University of Pittsburgh

Algorithms and Data Structures 2
CS 1501

Spring 2023

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(Slides are adapted from Dr. Ramirez's and Dr. Farnan's CS1501 slides)

Contact Info

- **Course website:** http://www.cs.pitt.edu/~skhattab/cs1501/
- **Instructor:** Sherif Khattab ksm73@pitt.edu
- **My Student Support Hours:** https://khattab.youcanbook.me
  - MW: 10:00-11:00; Th: 9:00-10:00 and 11:00-12:00; F by appointment
  - 6307 Sennott Square, Virtual Office: https://pitt.zoom.us/my/khattab
  - Please schedule at: https://khattab.youcanbook.me/
- **Communication**

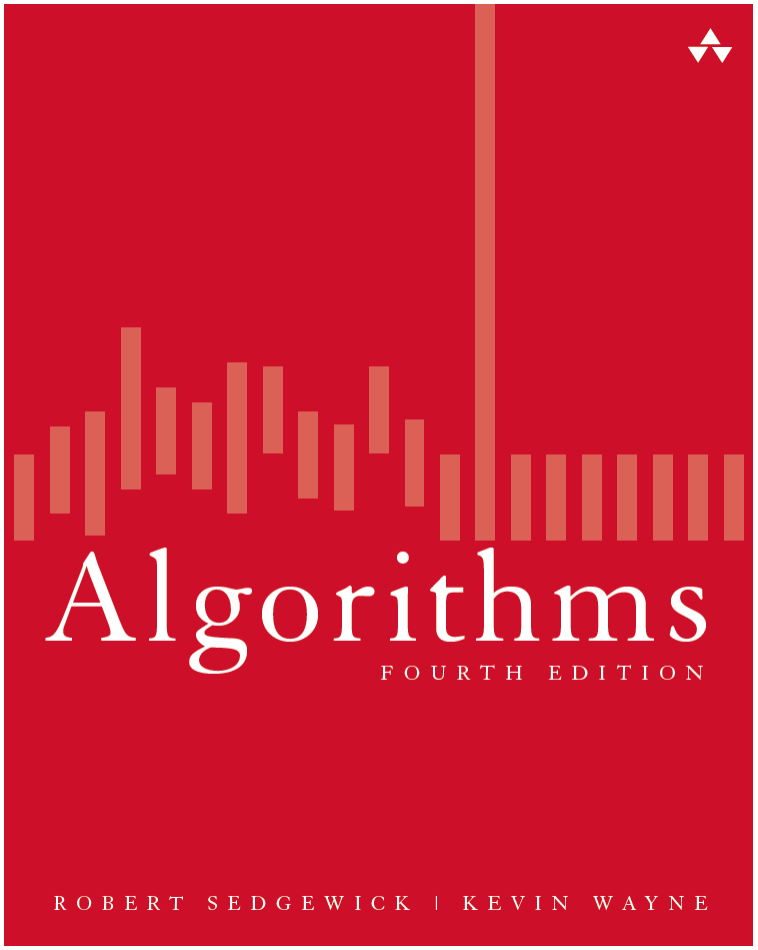
  Piazza (**Please expect a response within 72 hours)**

  Email not recommended!

Recitation Info

- **Teaching Team:**
  - Connor Sweeney, cps43@pitt.edu
  - Winston Osei-Bunso, wio6@pitt.edu
  - Thomas Brusilovsky, tpb22@pitt.edu
  - Taha Ayyaz Ahmad, taa95@pitt.edu
  - One more TA to be added
- **No recitations this week, but you got some work to do!**
  - Check Lab 0 on Canvas

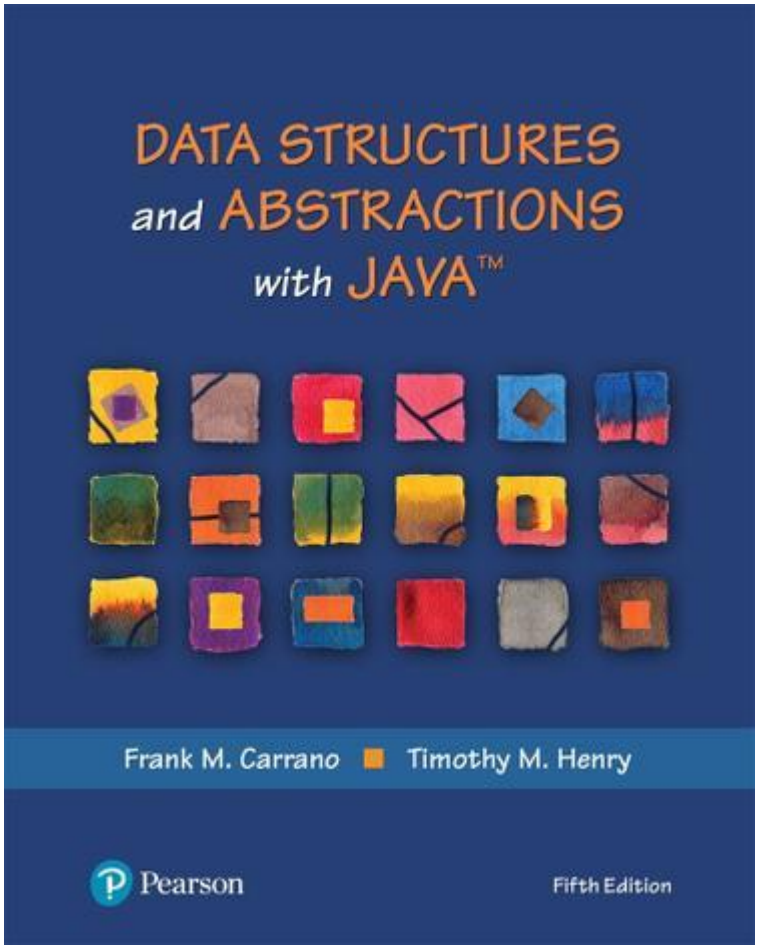
Textbooks



Algorithms (4th Edition)

Robert Sedgewick and Kevin Wayne

Online Resources: https://algs4.cs.princeton.edu/



Data Structures and Abstractions with Java (5th Edition)

Frank M. Carrano and Timothy M. Henry

Grades

- 40% on best four out of five programming assignments; mostly autograded
  - posted on Canvas, distributed using Github, and submitted on **Gradescope** from Github
- 30% on exams: 18% on higher grade and 12% on lower
- 10% on homework assignments on Gradescope
- 10% on lab exercises; mostly autograded
- 10% on in-class Top Hat questions

Canvas Walkthrough

- Lectures posted on Tophat
  - Draft PDF slides available on Github before class
- Lecture and recitation recordings
  - under Panopto Video
- RedShelf Inclusive Access for the Sedgewick Textbook
  - You can cancel and get a refund before Add/Drop
- Piazza for discussion and communication
- Gradescope and autograding policies
- Academic Integrity
- NameCoach

Expectations

- Your continuous feedback is important!
  - Anonymous Qualtrics survey
  - Midterm and Final OMET
- Your engagement is valued and expected with
  - classmates
  - teaching team
  - material

Lecture structure (mostly)

Time	Description
~5 min before and after class	Informal chat
~25 min	Announcements, review of muddiest points on previous lecture, and QA on assignments/labs/homework problems
~45 min	Lecturing with Tophat questions and/or activities
~5 minutes	QA and muddiest points/reflections

Why is this class (notoriously) hard?

- Lots of concepts
  - Attend lectures and recitations (if you absolutely cannot attend, watch the video recordings)
  - Study often!
  - Put effort into the weekly homework assignments
- Programming Assignments are relatively hard
  - Refresh your Java programming (CS 0445) and **debugging skills**
  - Start early and show up to student support hours!

Goals of the course

- To convert non-trivial *algorithms* and data structures into *programs*
  - Various issues will always pop-up during *implementation*
    - Such as?...
- To analyze algorithms and how they affect the run-times of the associated programs
  - Different solutions can be compared using many metrics

Announcements

- Lab 0 is due this Friday (not graded)
- Recitations start next week
- Homework 1 will be assigned this Friday
- JDB Example and VS Code Debugging Video will be available on Canvas
- Draft slides and handouts available on Canvas

Today's Agenda

- A technique for modeling runtime of algorithms
  - $\sum_{all\ statements} Cost * frequency$
- Determining the order of growth of a function
  - Ignoring lower-order terms and multiplicative constants
  - The Big O family

Let's consider the ThreeSum problem

- 3-sum Problem
  - Given a set of arbitrary integers find out how many **distinct** triples sum to exactly zero
  - do you have questions on the problem specification?
- Example input:
  - 5, -1, 2, -3, -2, 1, 0
  - what should the output be?

Brute-force solution

Enumerate all possible distinct triples and check their sums

```
cnt = 0

for each distinct triple

    if sum of triple equals zero

        increment cnt
```

- How would you enumerate all distinct triples?
- What if all the input integers are unique?

Brute-force solution

```
public static int count(int[] a) {
    int n = a.length;
    int cnt = 0;
    for (int i = 0; i < n; i++) {
        for (int j = i+1; j < n; j++) {
            for (int k = j+1; k < n; k++) {
                if (a[i] + a[j] + a[k] == 0) {
                    cnt++;
                }
            }
        }
    }
    return cnt;
}
```

- Why is it correct to start the j loop from i+1?
  - Would we miss a triple if we do so?
  - Is it correct to start the j loop from 0?

ThreeSum: brute-force, 3-loop solution

```
public static int count(int[] a) {
    int n = a.length;
    int cnt = 0;
    for (int i = 0; i < n; i++) {
        for (int j = i+1; j < n; j++) {
            for (int k = j+1; k < n; k++) {
                if (a[i] + a[j] + a[k] == 0) {
                    cnt++;
                }
            }
        }
    }
    return cnt;
}
```

Would that solution be correct if the input integers are not distinct?

Mathematically modelling runtime

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  -
  -

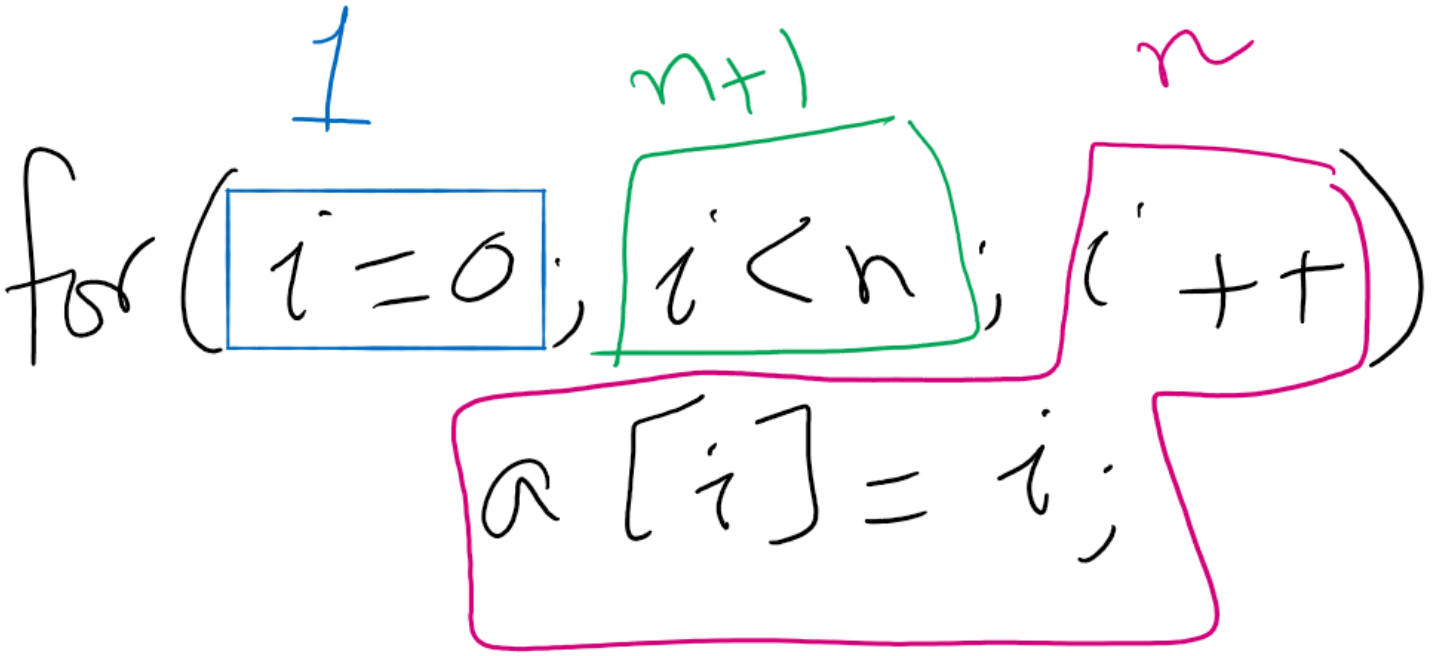
What is the runtime?

A technique for modeling runtime of algorithms

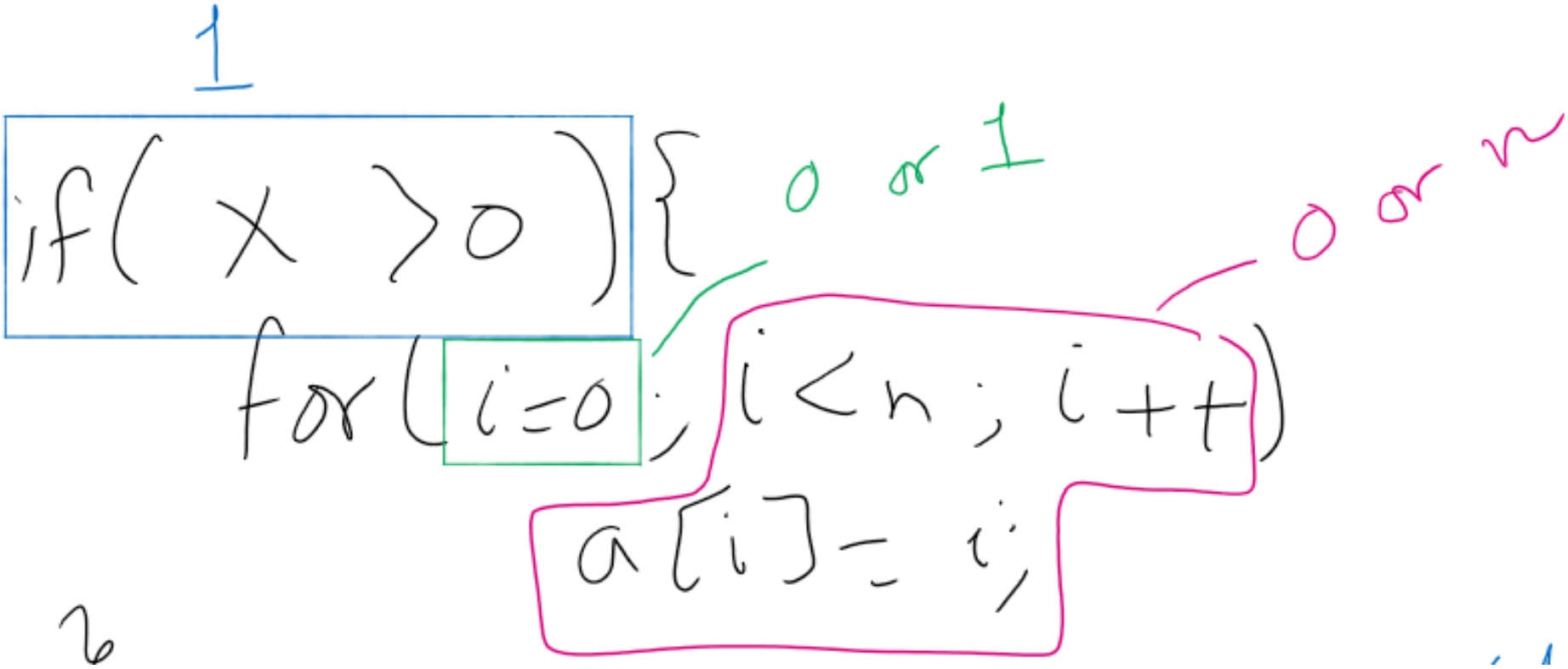
- $\sum_{all\ statements} Cost * frequency$

- Split the algorithm into blocks such that
  - the code statements in each block have the same frequency
- $\sum_{all\ blocks} Cost * frequency$

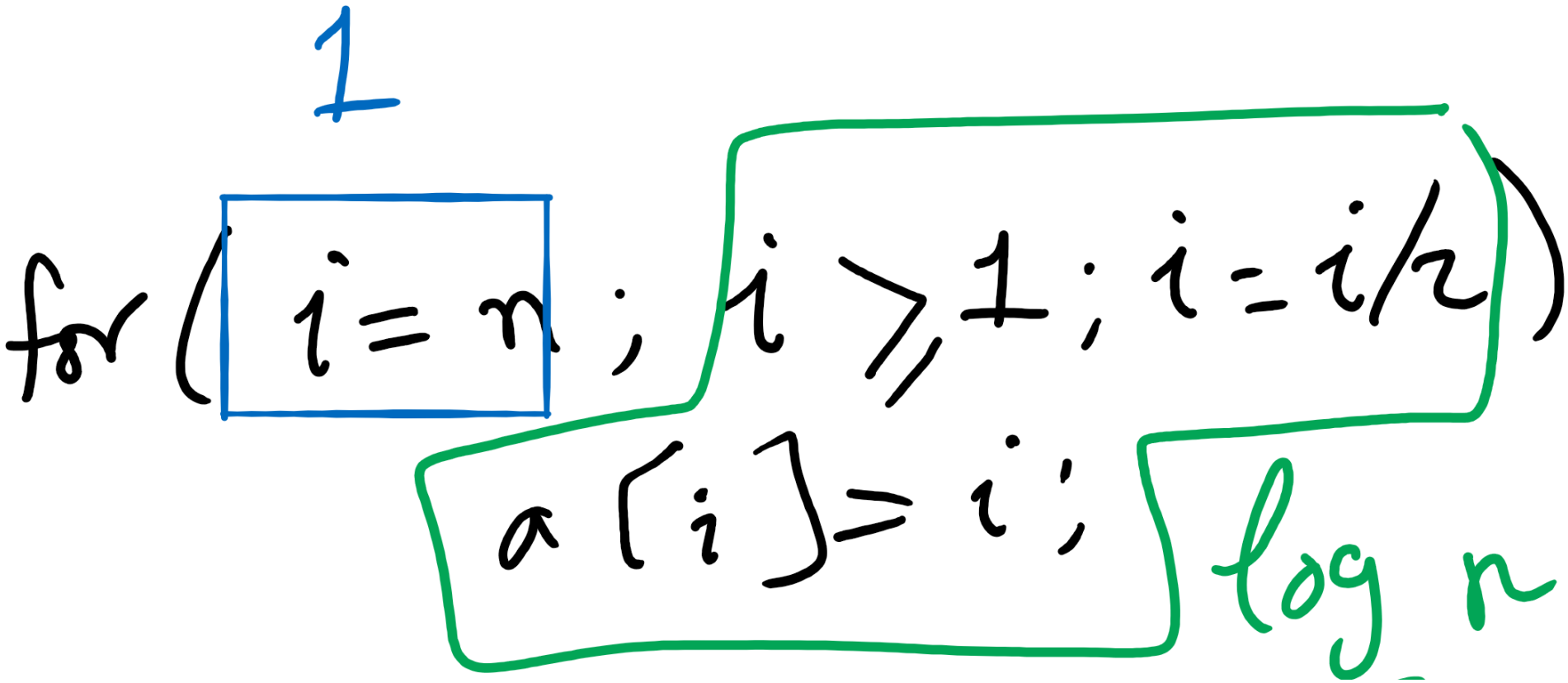
Algorithm Analysis Example 1



Algorithm Analysis Example 2



Algorithm Analysis Example 3



What is the runtime of ThreeSum?

```
public static int count(int[] a) {
   int n = a.length;
   int cnt = 0;
   for (int i = 0; i < n; i++) {
      for (int j = i+1; j < n; j++) {
         for (int k = j+1; k < n; k++) {
            if (a[i] + a[j] + a[k] == 0) {
               cnt++;
            }
         }
      }
   }
}
```

frequency: $f_0 = 1$
cost: t_0

freq: $f_1 = n$
cost: t_1

$f_4 = x$ (the number of triples that sum to 0 in the input array)
$0 \leq x \leq C(n, 3)$
cost: t_4

cost: t_3

$$\frac{(n-3)!6}{} \qquad \frac{6}{} \qquad \frac{6}{} \qquad \frac{2}{} + \frac{n}{3}$$

What is the runtime of ThreeSum?

frequency: $f_0 = 1$
cost: t_0

freq: $f_1 = n$
cost: t_1

$f_2 = (n-1) + (n-2) + (n-3) + \ldots + 1$
$= \frac{n-1}{2}(n - 1 + 1) = \frac{n^2}{2} - \frac{n}{2}$
cost = t_2

$f_4 = x$ (the number of triples that sum to 0 in the input array)
$0 \leq x \leq C(n, 3)$
cost: t_4

$f_3 = C(n, 3) = n_{C_3} = \frac{n!}{(n-3)!3!}$
$= \frac{n(n-1)(n-2)(n-3)!}{(n-3)!6} = \frac{n(n-1)(n-2)}{6} = \frac{n^3}{6} - \frac{n^2}{2} + \frac{n}{3}$
cost: t_3

Grand total = $\sum_{i=0}^{4} f_i * t_i$

$$= \frac{t_3}{6}n^3 + (\frac{t_2}{2} - \frac{t_3}{2})n^2 + (\frac{t_3}{3} - \frac{t_2}{2} + t_1)n + t_0 + t_4 x$$

What is the runtime of ThreeSum?

$$\frac{t_3}{6}n^3 + (\frac{t_2}{2} - \frac{t_3}{2})n^2 + (\frac{t_3}{3} - \frac{t_2}{2} + t_1)n + t_0 + t_4x$$

- Remember that $0 \leq x \leq C(n, 3)$
- If x = 0 → best-case runtime

$$\frac{t_3}{6}n^3 + (\frac{t_2}{2} - \frac{t_3}{2})n^2 + (\frac{t_3}{3} - \frac{t_2}{2} + t_1)n + t_0$$

- If x = $C(n, 3)$ → worst-case runtime

$$\frac{t_3}{6}n^3 + (\frac{t_2}{2} - \frac{t_3}{2})n^2 + (\frac{t_3}{3} - \frac{t_2}{2} + t_1)n + t_0 + t_4(\frac{n^3}{6} - \frac{n^2}{2} + \frac{n}{3})$$