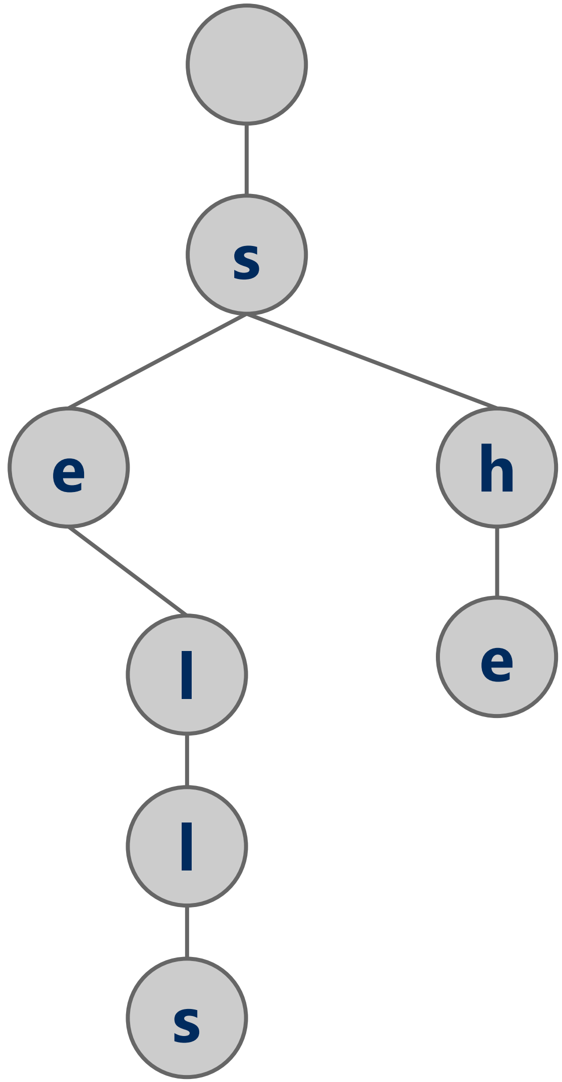
Another trie example



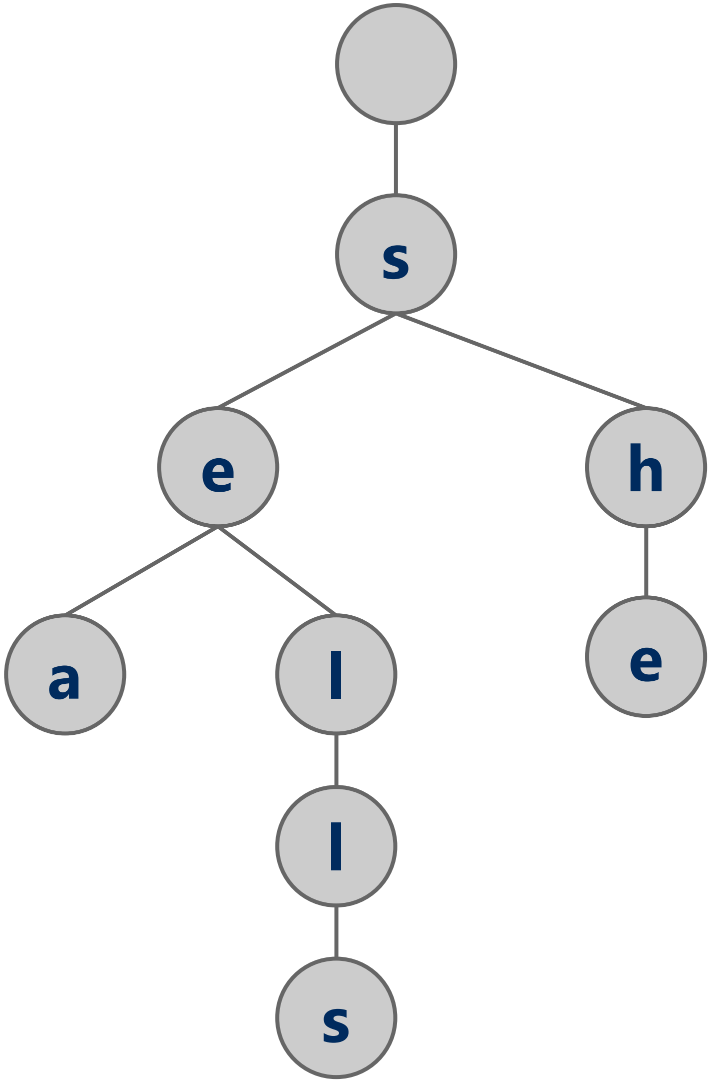
sells

Another trie example

sells



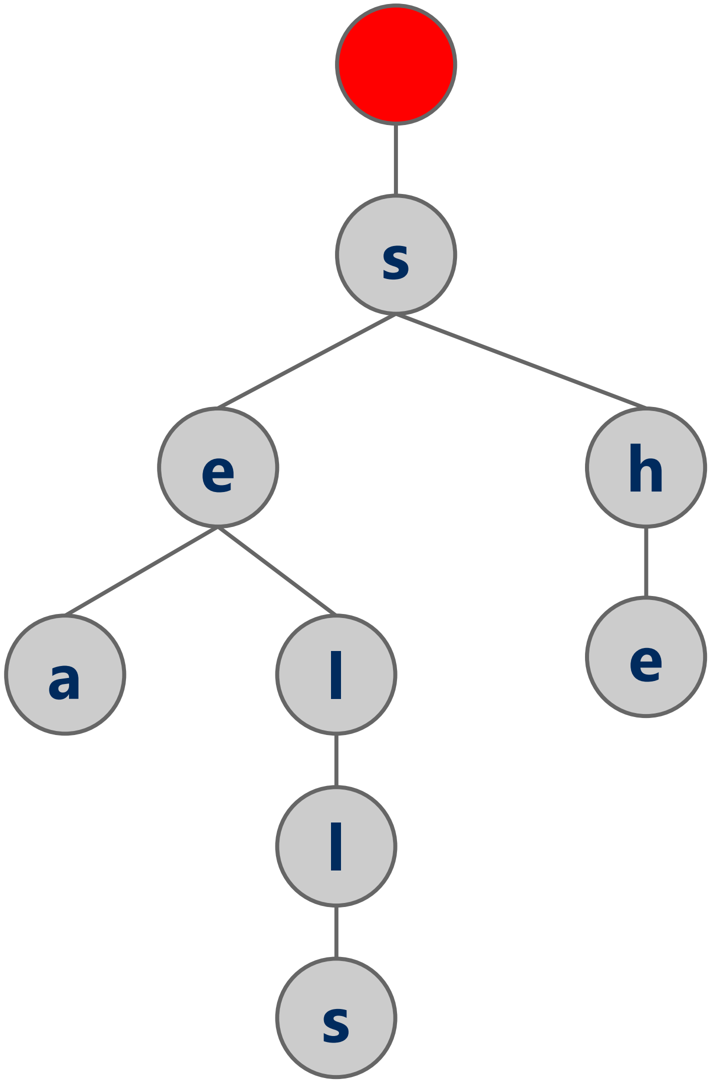
Another trie example



sea

Another trie example

shells



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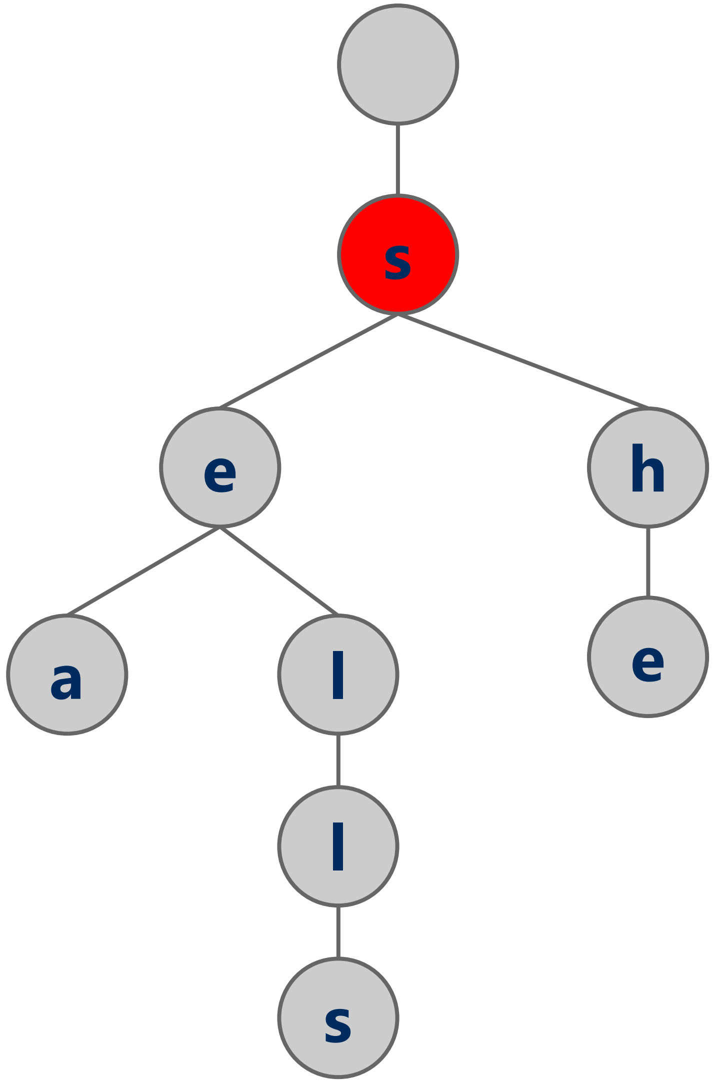
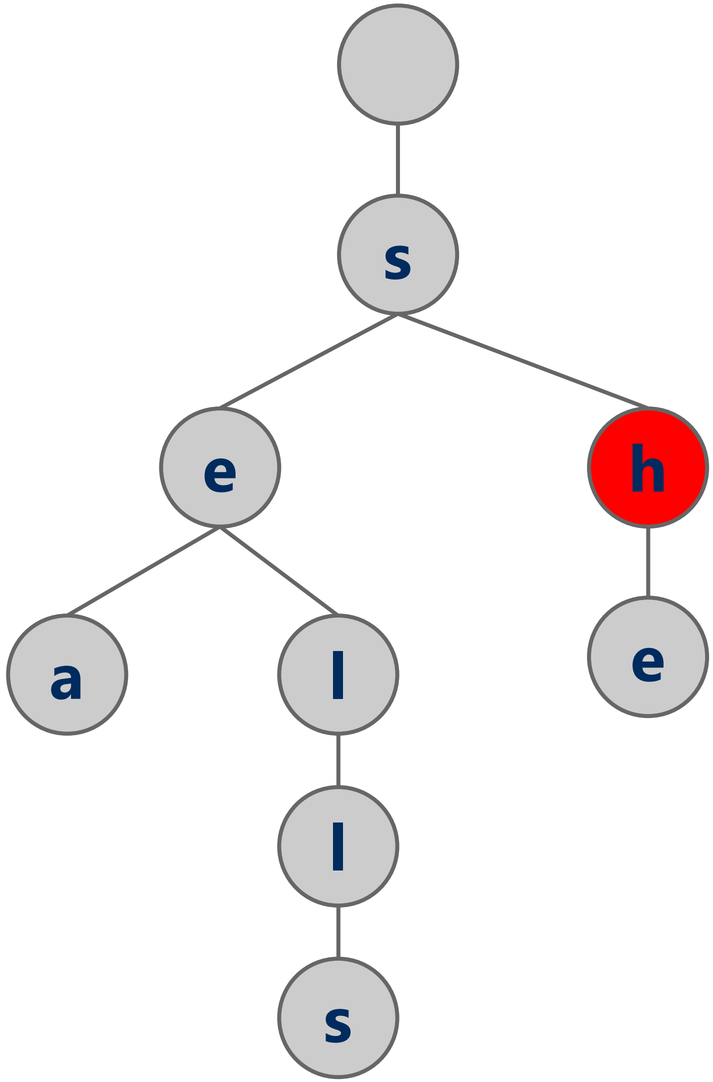


shells

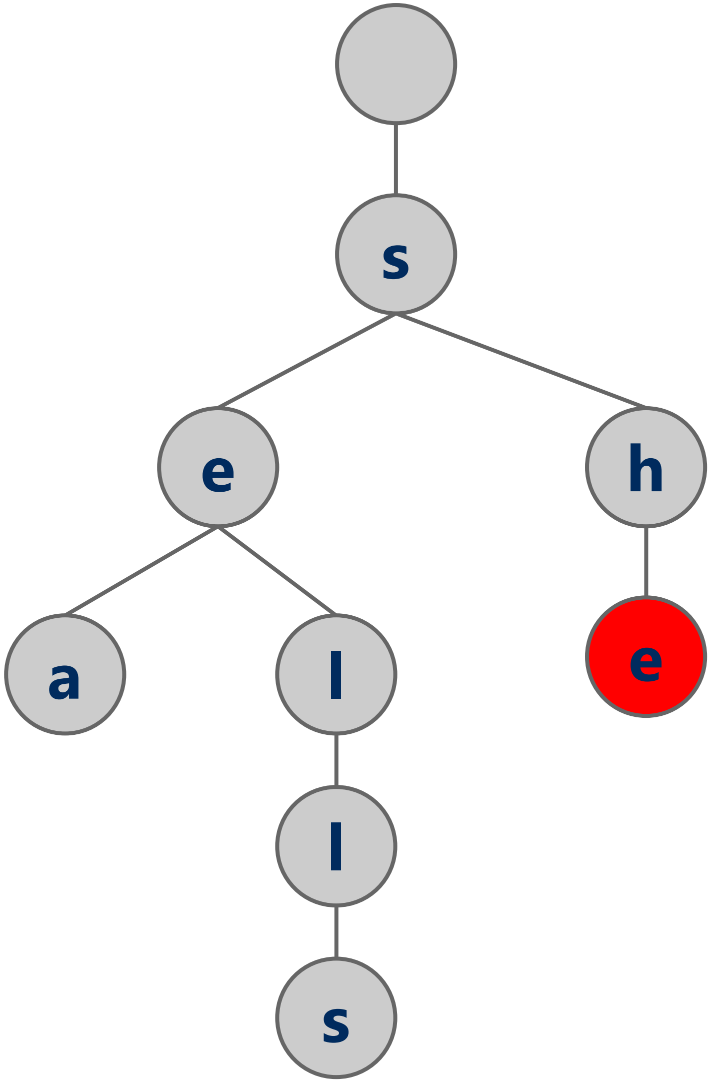
Another trie example

shells



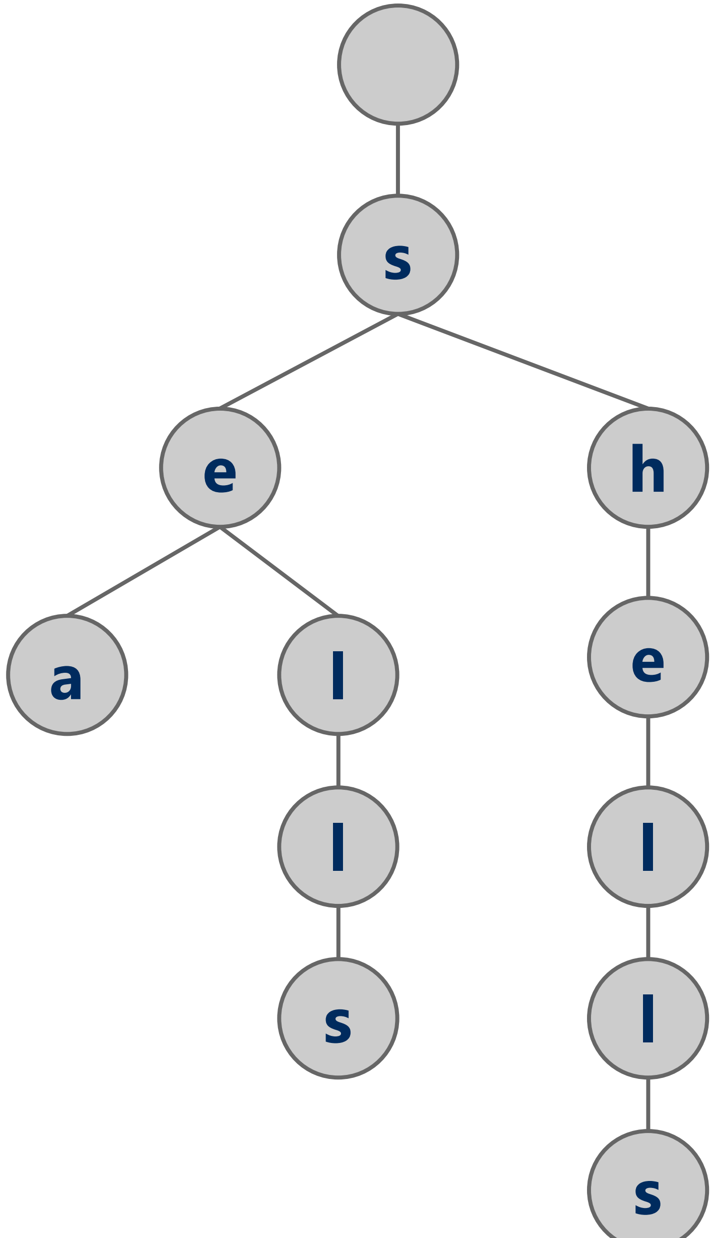
Another trie example

shells

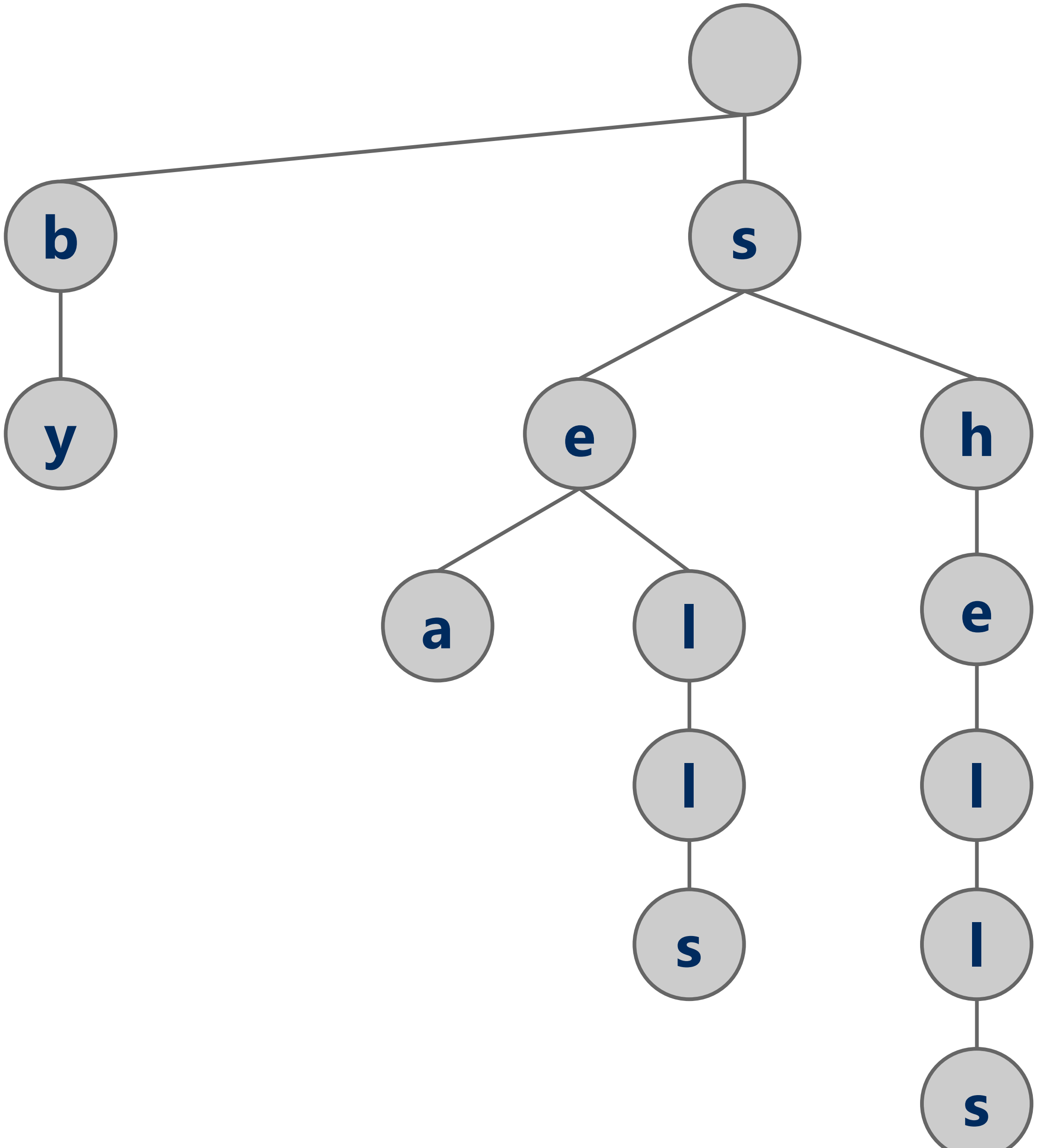


Another trie example

shells

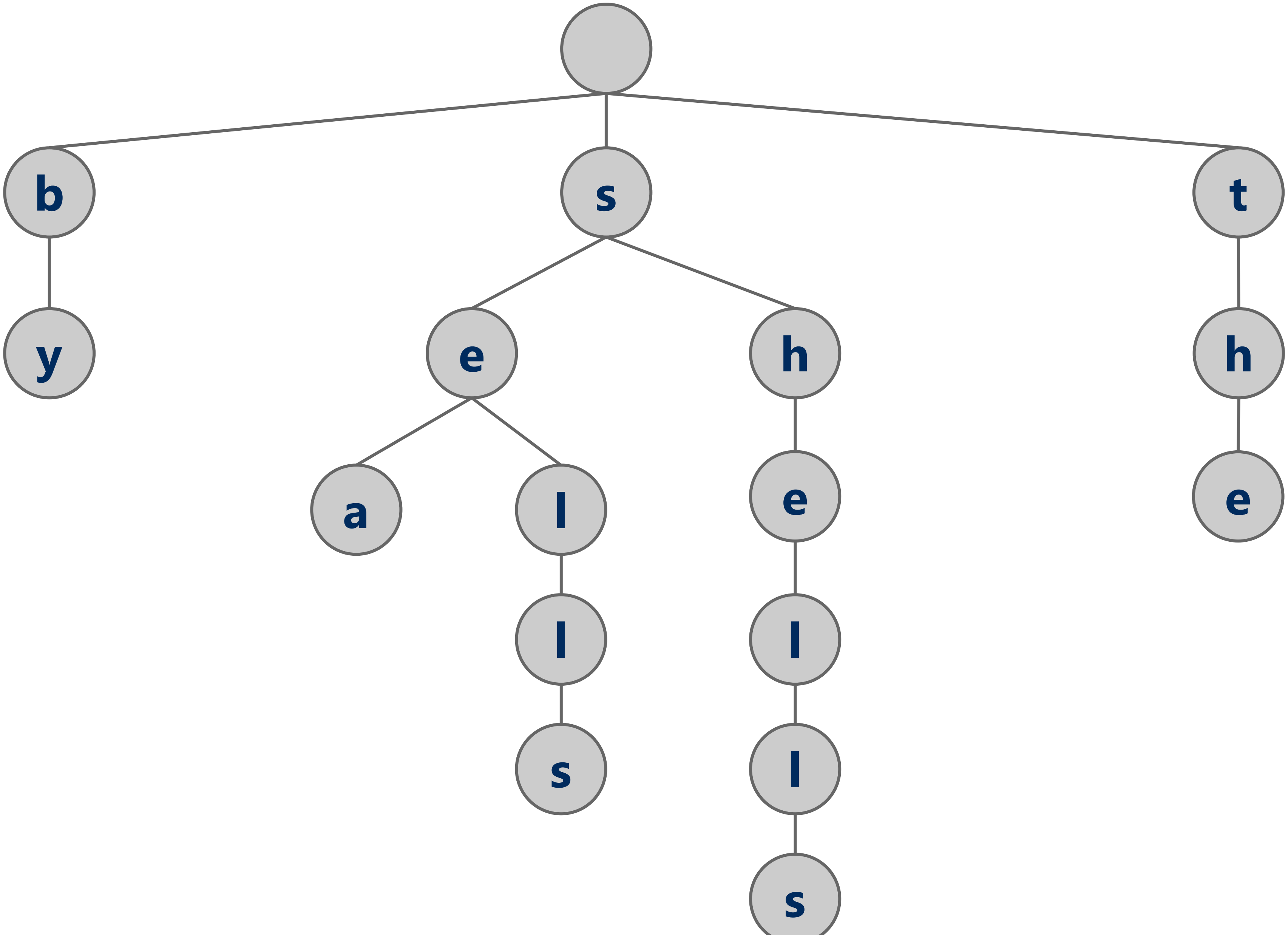


Another trie example



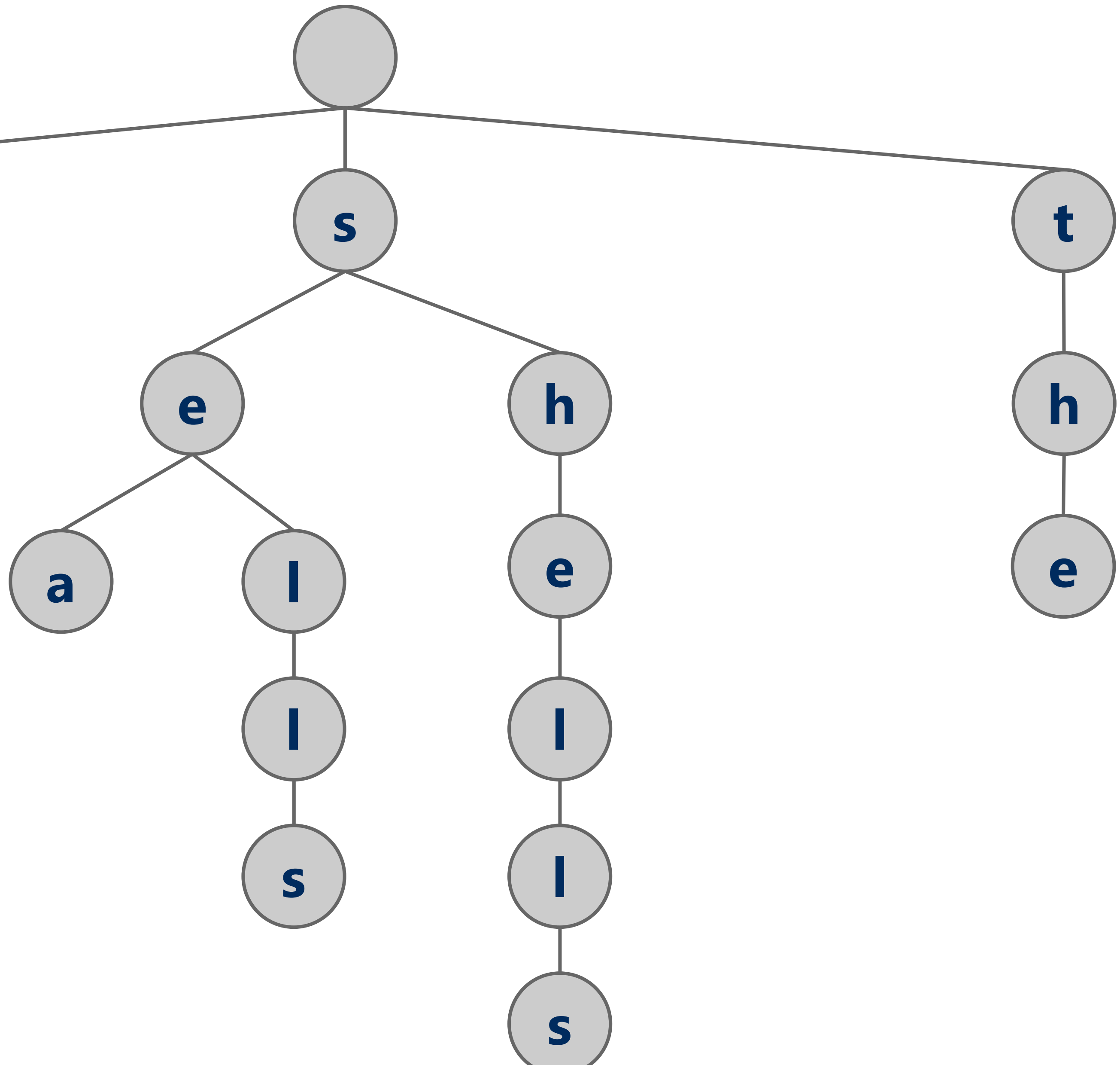
by

Another trie example



the

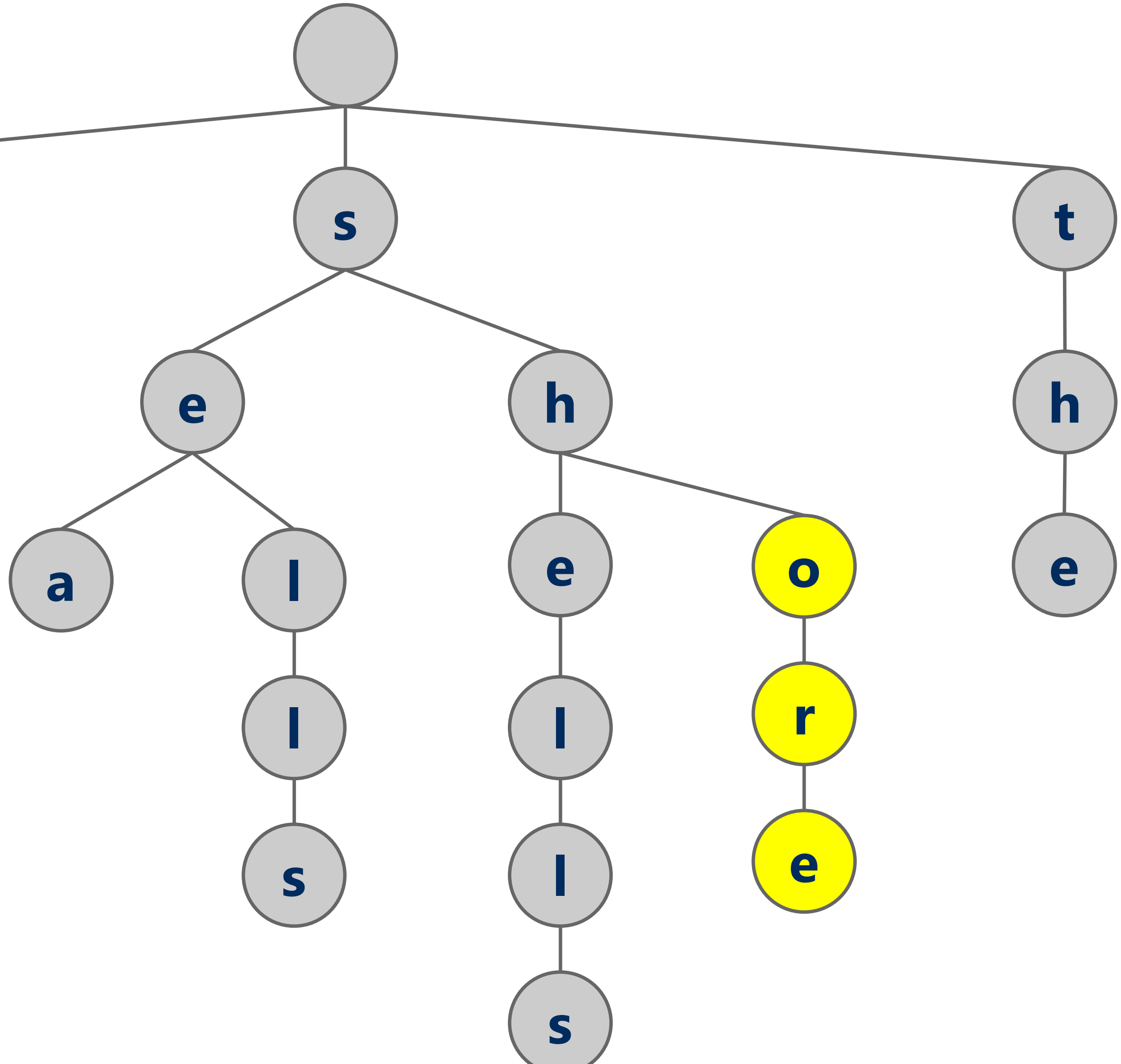
Another trie example



sea

Another trie example

shore



Analysis

- Runtime of add and *search hit*?
- O(w) where w is the character length of the string
  - So, what do we gain over RSTs?
    - $w < b$
    - e.g., assuming fixed-size encoding $w = \dfrac{b}{\lceil \log R \rceil}$
    - tree height is reduced

Search Miss

- Search Miss time for R-way RST
  - Require an average of $log_R(n)$ nodes to be examined
    - Proof in Proposition H of Section 5.2 of the text
- Average tree height with 2^{20} keys in an RST?
  - $\log_2 n = \log_2 2^{20} = 20$
- With 2^{20} keys in a large branching factor trie, assuming 8-bits at a time?
  - $\log_R n = \log_{256} 2^{20} = \log_{256} (2^8)^{2.5} = = \log_{256} 256^{2.5} = 2.5$

Implementation Concerns

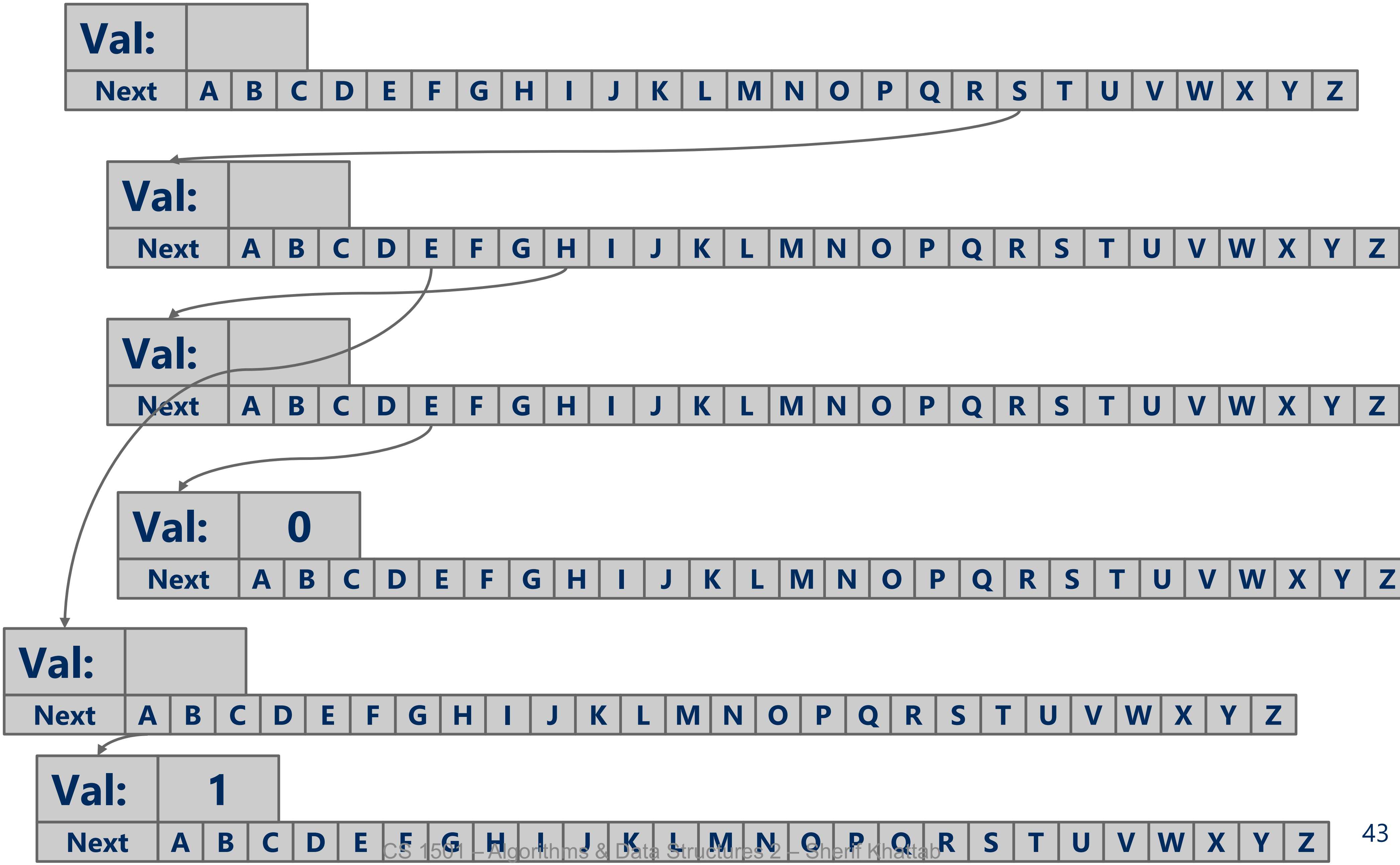
- See TrieSt.java
    - Implements an R-way trie
- Basic node object:

Where R is the branching factor

```
private class Node {
    private Object val;
    private Node[] next;
    private Node(){
      next = new Node[R];
    }
}
```

- Non-null **val** means we have traversed to a valid key
- Again, note that keys are not directly stored in the trie at all

R-way trie example

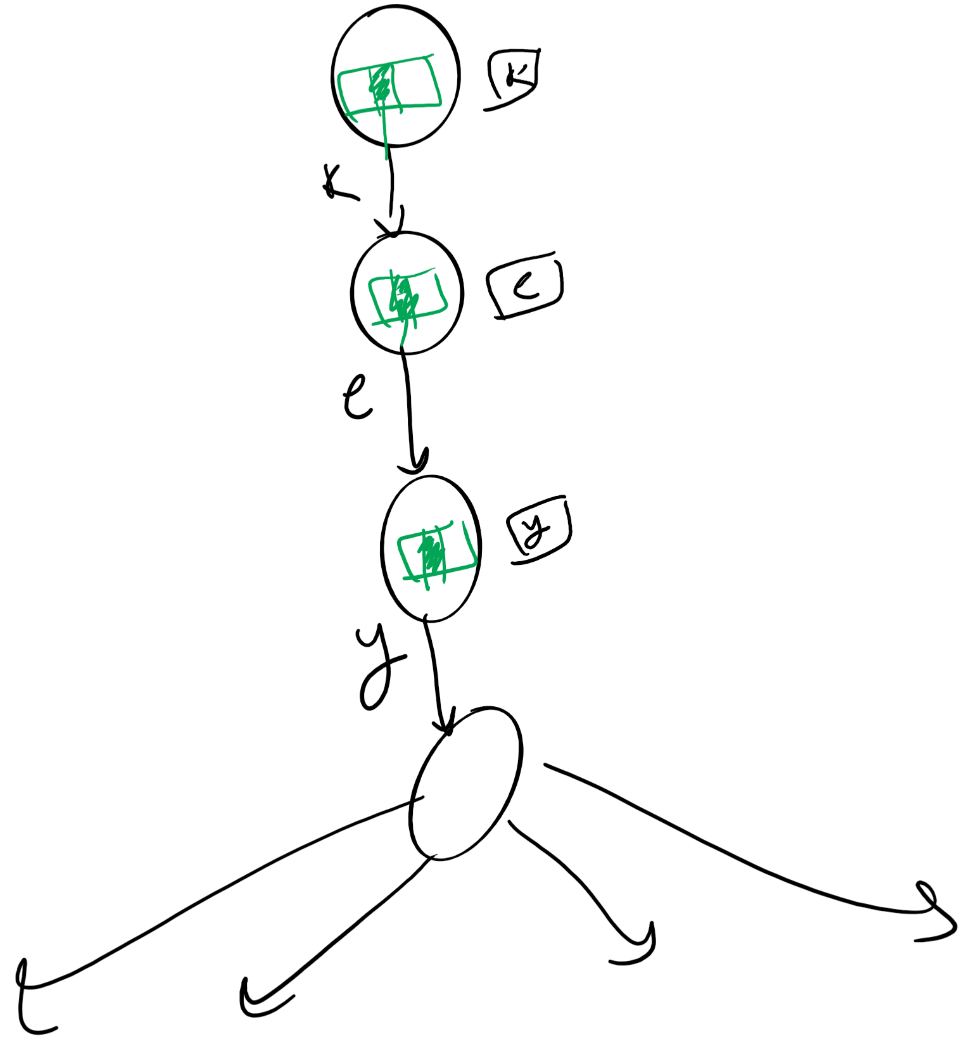


Summary of running time

	insert	Search hit	Search miss
binary RST	$\Theta(b)$	$\Theta(b)$	$\Theta(\log_2 n)$ on average
multi-way RST	$\Theta(w)$	$\Theta(w)$	$\Theta(\log_R n)$

R-way RST's nodes are large!

- Considering 8-bit ASCII, each node contains 2^8 references!
- This is especially problematic as in many cases, a lot of this space is wasted
  - Common paths or prefixes for example, e.g., if all keys begin with "key", thats 255*3 wasted references!
  - At the lower levels of the trie, most keys have probably been separated out and reference lists will be sparse



Solution: De La Briandais tries (DLBs)

Main idea: replace the array inside the node of the R-way trie with a linked-list

DLB Nodelets

Two alternative implementations:

```java
private class DLBNode {
    private Object val;
    private T character;
    private Node sibling;
    private Node child;
}
```

If search terminates on a node with non-null value, key is found; otherwise, not found.

```java
private class DLBNode {
    private Object val;
    private Character character;
    private Node sibling;
    private Node child;
}
```

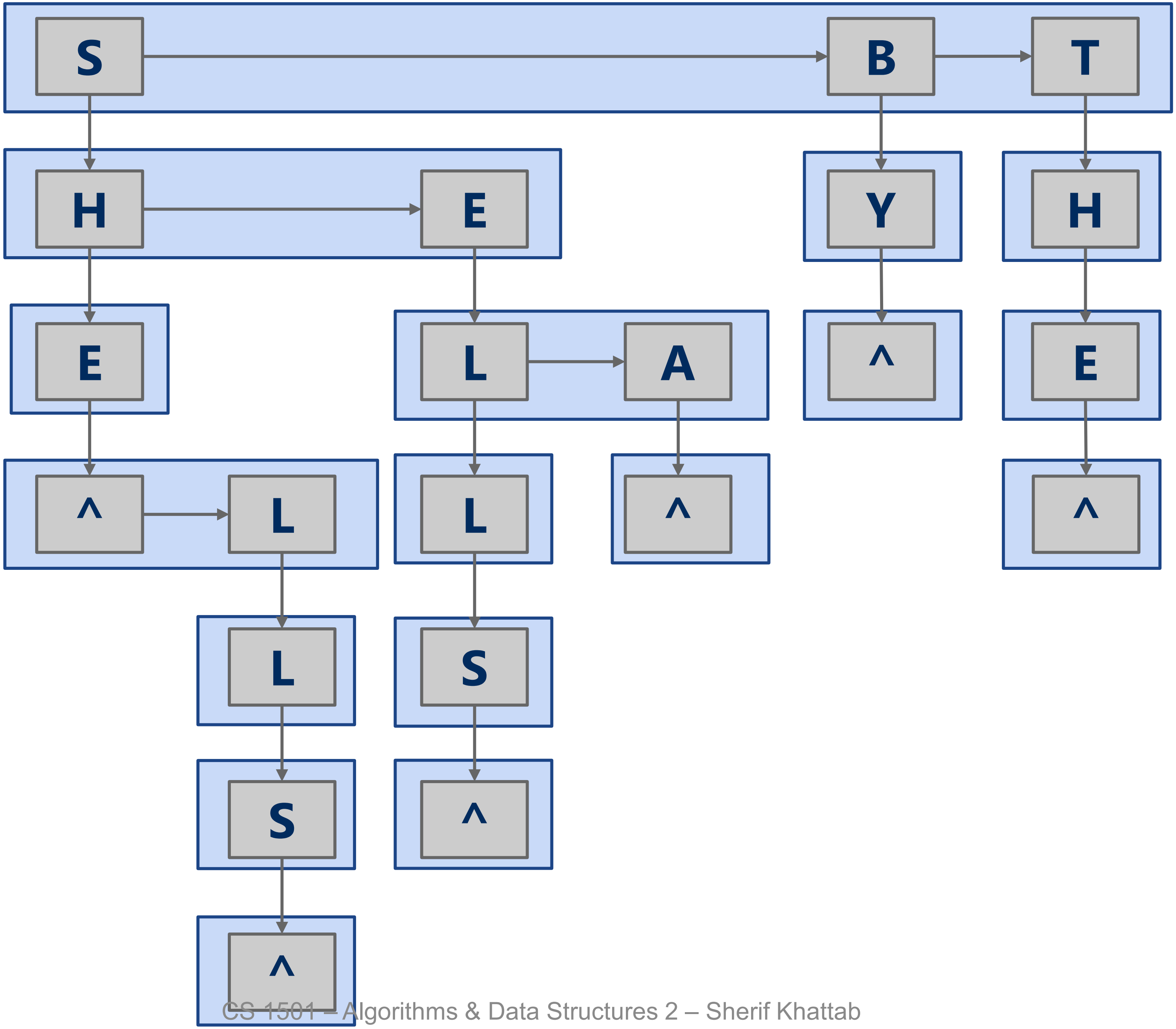
Add a sentinel character (e.g., ^) to each key before add and search

If search encounters null, key not found; otherwise, key is found

Adding to DLB Trie

- if root is null, set root ← new node
- current node ← root
- for each *character c* in the key
  - Search for *c* in the linked list headed at current using sibling links
    - if not found, create a new node and attach as a sibling to the linked list
  - move to child of the found node
    - either recursively or by current ← child
- if at last character of key, insert value into current node and return

DLB Example



DLB analysis

- How does DLB performance differ from R-way tries?
- Which should you use?

	search hit insert	
R-way RST	$\theta(w)$	
DLB	$\theta(wR)$	

Runtime Comparison for Search Trees/Tries

	Search hit	Search miss (average)	insert
BST	$\Theta(n)$	$\Theta(\log n)$	$\Theta(n)$
RB-BST	$\Theta(\log n)$	$\Theta(\log n)$	$\Theta(\log n)$
DST	$\Theta(b)$	$\Theta(\log n)$	$\Theta(b)$
RST	$\Theta(b)$	$\Theta(\log n)$	$\Theta(b)$
R-way RST	$\Theta(w)$	$\Theta(\log_R n)$	$\Theta(w)$
DLB	$\Theta(wR)$	$\Theta(\log_R n \cdot R)$	$\Theta(w \cdot R)$

Final notes on Search Tree/Tries

- We did not present an exhaustive look at search trees/tries, just the sampling that we're going to focus on
- Many variations on these techniques exist and perform quite well in different circumstances
  - Ternary search Tries
  - R-way tries without 1-way branching
- See the table at the end of Section 5.2 of the text