Algorithms and Data Structures 2
CS 1501

Spring 2023

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(Slides are adapted from Dr. Ramirez's and Dr. Farnan's CS1501 slides.)

Announcements

- Homework 1 is due this Friday
- Lab 1 posted on Canvas
  - Explained in recitations of this week
  - Distributed using Github Classroom
  - Due on Tuesday 1/31
- Assignment 1 will be posted this Friday

Previous lectures …

- A technique for modeling runtime of algorithms
  - $\sum_{all\ algorithm\ steps} Cost * frequency$
- Split the algorithm into blocks
  - steps in each block have the same frequency
  - $\sum_{all\ blocks} Cost * frequency$
- Asymptotic analysis
  - Focus on the order of growth of running time functions
  - Ignore lower order terms
  - Big O family of functions
    - and constant factors
  - Tilde Approximation
    - Keep the constant factor of the highest order term

Bonus Alert!

- Modify ThreeSum to work correctly with duplicate values in the input
  - The triples must be distinct in value
- Write a Theta(n^2) solution to the 3-sum problem

Send your solution using Piazza in a private message to all instructors labeled with the "bonus" tag

The Boggle Game

- Given a 4x4 board of letters, find all words with at least 3 adjacent letters
- Adjacent letters are horizontally, vertically, or diagonally neighboring
- Any cube in the board can only be used once per word
  - but can be used fo multiple words

How to move through the Boggle letters?

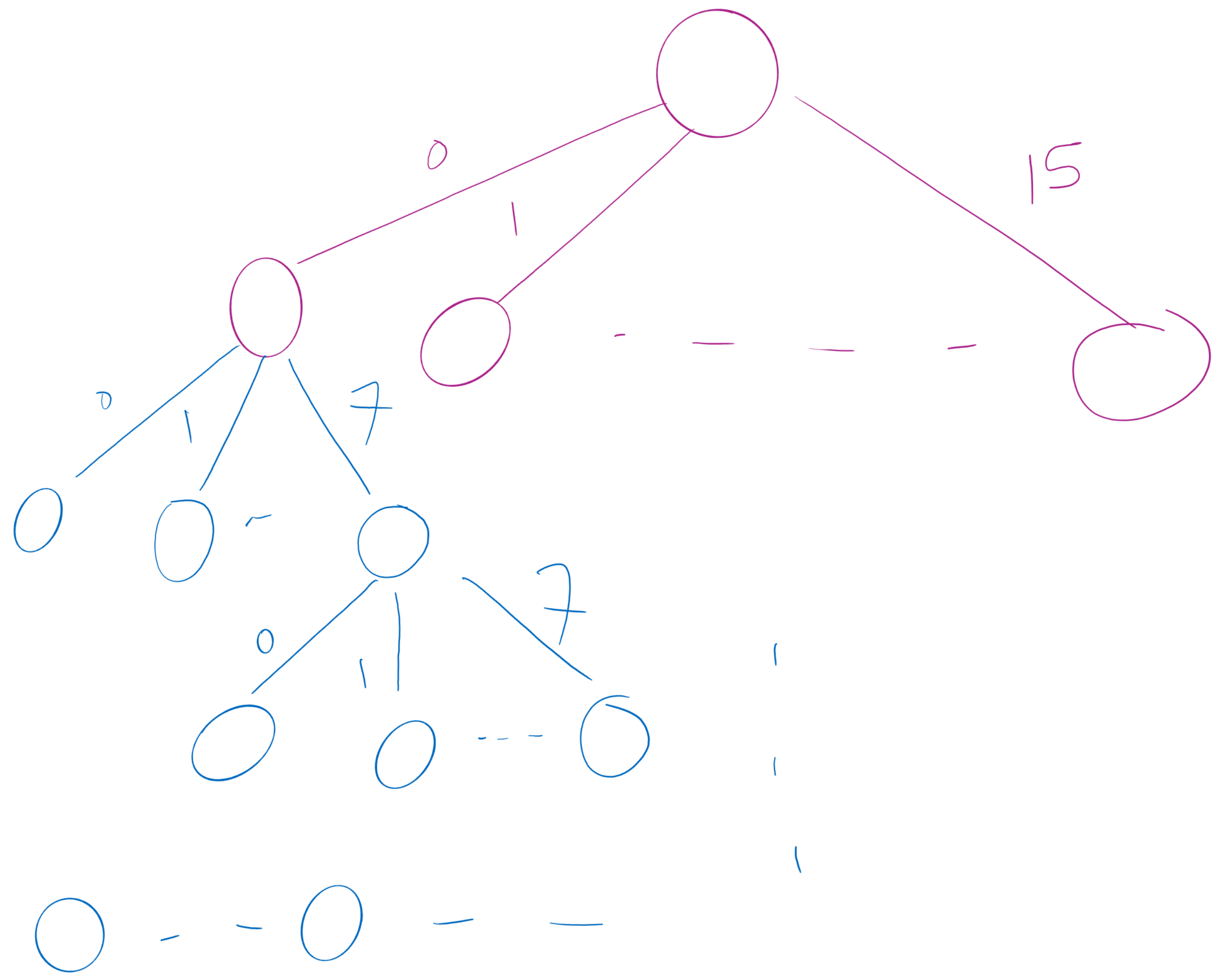
- Have 16 different options to start from
- Have 8 different options from each cube
  - From B[i][j]:
    - B[i-1][j-1]
    - B[i-1][j]
    - B[i-1][j+1]
    - B[i][j-1]
    - B[i][j+1]
    - B[i+1][j-1]
    - B[i+1][j]
    - B[i+1][j+1]

Boggle Game: Decisions and Options

- For the first decision (which cube to start from), how many possible options do we have?
  - 16
- Starting from a cube, the next decision to make is which adjacent cube to move to.
  - There are <u>**at most**</u> 8 possible options to choose from
- There is a decision to make at each cube that we go through
  - A maximum of 15 decisions
  - Some decisions may be trivial
    - the number of *valid* options < 2

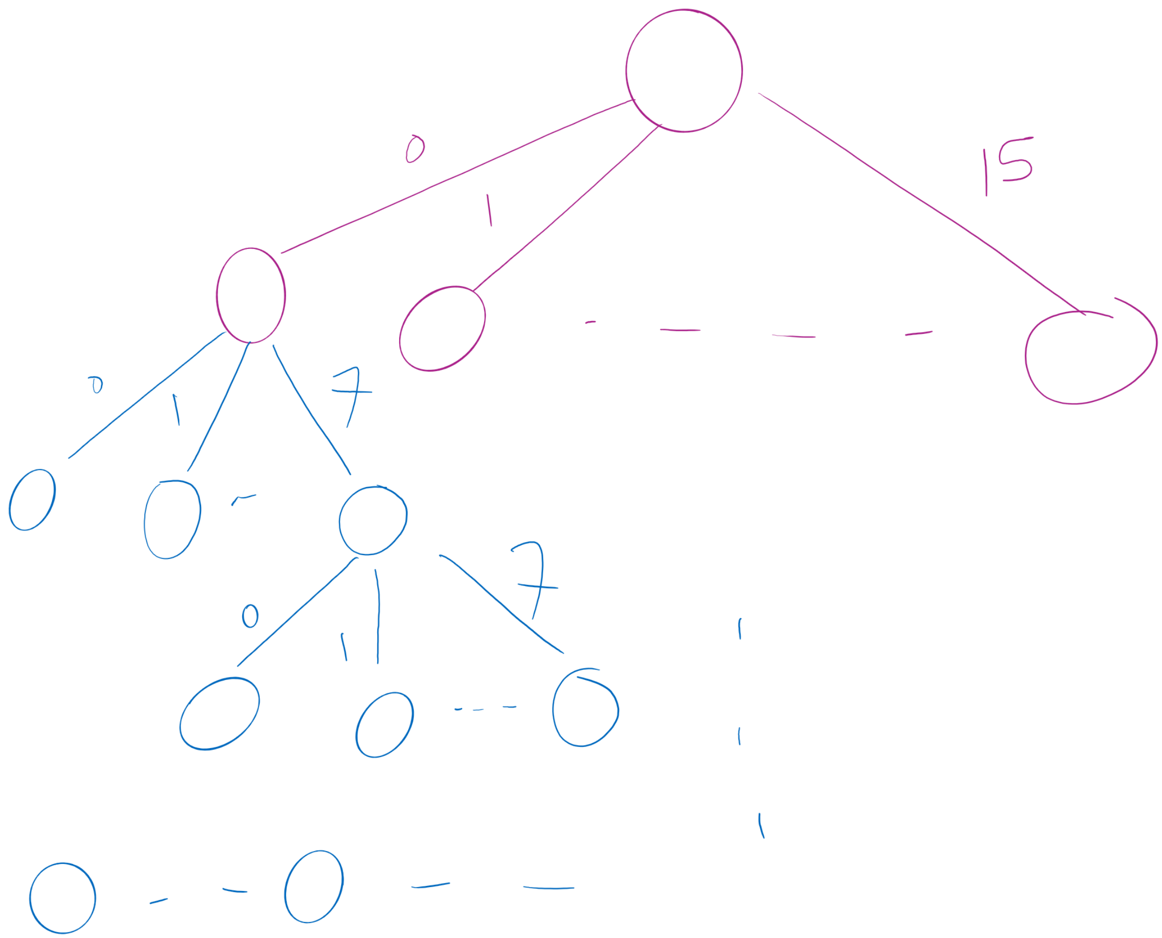
Search Space

Each node (circle) that has branches corresponds to a decision



Exhaustive Search Algorithm

- In an exhaustive search algorithm, we try all possible options for all possible decisions
  - Looking for one or all solutions



First decision

Are all 16 options *valid*?

Yes!

For next decisions, are all 8 options valid?

- No!
- Can’t go past the edge of the board
- Can’t reuse letter cubes
- Each letter added must lead to the prefix of an actual word
  - check the dictionary as we add each letter, if the currently constructed string does not start any word (i.e., not a <u>prefix</u>) don’t go further down that way
  - Practically, this can be used for huge savings
- This is called pruning!

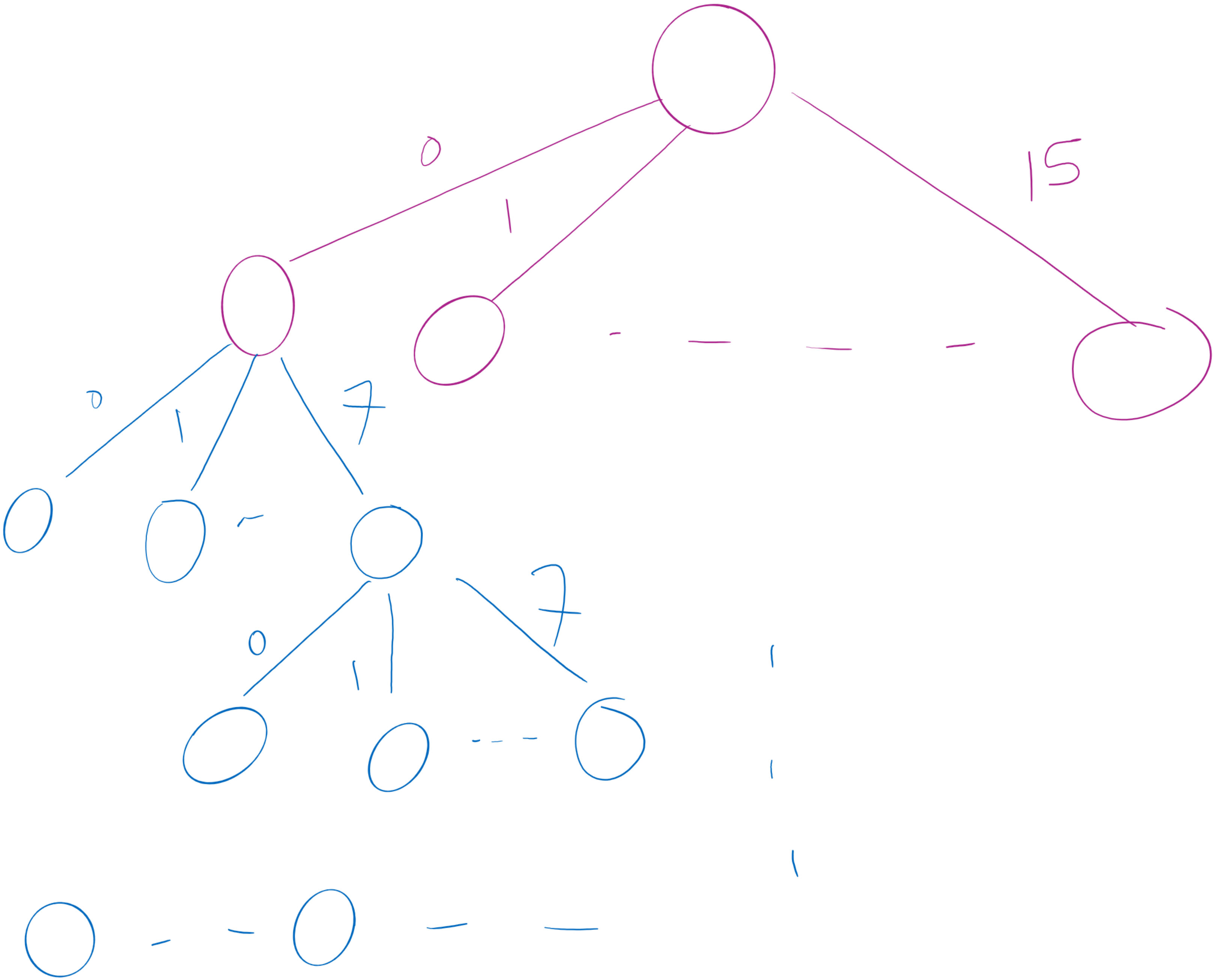
What happens when an options is valid?

- The same process pretty much repeats
- Check all 8 options and find a valid one to go down
- Do we need to go down just one valid option?
  - Or down all valid options?
- We need to somehow go back and try the other valid options
  - This is called backtracking

What happens when no valid options?

- We need to somehow go back to the previous decision and try other valid options if any
  - What if no more valid options for the previous decision?
  - Go back to the previous decision and so on …
  - This is again called backtracking

Backtracking on the Search Space



Then what?

How can we code up such exhaustive search with backtracking and pruning?

Backtracking Framework

```
void solve(current decision, partial solution) {
   for each option at the current decision {
      if option is valid {
         apply option to partial solution
         if partial solution a valid solution
            report partial solution as a final solution
         if more decisions possible
            solve(next decision, updated partial solution)
          undo changes to partial solution
       }
    }
}
```

Application to Boggle Game

void solve(current decision, ….

Which cube are we at:

row number (0..3) and column number (0..3)

Backtracking Framework

```
void solve(….., partial solution) {

    ….

}
```

The letters that we have seen so far

Backtracking Framework

```
void solve(….) {
  for each option at the current decision {
      …
    }
}
```

We have 8 options (except for the first decision, which has 16 options)

- From B[i][j]:
  - B[i-1][j-1]
  - B[i-1][j]
  - B[i-1][j+1]
  - B[i][j-1]
  - B[i][j+1]
  - B[i+1][j-1]
  - B[i+1][j]
  - B[i+1][j+1]

Backtracking Framework

```
void solve(current decision, partial solution) {
  for each option at the current decision {
    if option is valid {
      ...
    }
  }
}
```

Invalid options send us outside the board, reuse a cube, result in a non-prefix

Backtracking Framework

```
void solve(current decision, partial solution) {
   for each option at the current decision {
      if option is valid {
         apply option to partial solution
         …
      }
   }
}
```

Append the letter to the partial solution and mark the cube as used

Backtracking Framework

...

if partial solution a valid solution

report partial solution as a final solution

...

If the partial solution is a 3+ letter word in the dictionary

Backtracking Framework

...

if more decisions possible

...

A decision is possible if partial solution is a prefix of a word in the dictionary

Backtracking Framework

…

solve(next decision, updated partial solution)

…

a recursive call.

next decision is the next cube down the direction that we chose

Backtracking Framework

…

undo changes to partial solution

…

Remove the last letter that we appended from partial solution and mark cube as unused

Backtracking Framework

```
void solve(current decision, partial solution) {
  for each option at the current decision {
     if option is valid {
        apply option to partial solution
        if partial solution a valid solution
          report partial solution as a final solution
        if more decisions possible
          solve(next decision, updated partial solution)
         undo changes to partial solution
      }
  }
}
```

You will implement this algorithm in Lab 1 next week!

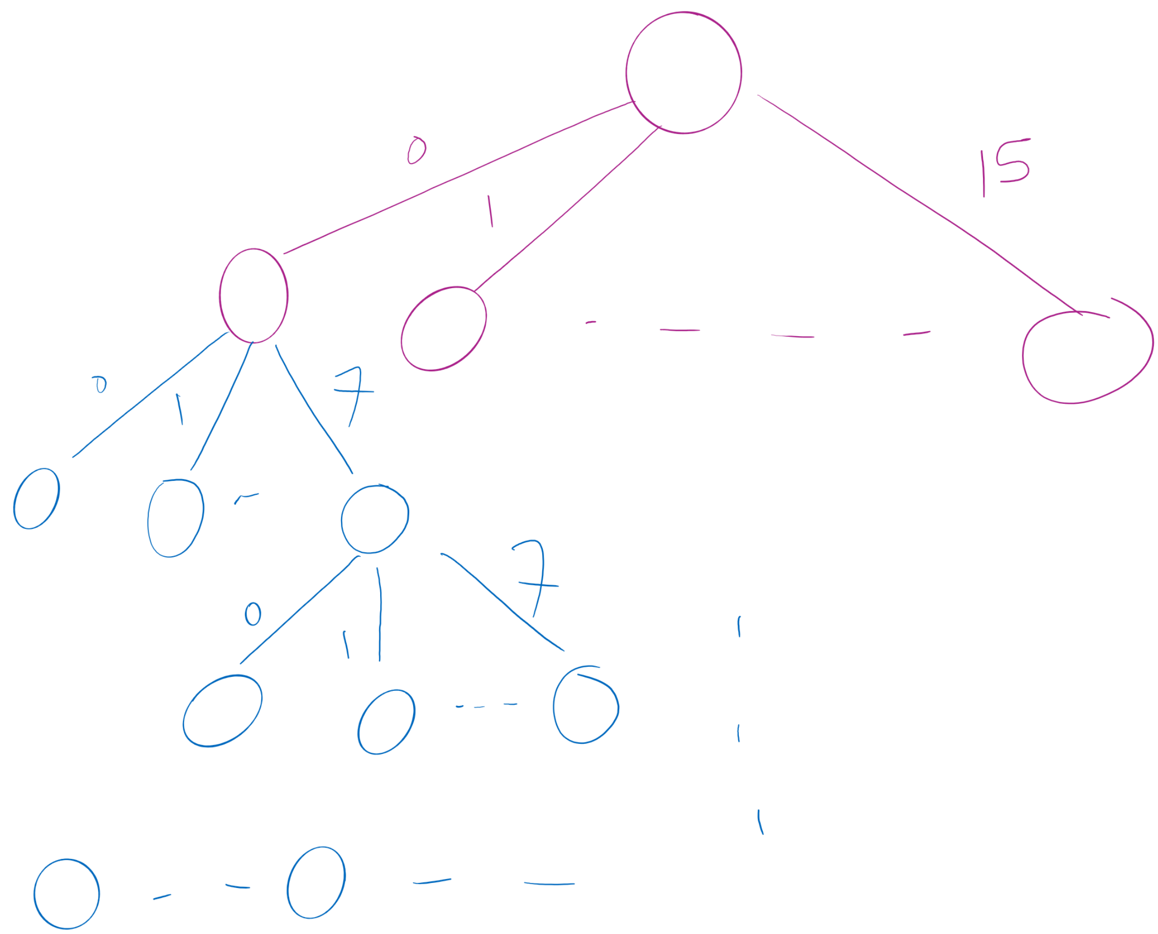
Backtracking Algorithm for Boggle

```
void solve(row and column of current cell, word string so far) {
  for each of the eight directions {
     if neighbor down the direction is a in the board and hasn't been used {
        append neighbor's letter to word string and mark neighbor
        as used
        if word string a word with 3+ letters
           add word string to set of solutions
        if word string is a prefix
           solve(row and column of neighbor, word string)
         delete last letter of word string and mark neighbor as unused
      }
    }
}
```

How many recursive calls are made by solve?

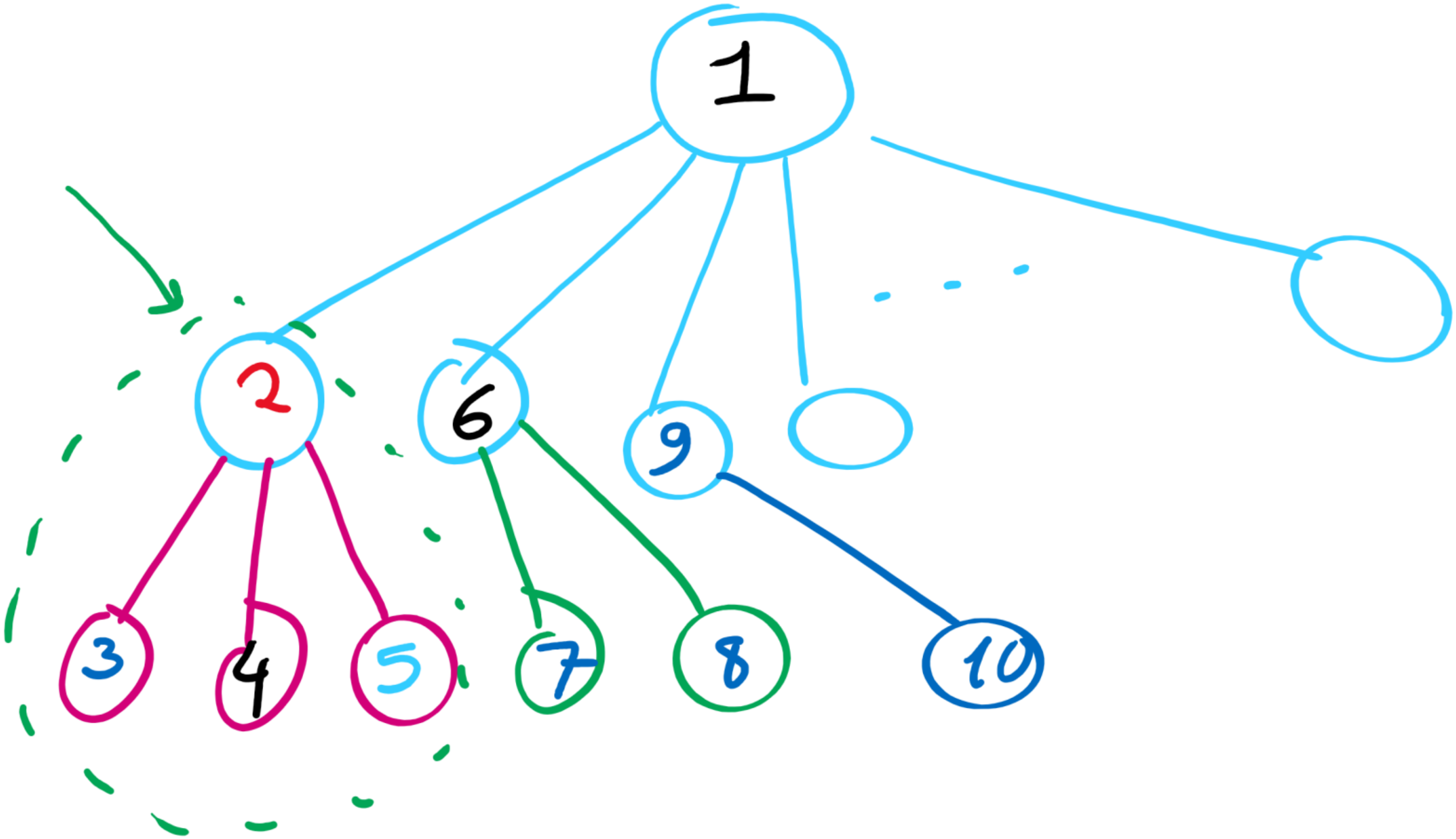
Search Space for Boggle

- The search space can be modeled as a *tree*
- Each circle (aka node) represents one call to solve
  - except the root node (why?)



Search Tree Traversal Order in Backtracking

- Each node (circle) corresponds to a recursive call
- The runtime cost per node is the cost of all statements except the recursive call



Moving down the tree

```
void solve(current decision, partial solution) {
  for each option at the current decision {
    if option is valid {
      apply option to partial solution
      if partial solution a valid solution
        report partial solution as a final solution
      if more decisions possible
        solve(next decision, updated partial solution)
       undo changes to partial solution
     }
   }
}
```

Backtracking (moving up the tree)

```
void solve(current decision, partial solution) {
  for each option at the current decision {
    if option is valid {
      apply option to partial solution
      if partial solution a valid solution
        report partial solution as a final solution
      if more decisions possible
        solve(next decision, updated partial solution)
      undo changes to partial solution
    }
  }
  return
}
```

What is the running time?

- Can't easily use the frequency and cost technique because of the recursive call(s)
- In the worst case, the backtracking algorithm must visit each node in the search space tree
  - Why?
- At each node, the *non-recursive* part of solve executes
- The worst-case runtime =

  number of nodes *

  work per node (non-recursive part of solve)

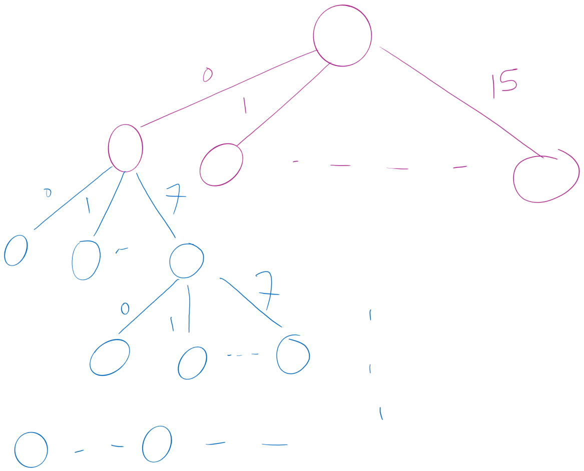
Non-recursive part of solve

Everything but the recursive calls

```
void solve(current decision, partial solution) {
   for each option at the current decision {
       if option is valid {
          apply option to partial solution
          if partial solution a valid solution
             report partial solution as a final solution
          if more decisions possible
             solve(next decision, updated partial solution)
           undo changes to partial solution
        }
     }
}
```

Search Space Size

- How many nodes are there?
- **Maximum number of nodes** = 1 + 16 * (??)
- = 1 + 16 * (1 + 8 + 8^2 + 8^3 + … + 8^{15})
- = 1 + 16 * $\theta(largest\ term)$
- = 1 + 16 * $\theta\left(8^{15}\right)$
- In terms of board size (*n*)
  - **Maximum number of nodes =** 1 + n * $\theta(8^{n-1})$
  - = $\theta(n * \ 8^{n-1})$ = $\theta(n * \ 8^{n})$
- Exponential!



Running time

- Worst-case runtime of Backtracking for Boggle = $\Omega(n*8^n)$, where *n* is the board size (# cells)
  - if the non-recursive work is constant, that is *O(1)*, then
    - worst-case runtime = $O(n*8^n)$
  - We will see we make it constant
- Worst-case runtime of backtracking algorithm is exponential!
  - Pruning has practical savings in runtime, but doesn't significantly reduce the runtime
    - Still exponential in the worst case

Non-recursive Work

```
void solve(row and column of current cell, word string so far) {
  for each of the eight directions {
     if neighbor down the direction is a in the board and hasn't been used {
        append neighbor's letter to word string and mark neighbor used
        if word string a word with 3+ letters
           add word string to set of solutions
        if word string is a prefix
           solve(row and column of neighbor, word string)
         delete last letter of word string and mark neighbor as unused
     }
  }
}
```

How to make the non-recursive work constant?

- How can we make the dictionary lookup (for prefixes and full words) *O(1)?*
  - Hash table? runtime?
  - Later in the course, we will see an efficient way to perform this task using a *tree*
- How about the time to append and delete letters from the word string?

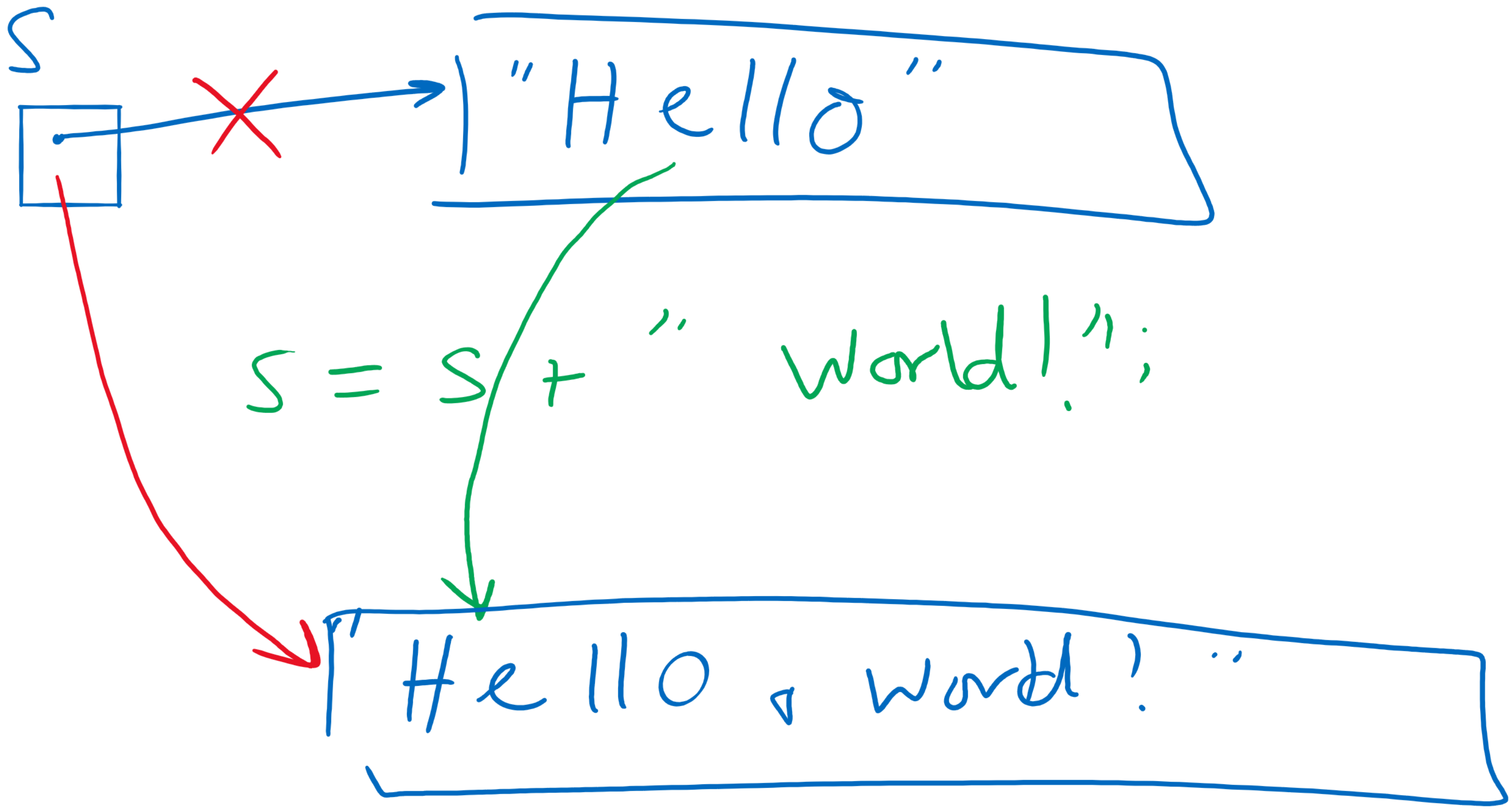
How to make the non-recursive work constant?

- Constructing the words over the course of recursion will mean building up and tearing down strings
  - Moving down the tree adds a new character to the current word string
  - Backtracking removes the most recent character
  - Basically pushing/popping to/from a string stack
- Push/Pop stack operations are generally $\Theta(1)$
  - Unless you need to resize, but that cost can be **amortized**

How to make the non-recursive work constant?

- What if we use String to hold the current word string?
- Java Strings are *immutable*
  - s = new String("Here is a basic string");
  - s = s + " this operation allocates and initializes all over again";
  - Becomes essentially a $\Theta(n)$ operation
    - Where n is the length() of the string

Concatenating to String Objects



StringBuilder to the rescue

- For StringBuilder objects
  - append() and deleteCharAt() can be used to push and pop
  - Back to $\Theta(1)$!
    - Still need to account for resizing, though…
- StringBuffer can also be used for this purpose
  - Differences?