University of Pittsburgh

Algorithms and Data Structures 2
CS 1501

Pitt SCI

Spring 2023

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(Slides are adapted from Dr. Ramirez's and Dr. Farnan's CS1501 slides.)

Announcements

- Upcoming Deadlines
  - Homework 4: this Friday @ 11:59 pm
  - Lab 3: Tuesday 2/14 @ 11:59 pm
  - Assignment 1: Friday 2/17 @ 11:59 pm
- Please make your Piazza posts public as much as possible

Previous lecture

- Red-Black BST (self-balancing BST)
  - add
  - delete
  - runtime of operations
- Turning recursive tree traversals to iterative

This Lecture

- Binary Search Tree uses comparisons between keys to guide the searching
- What if we use the digital representation of keys for searching instead?
  - Keys are represented as a sequence of digits (e.g., bits) or alphabetic characters
- Digital Searching Problem

Digital Searching Problem

- Input:
  - a (large) dynamic set of data items in the form of
    - *n* (key, value) pairs; key is a string from an alphabet of size *R*
    - Each key has *b* bits or *w* characters (the chars are from the alphabet)
    - What is the relationship between *b* and *w*?
  - a *target key (k)*
- Output:
  - The corresponding value to *k* if target key found
  - Key not found otherwise

Digital Search Trees (DSTs)

Instead of looking at less than/greater than, lets go left or right based on the bits of the key

So, we again have 4 options:

- current node is null, k not found
- k is equal to the current node's key, k is found, return corresponding value
- current bit of k is 0, continue to left child
- current bit of k is 1, continue to right child

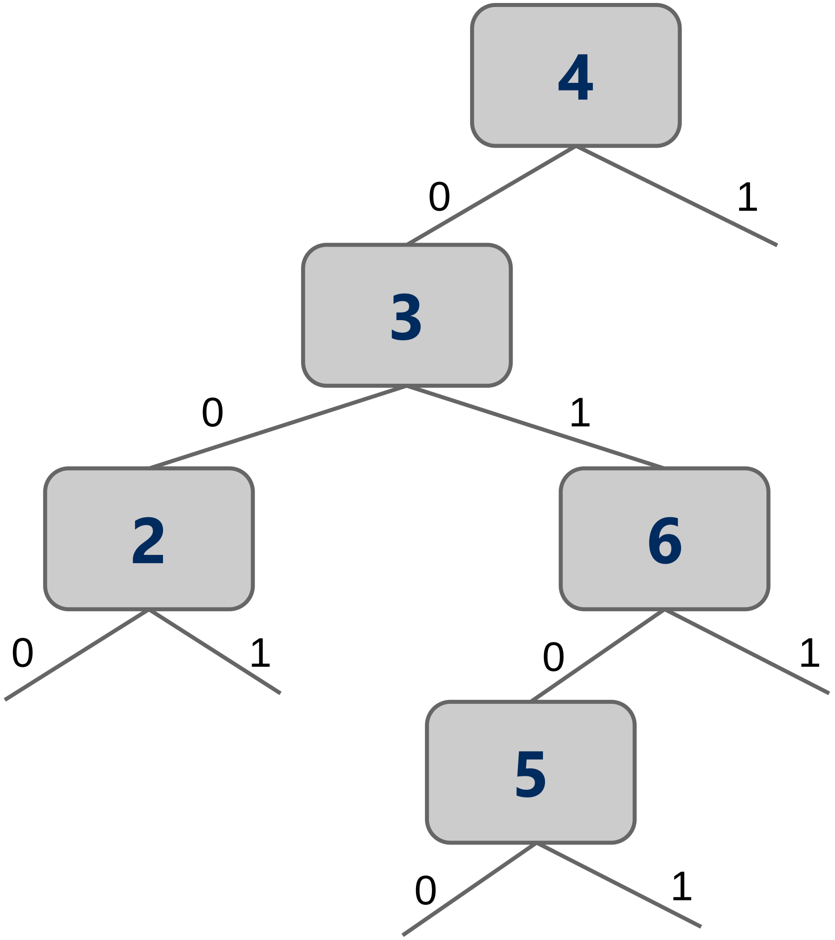
DST example: Insert and Search

Insert:

4	0100
3	0011
2	0010
6	0110
5	0101

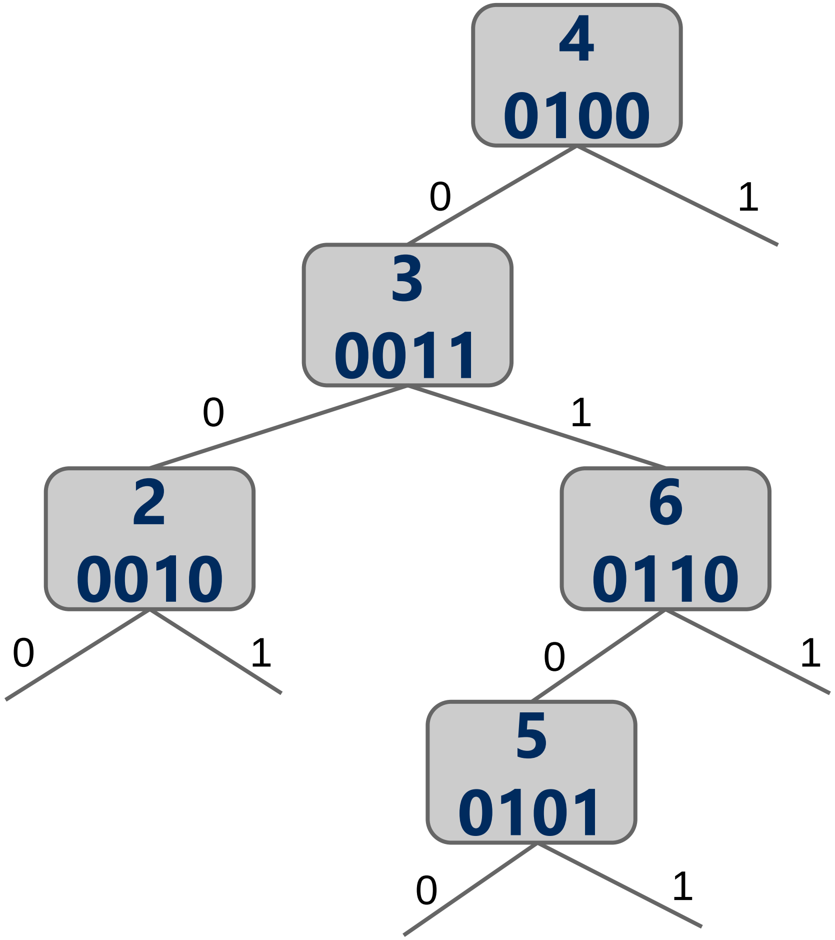
Search:

3	0011
7	0111



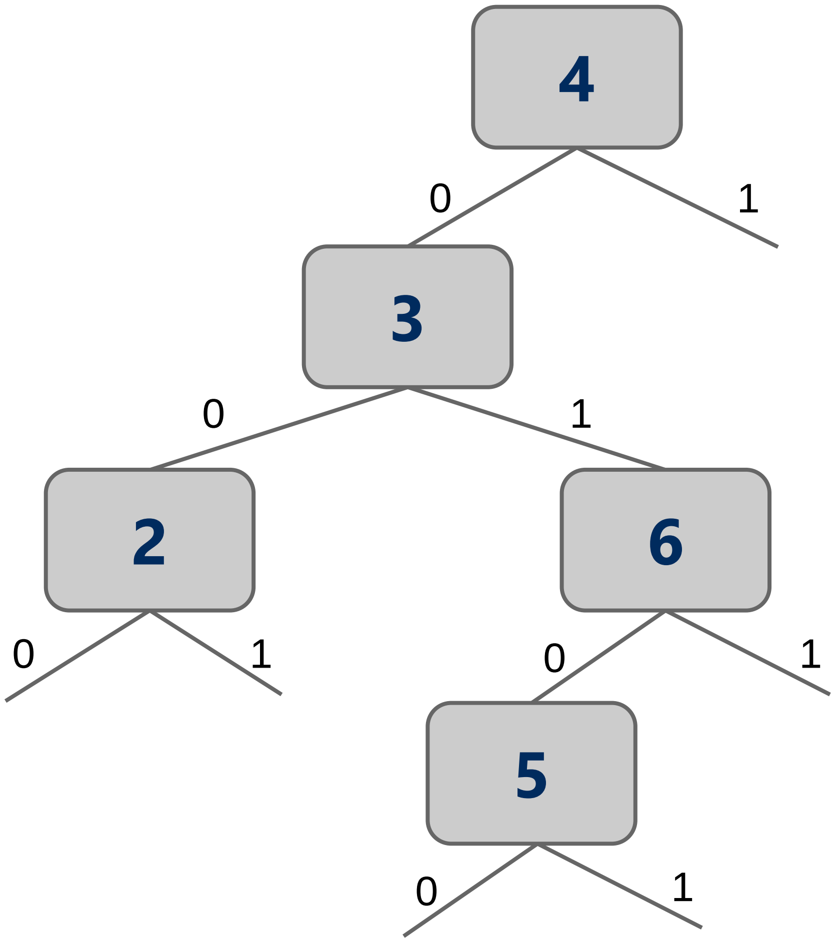
DST and Prefixes

- In a DST, each node shares a **<u>common prefix of length depth(node)</u>** with all nodes in its subtree
  - E.g., 6 shares the prefix "01" with 5
- In-order traversal doesn't produce a sorted order of the items
  - Insertion algorithm can be modified to make a DST a BST at the same time



DST example: Delete

- Delete 3
- Can replace it with any leaf in its subtree
- Let's replace it with 2
- OK because 2 shares "0" as a prefix with 3, so it also shares "0" as a prefix with 6 and 5



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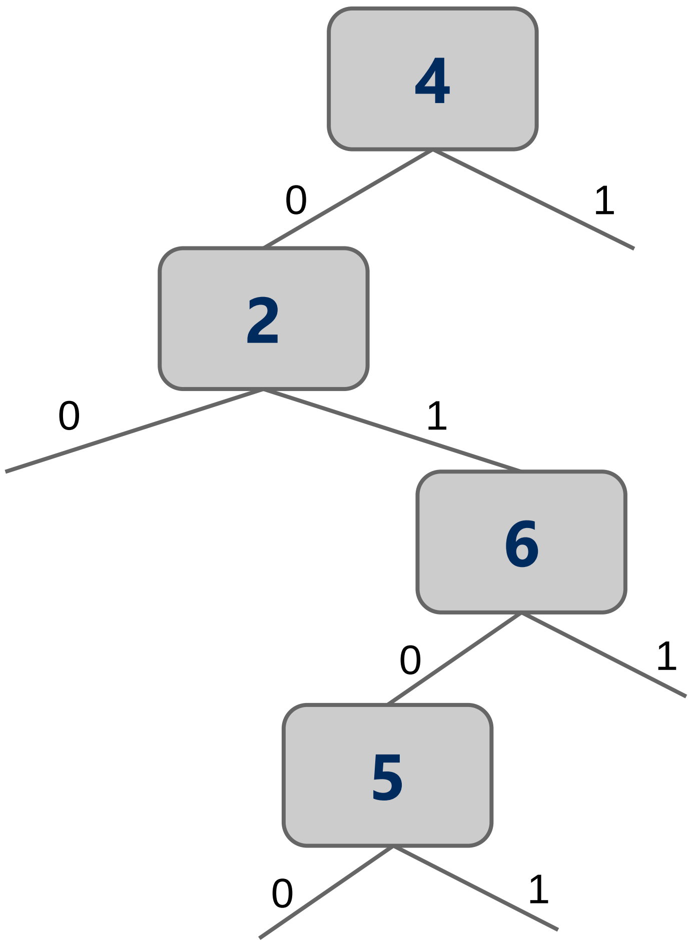


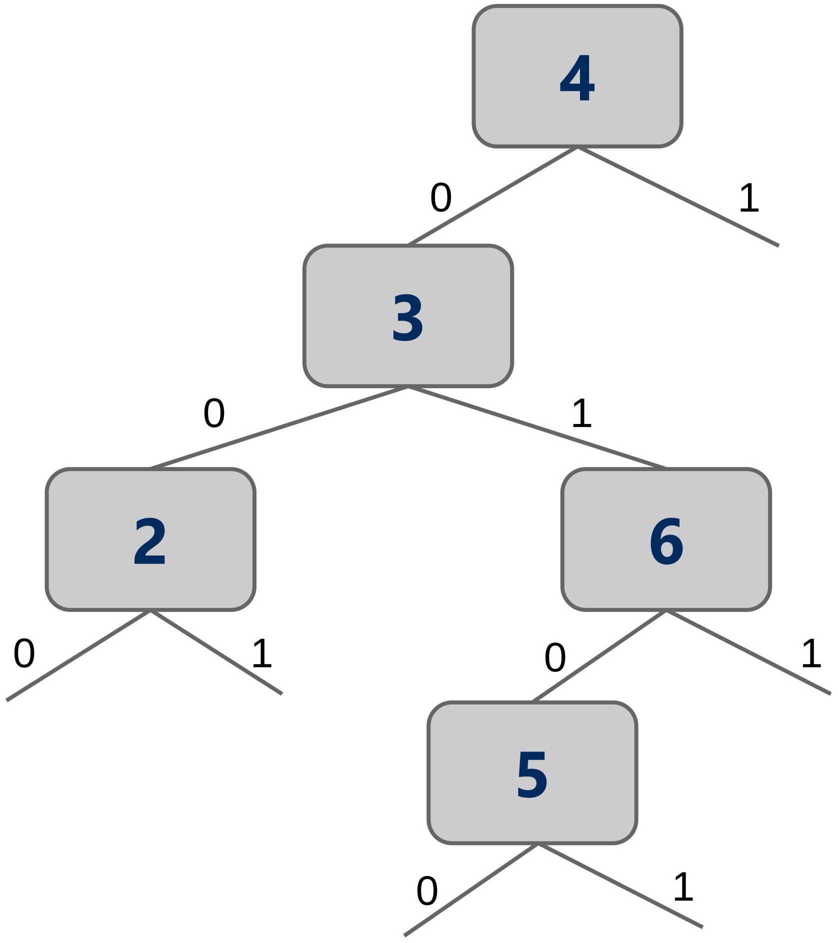
DST example: Variable length keys

- Insert

1 01

- Must be in place of 6

- Replace 6 by 1 and re-insert 6



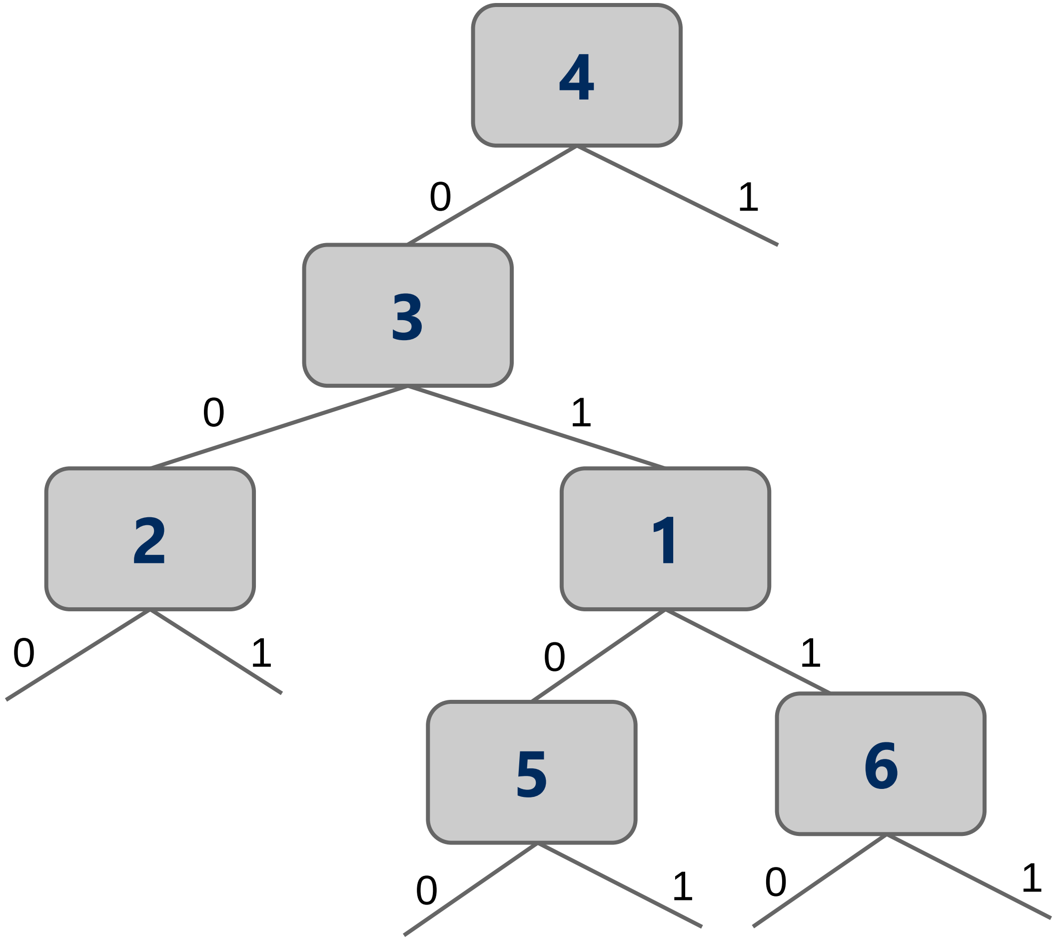
DST example: Variable length keys

- Insert

1 01

- Must be in place of 6
- Replace 6 by 1 and re-insert

6 0110



Analysis of digital search trees

average case runtime $= \sum_{\text{all cases}} Pr(\text{Case}_i) \times$ runtime for Case_i

- Runtime?
  - O(b), b is the bit length of the target or inserted key
  - On average, b = log(n)
  - When branching according to a 0 or 1 is equally likely
  - In general b >= $\lceil \log n \rceil$
- We end up doing many **equality** comparisons against the full key
- This is better than less than/greater than comparison in BST
- Can we improve on this?

Radix search tries (RSTs)

- Trie as in retrieve, pronounced the same as "try"
- Instead of storing keys inside nodes in the tree, we store them implicitly as paths down the tree
  - Interior nodes of the tree only serve to direct us according to the bitstring of the key
  - Values can then be stored at the end of key's bitstring path (i.e., at leaves)
  - RST uses less space than BST and DST

Adding to Radix Search Trie (RST)

- Input: key and corresponding value
- if root is null, set root ← new node
- current node ← root
- for each *bit* in the key
  - if bit == 0,
    - if left child of current node is null, create a new node and attach as the left child
    - move to left child
      - either recursively or by setting current ← current.left
  - if bit == 1,
    - if right child of current node is null, create a new node and attach as the right child
    - move to right child
      - either recursively or by setting current ← current.right
- insert corresponding value into current node

RST example

Insert:

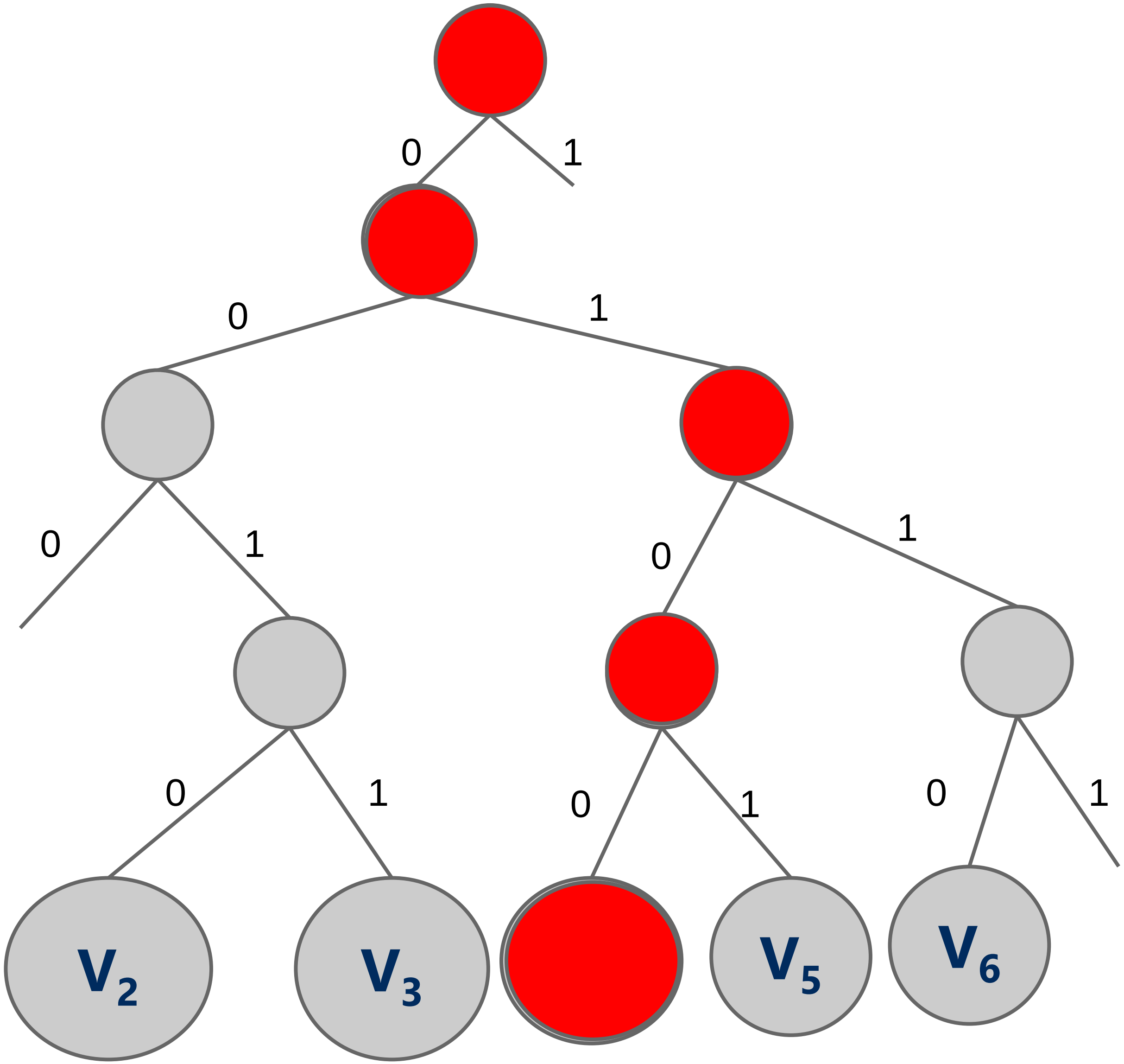
4 0100

3 0011

2 0010

6 0110

5 0101



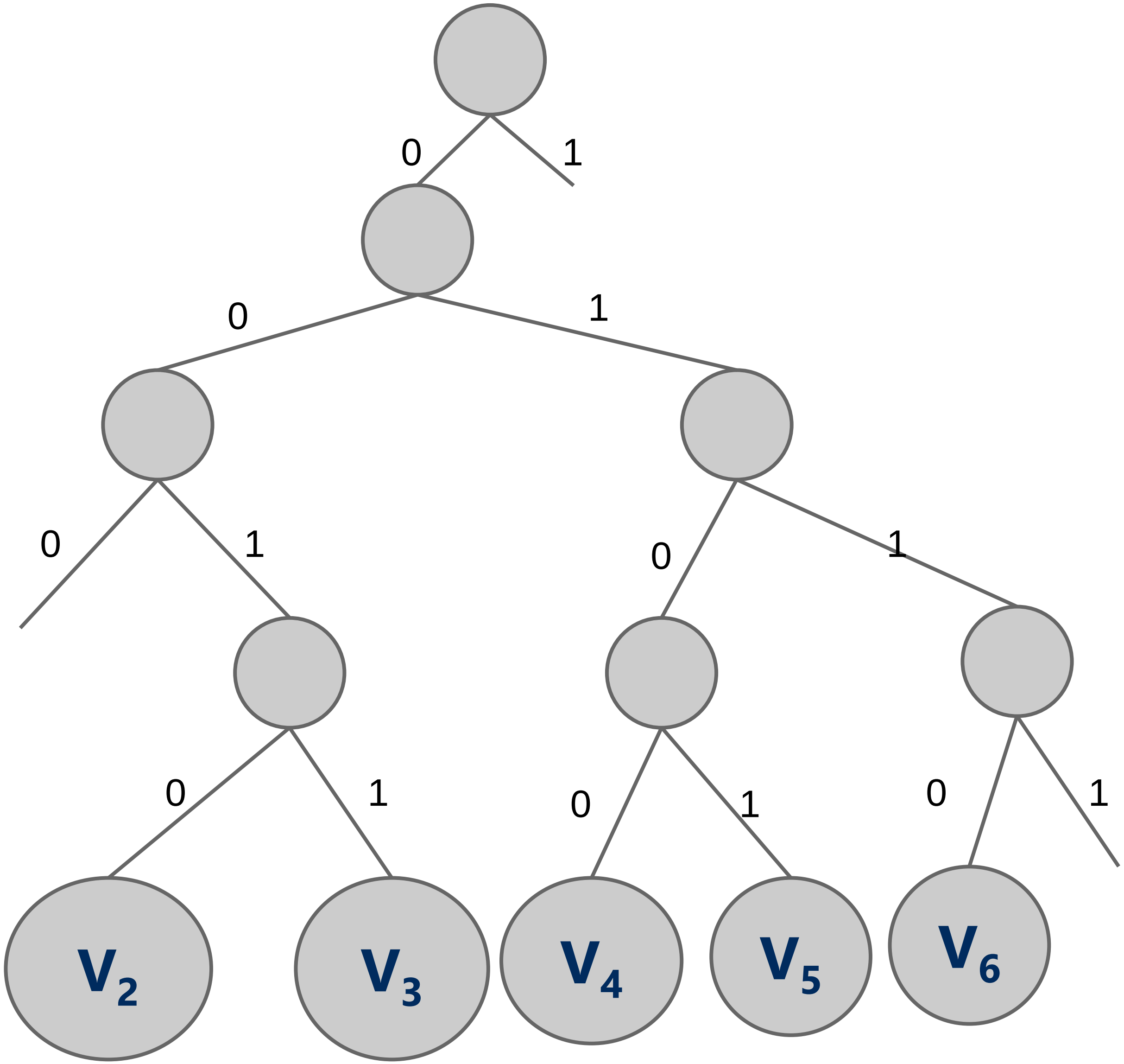
Searching in Radix Search Trie (RST)

- Input: key
- current node ← root
- for each *bit* in the key
  - if current node is null, return *key not found*
  - if bit == 0,
    - move to left child
      - either recursively or by setting current ← current.left
  - if bit == 1,
    - move to right child
      - either recursively or by setting current ← current.right
- if current node is null or the value inside is null
  - return key *not found*
- else return the value stored in current node

RST example

Search:

3 0011

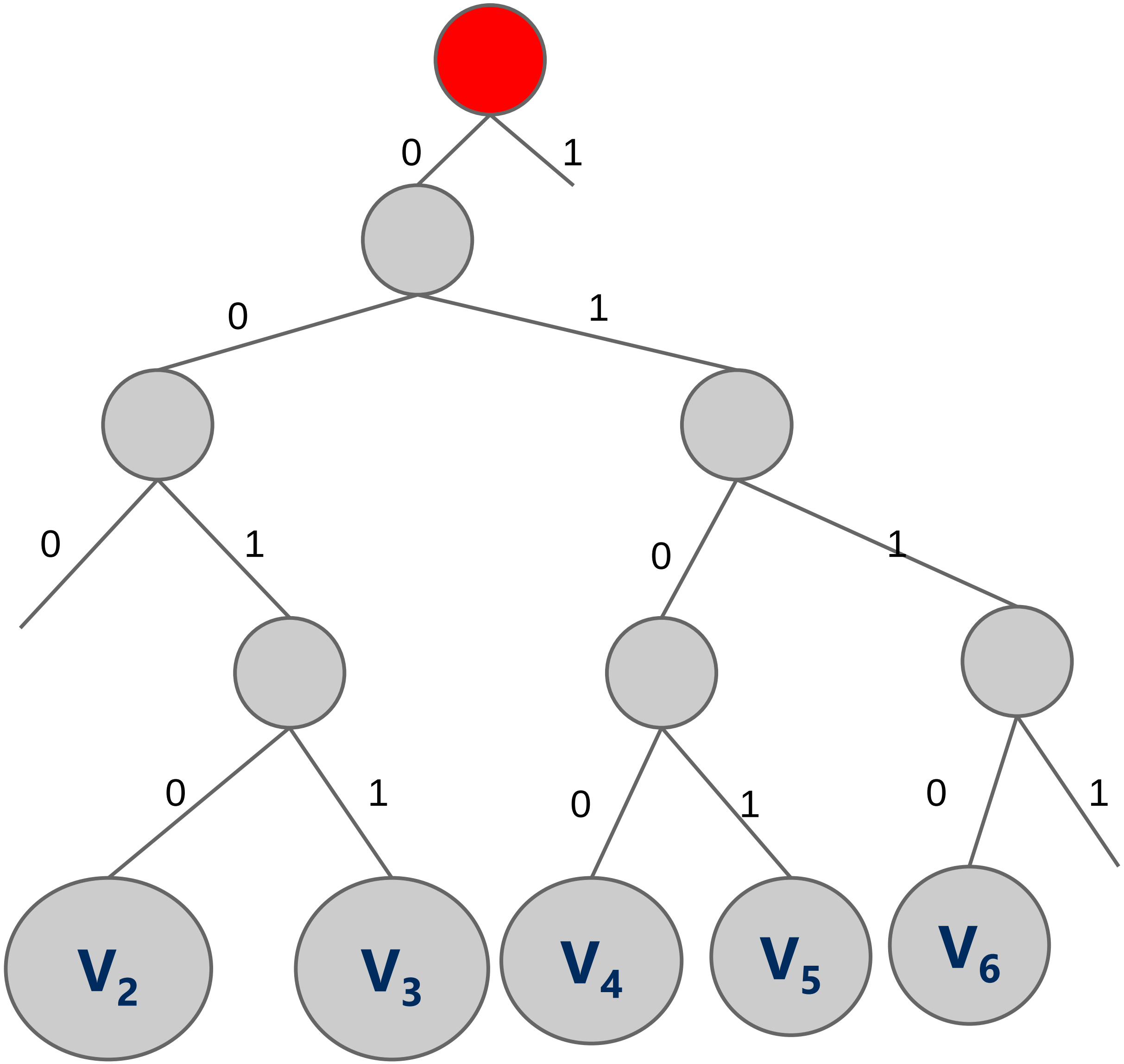


RST example

Search:

3 0011

7 0111

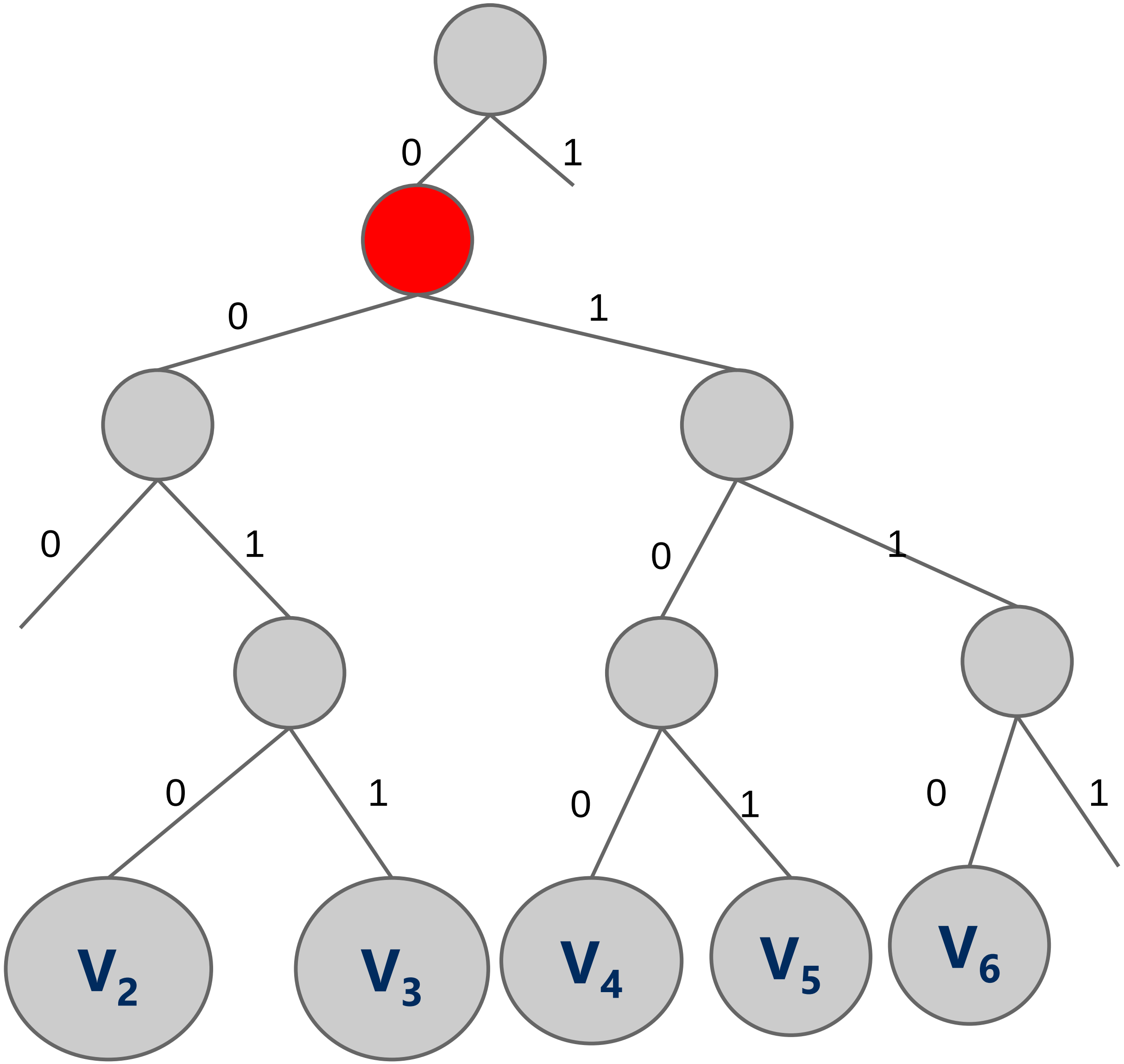


RST example

Search:

3 0011

7 0111



RST example

Search:

3 0011

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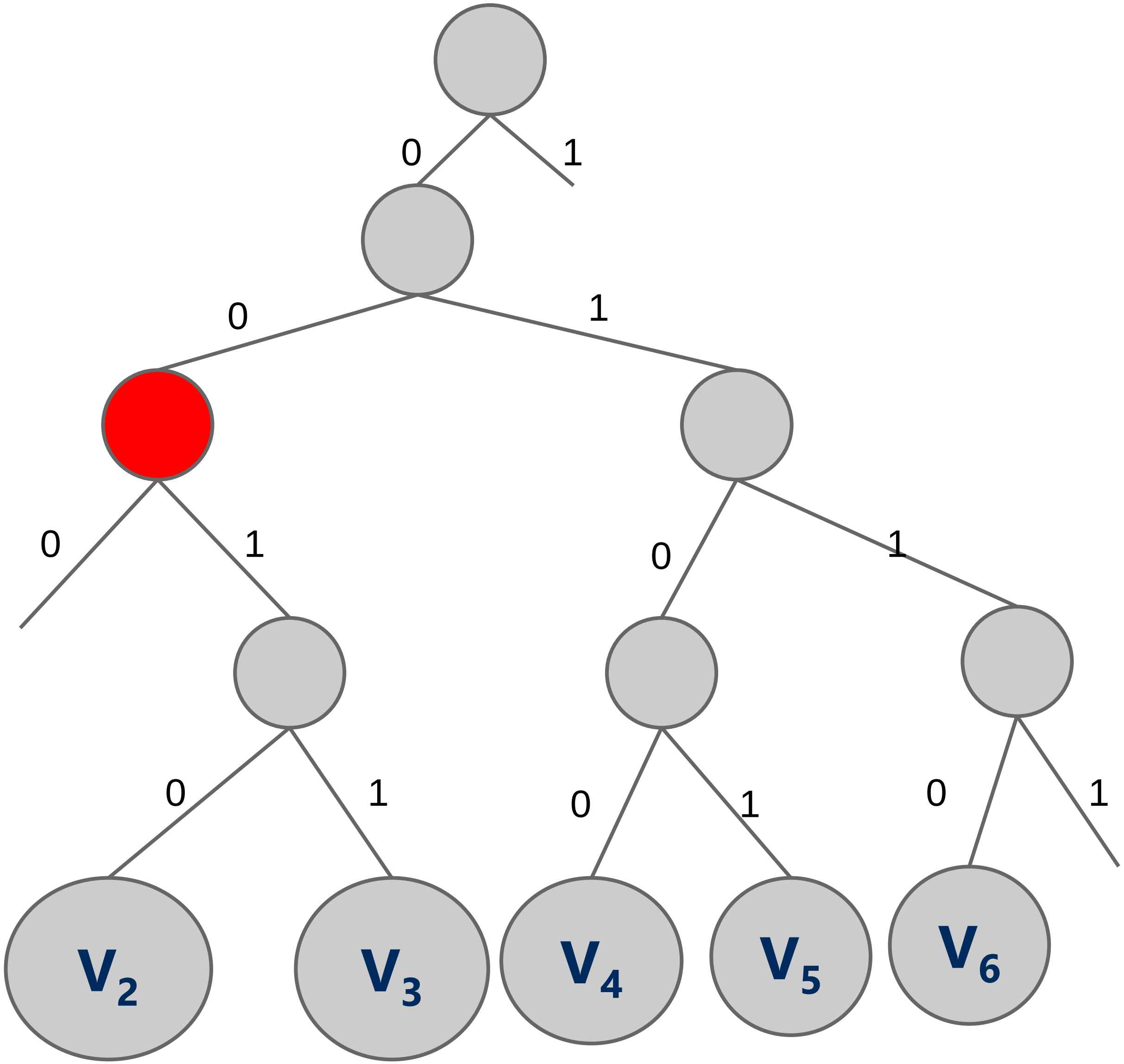
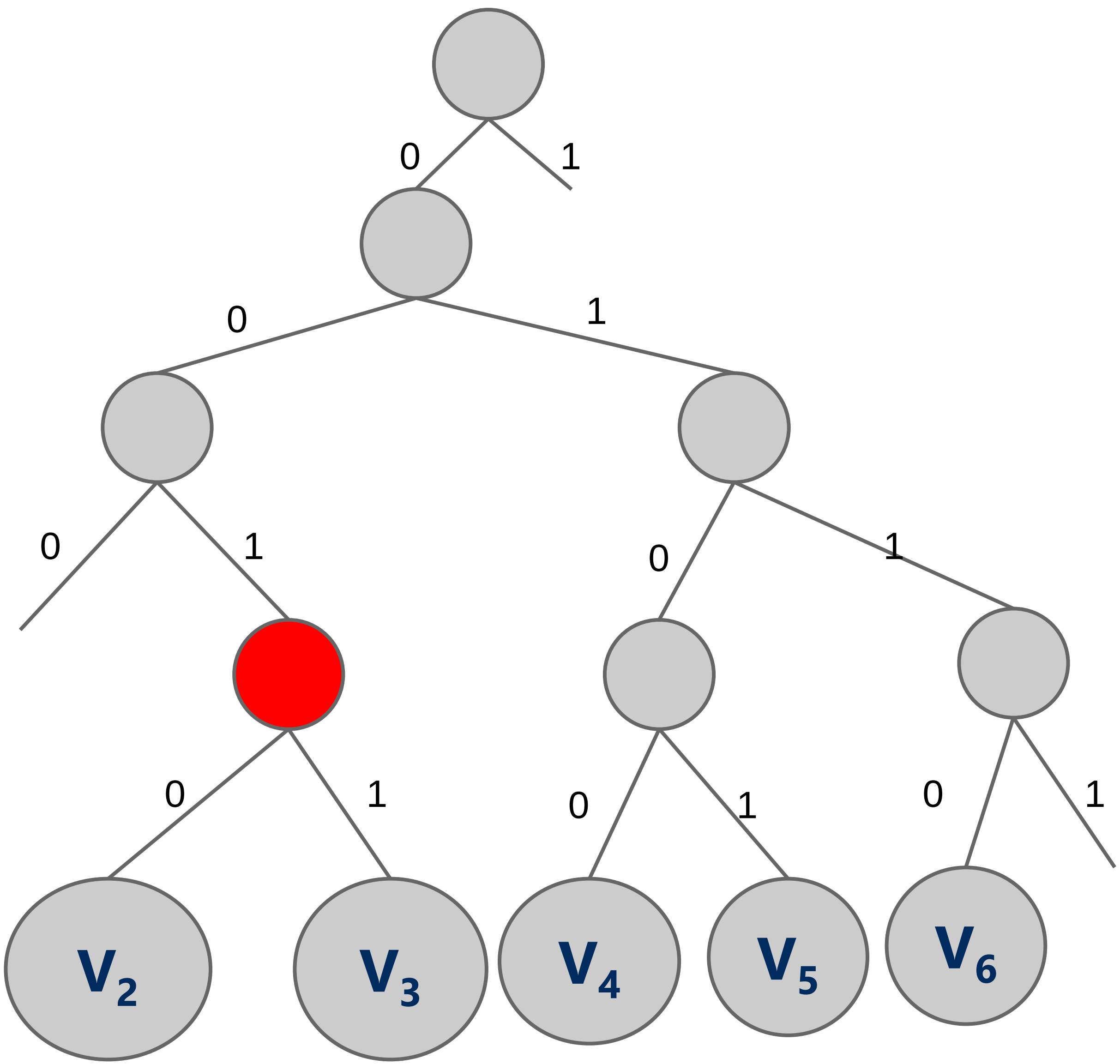


RST example

Search:

3 0011

7 0111

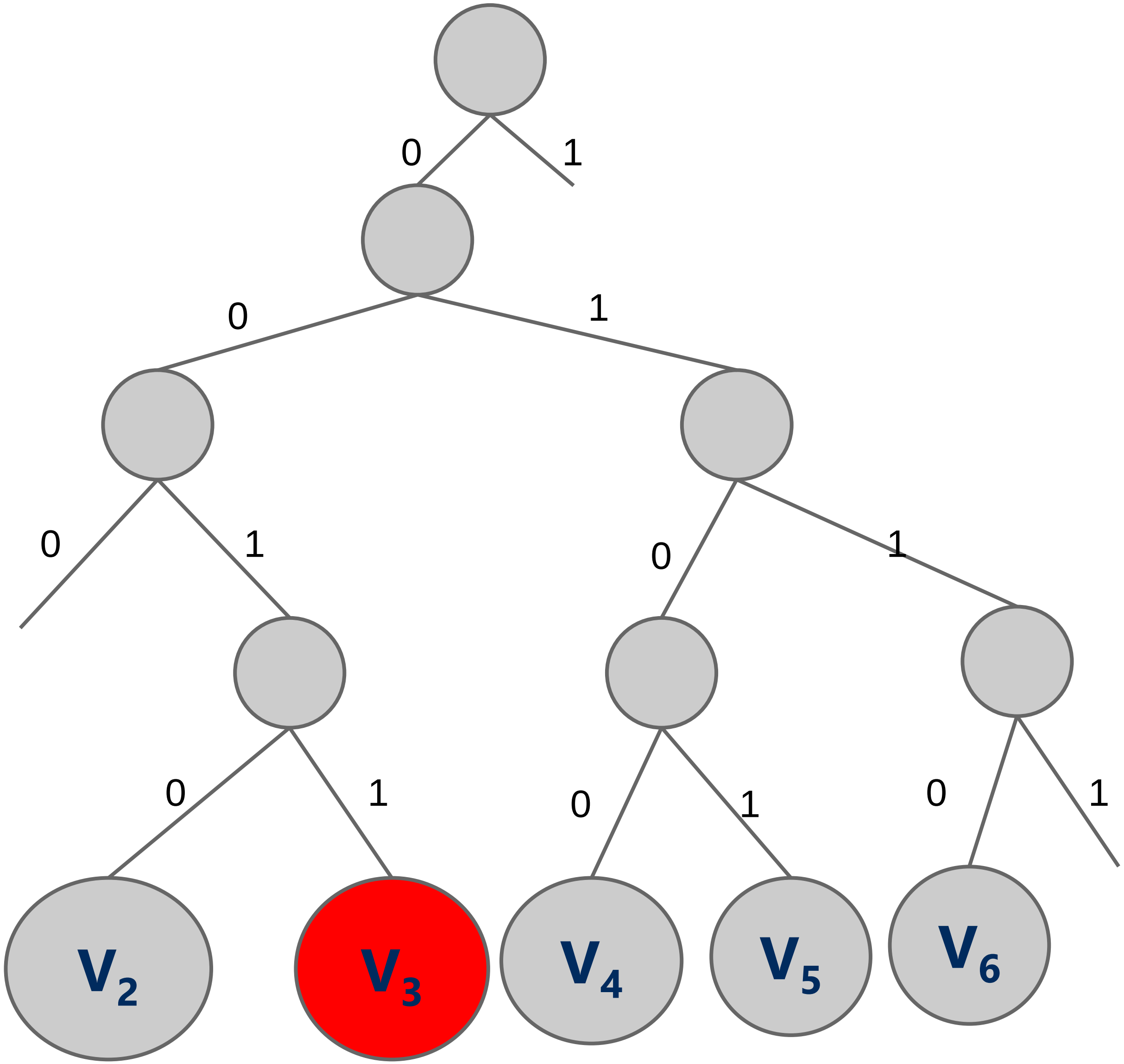


RST example

Search:

3 0011

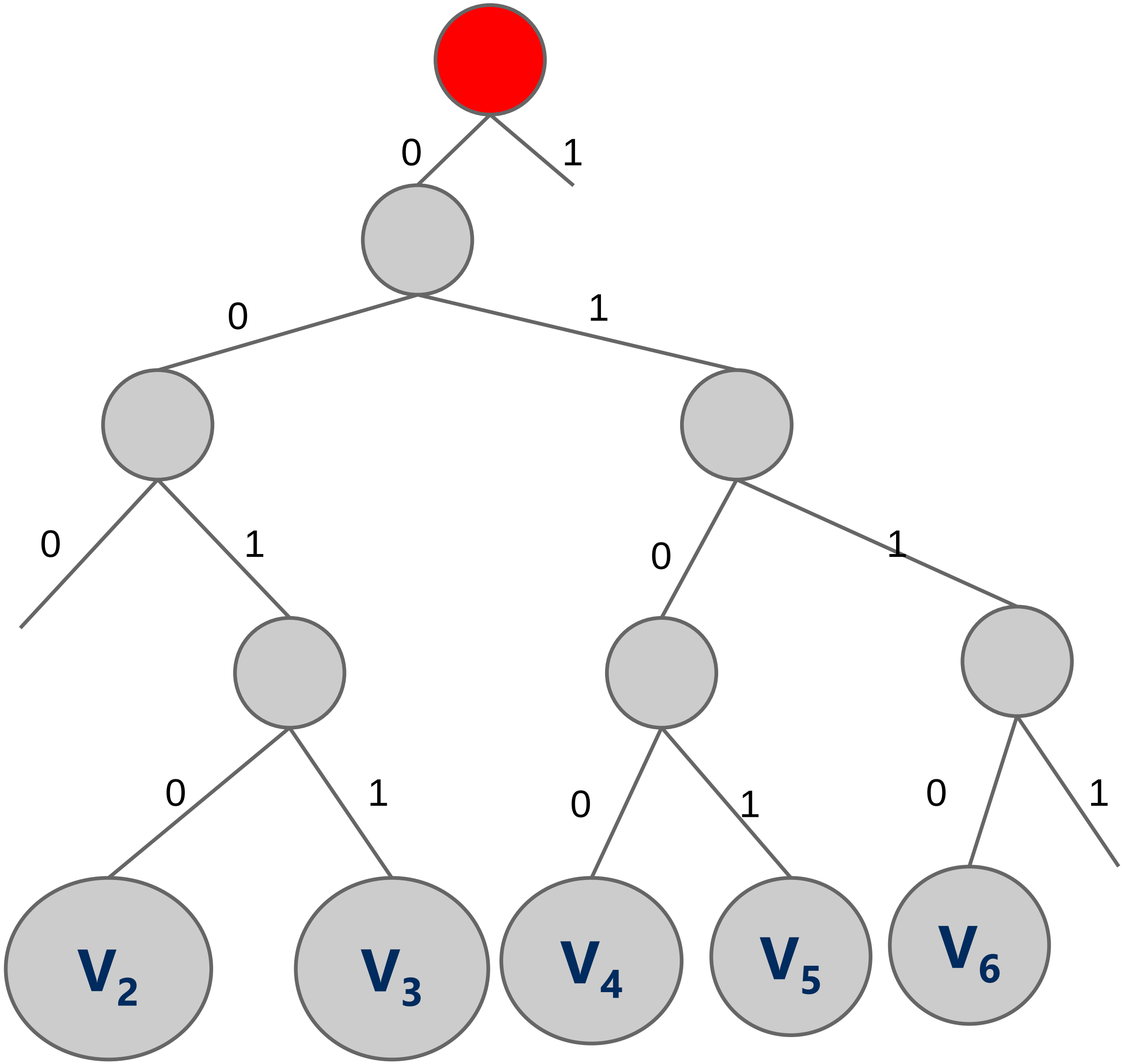
7 0111



RST example

Search:

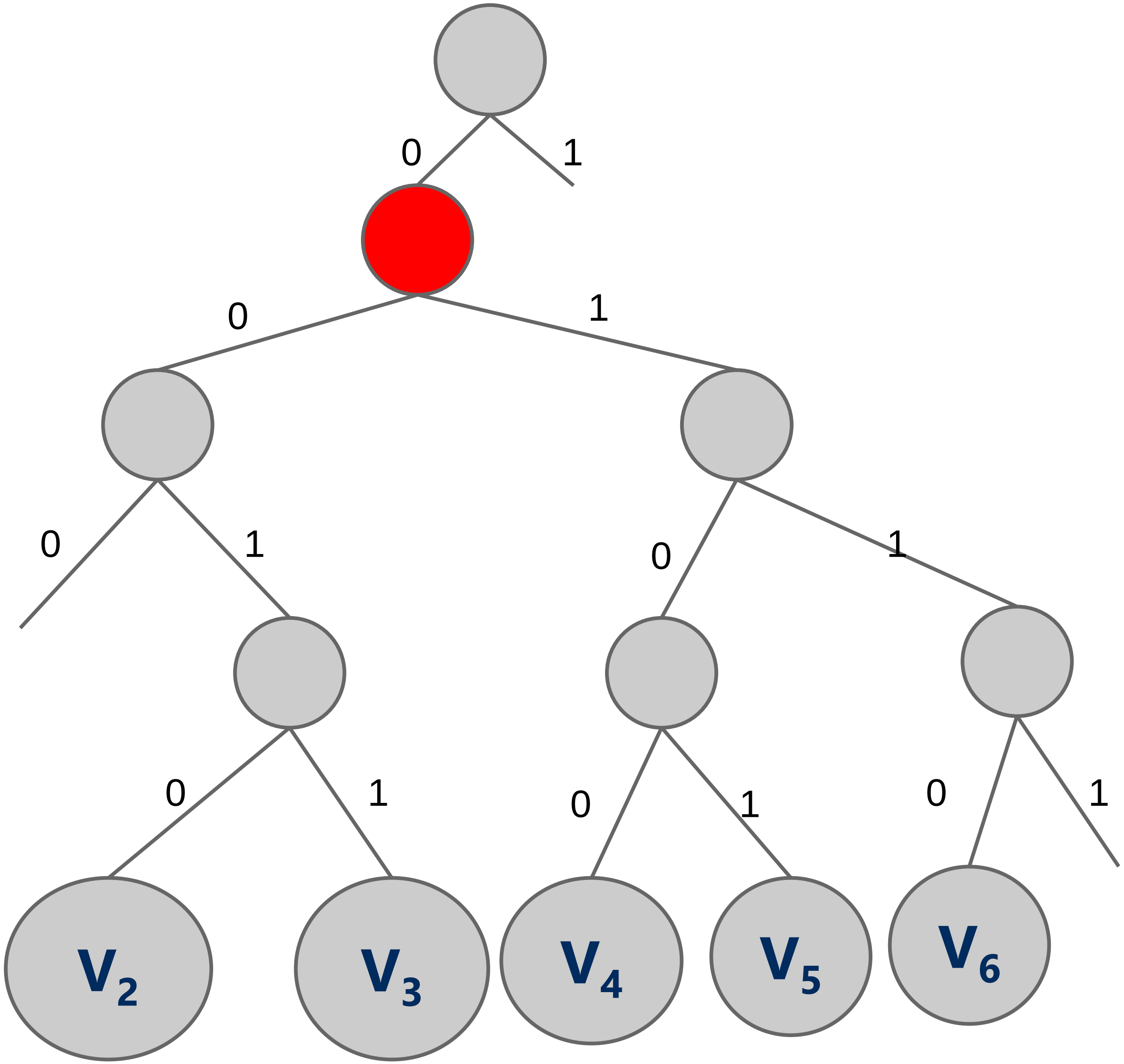
7 0111



RST example

Search:

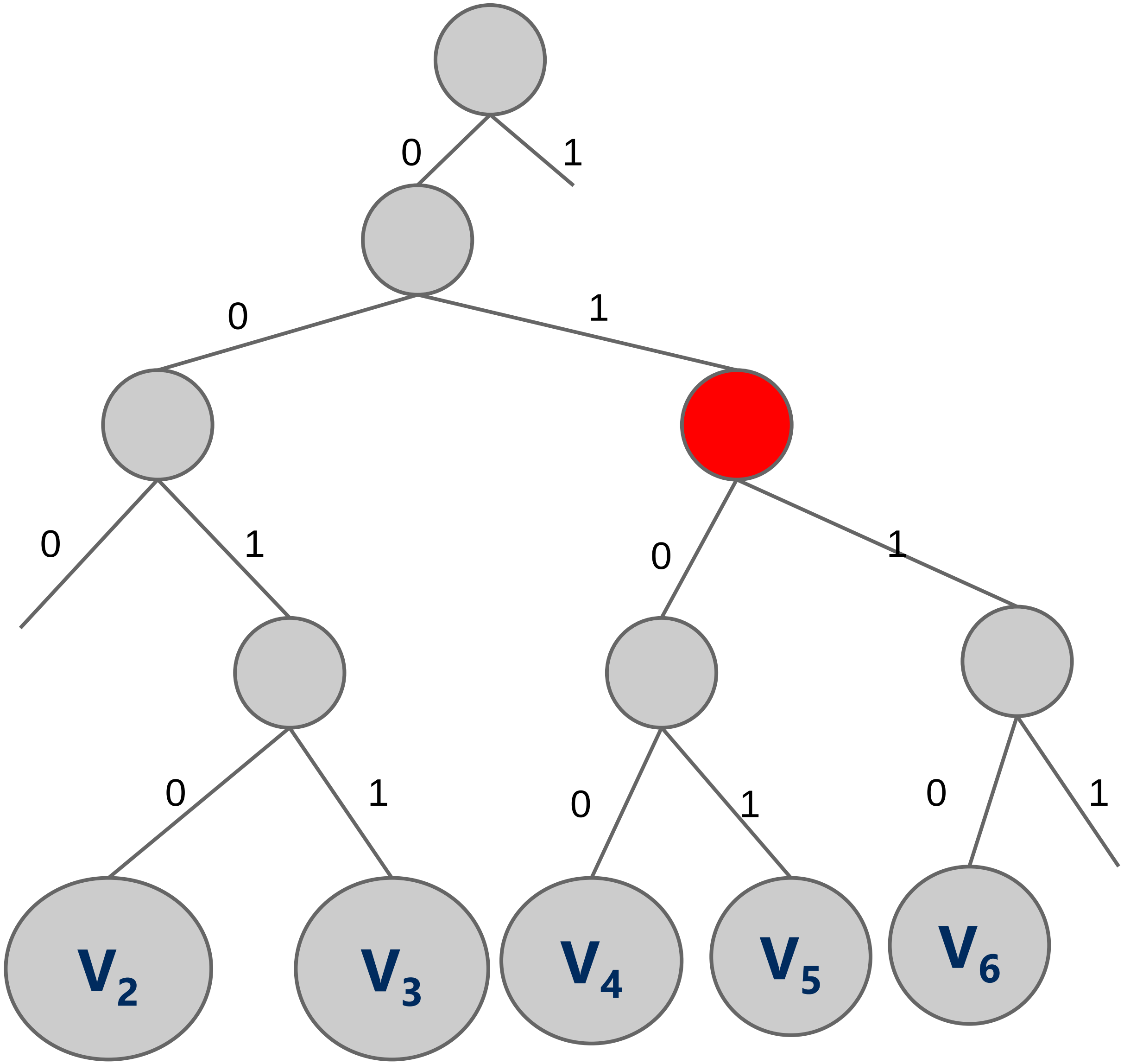
7 0111



RST example

Search:

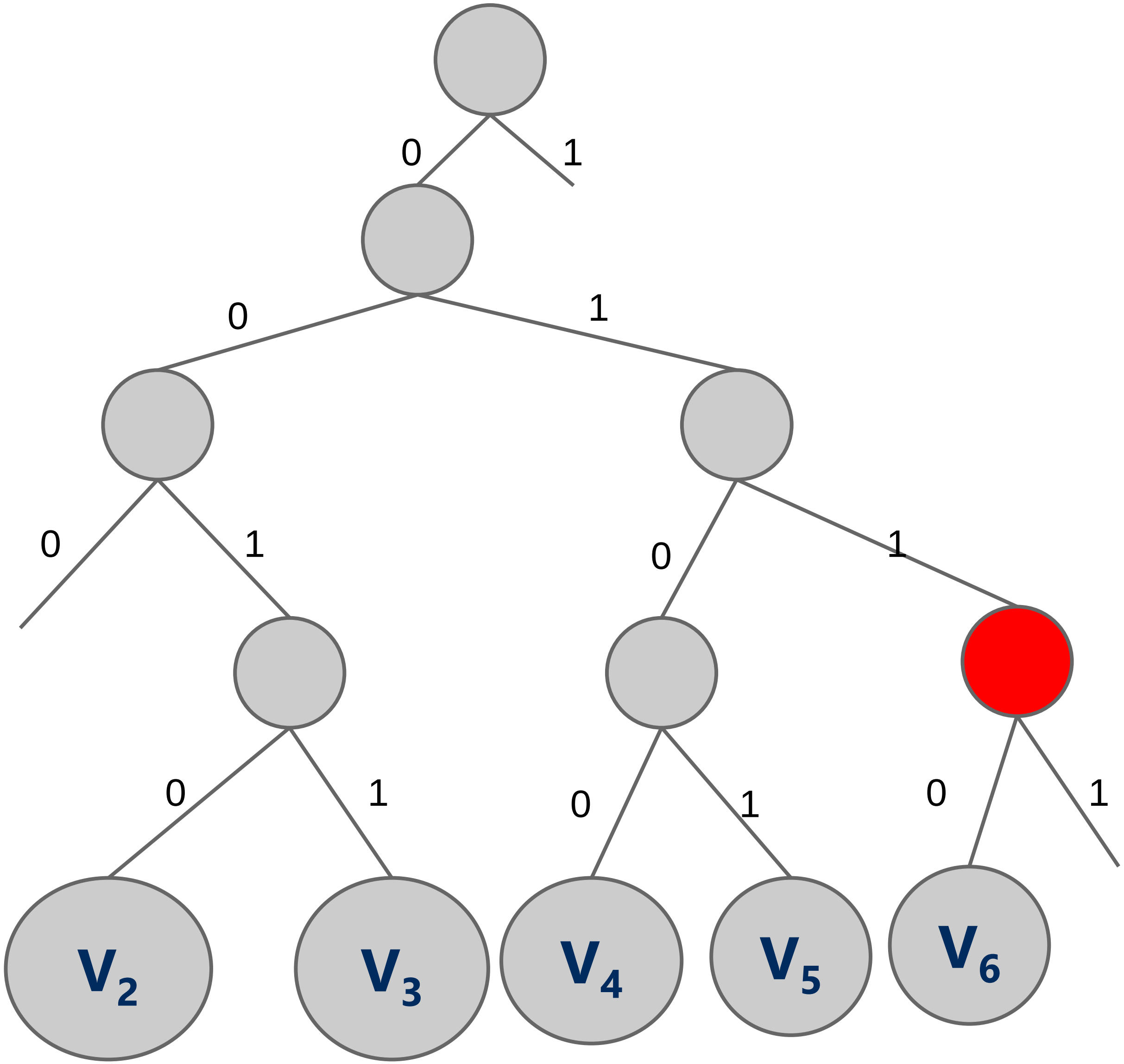
7 0111



RST example

Search:

7 0111



RST example

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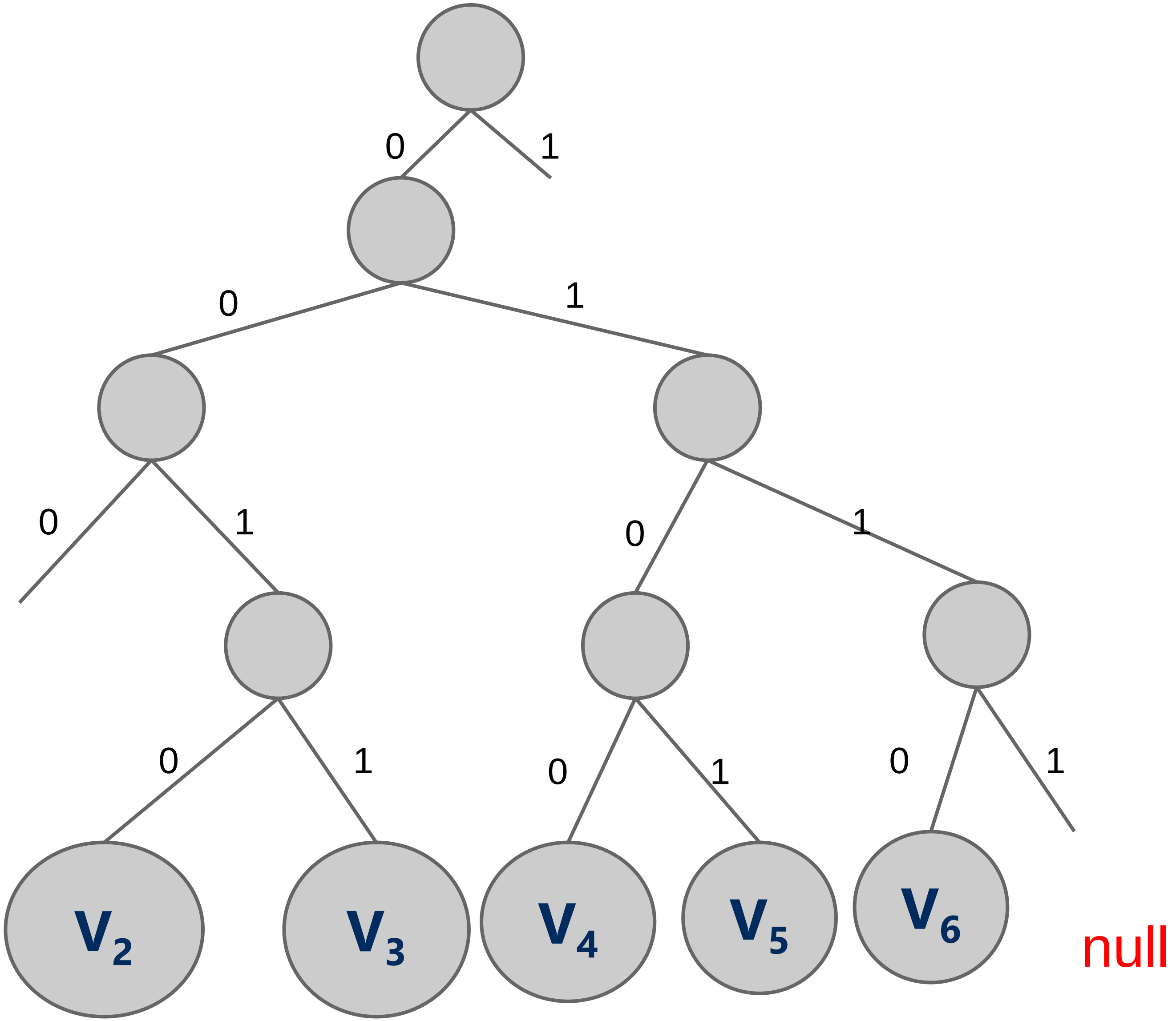
7 0111



RST analysis

- Runtime?
- O(b), the bit length of the key
  - However, this time we don't have full key comparisons
- Would this structure work as well for other key data types?
- Characters?
  - Characters are the same as 8-bit ints (assuming simple ascii)
- Strings?
- May have huge bit lengths
- How to store Strings?

Larger branching factor tries

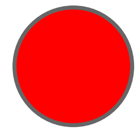
- In our binary-based Radix search trie, we considered one bit at a time
- What if we applied the same method to characters instead of bits in a string?
    - What would this new structure look like?
    - How many children per node?
        - up to R (the alphabet size)
    - Also called R-way radix search tries

Adding to R-way Radix RST

- if root is null, set root ← new node
- current node ← root
- for each *character c* in the key
  - Find the c*th* child
    - if child is null, create a new node and attach as the c*th* child
    - move to child
      - either recursively or by current ← child
- if at last character of key, insert value into current node

Another trie example

she



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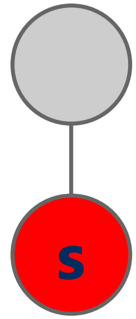

she

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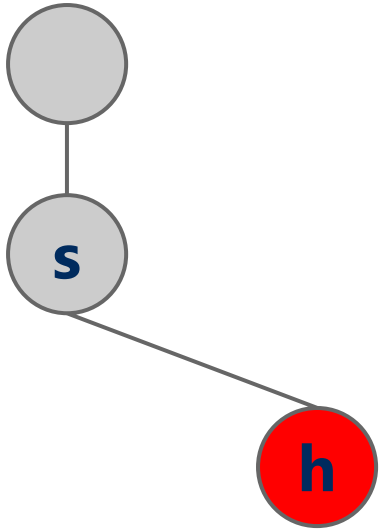

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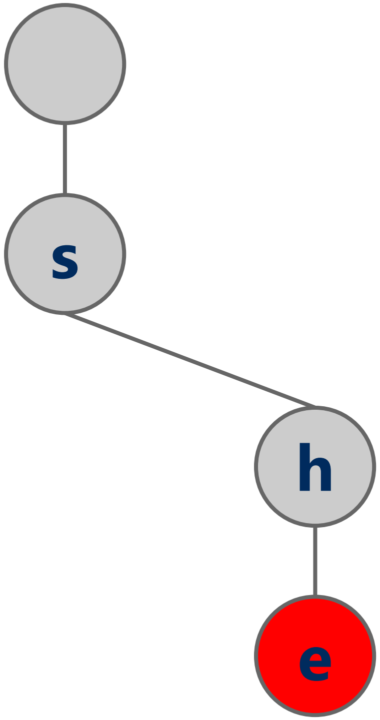

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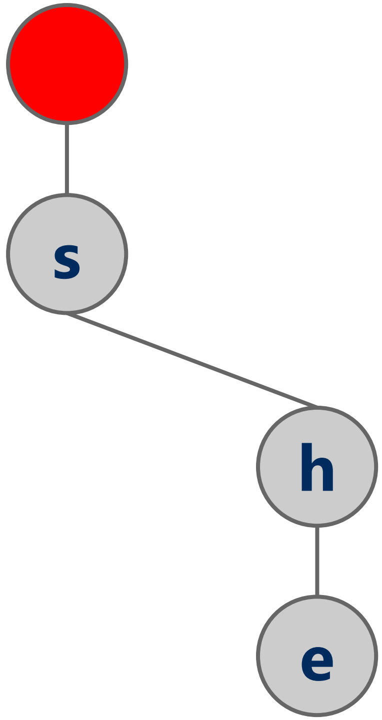
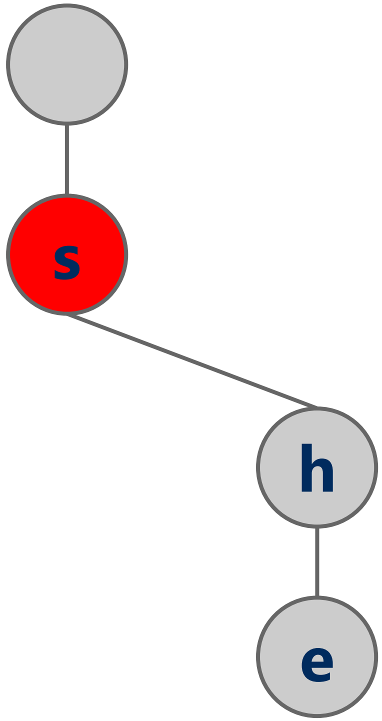


sells

Another trie example



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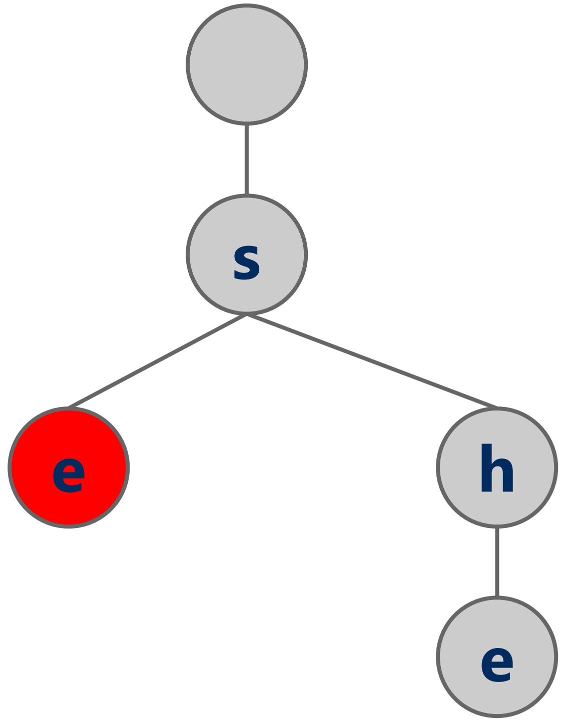

sells

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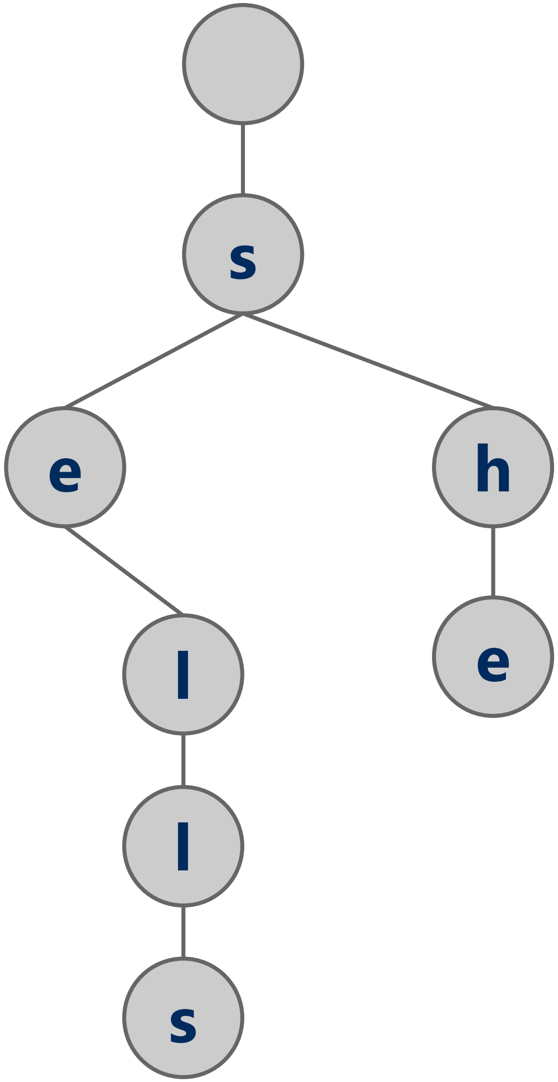

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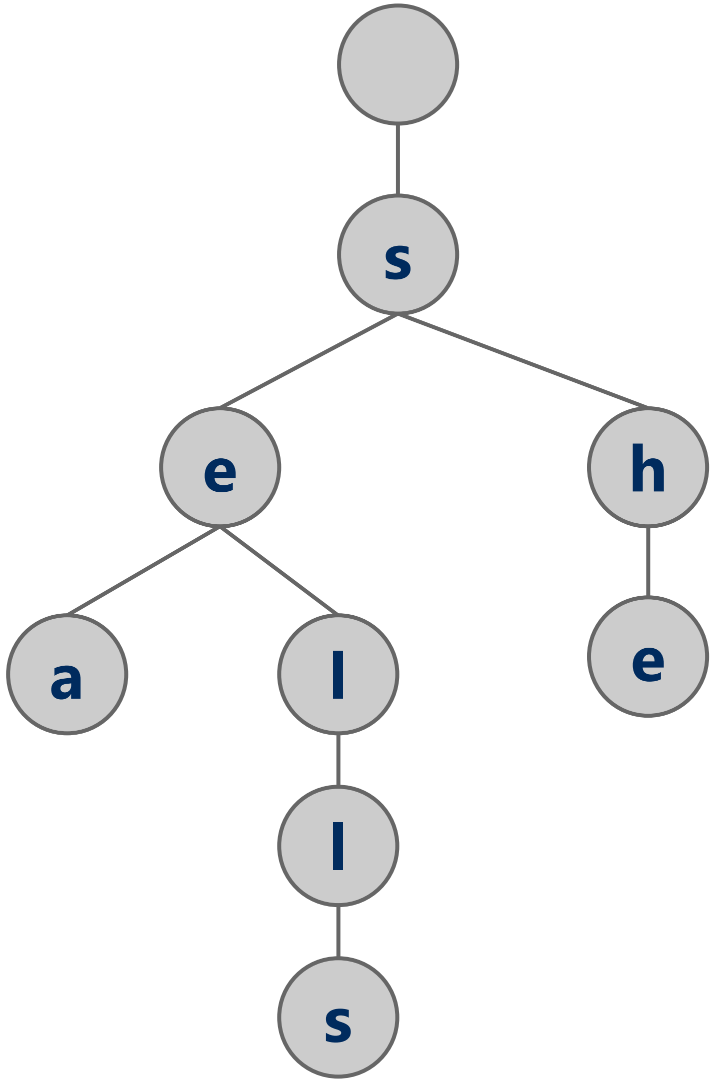
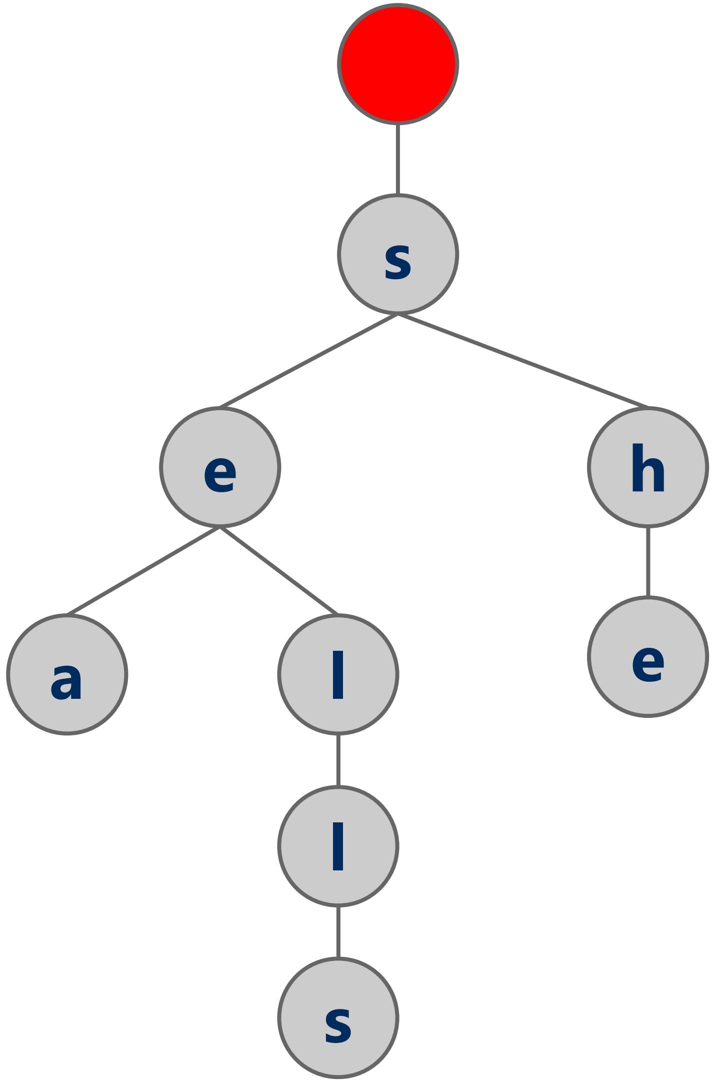


sea

Another trie example

shells



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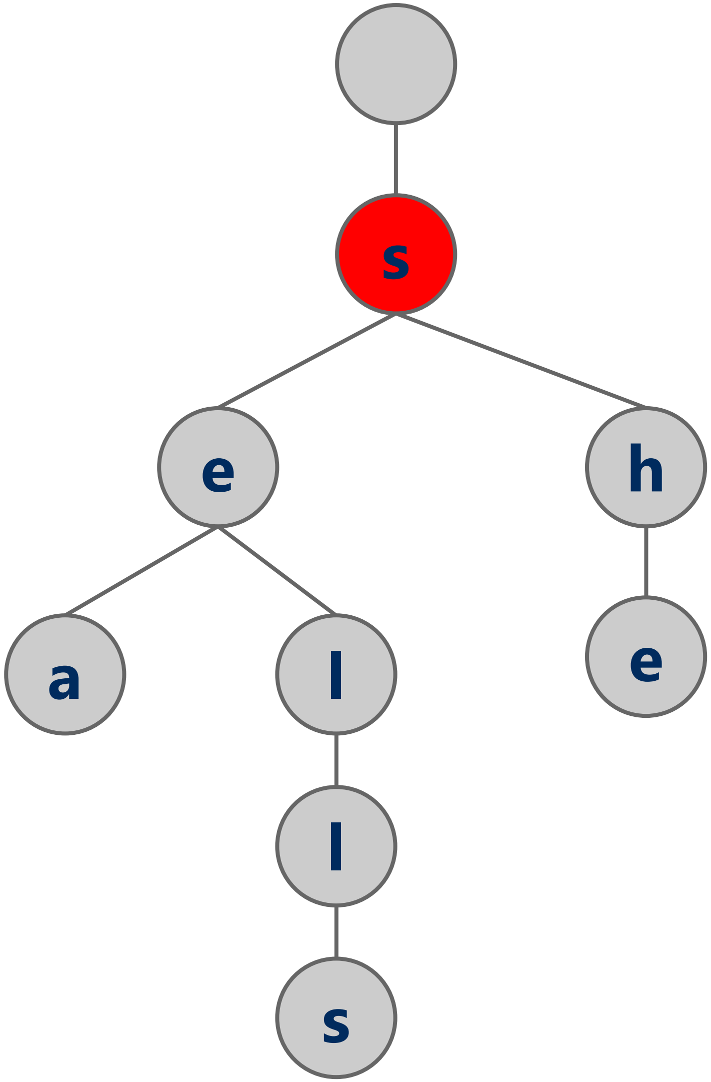

shells

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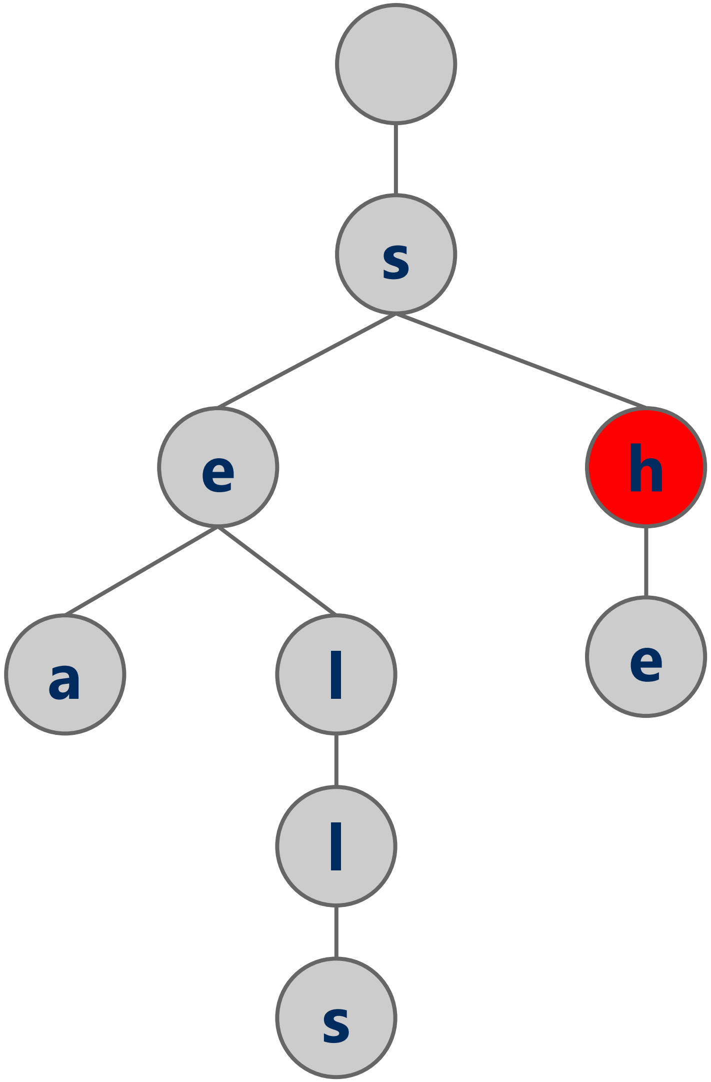

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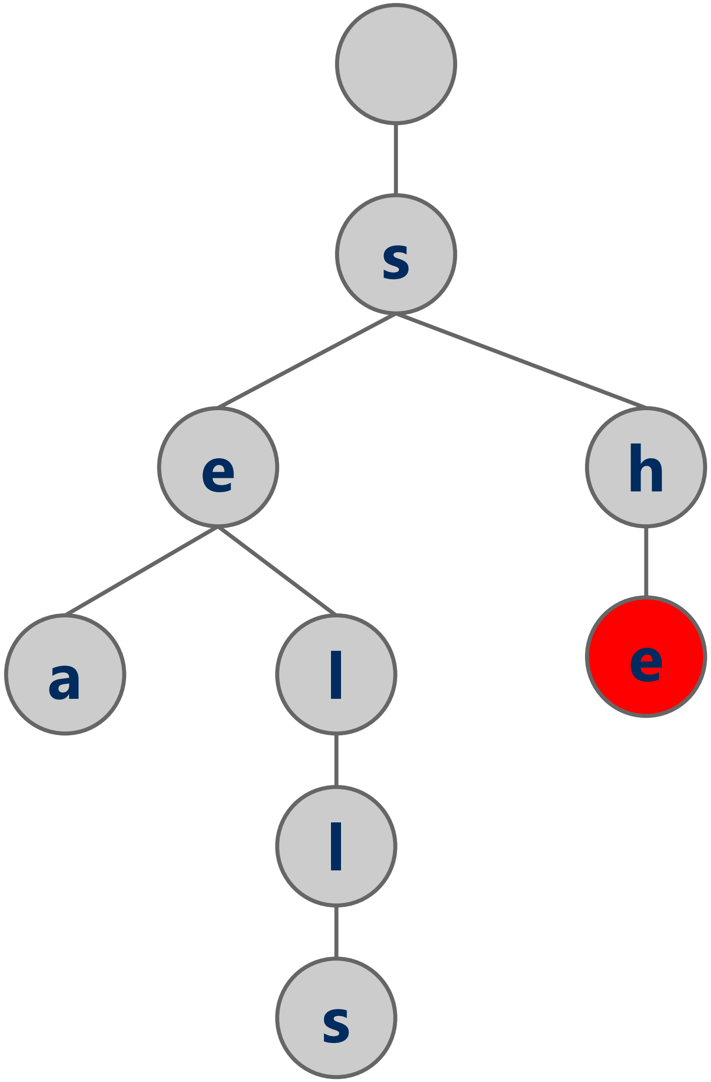

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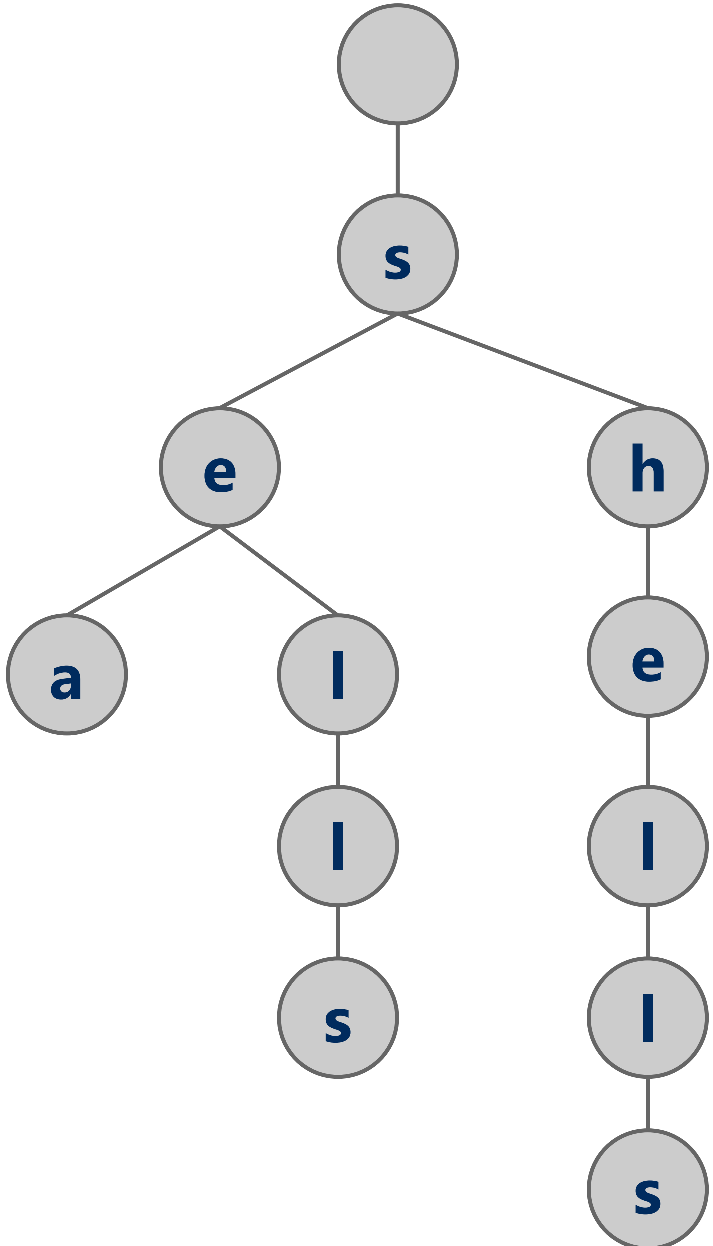

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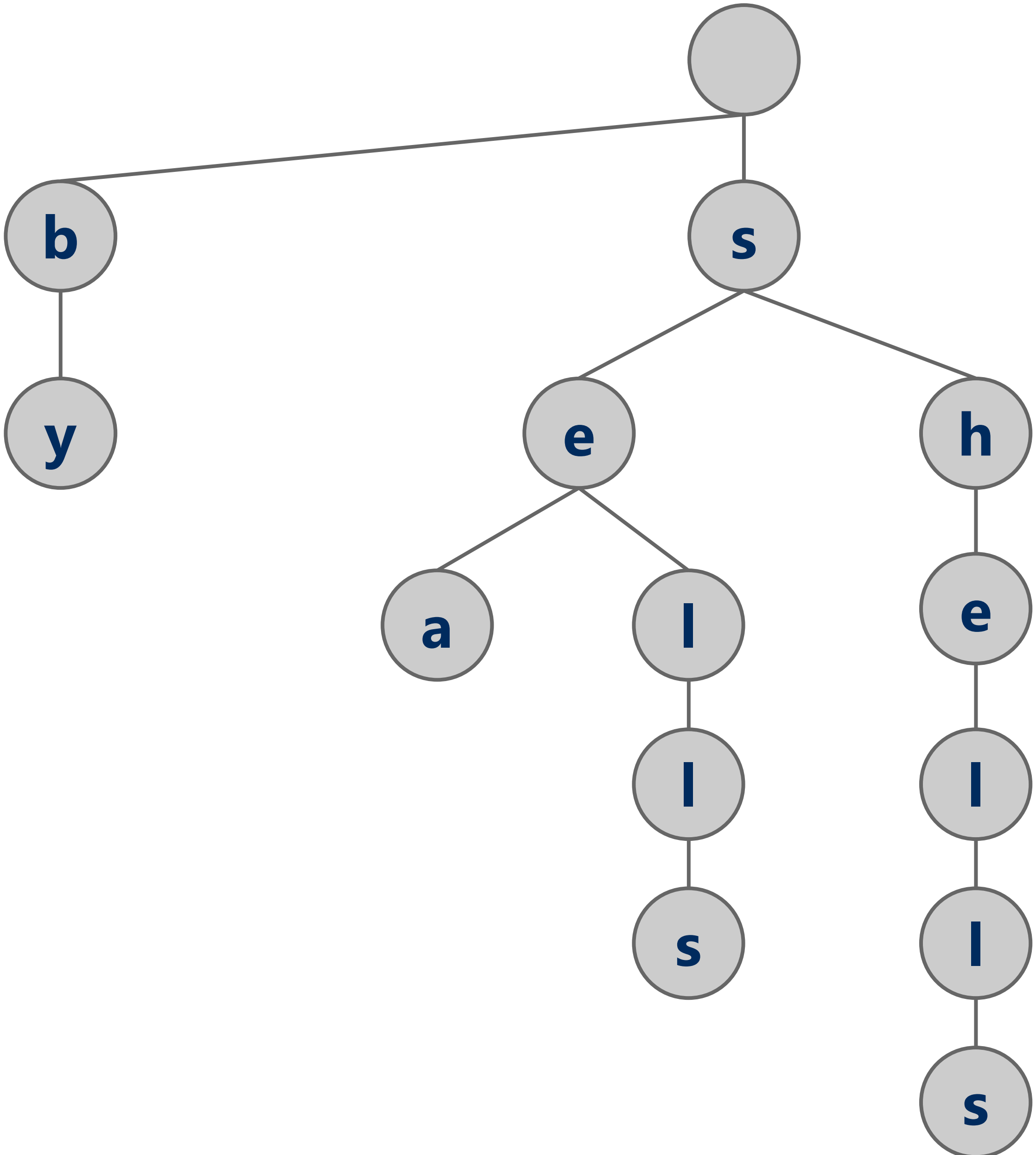

by

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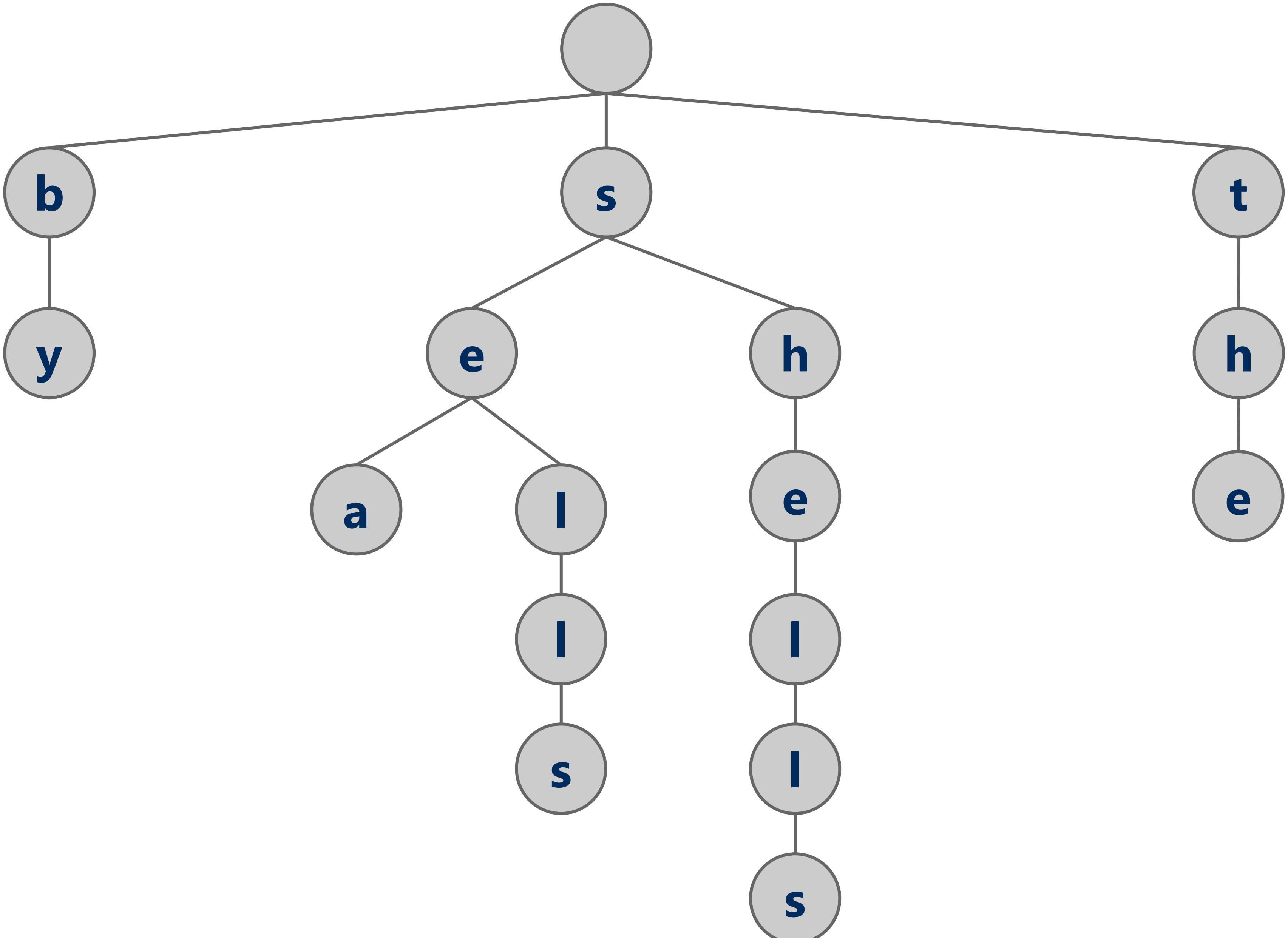

the

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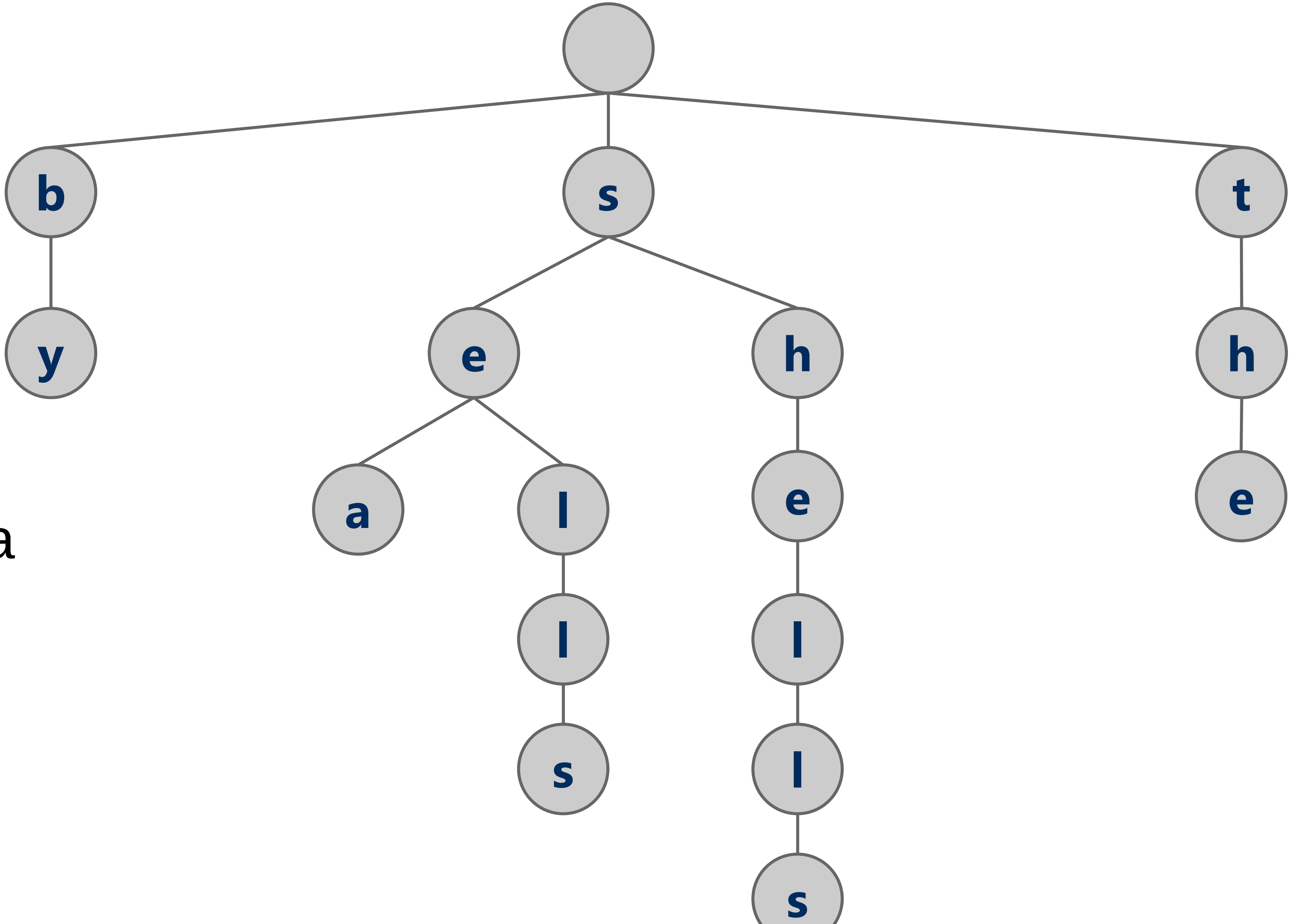
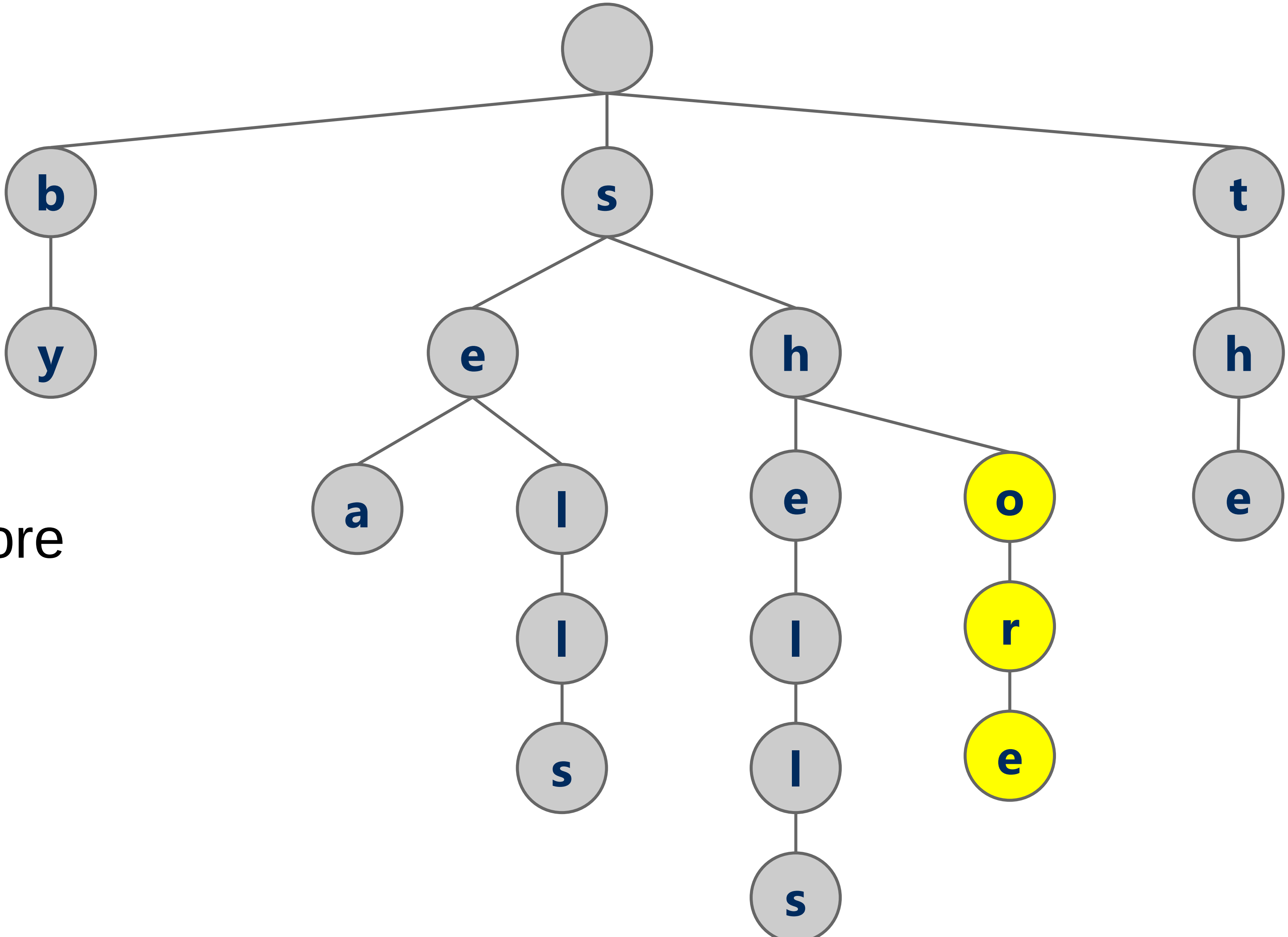


sea

Another trie example



shore

Analysis

- Runtime of add and *search hit*?
- O(w) where w is the character length of the string
  - So, what do we gain over RSTs?
    - $w < b$
    - e.g., assuming fixed-size encoding $$w = \frac{b}{\lceil \log R \rceil}$$
    - tree height is reduced

Search Miss

- Search Miss time for R-way RST
  - Require an average of $log_R(n)$ nodes to be examined
    - Proof in Proposition H of Section 5.2 of the text
- Average tree height with 2^{20} keys in an RST?
  - $\log_2 n = \log_2 2^{20} = 20$
- With 2^{20} keys in a large branching factor trie, assuming 8-bits at a time?
  - $\log_R n = \log_{256} 2^{20} = \log_{256} (2^8)^{2.5} = = \log_{256} 256^{2.5} = 2.5$

Implementation Concerns

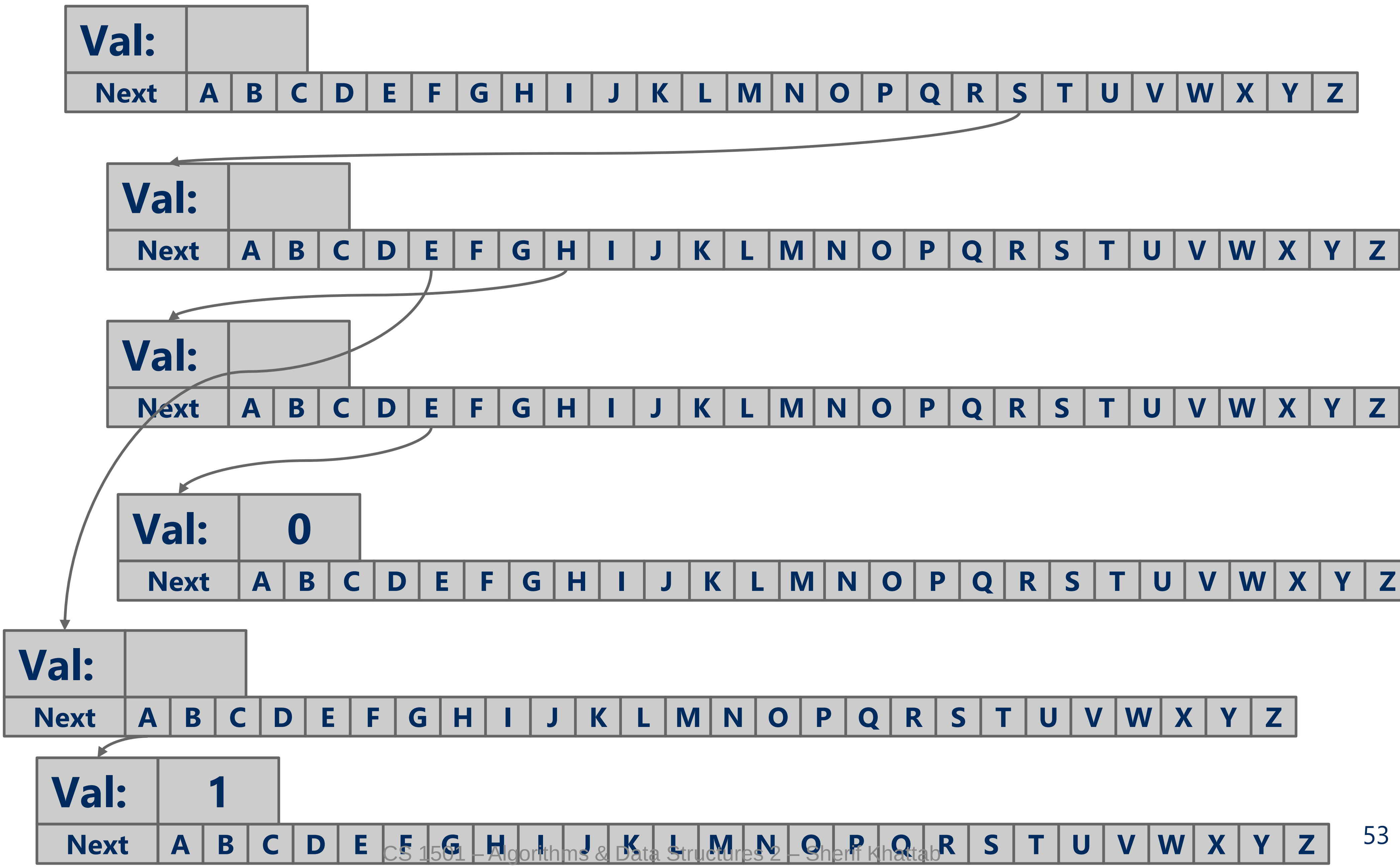
- See TrieSt.java
  - Implements an R-way trie
- Basic node object:

Where R is the branching factor

```
private class Node {
    private Object val;
    private Node[] next;
    private Node(){
      next = new Node[R];
    }
}
```

- Non-null **val** means we have traversed to a valid key
- Again, note that keys are not directly stored in the trie at all

R-way trie example

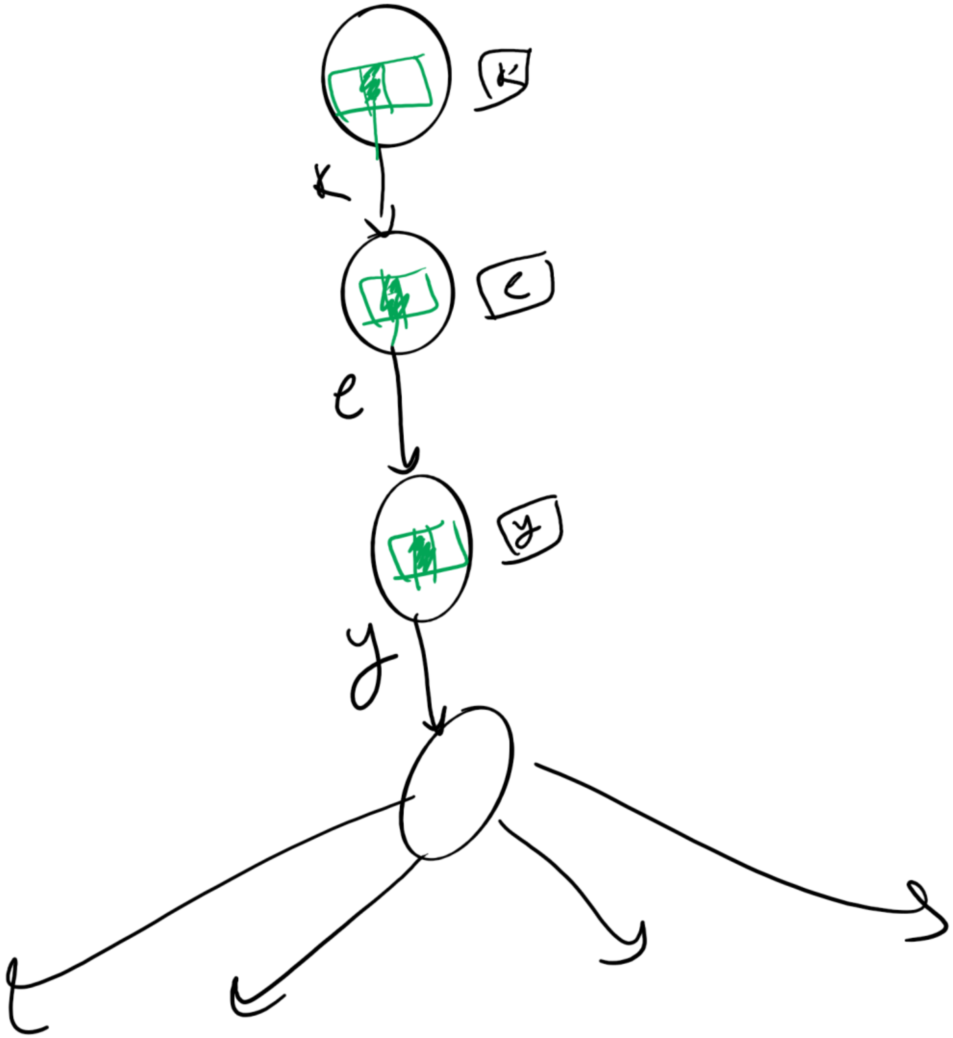


Summary of running time

	insert	search hit	search miss
binary RST	$\Theta(b)$	$\Theta(b)$	$\Theta(\log_2 n)$ on average
multi-way RST	$\Theta(w)$	$\Theta(w)$	$\Theta(\log_R n)$

R-way RST's nodes are large!

- Considering 8-bit ASCII, each node contains 2^8 references!
- This is especially problematic as in many cases, a lot of this space is wasted
  - Common paths or prefixes for example, e.g., if all keys begin with "key", thats 255*3 wasted references!
  - At the lower levels of the trie, most keys have probably been separated out and reference lists will be sparse



Solution: De La Briandais tries (DLBs)

Main idea: replace the array inside the node of the R-way trie with a linked-list

DLB Nodelets

Two alternative implementations:

```java
private class DLBNode {
    private Object val;
    private T character;
    private Node sibling;
    private Node child;
}
```

If search terminates on a node with non-null value, key is found; otherwise, not found.

```java
private class DLBNode {
    private Object val;
    private Character character;
    private Node sibling;
    private Node child;
}
```

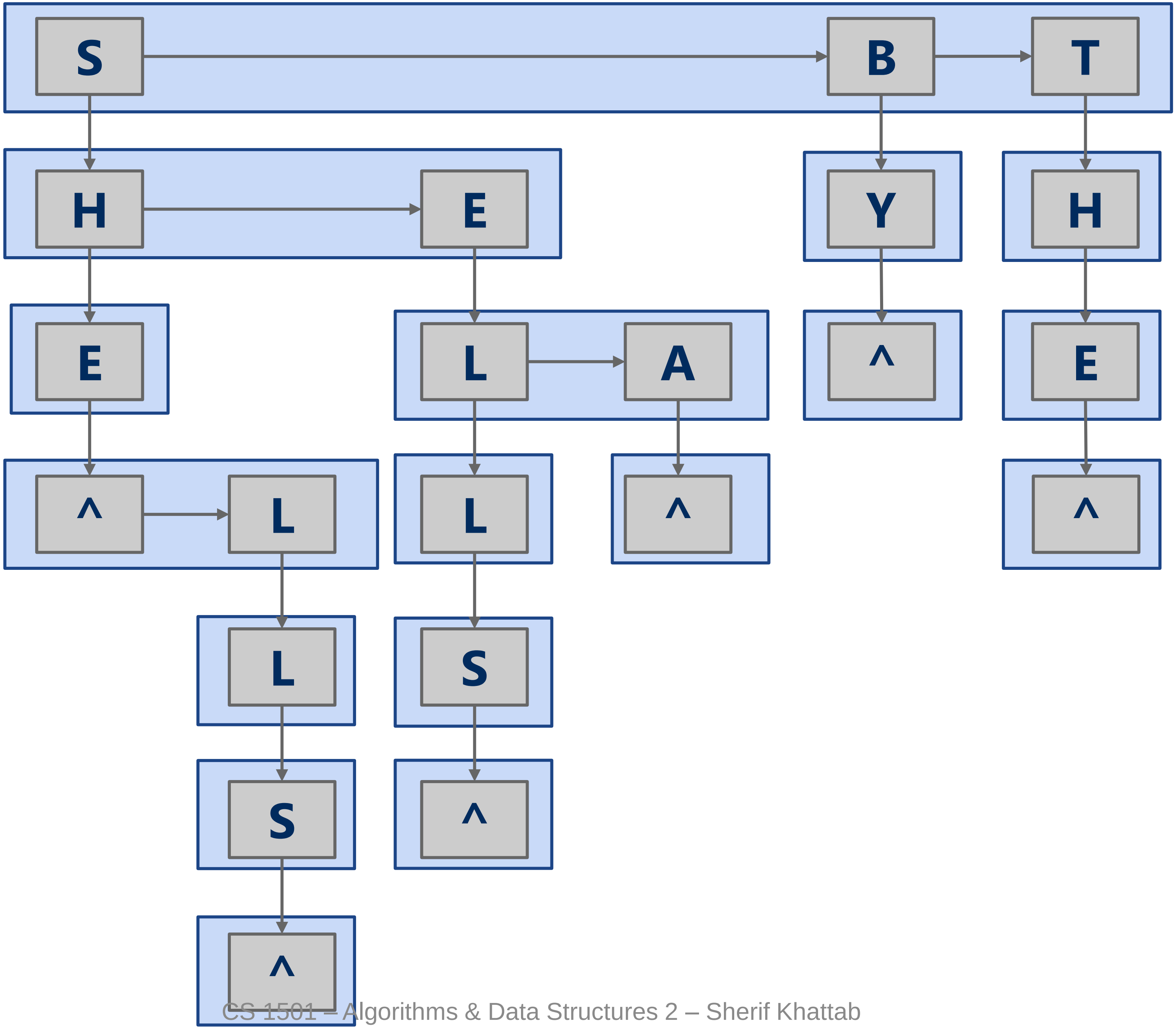
Add a sentinel character (e.g., ^) to each key before add and search

If search encounters null, key not found; otherwise, key is found

Adding to DLB Trie

- if root is null, set root ← new node
- current node ← root
- for each *character c* in the key
  - Search for *c* in the linked list headed at current using sibling links
    - if not found, create a new node and attach as a sibling to the linked list
  - move to child of the found node
    - either recursively or by current ← child
- if at last character of key, insert value into current node and return

DLB Example



DLB analysis

- How does DLB performance differ from R-way tries?
- Which should you use?

	search hit insert	
R-way RST	$\Theta(w)$	
DLB	$\Theta(wR)$	

Runtime Comparison for Search Trees/Tries

	Search hit	Search miss (average)	insert
BST	$\Theta(n)$	$\Theta(\log n)$	$\Theta(n)$
RB-BST	$\Theta(\log n)$	$\Theta(\log n)$	$\Theta(\log n)$
DST	$\Theta(b)$	$\Theta(\log n)$	$\Theta(b)$
RST	$\Theta(b)$	$\Theta(\log n)$	$\Theta(b)$
R-way RST	$\Theta(w)$	$\Theta(\log_R n)$	$\Theta(w)$
DLB	$\Theta(wR)$	$\Theta(\log_R n \cdot R)$	$\Theta(w \cdot R)$

Final notes on Search Tree/Tries

- We did not present an exhaustive look at search trees/tries, just the sampling that we're going to focus on
- Many variations on these techniques exist and perform quite well in different circumstances
  - Ternary search Tries
  - R-way tries without 1-way branching
- See the table at the end of Section 5.2 of the text