University of Pittsburgh

Algorithms and Data Structures 2
CS 1501

Spring 2023

Sherif Khattab

ksm73@pitt.edu

(Slides are adapted from Dr. Ramirez's and Dr. Farnan's CS1501 slides.)

Announcements

- Upcoming Deadlines
  - Homework 6: this Friday @ 11:59 pm
  - Lab 5: Tuesday 2/28 @ 11:59 pm
  - Assignment 2: Friday 3/17 @ 11:59 pm
    - Support video and slides on Canvas
- Midterm Exam on Wednesday 3/1
    - in-person, closed-book
    - Study guide and old exams on Canvas
    - Practice questions on GradeScope (with answers)
- Lost points because autograder or simple mistake?
  - please reach out to Grader TA over Piazza
- Navigating the Panopto Videos
  - Video contents
  - Search in captions

Previous lecture

- Huffman Compression

This Lecture

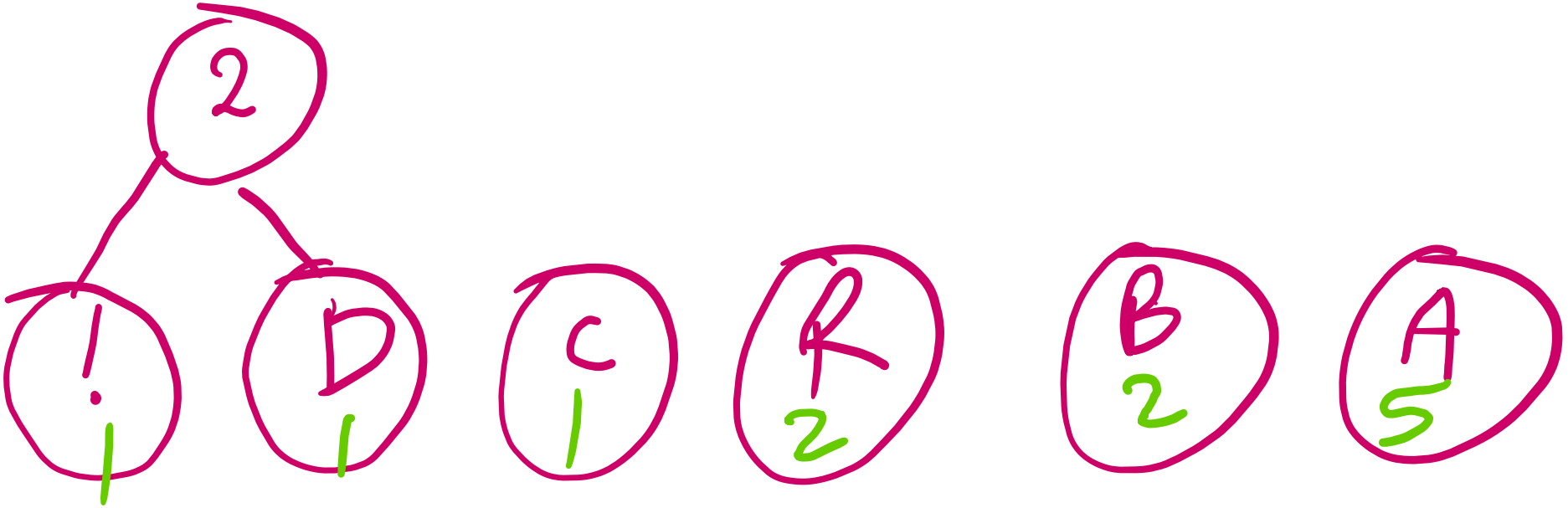
- Huffman Compression
- Run-length Encoding
- LZW

Huffman Compression Example

- Build a tree for “ABRACADABRA!”
- file size: *n* = *12*
- no. of unique characters: *K* = *6*

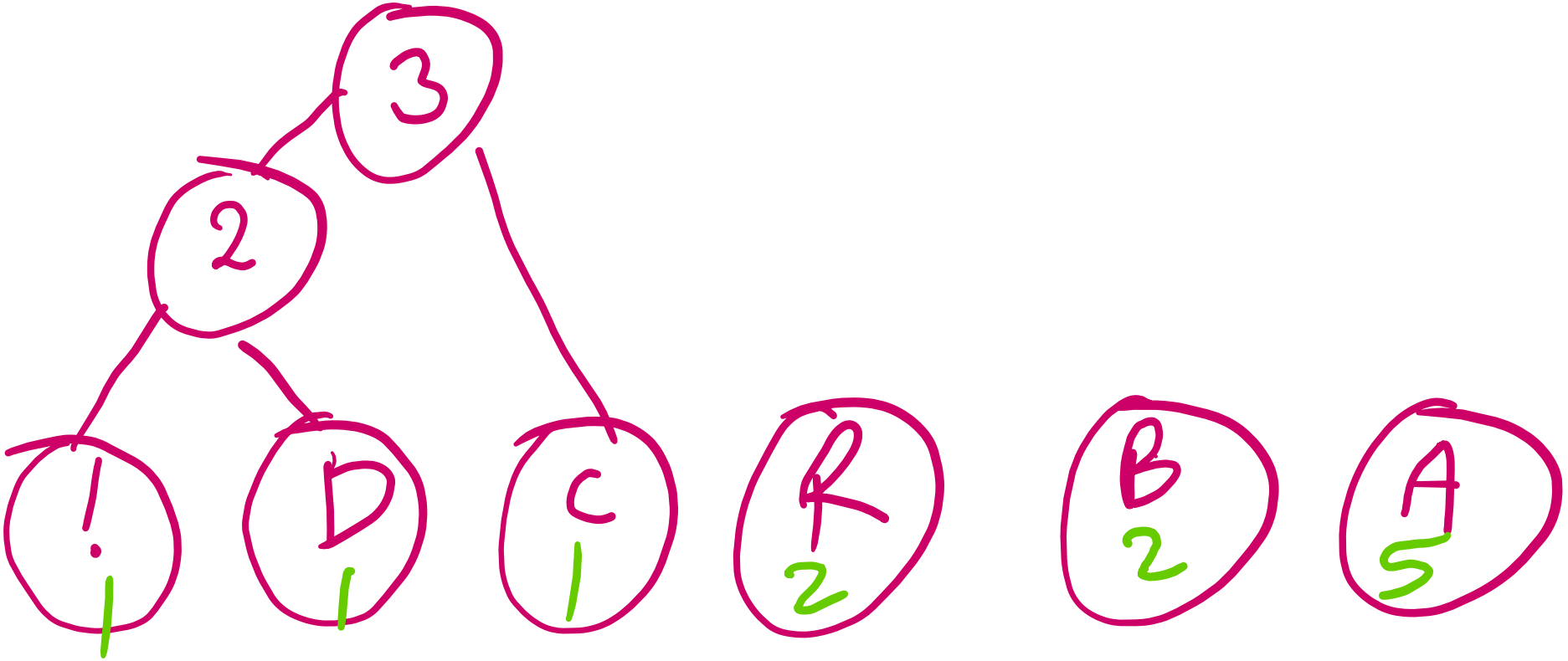
Example

- Build a tree for “ABRACADABRA!”



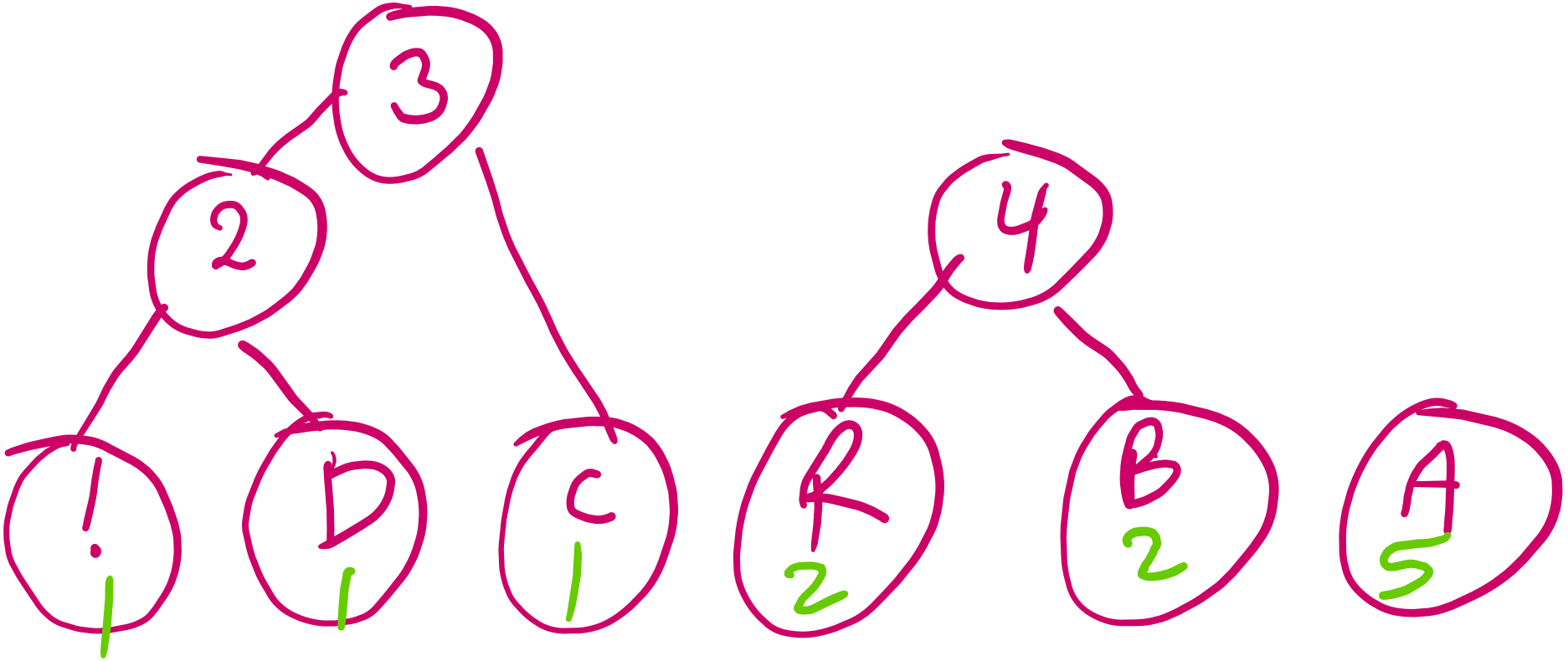
Example

- Build a tree for “ABRACADABRA!”



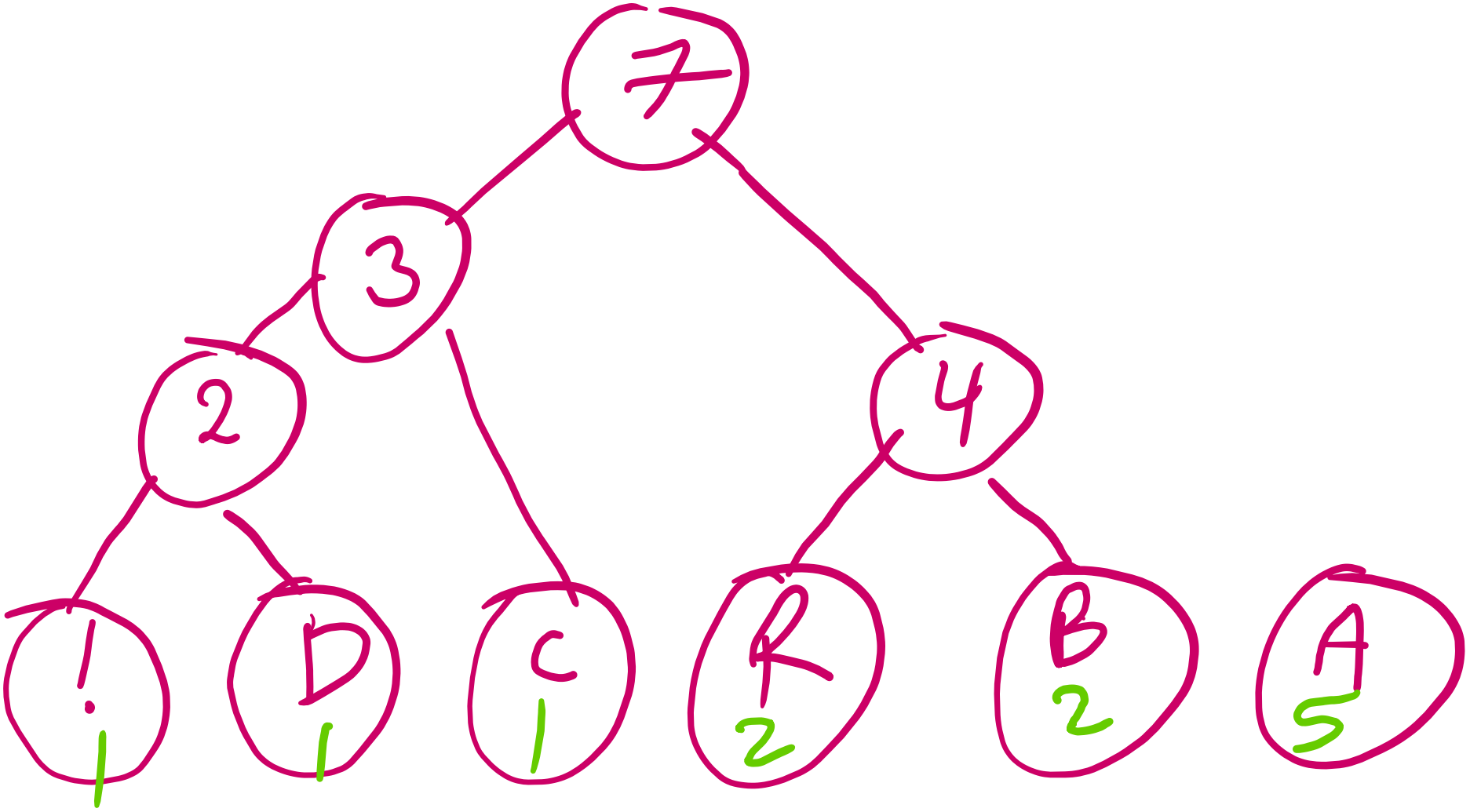
Example

- Build a tree for “ABRACADABRA!”



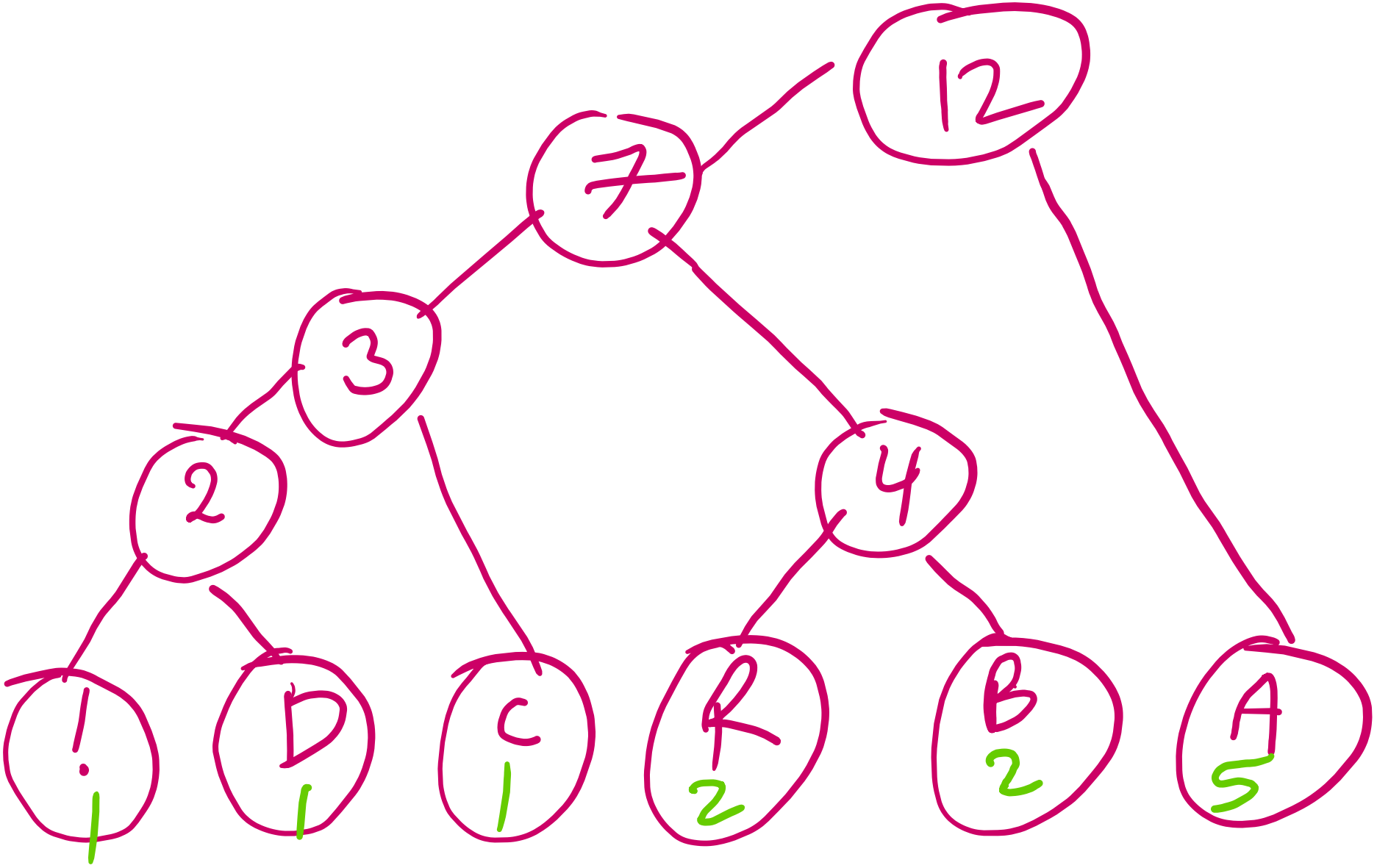
Example

- Build a tree for “ABRACADABRA!”



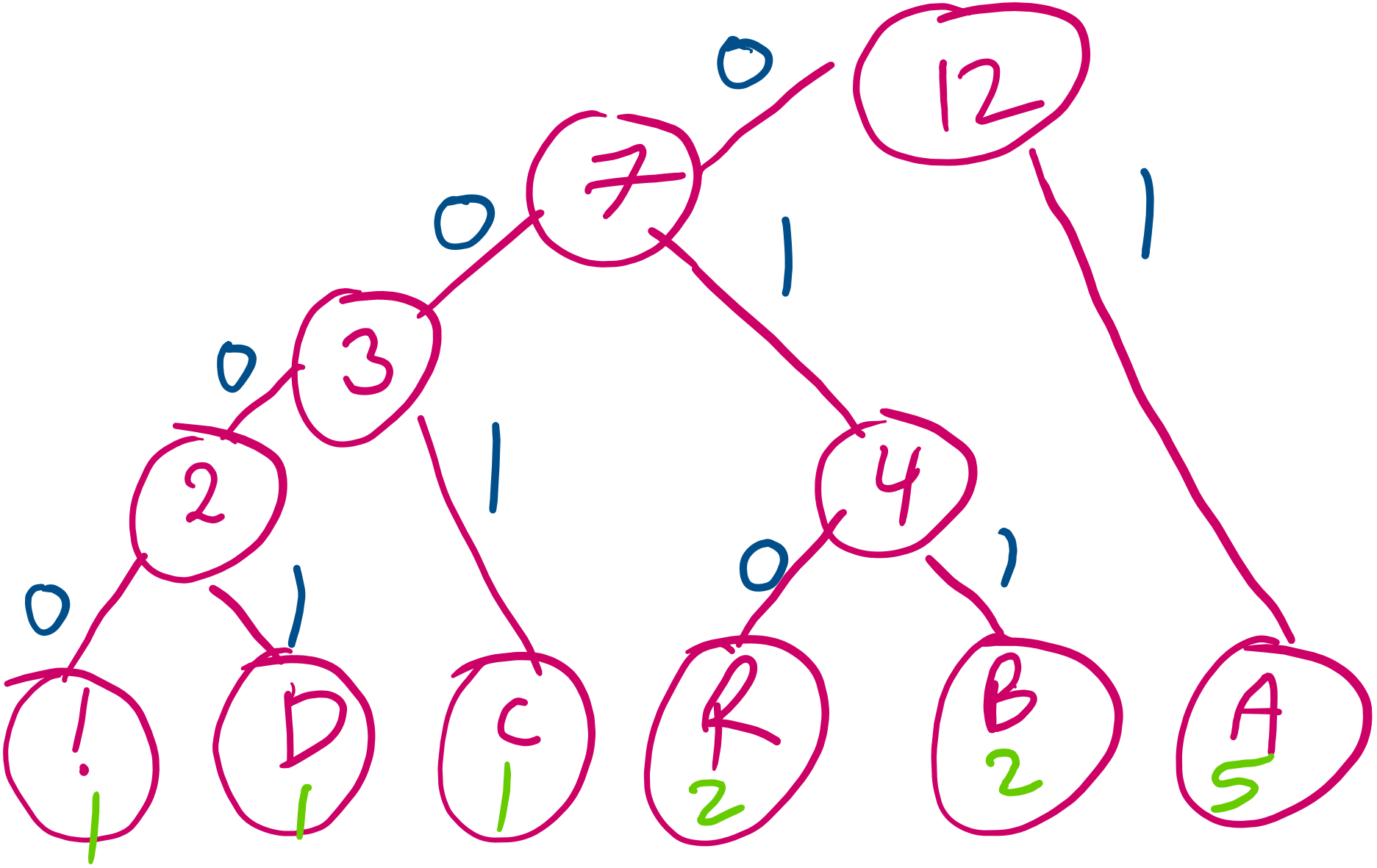
Example

- Build a tree for “ABRACADABRA!”



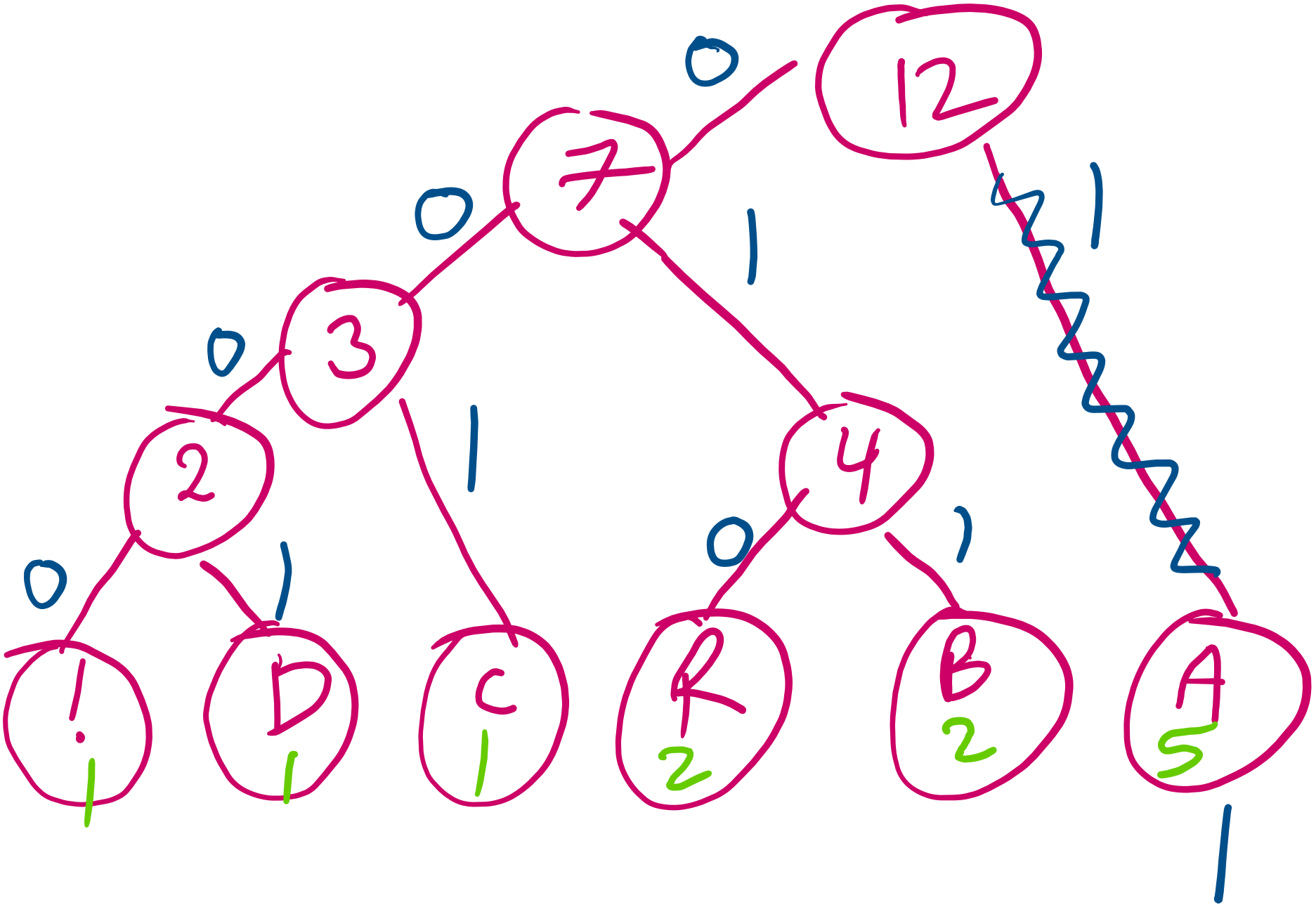
Example

- Build a tree for “ABRACADABRA!”



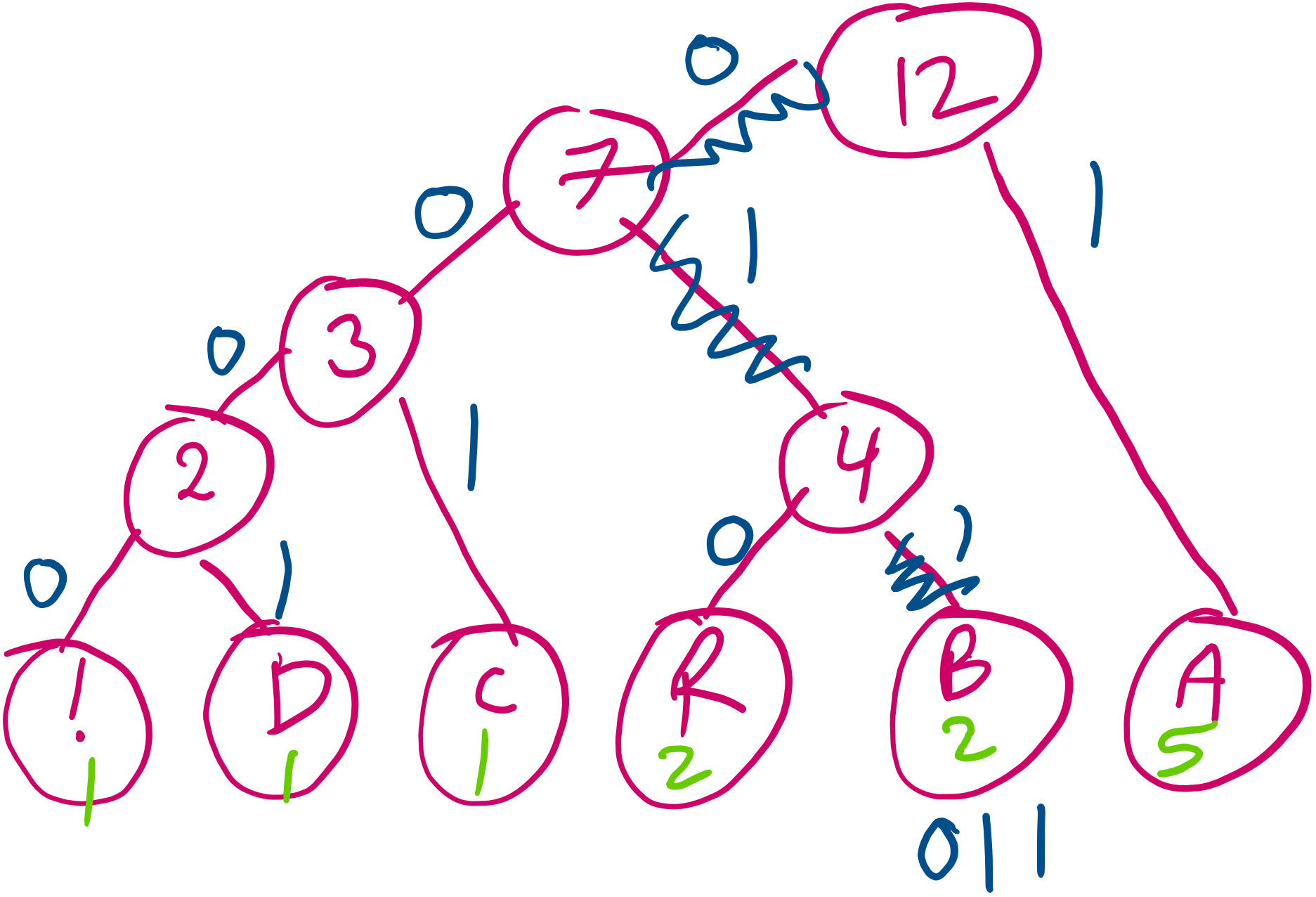
Example

- Build a tree for “ABRACADABRA!”



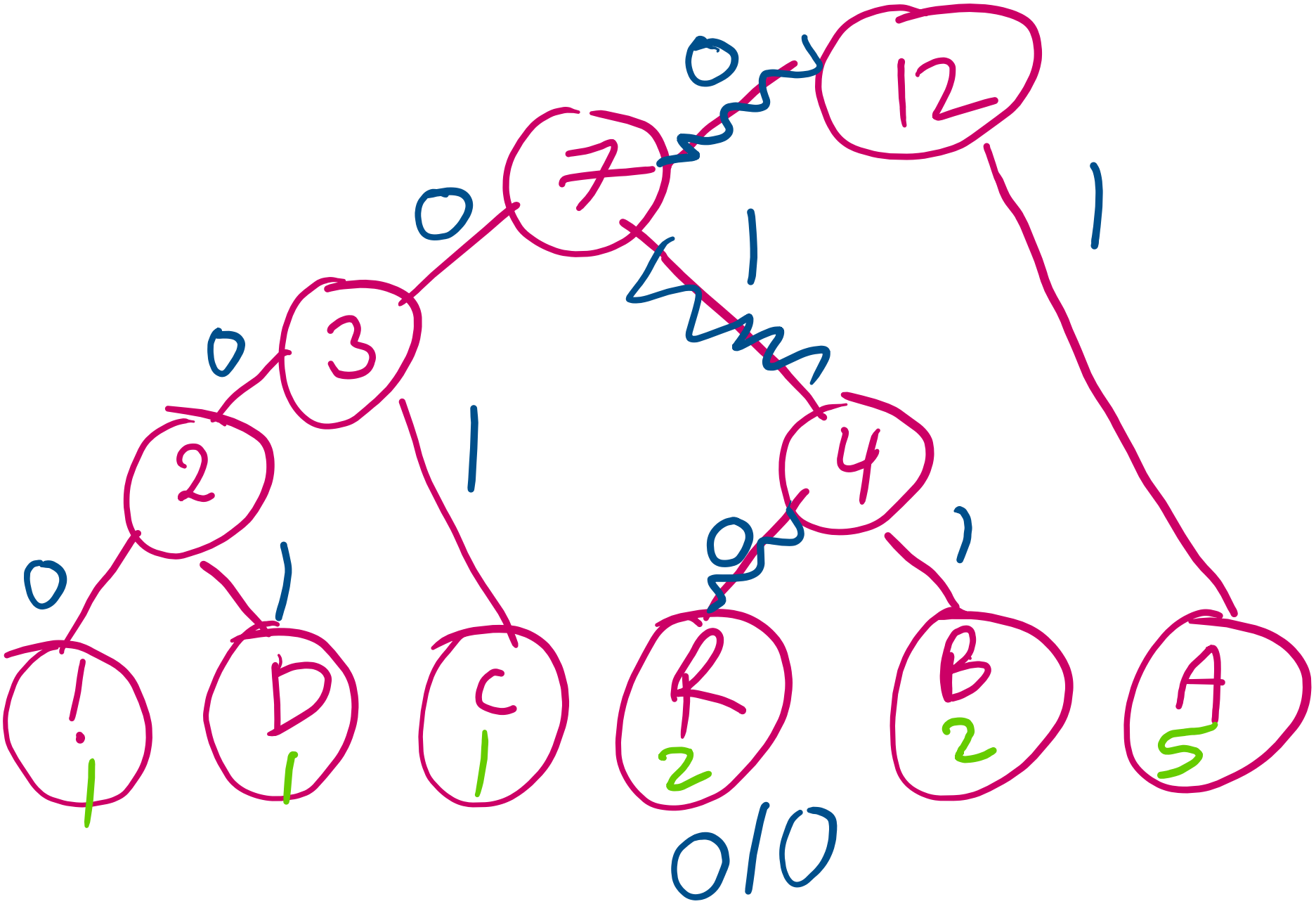
Example

- Build a tree for “ABRACADABRA!”



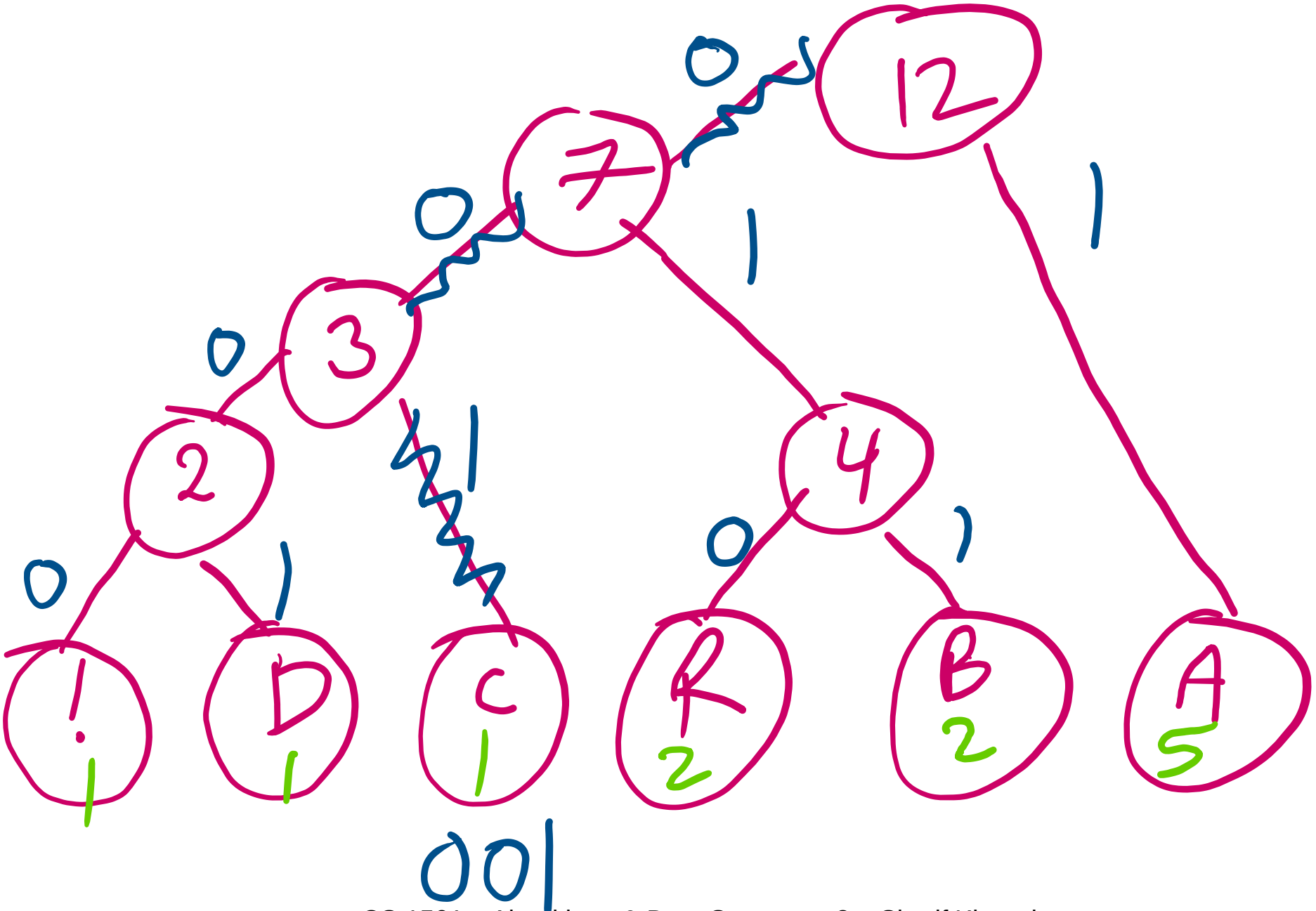
Example

- Build a tree for “ABRACADABRA!”



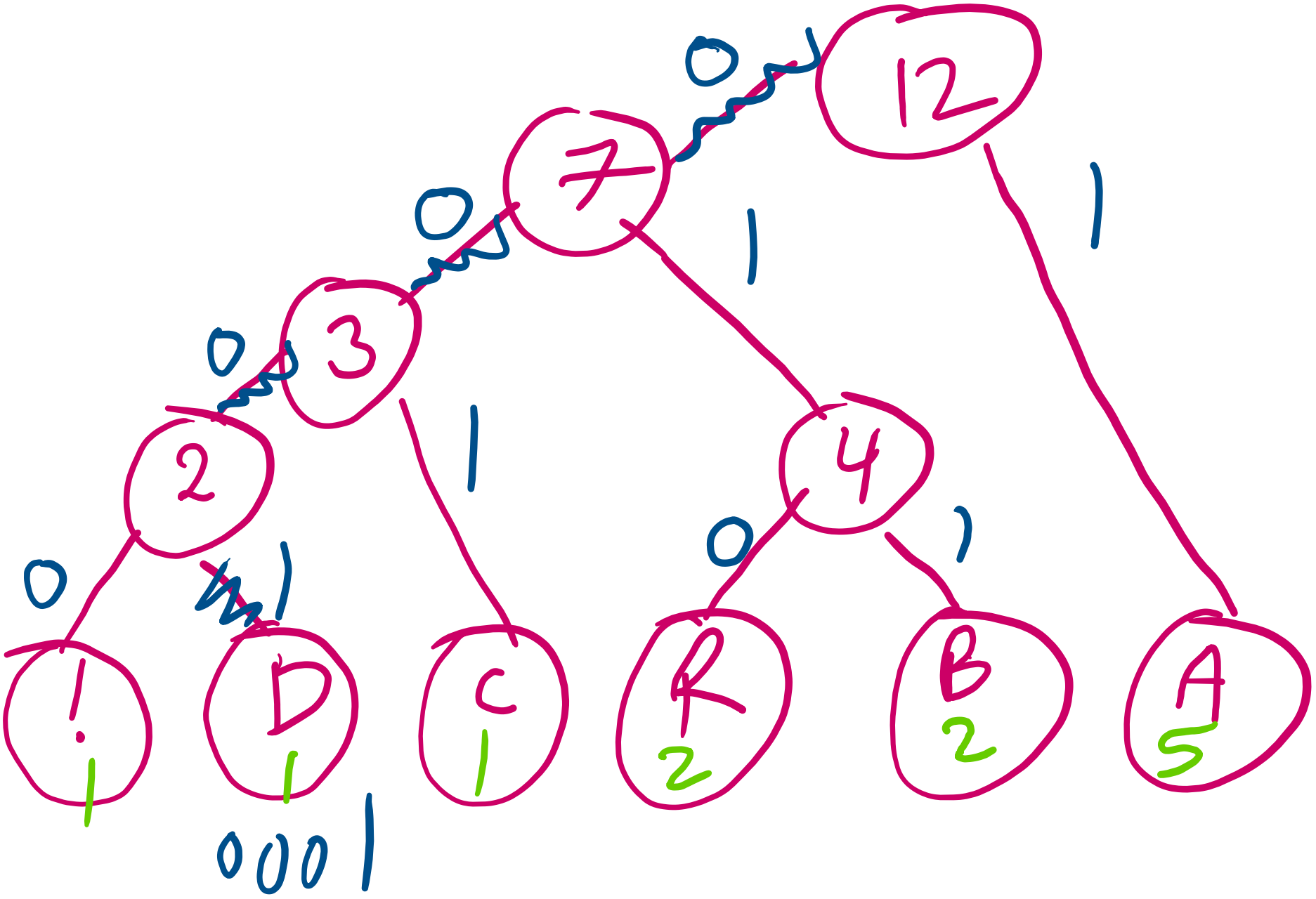
Example

- Build a tree for “ABRACADABRA!”



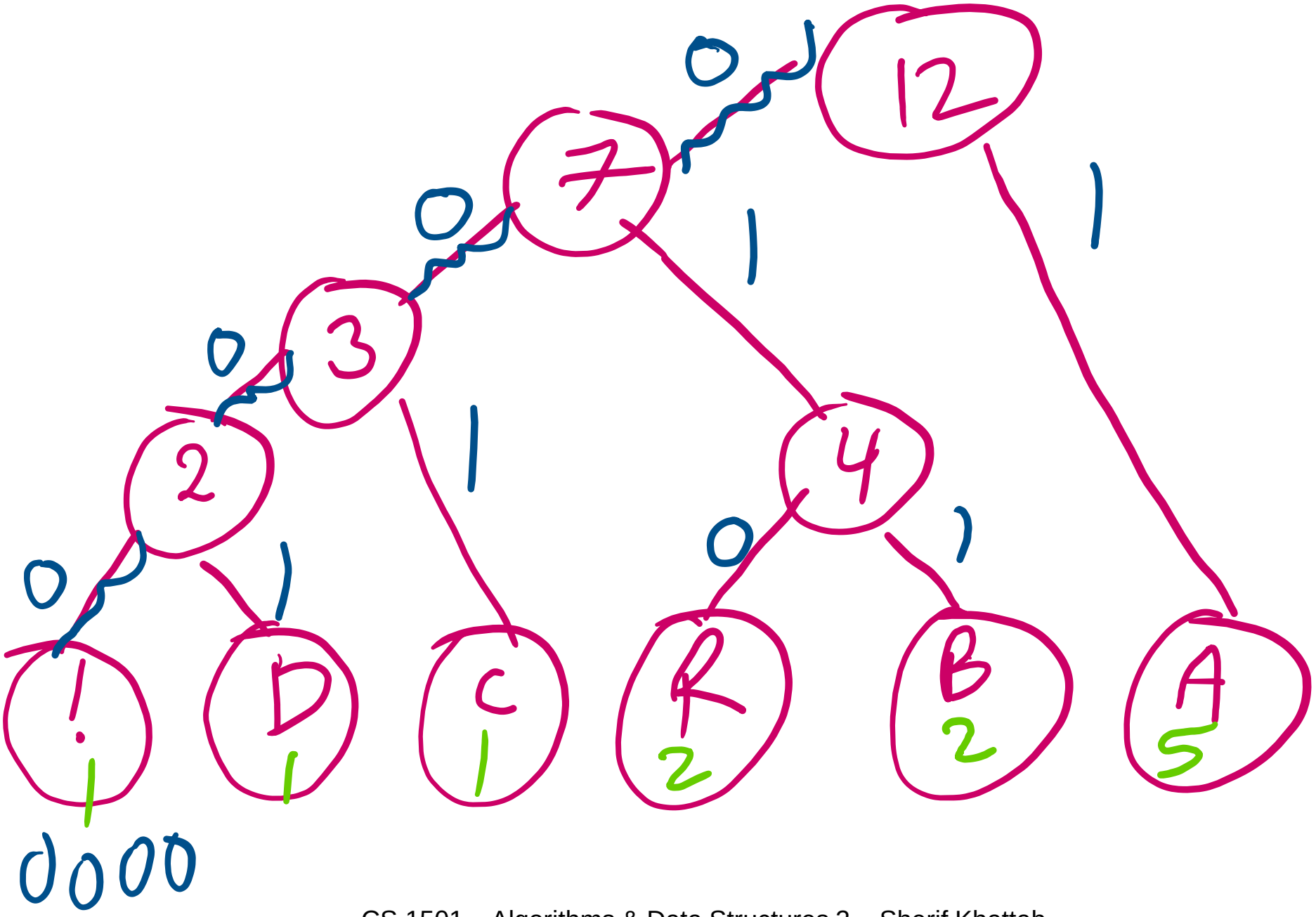
Example

- Build a tree for “ABRACADABRA!”



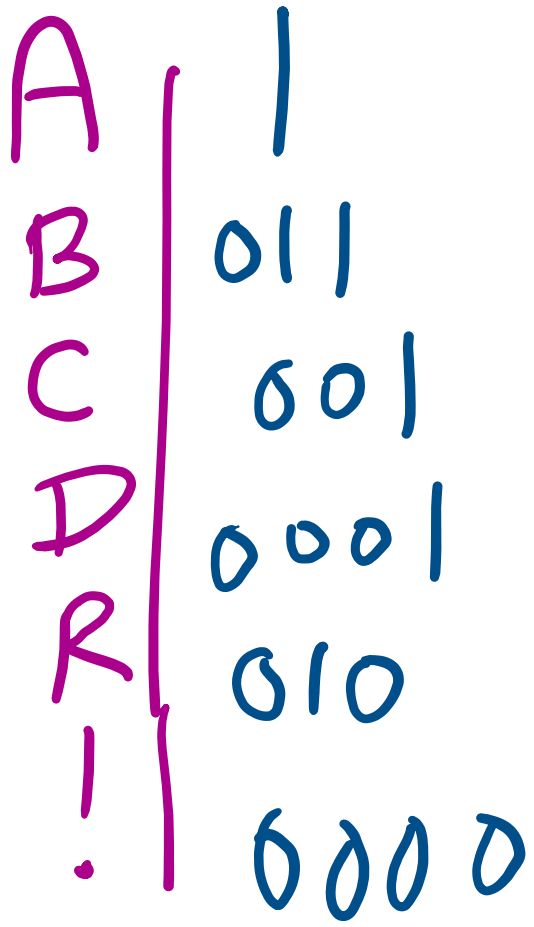
Example

- Build a tree for “ABRACADABRA!”



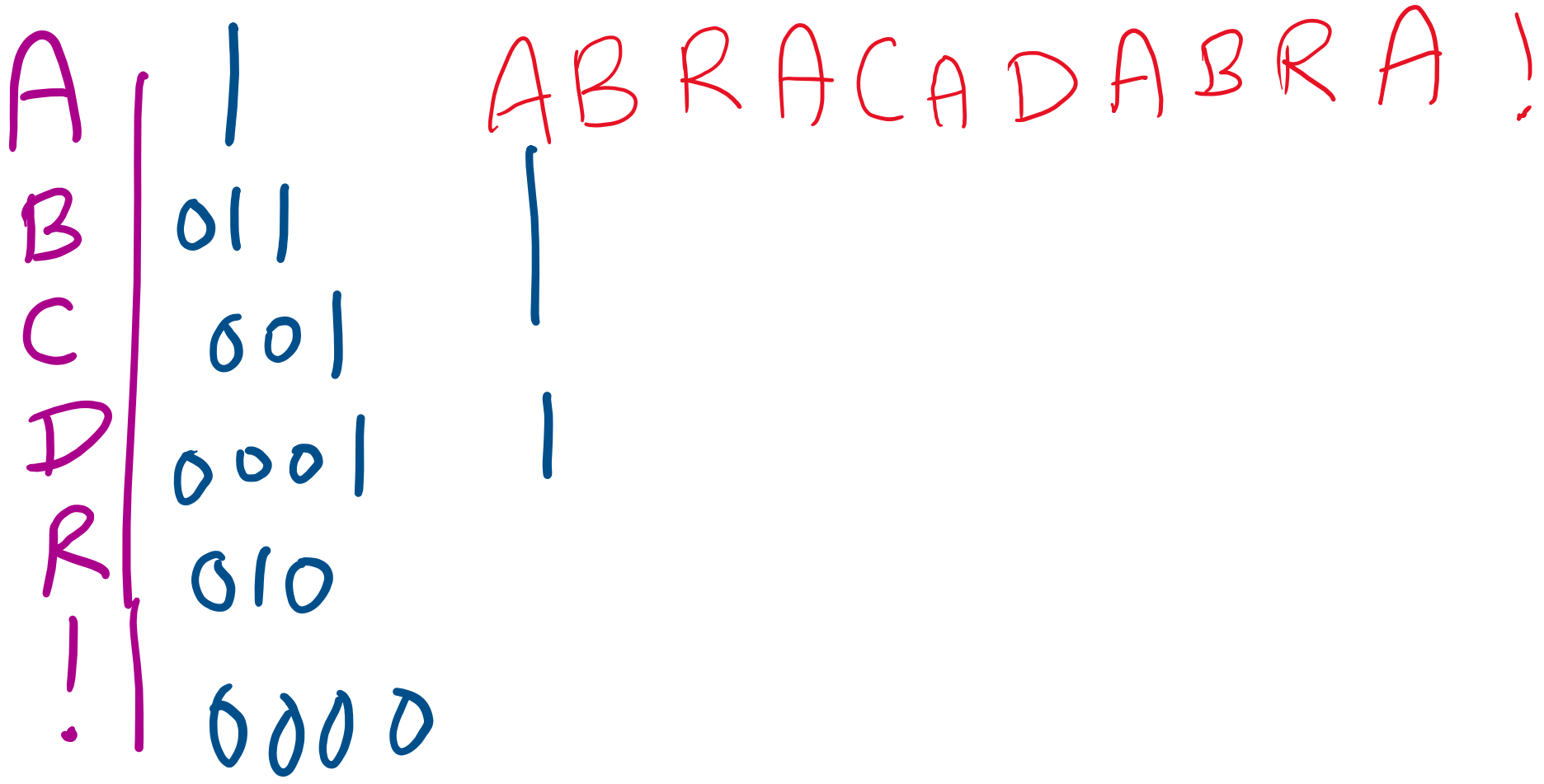
Example

- Build a tree for “ABRACADABRA!”



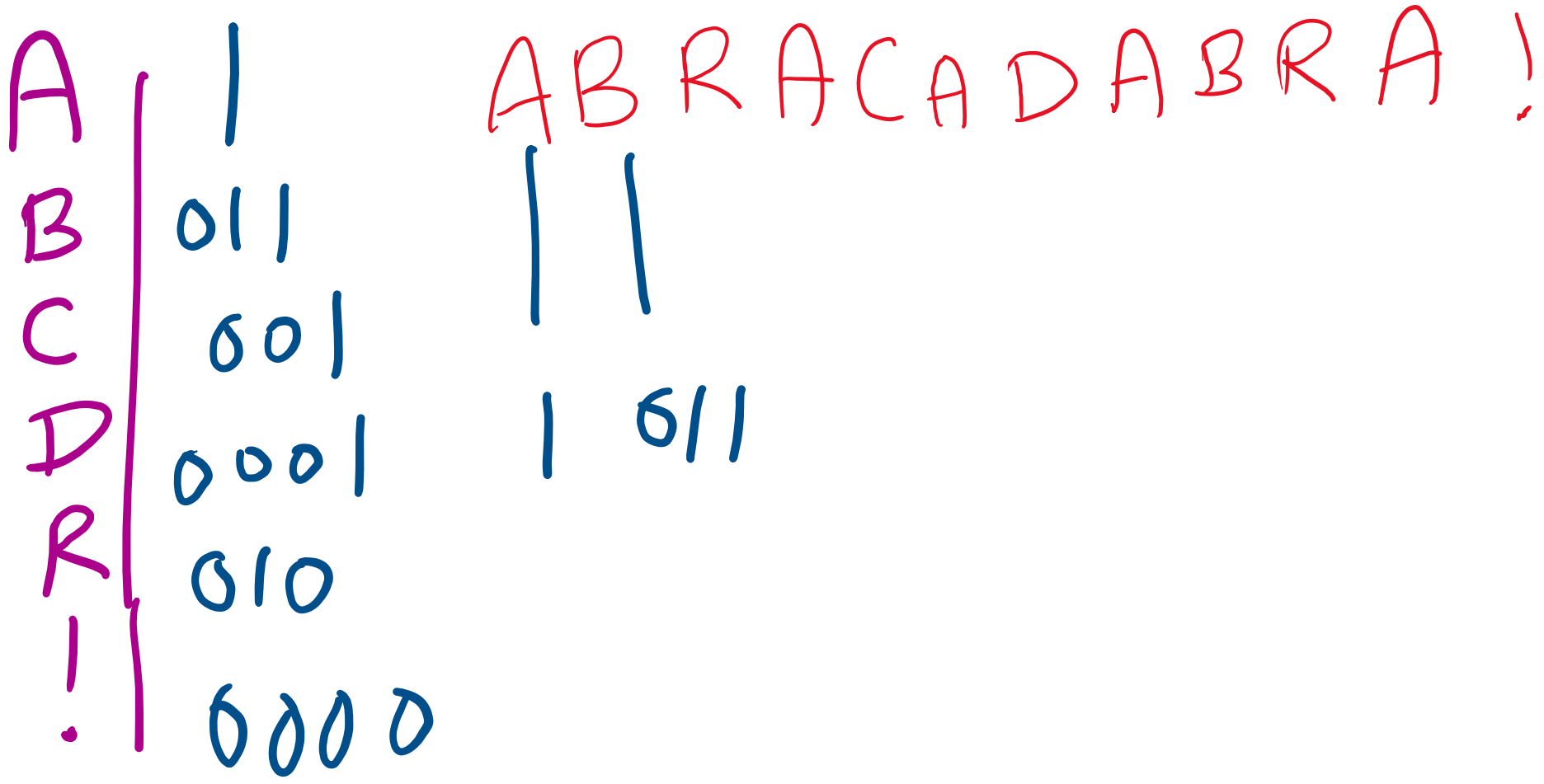
Example

- Build a tree for “ABRACADABRA!”



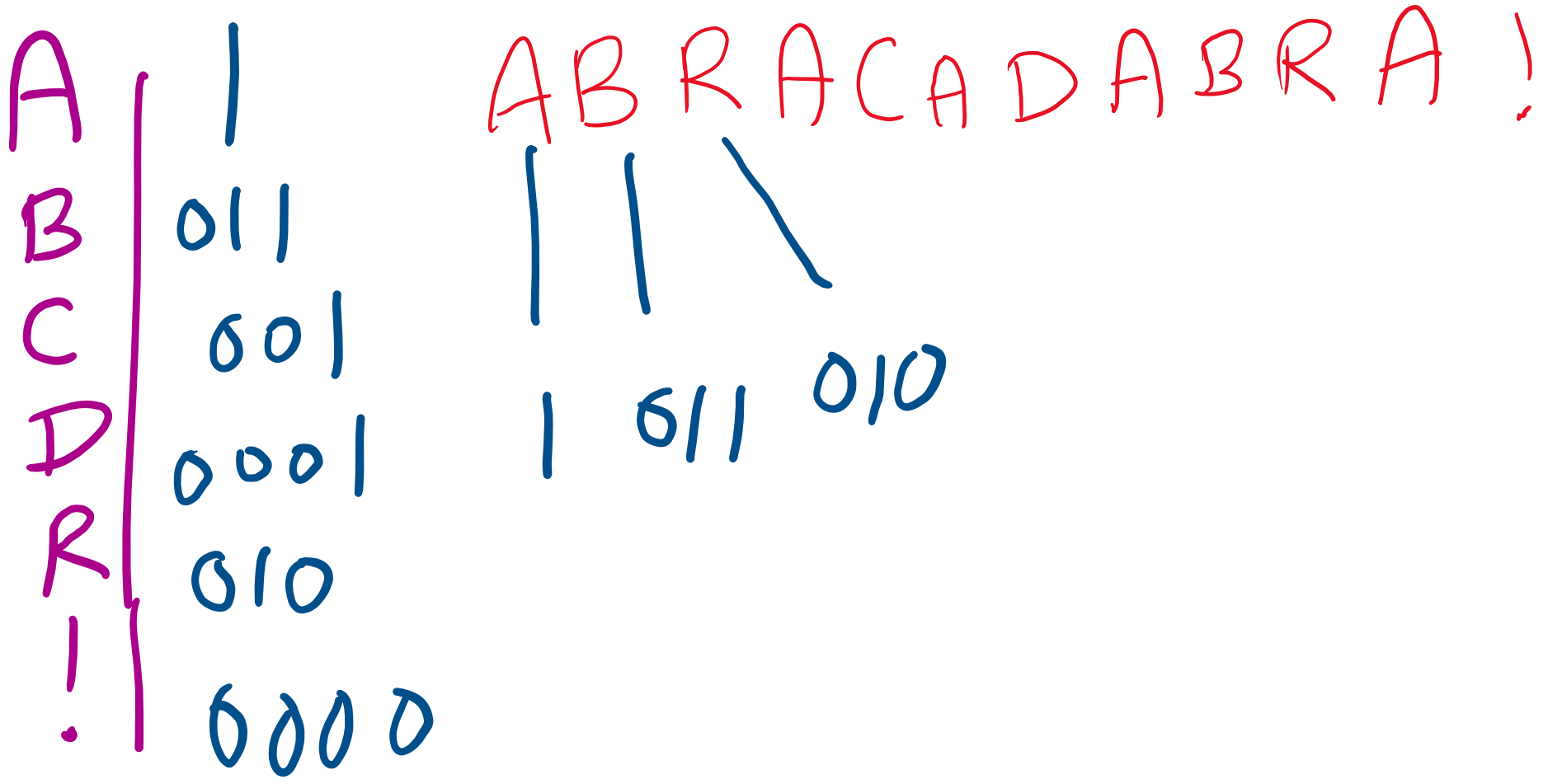
Example

- Build a tree for “ABRACADABRA!”



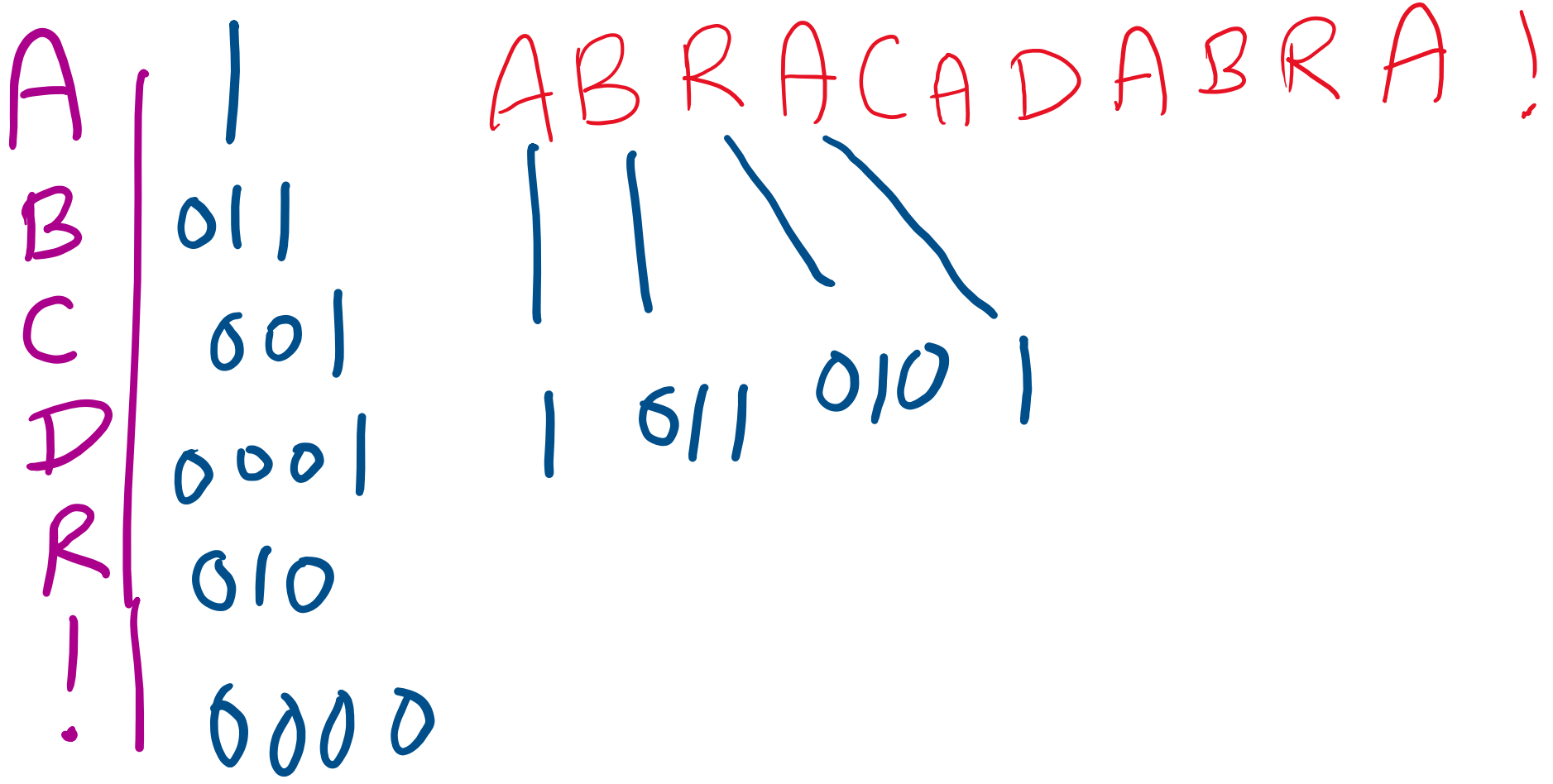
Example

- Build a tree for “ABRACADABRA!”



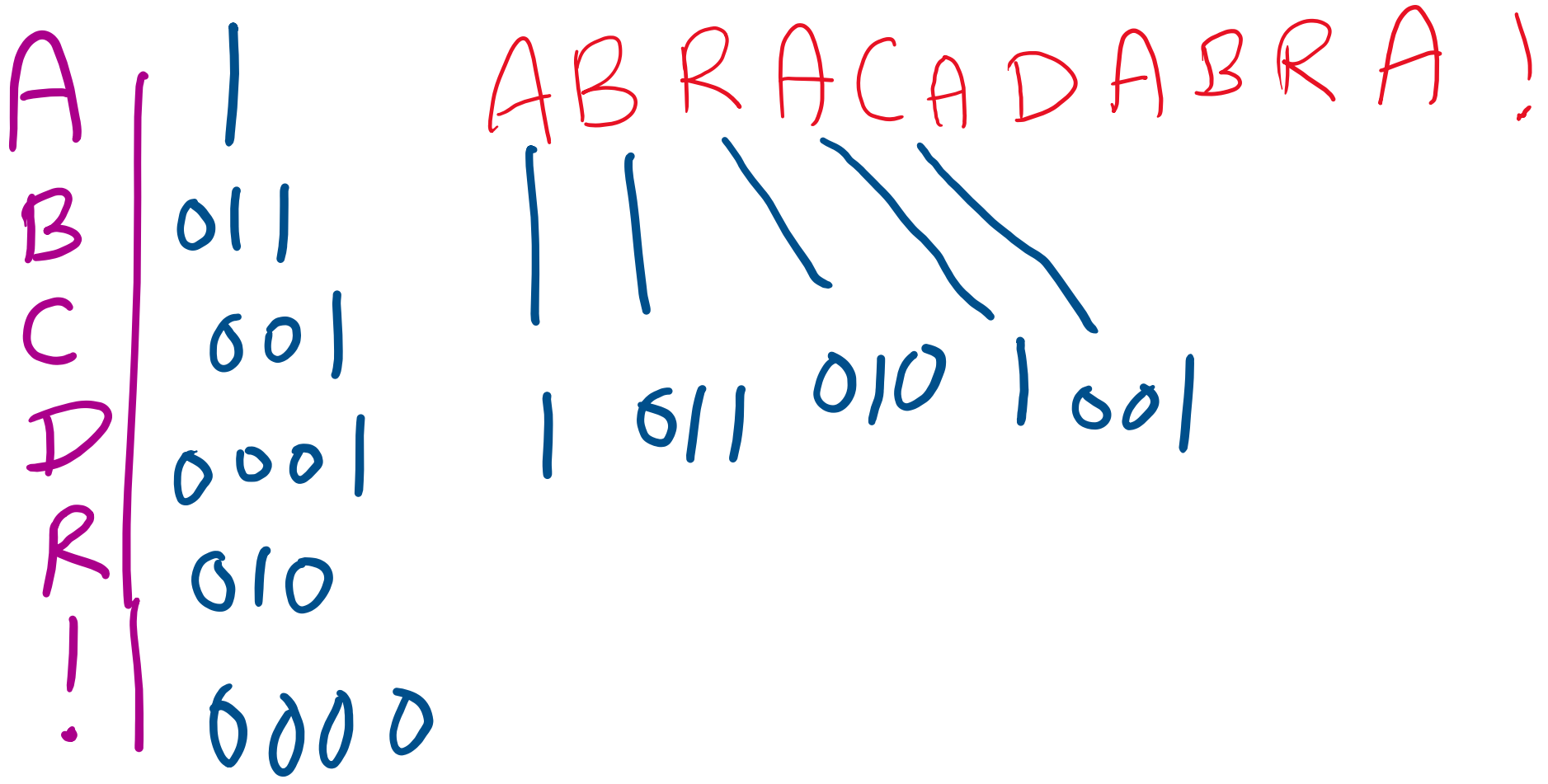
Example

- Build a tree for “ABRACADABRA!”



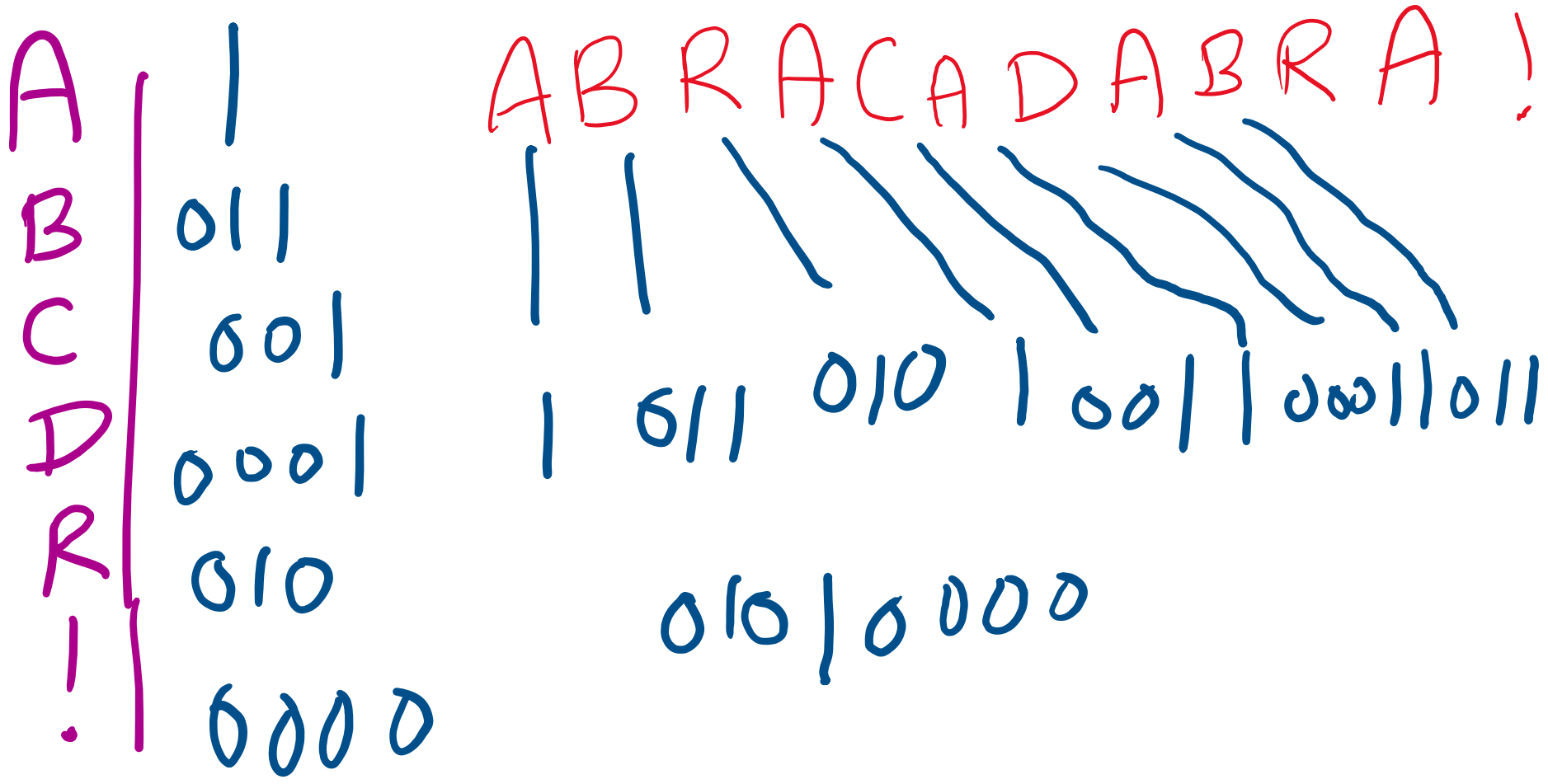
Example

- Build a tree for “ABRACADABRA!”



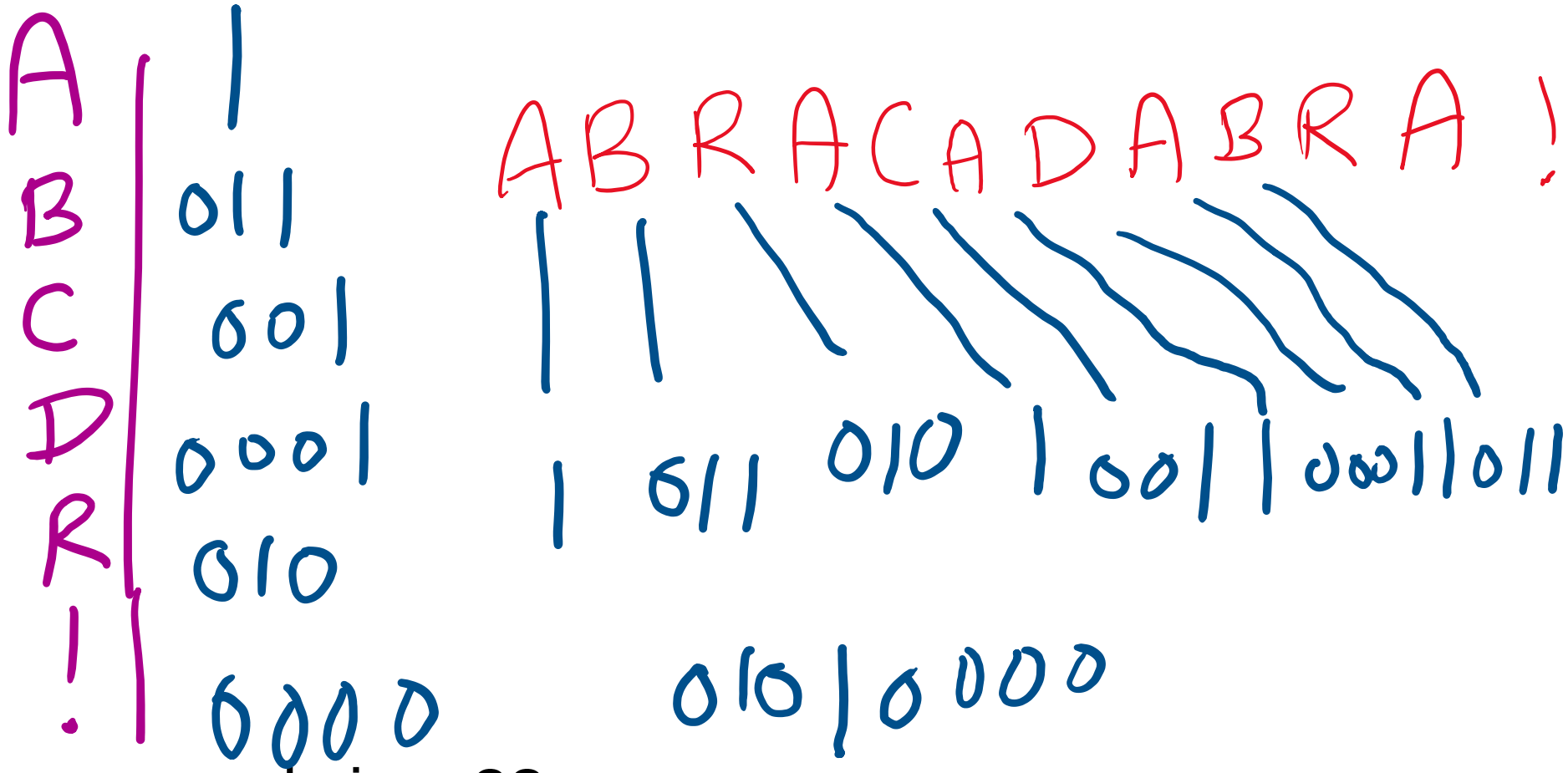
Example

- Build a tree for “ABRACADABRA!”



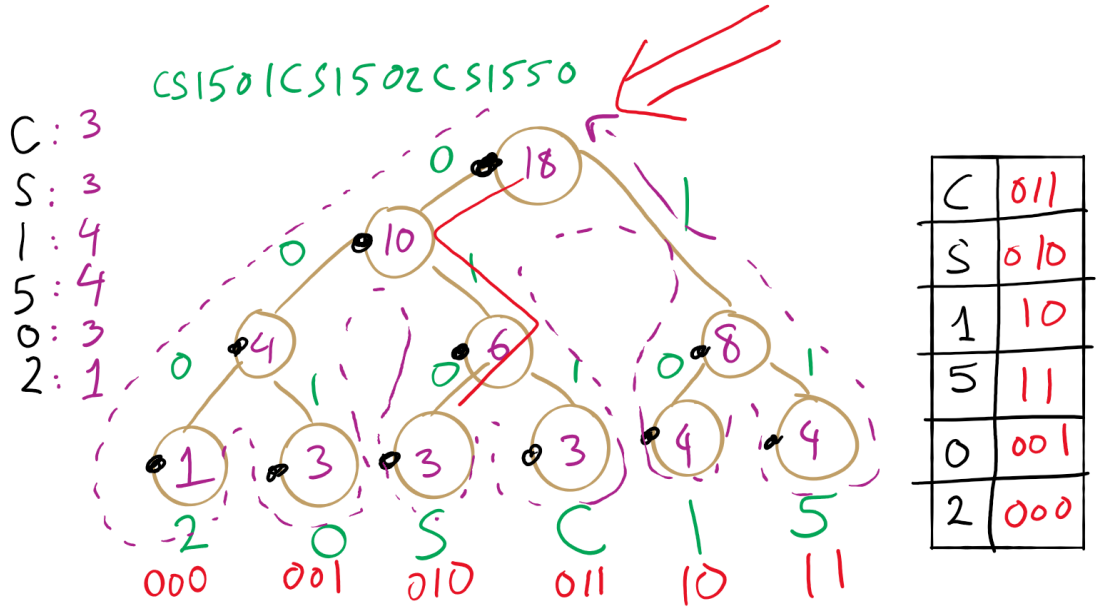
Example

- Build a tree for “ABRACADABRA!”



- Compressed size: 28
- Original size: 12*8 = 96 bits
- Compression ratio = 28/96

Huffman Compression Example



C	011
S	010
1	10
5	11
0	001
2	000

C S 1 5 0 ! C S 1 5 0 2 C S 1 5 5 0

011 010 10 11 001 10 011 010 10 11 001 000 011 010 10 11 11 001

Trie

0001 [ASCII for 2] 1 [ASCII for 0] 1 [ASCII for S] - - — — —

CS1

Implementation concerns

- Need to efficiently be able to select lowest weight trees to merge when constructing the trie
  - Can accomplish this using a *priority queue*
- Need to be able to read/write bitstrings!
  - Unless we pick multiples of 8 bits for our codewords, we will need to read/write *fractions* of bytes for our codewords
    - We're not actually going to do I/O on fraction of bytes
    - We'll maintain a buffer of bytes and perform bit processing on this buffer
    - See BinaryStdIn.java and BinaryStdOut.java

We need to read and write individual bits: Binary I/O

```
private static void writeBit(boolean bit) {
      // add bit to buffer by shifting in a zero
      buffer <<= 1;
      if (bit) buffer |= 1;  //then turning it to one if needed
      // if buffer is full (8 bits), write out as a single byte
      N++;
      if (N == 8) clearBuffer();
}
```

```
writeBit(true);
writeBit(false);
writeBit(true);
writeBit(false);
writeBit(false);
writeBit(false);
writeBit(false);
writeBit(true);
```

buffer: **00000000**

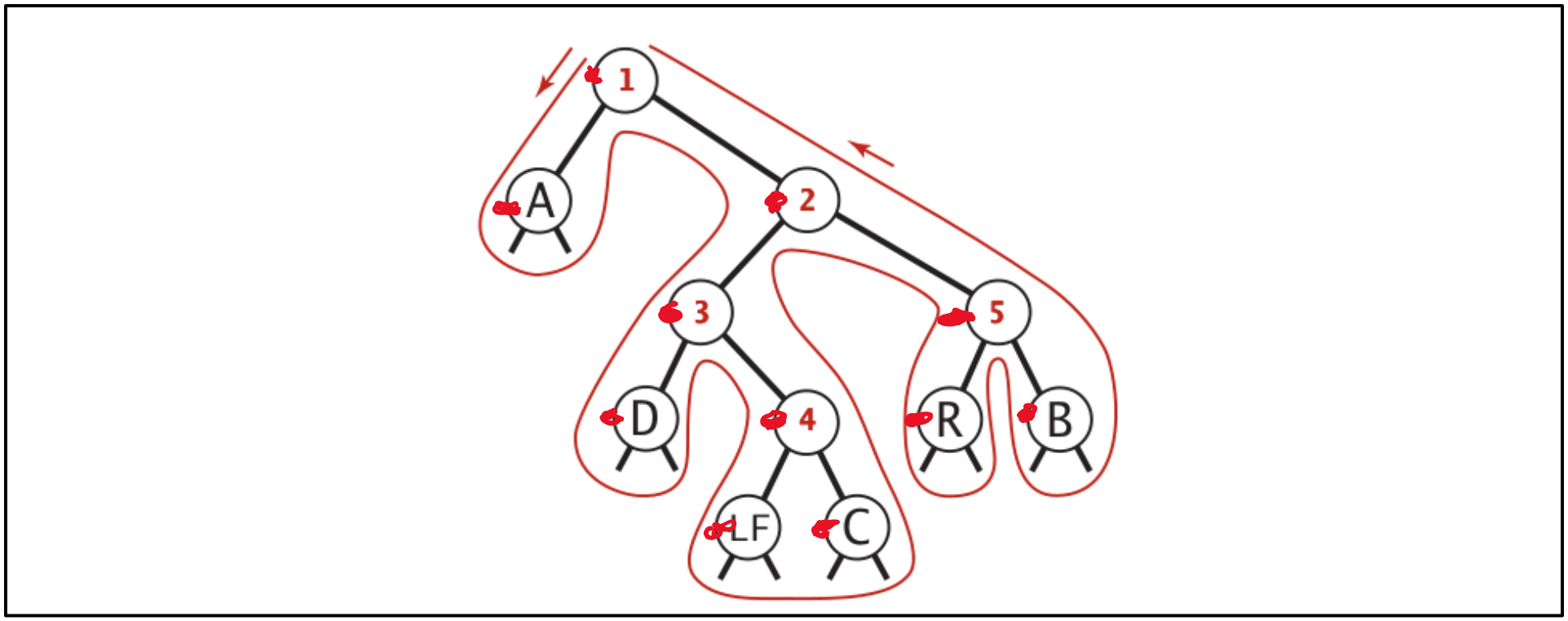
N: **0**

Further implementation concerns

- To encode, we'll need to read in characters and output codes
- To decode, we'll need to read in codes and output characters
- Sounds like we'll need a symbol table!
  - What implementation would be best?
    - Same for encoding and decoding?
  - Note that this means we need access to the trie to expand a compressed file!
    - Let's store the trie in the compressed file
    - how?

Representing tries as bitstrings

Preorder traversal

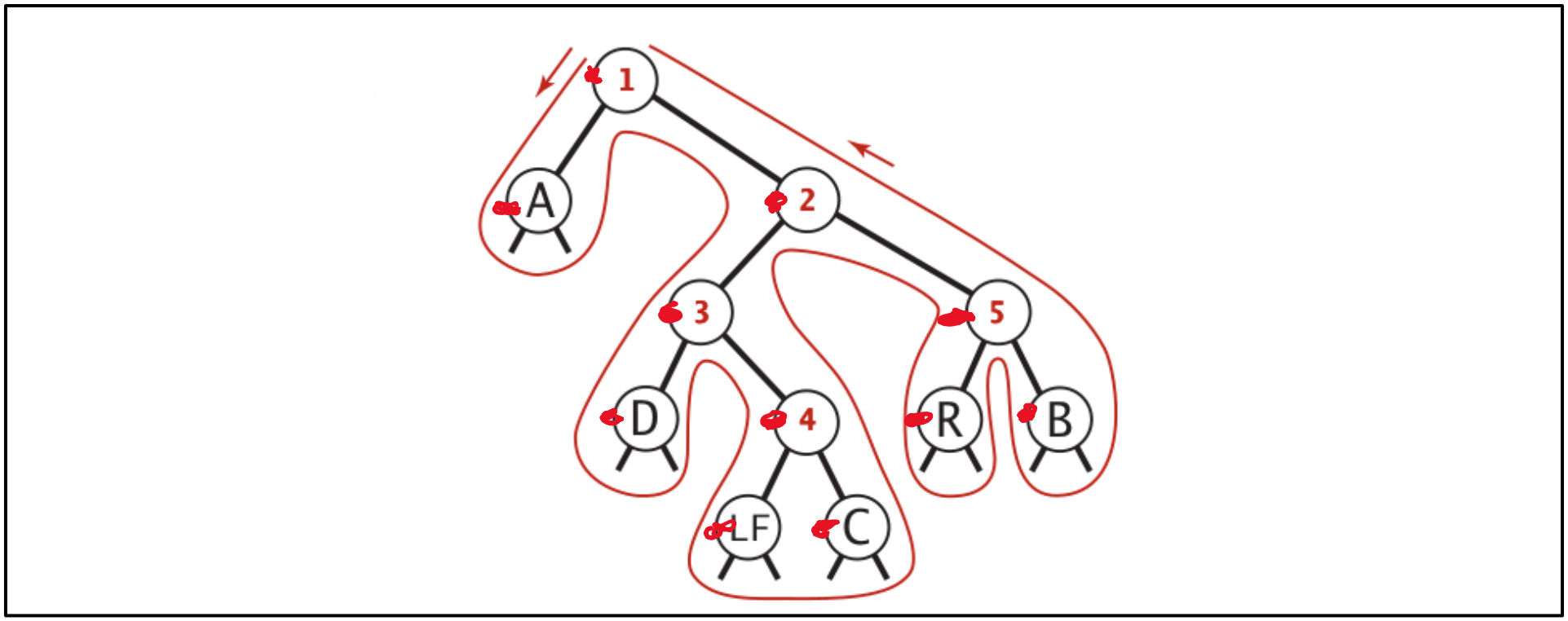


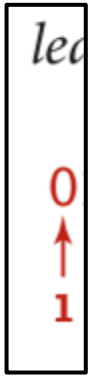
internal node → 0

leaf node → 1 followed by ASCII code of char inside

Representing tries as bitstrings

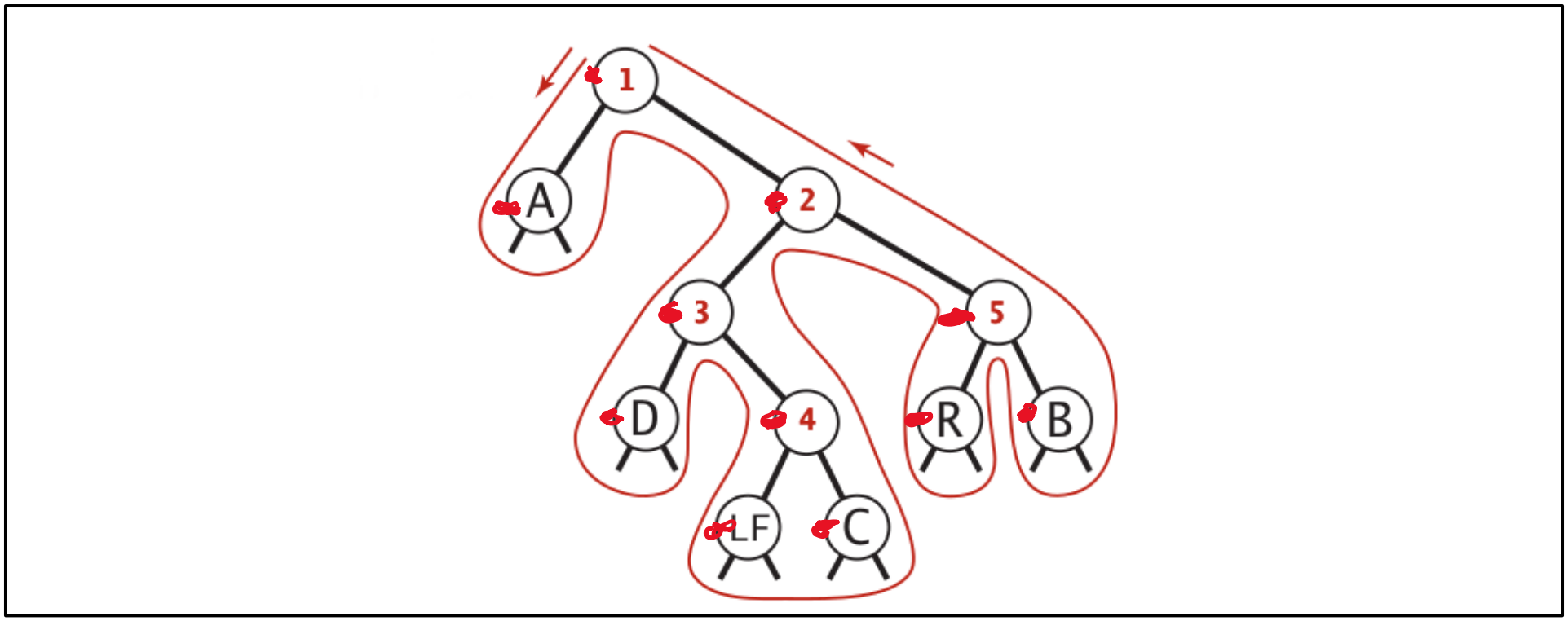
Preorder traversal

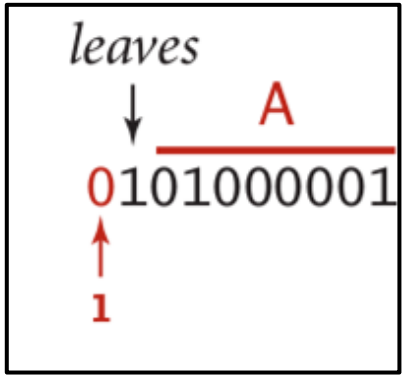




Representing tries as bitstrings

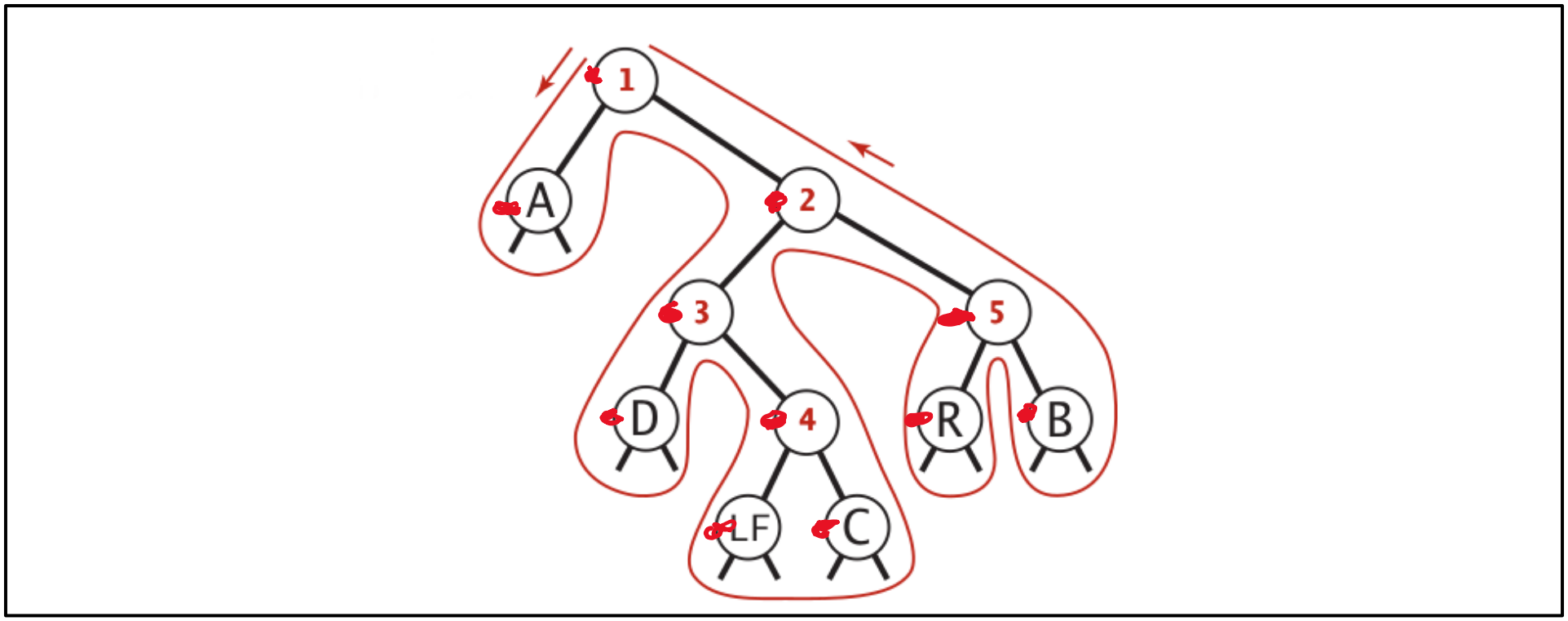
Preorder traversal

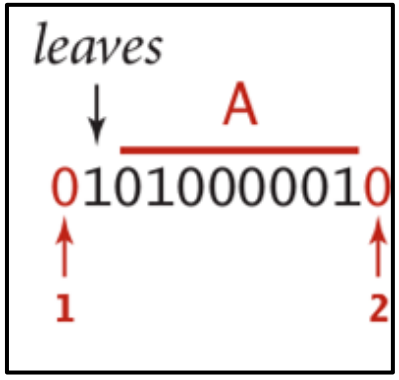




Representing tries as bitstrings

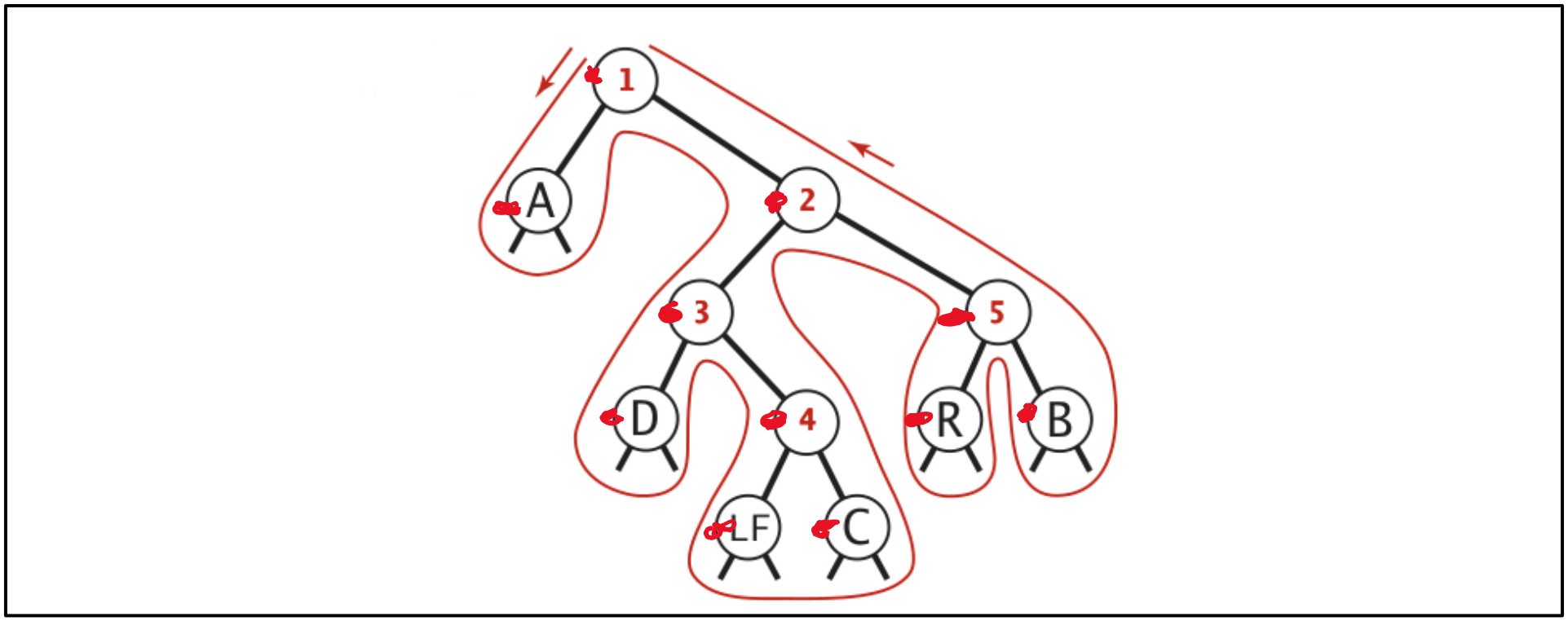
Preorder traversal

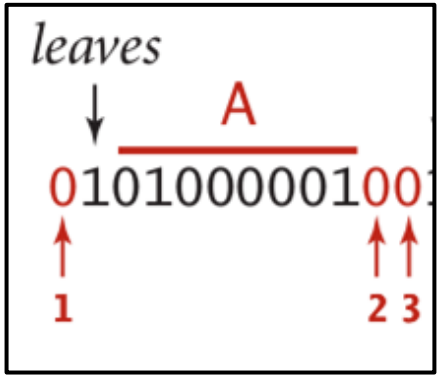




Representing tries as bitstrings

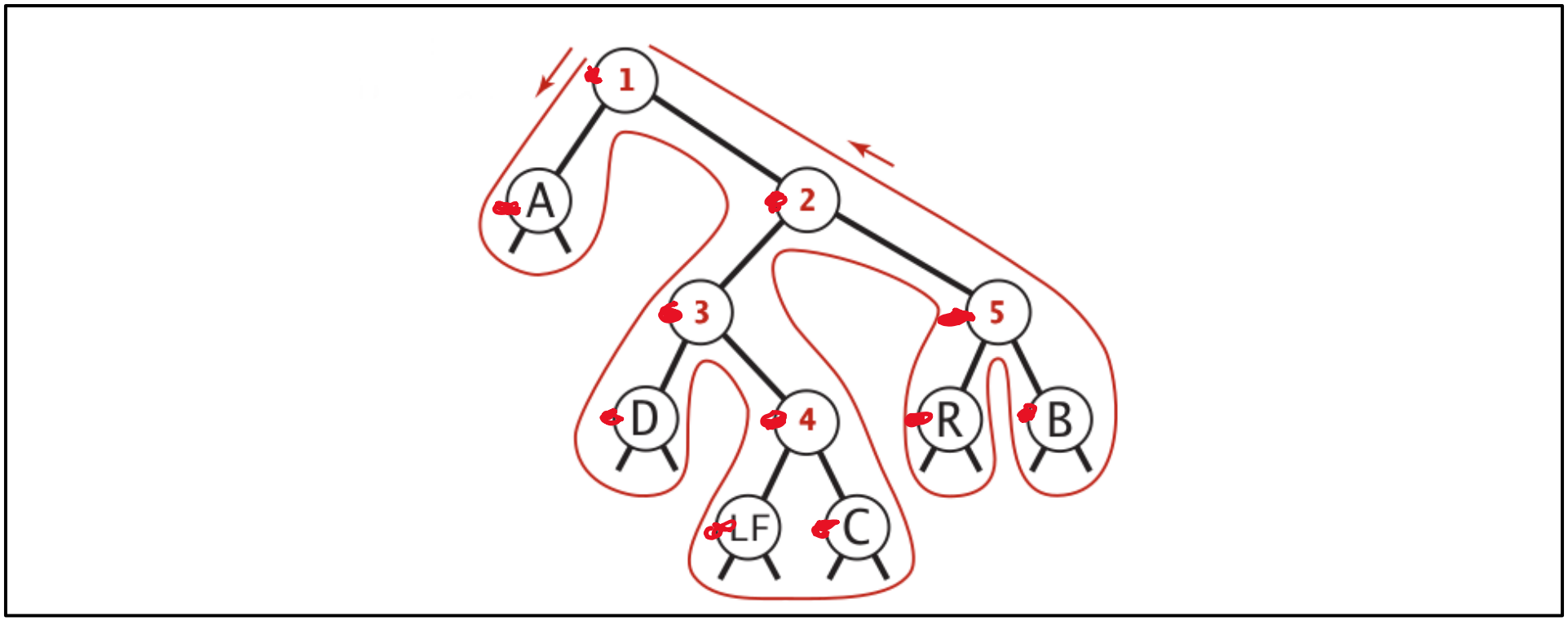
Preorder traversal

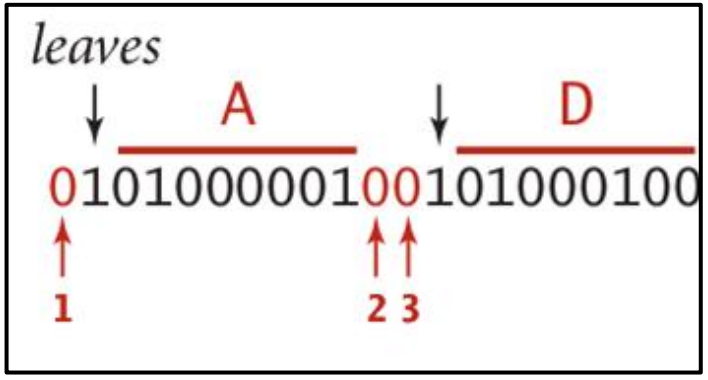




Representing tries as bitstrings

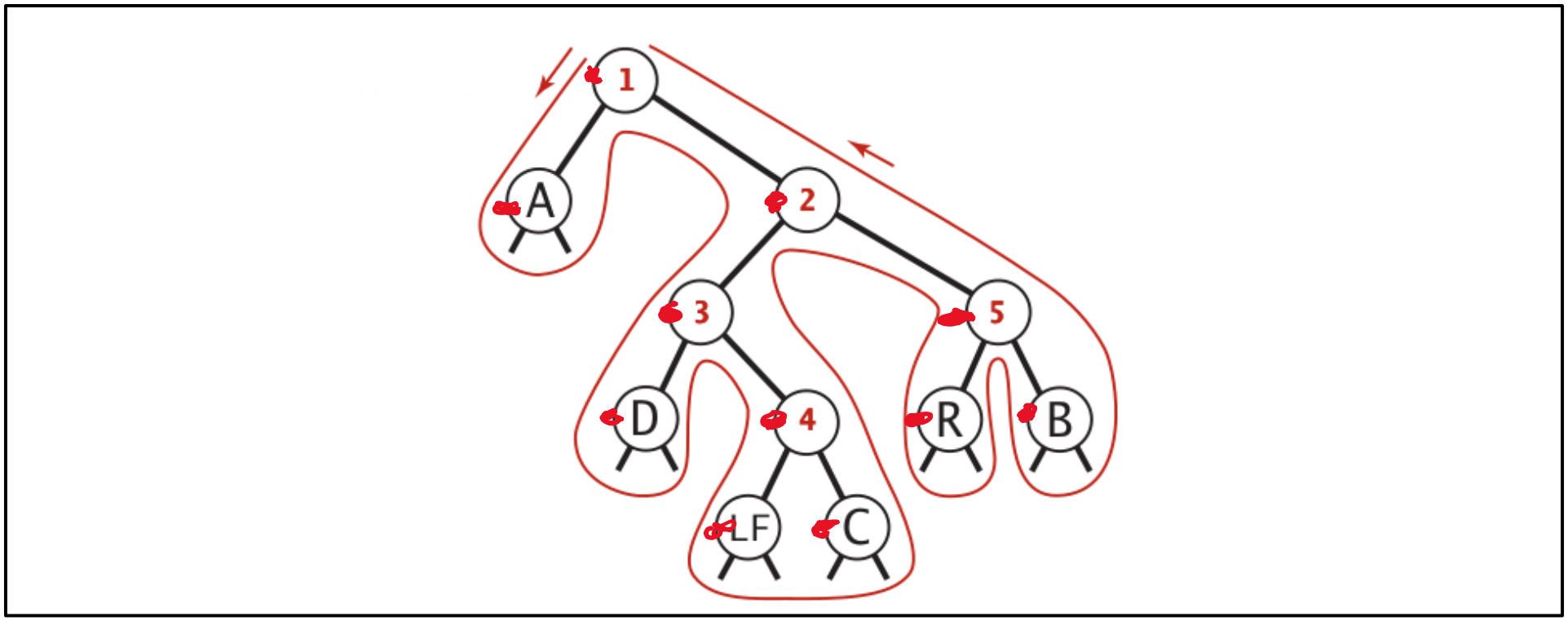
Preorder traversal

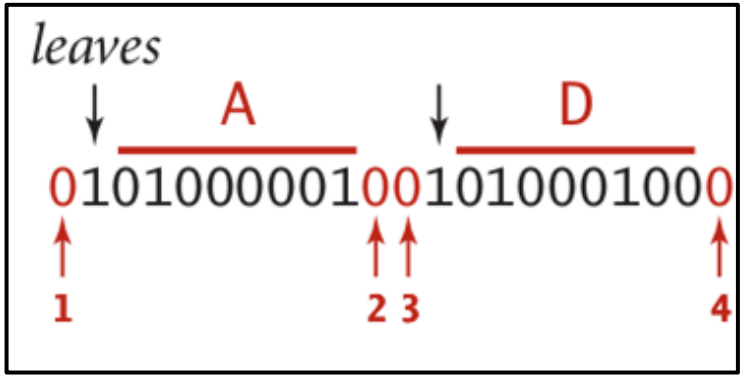




Representing tries as bitstrings

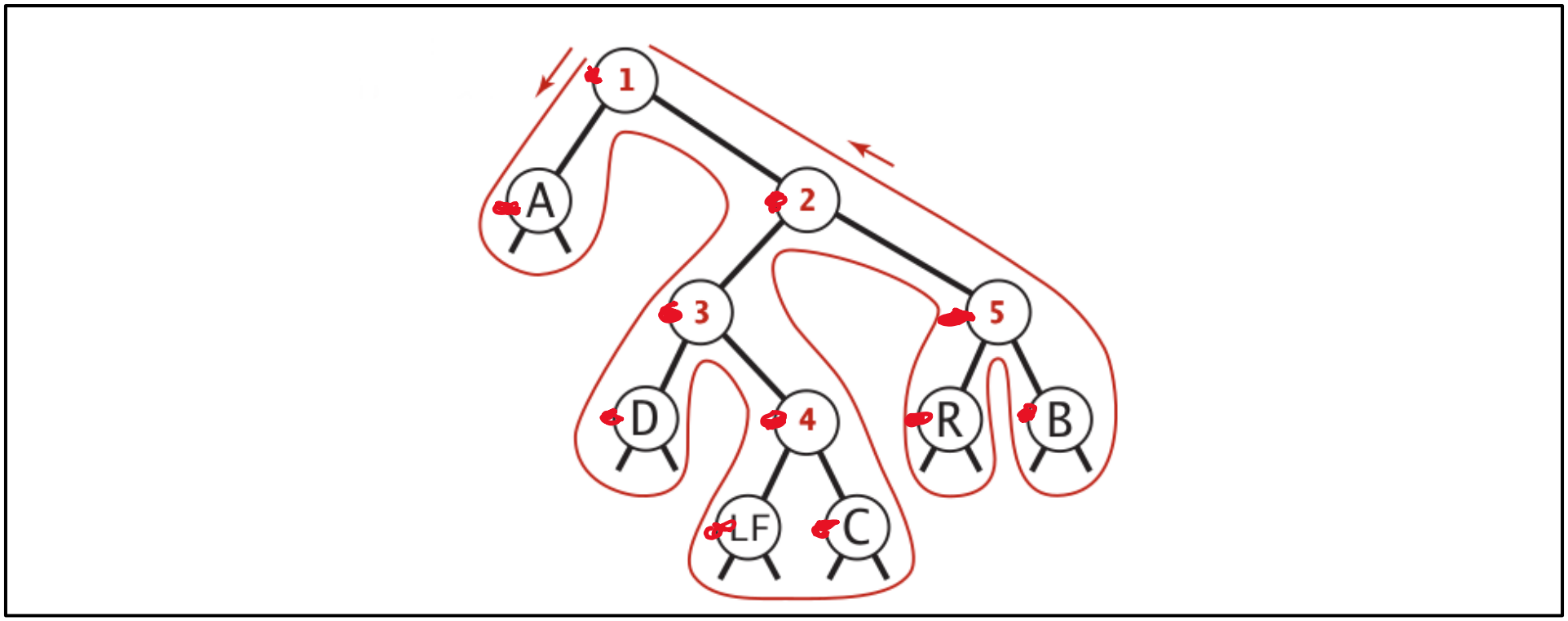
Preorder traversal

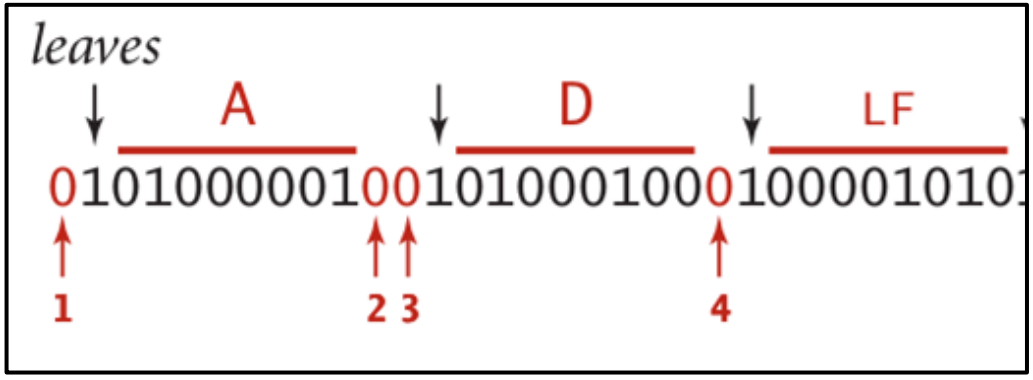




Representing tries as bitstrings

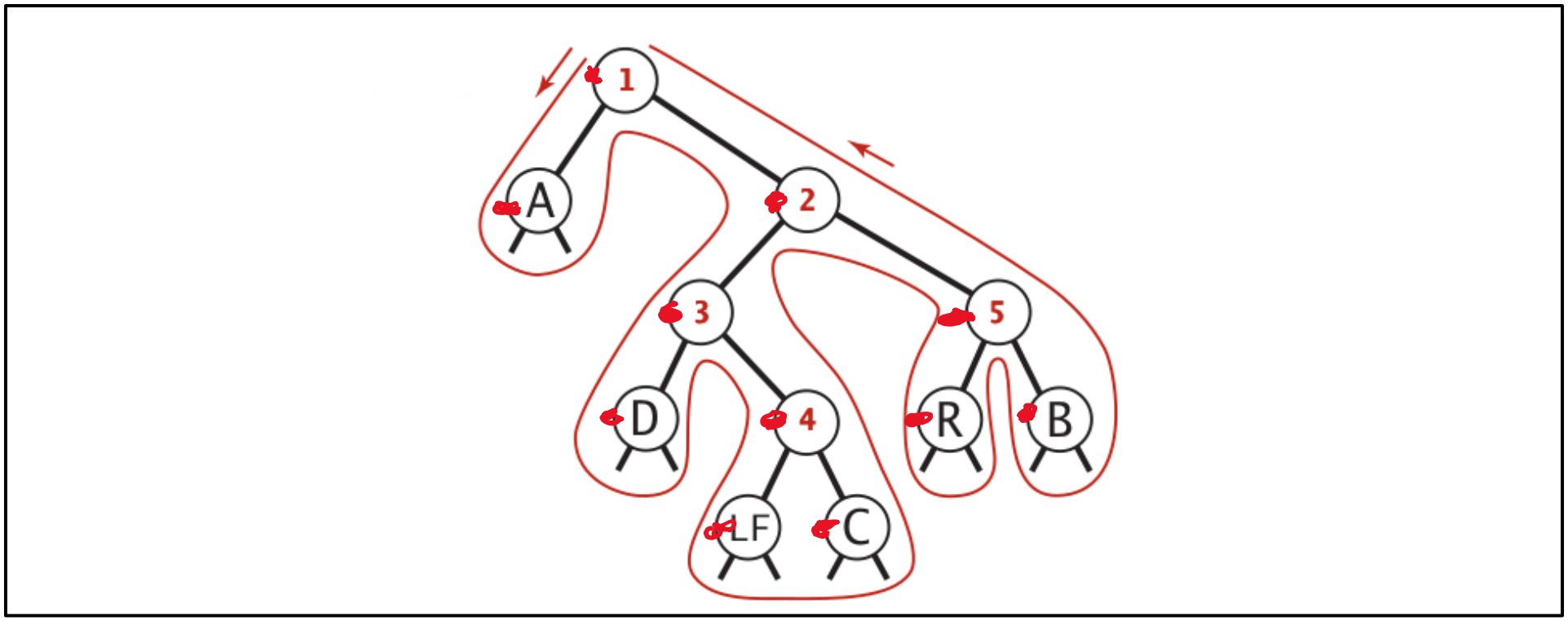
Preorder traversal

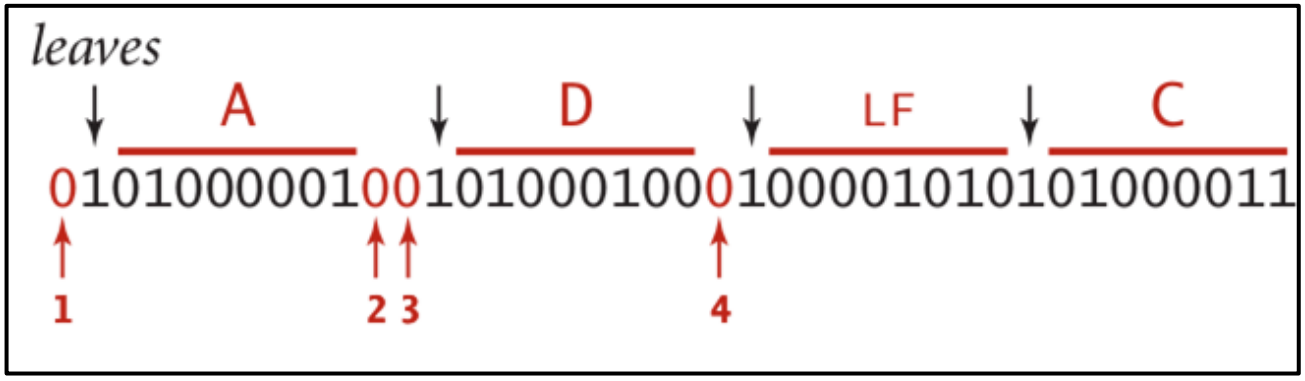




Representing tries as bitstrings

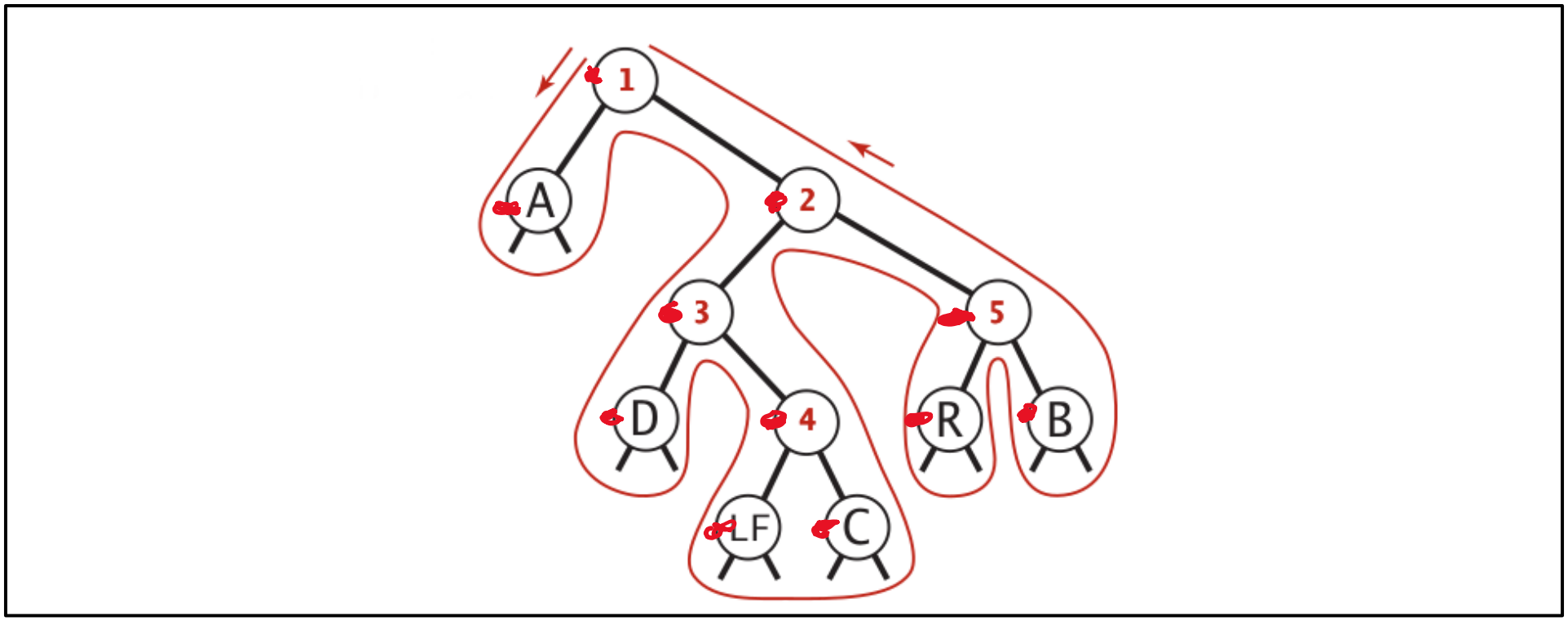
Preorder traversal

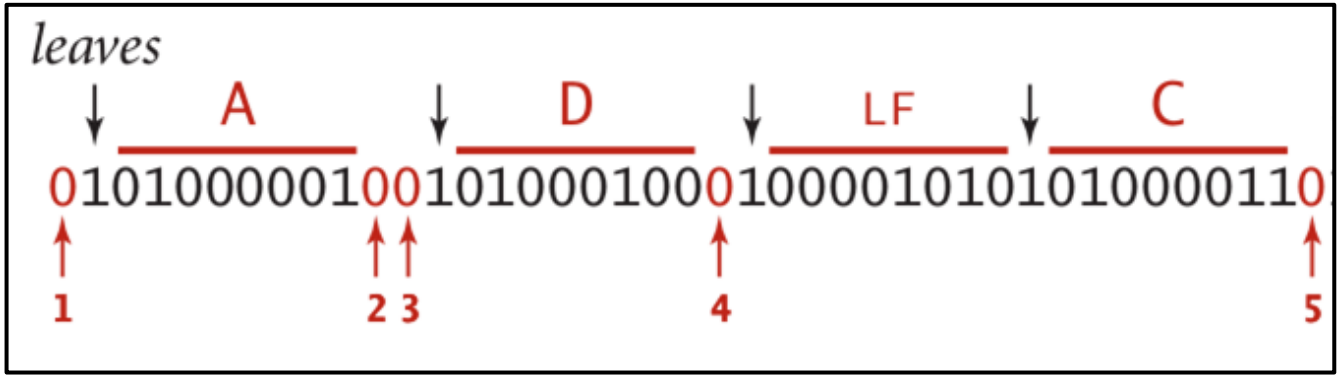




Representing tries as bitstrings

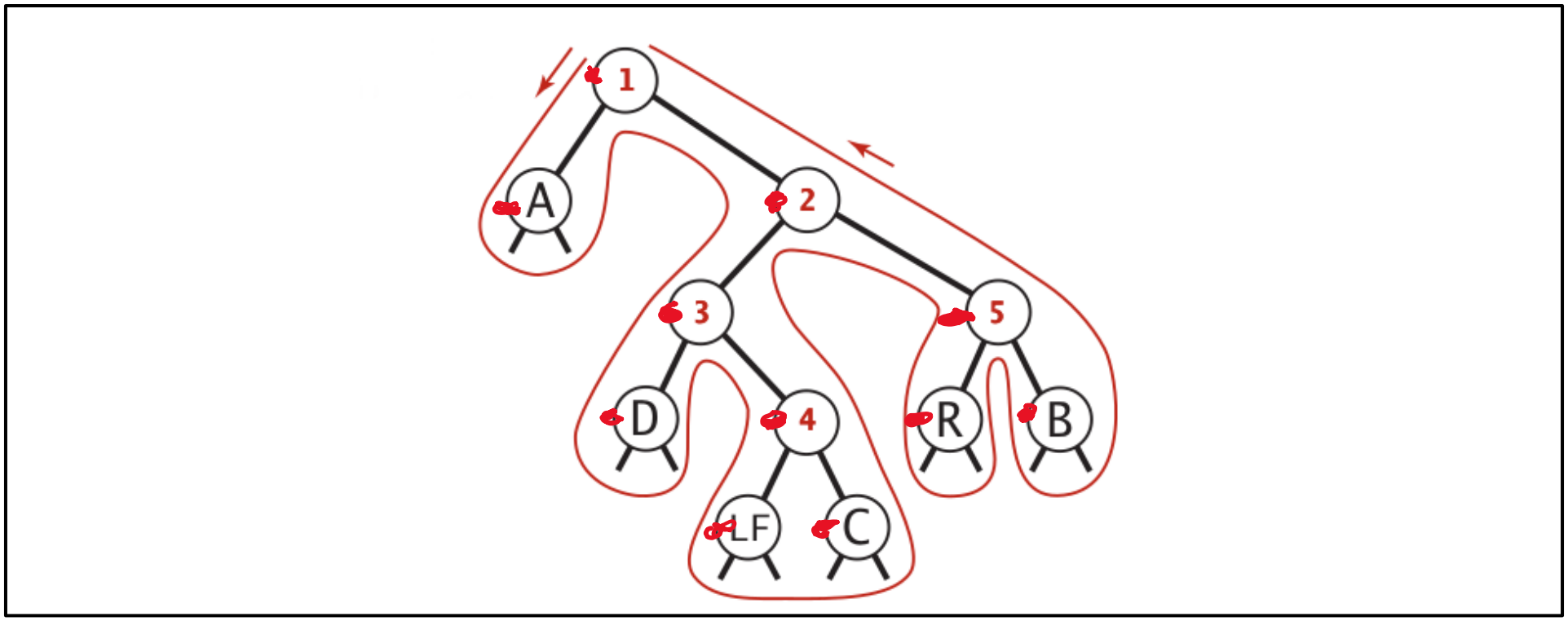
Preorder traversal

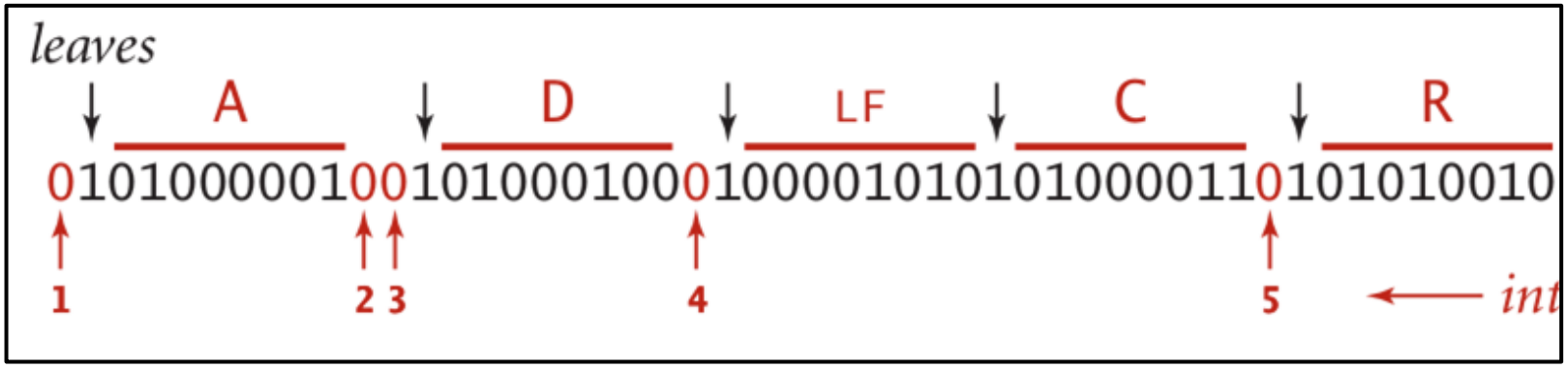




Representing tries as bitstrings

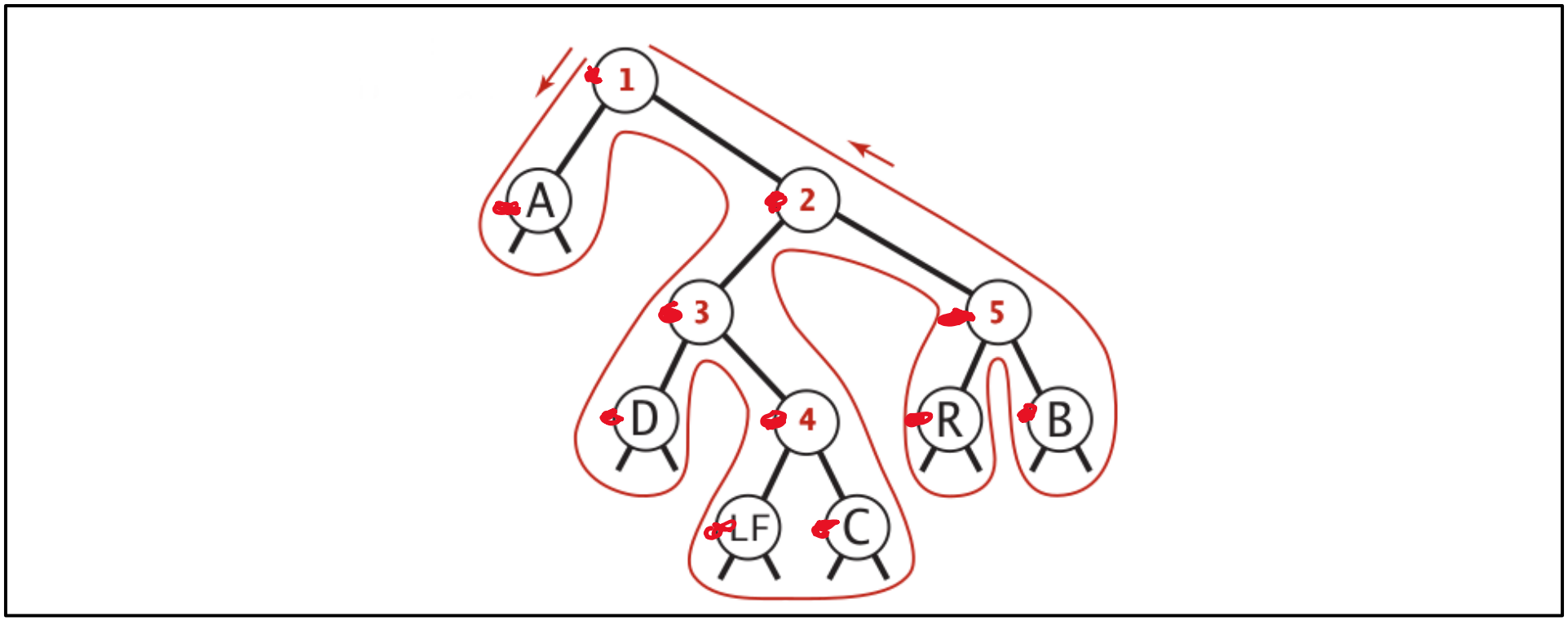
Preorder traversal

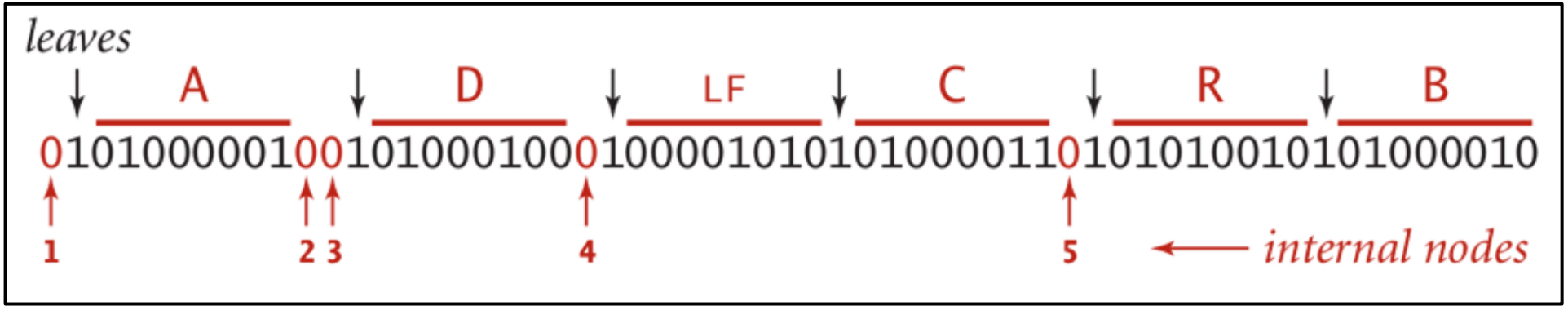




Representing tries as bitstrings

Preorder traversal





Writing a trie

```
private static void writeTrie(Node x){
   if (x.isLeaf()) {
      BinaryStdOut.write(true);
      BinaryStdOut.write(x.ch);
      return;
   }
   BinaryStdOut.write(false);
   writeTrie(x.left);
   writeTrie(x.right);
}
```

Reading a trie back

Similar to the read tree lab!

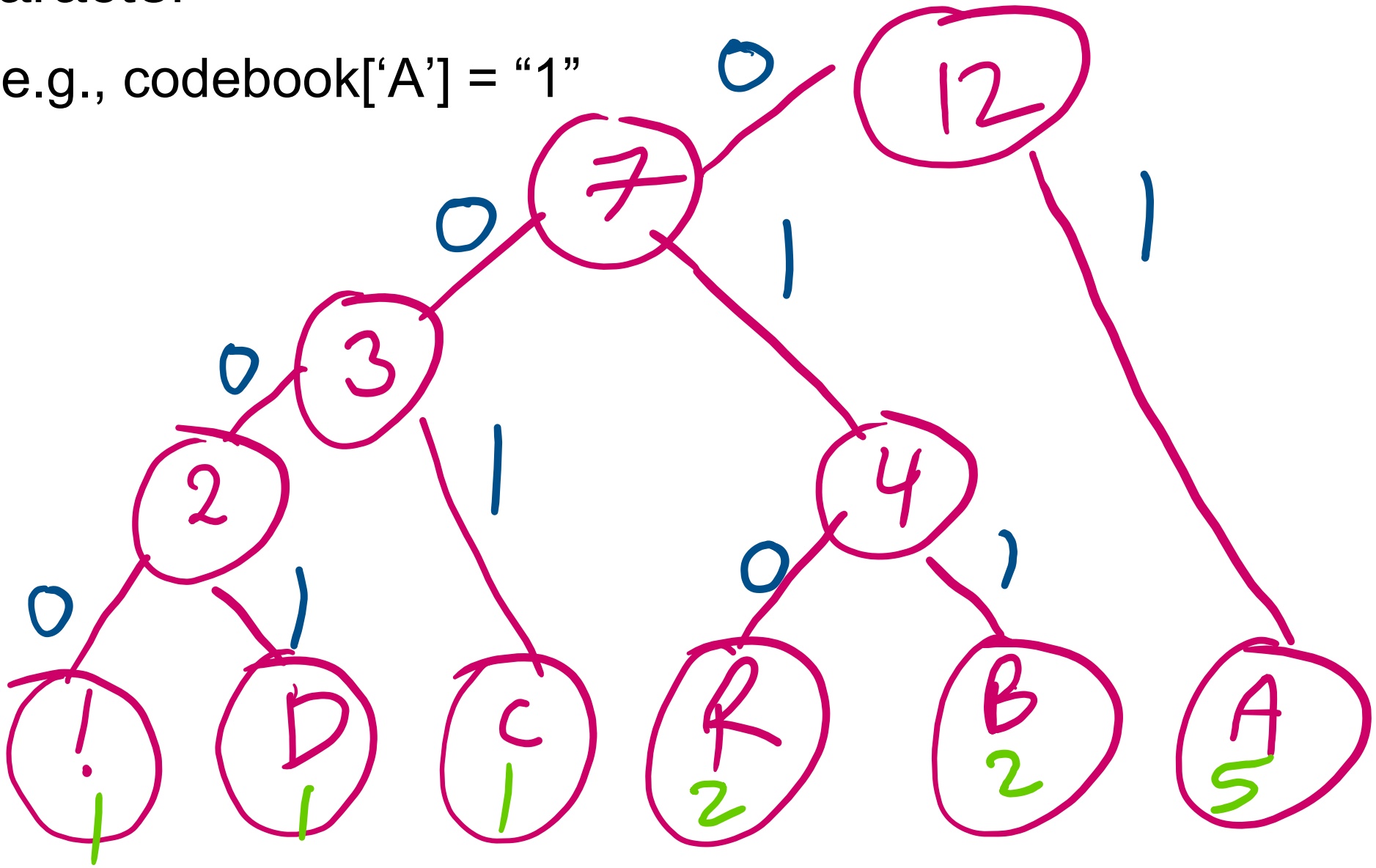
```
private static Node readTrie() {
  if (BinaryStdIn.readBoolean())
    return new
  Node(BinaryStdIn.readChar(), 0, null,
  null);
  return new Node('\0', 0, readTrie(),
readTrie());
}
```

Huffman pseudocode

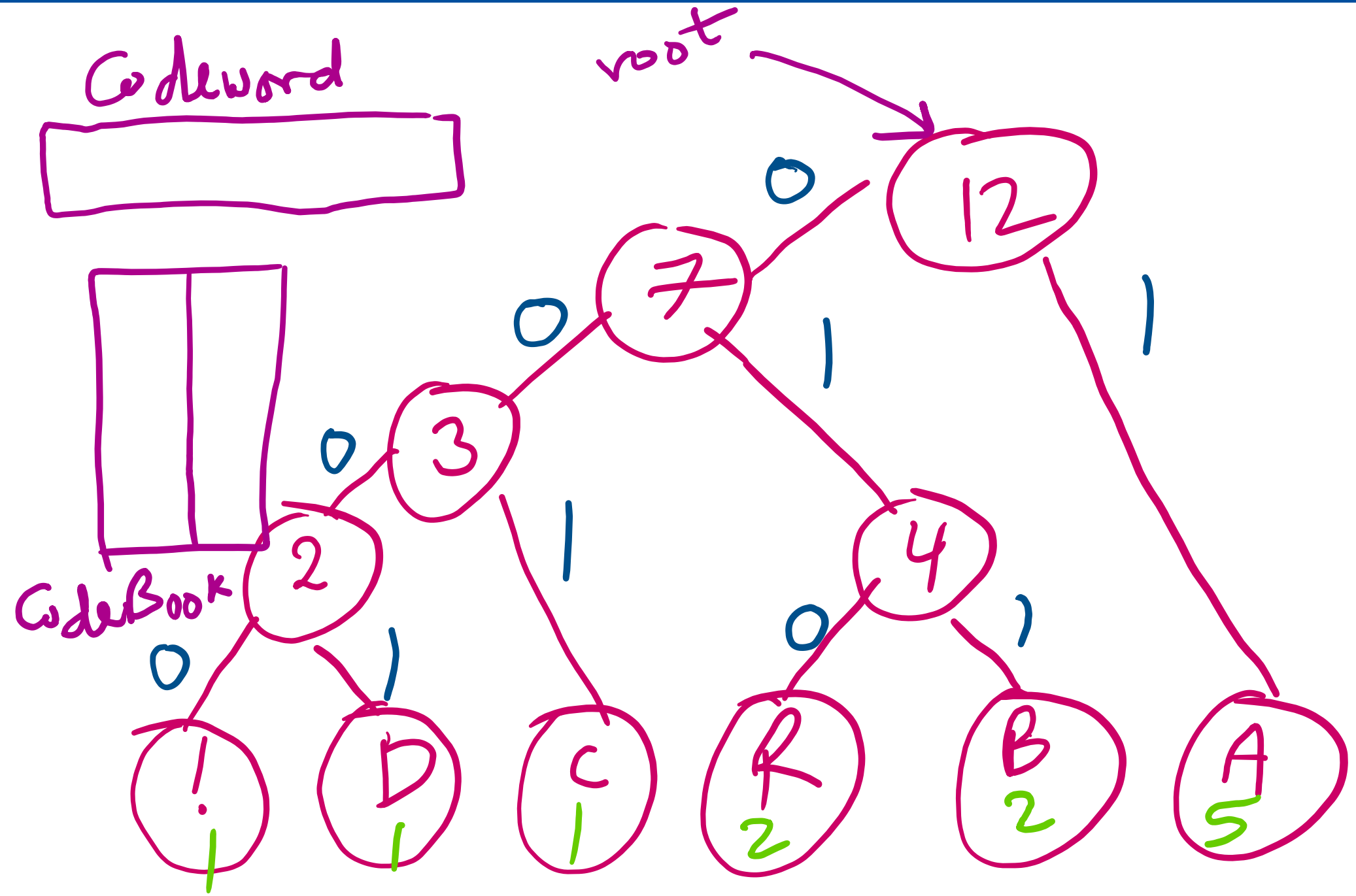
- Encoding approach:
  - Read input
  - Compute frequencies
  - Build trie/codeword table
  - Write out trie as a bitstring to compressed file
  - Write out character count of input (**why is that necessary**?)
  - Use table to write out the codeword for each input character
- Decoding approach:
  - Read trie
  - Read character count
  - Use trie to decode bitstring of compressed file

Huffman Compression: Generating the *Codebook*

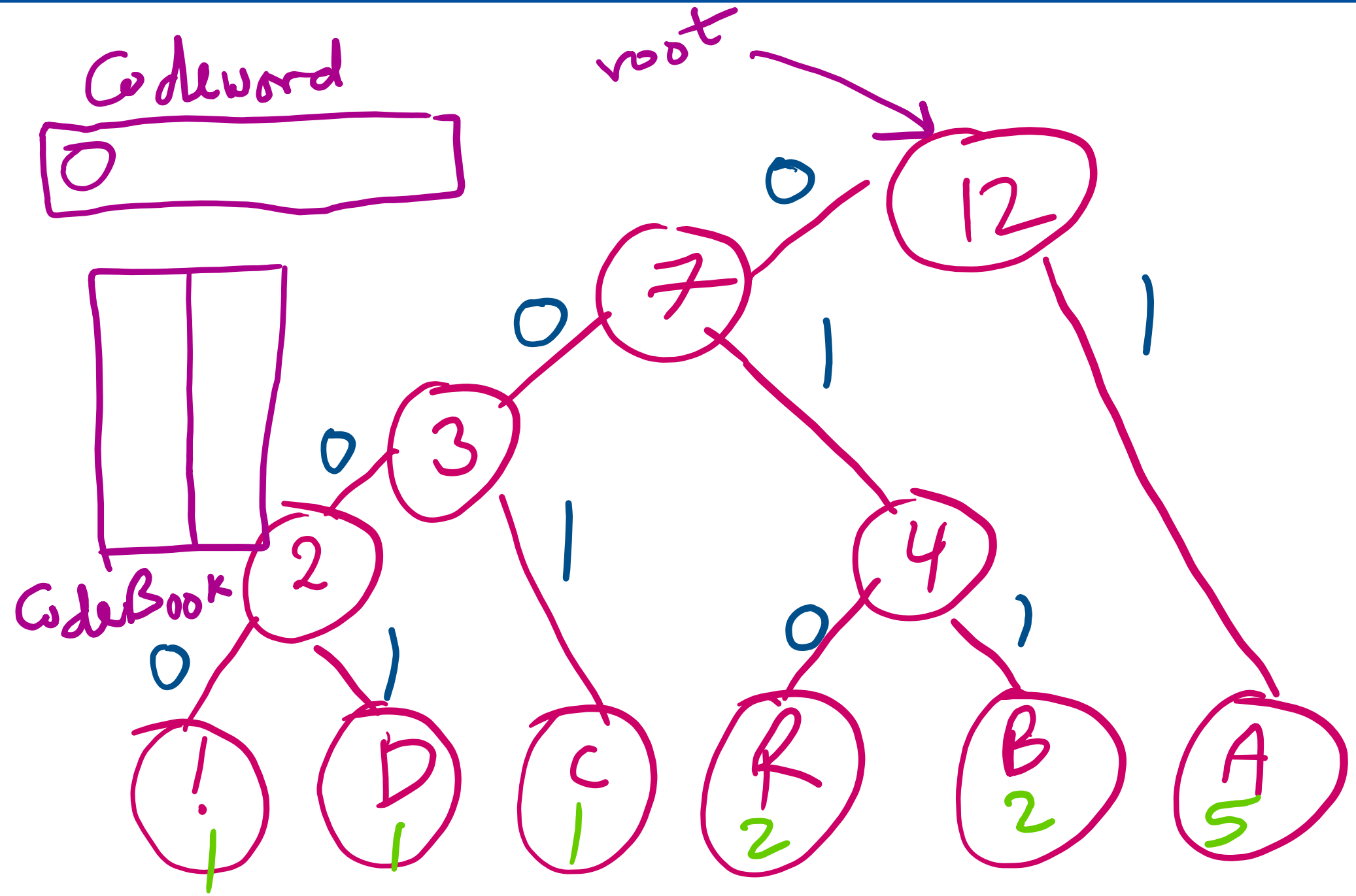
- The codebook is a table of codewords indexed by character
  - e.g., codebook[‘A’] = “1”



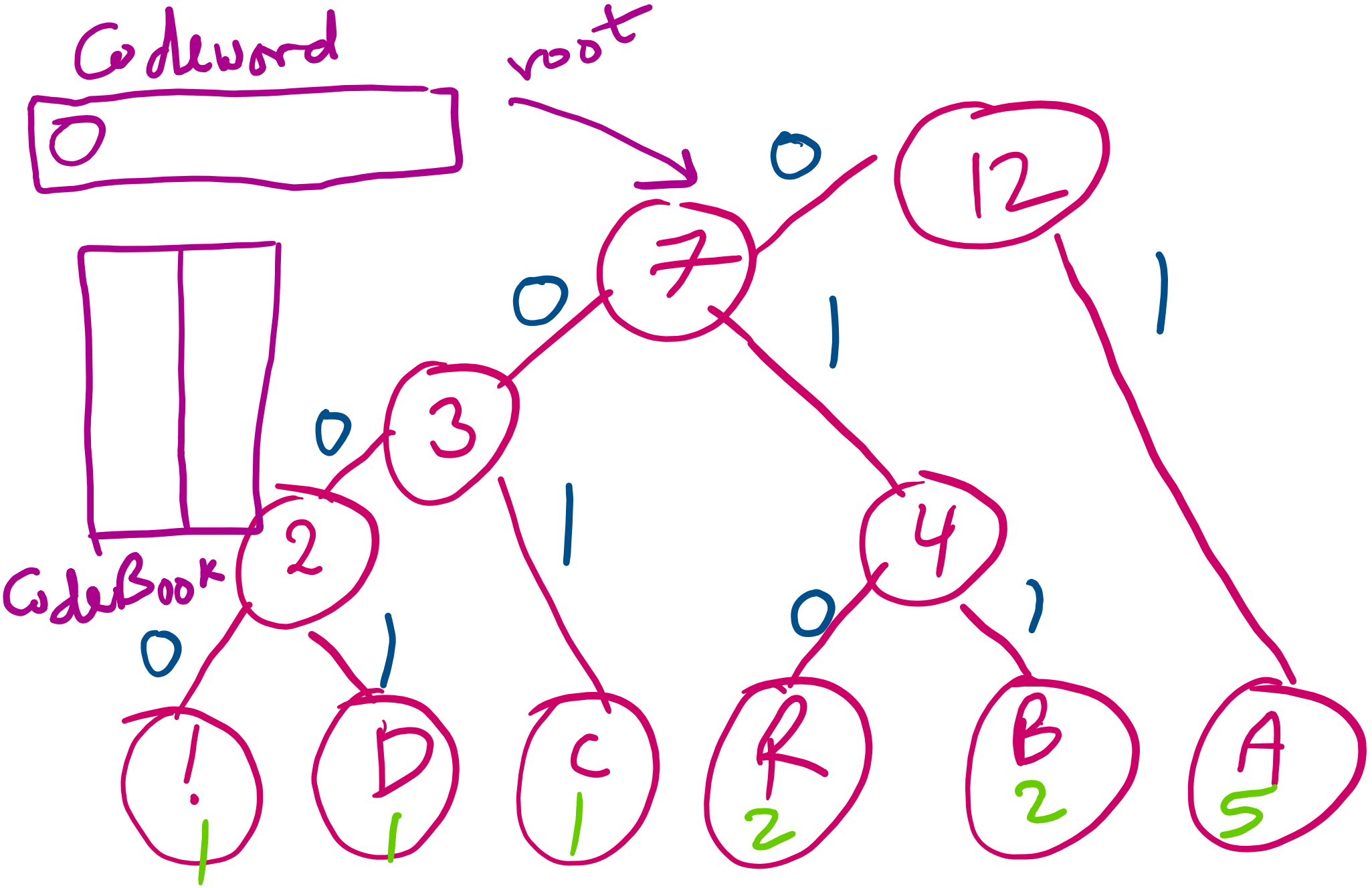
Huffman Compression: Generating the *Codebook*



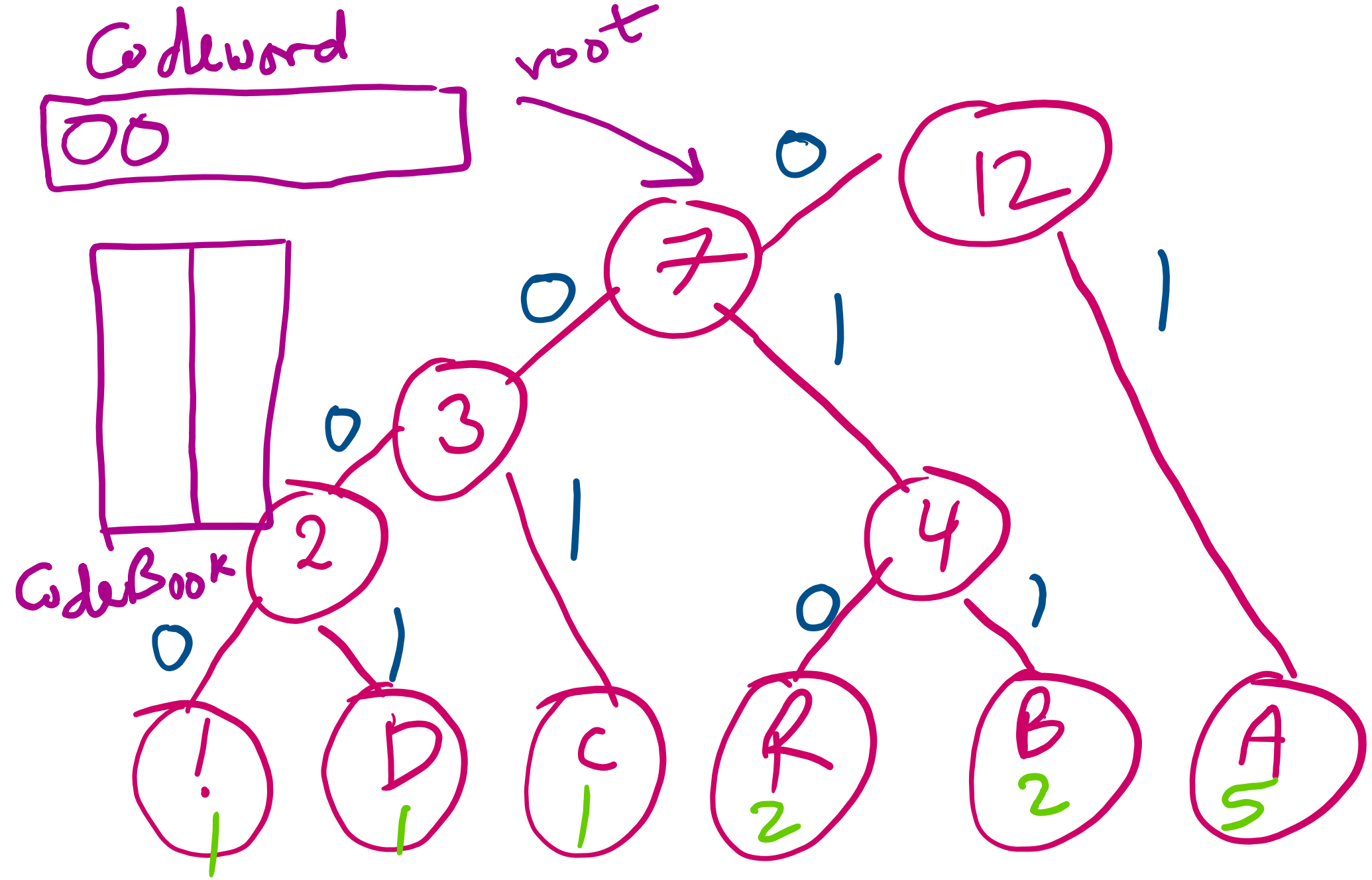
Huffman Compression: Generating the *Codebook*



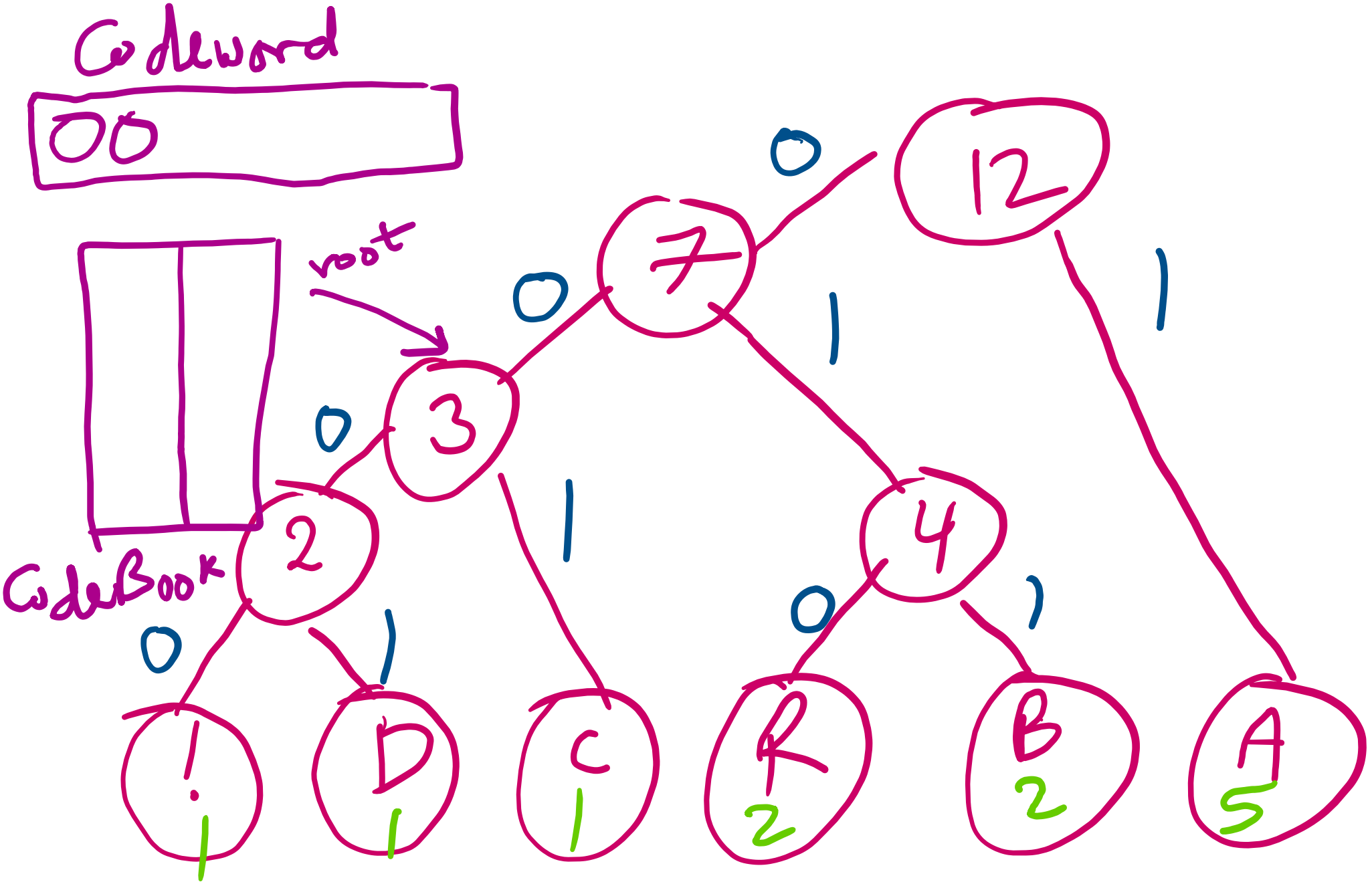
Huffman Compression: Generating the *Codebook*



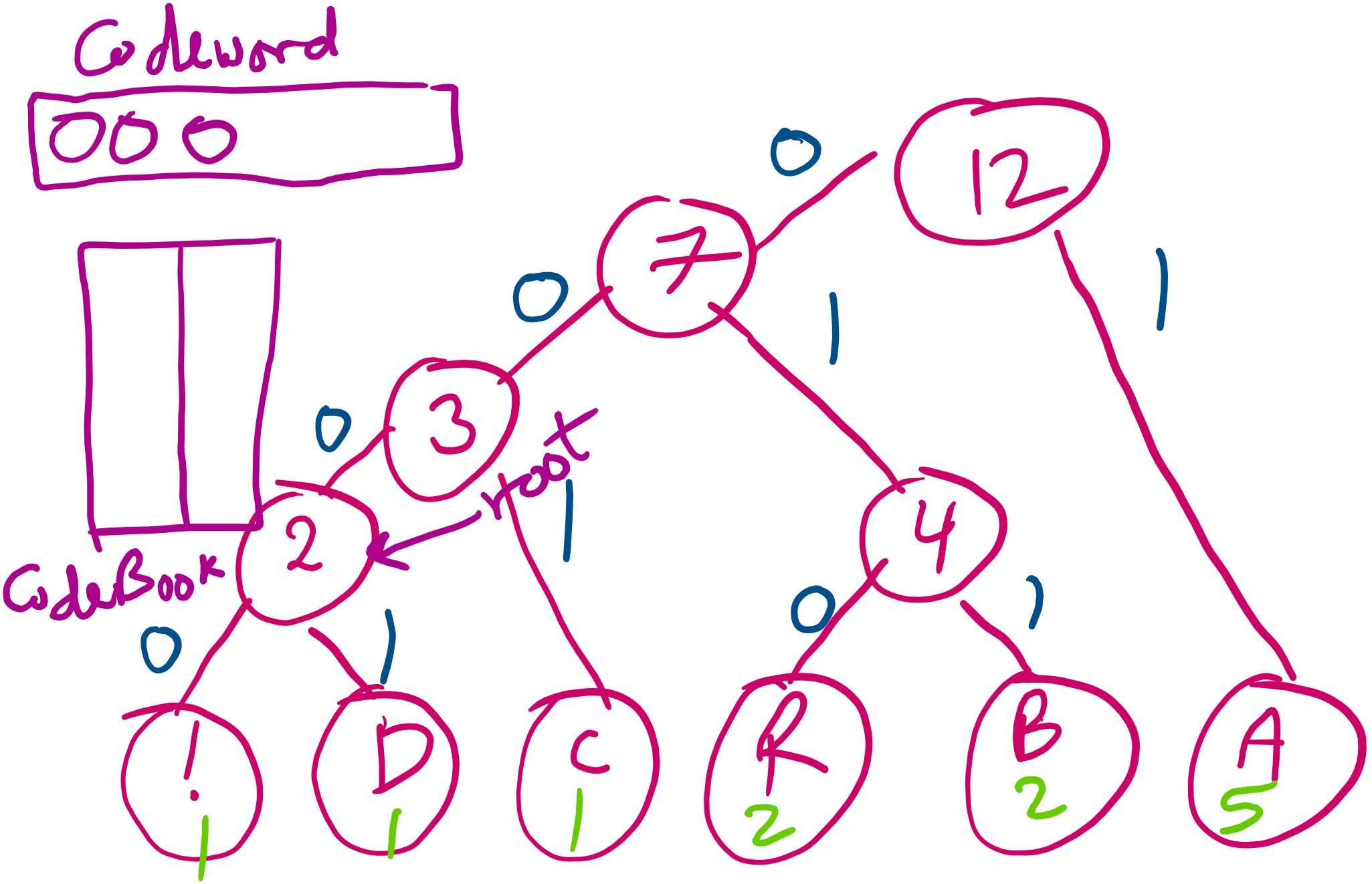
Huffman Compression: Generating the *Codebook*



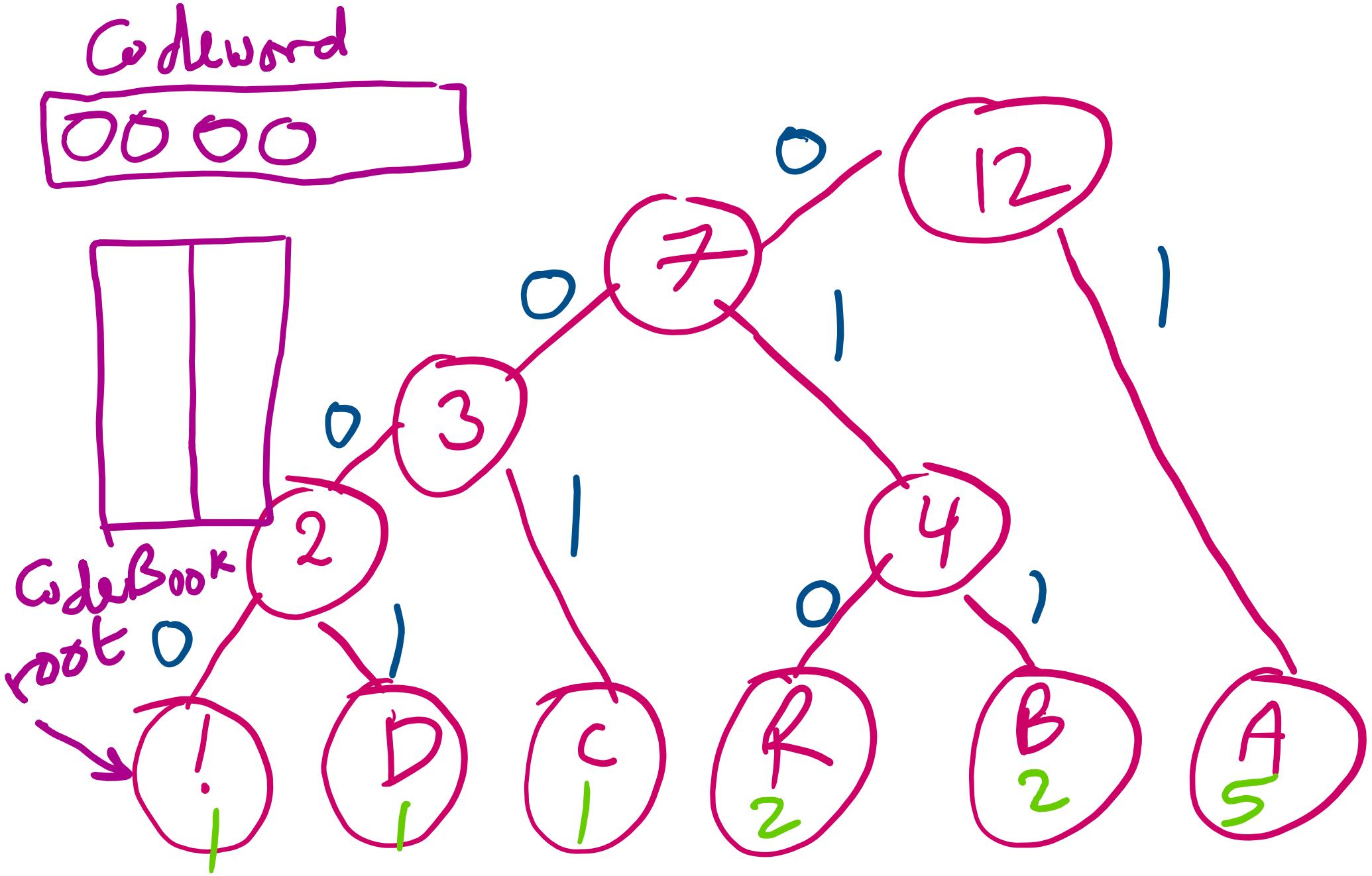
Huffman Compression: Generating the *Codebook*



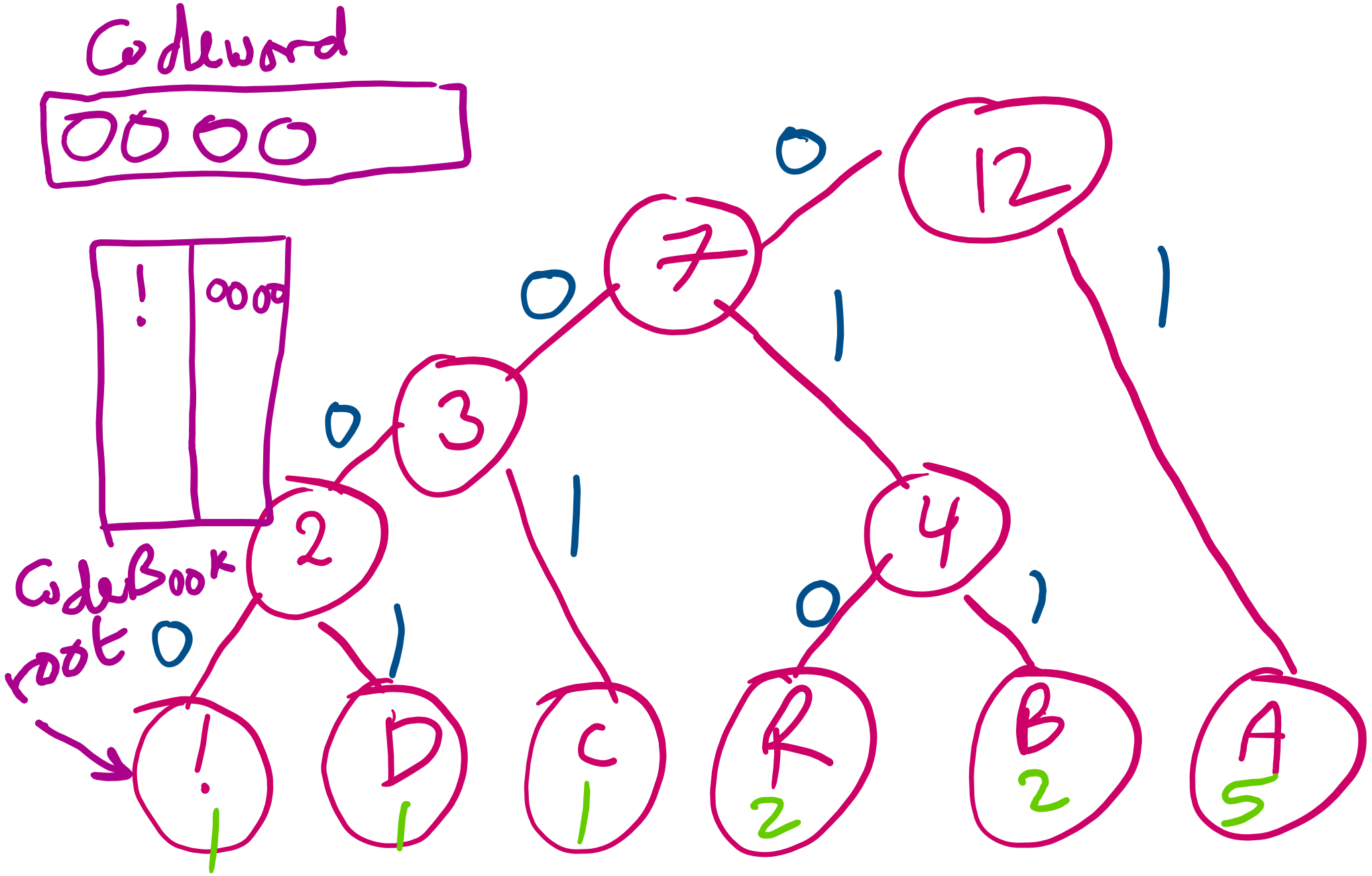
Huffman Compression: Generating the *Codebook*



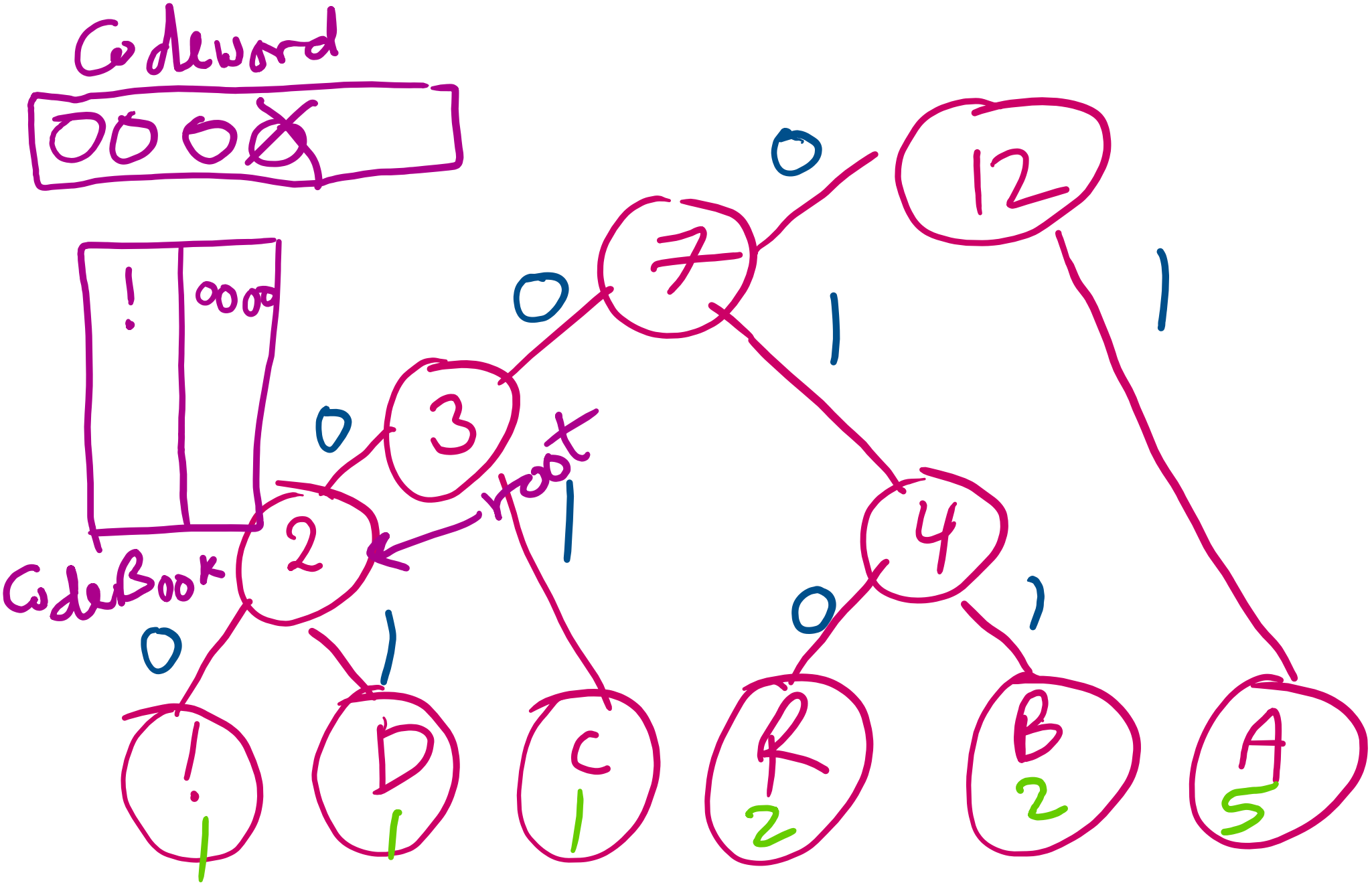
Huffman Compression: Generating the *Codebook*



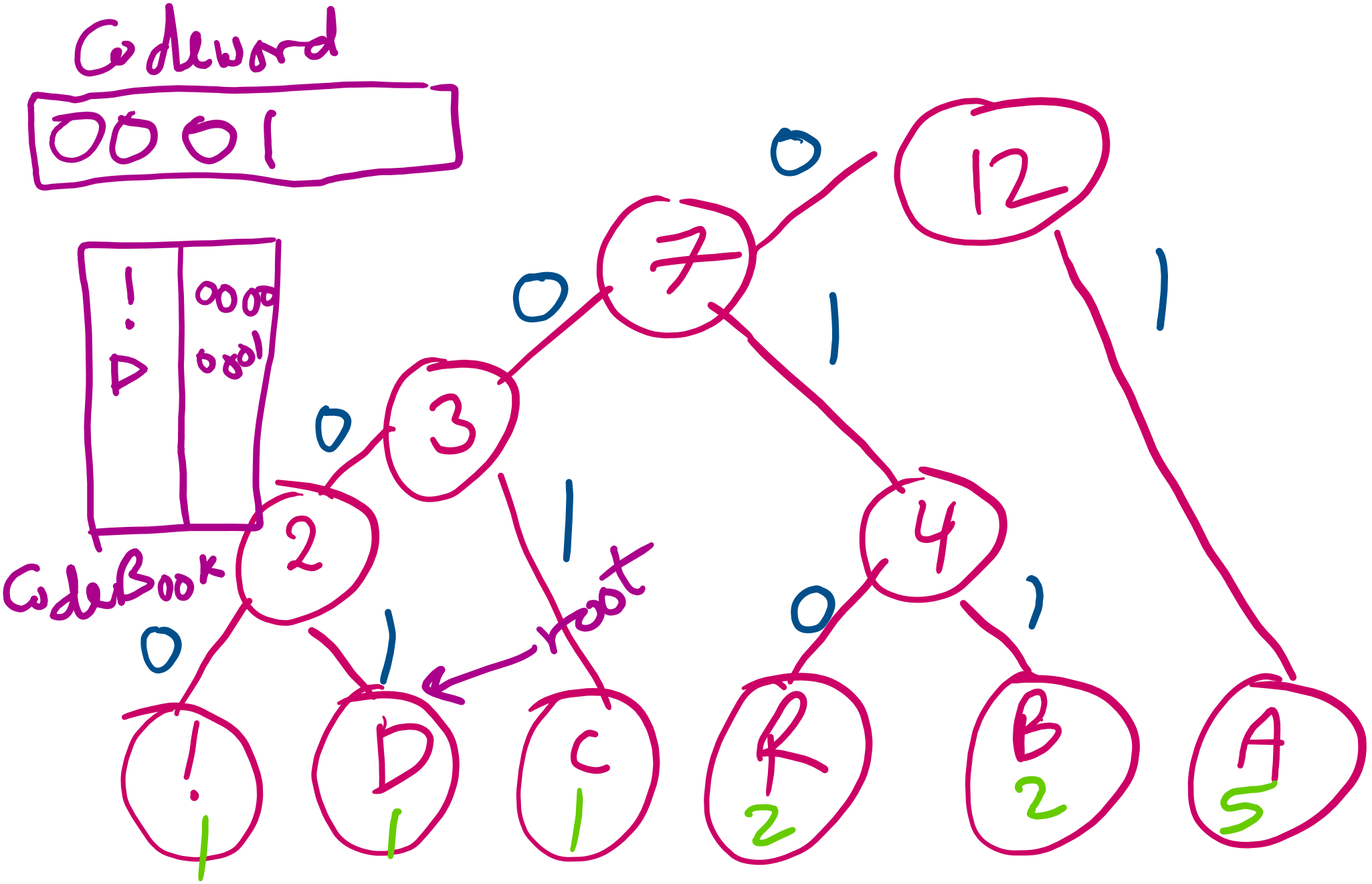
Huffman Compression: Generating the *Codebook*



Huffman Compression: Generating the *Codebook*



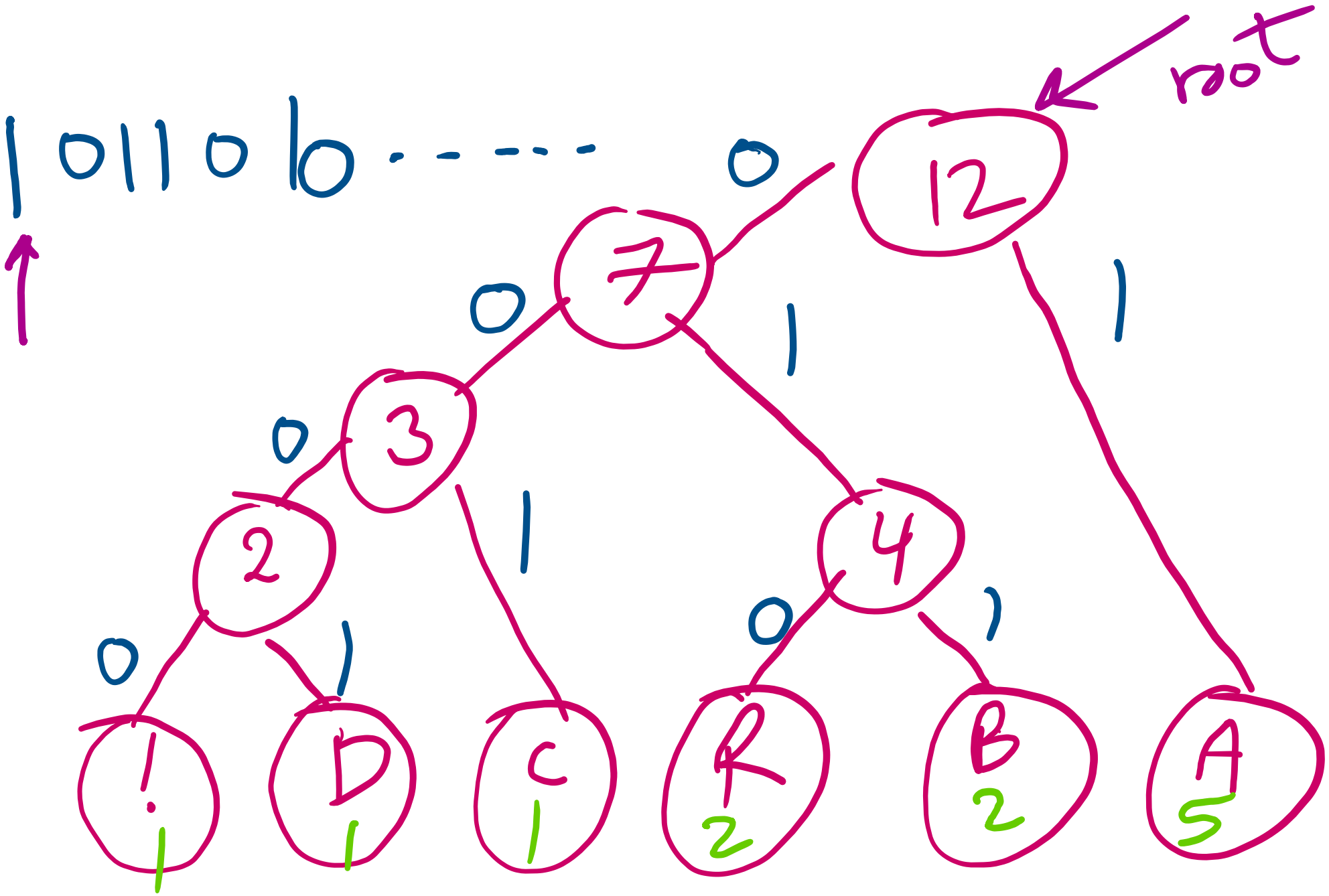
Huffman Compression: Generating the *Codebook*



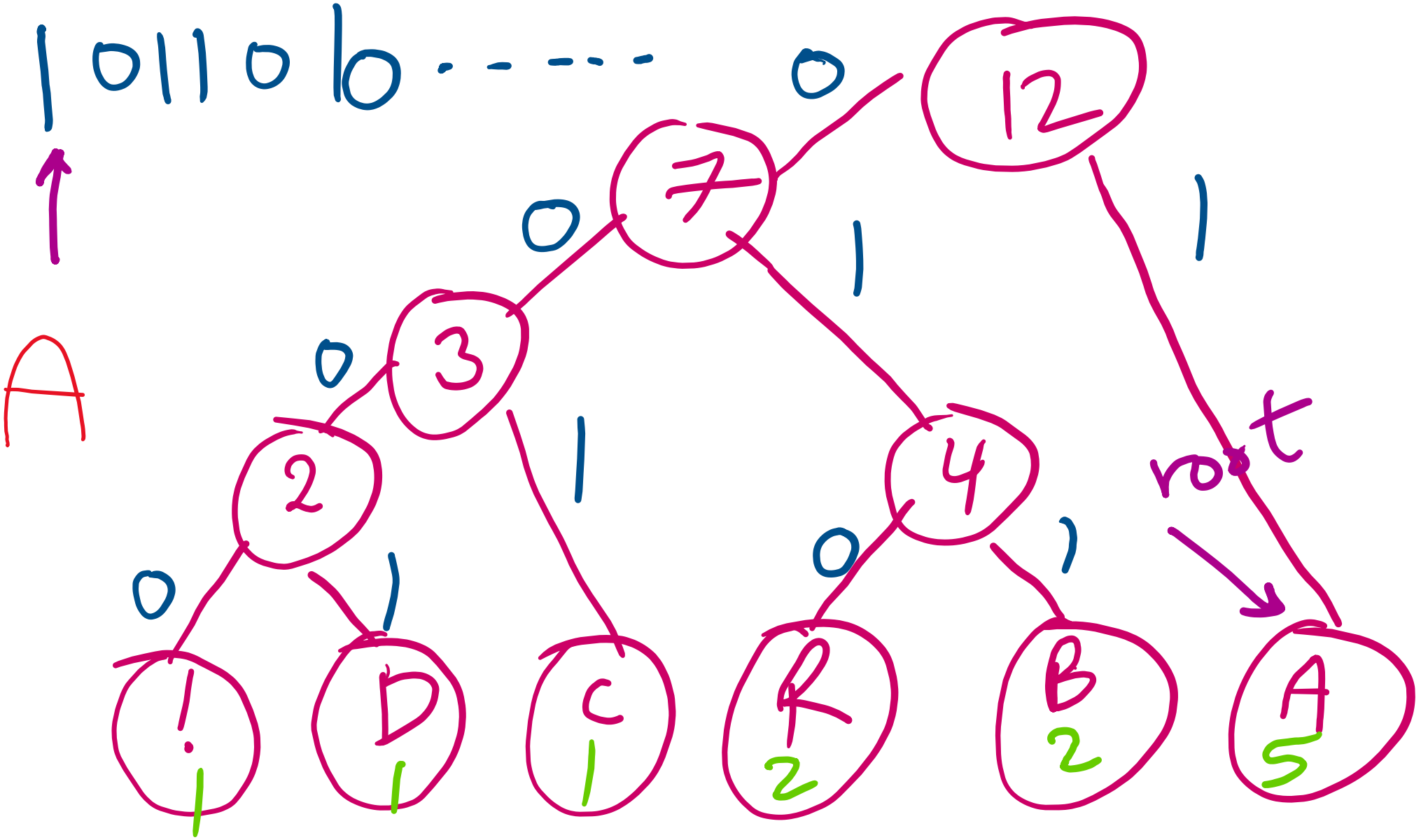
Huffman Compression: Generating the *Codebook*

```
void generateCodeBook(Node root, StringBuilder codeword) {
    if(root.isLeaf()){
        codebook[root.data] = codeword.toString();
    }
    if(root.left != null){
        //append 0 to codeword, move left, pop last character in codeword
    }
    if(root.right != null){
        //append 1 to codeword, move right, pop last character in codeword
    }
}
```

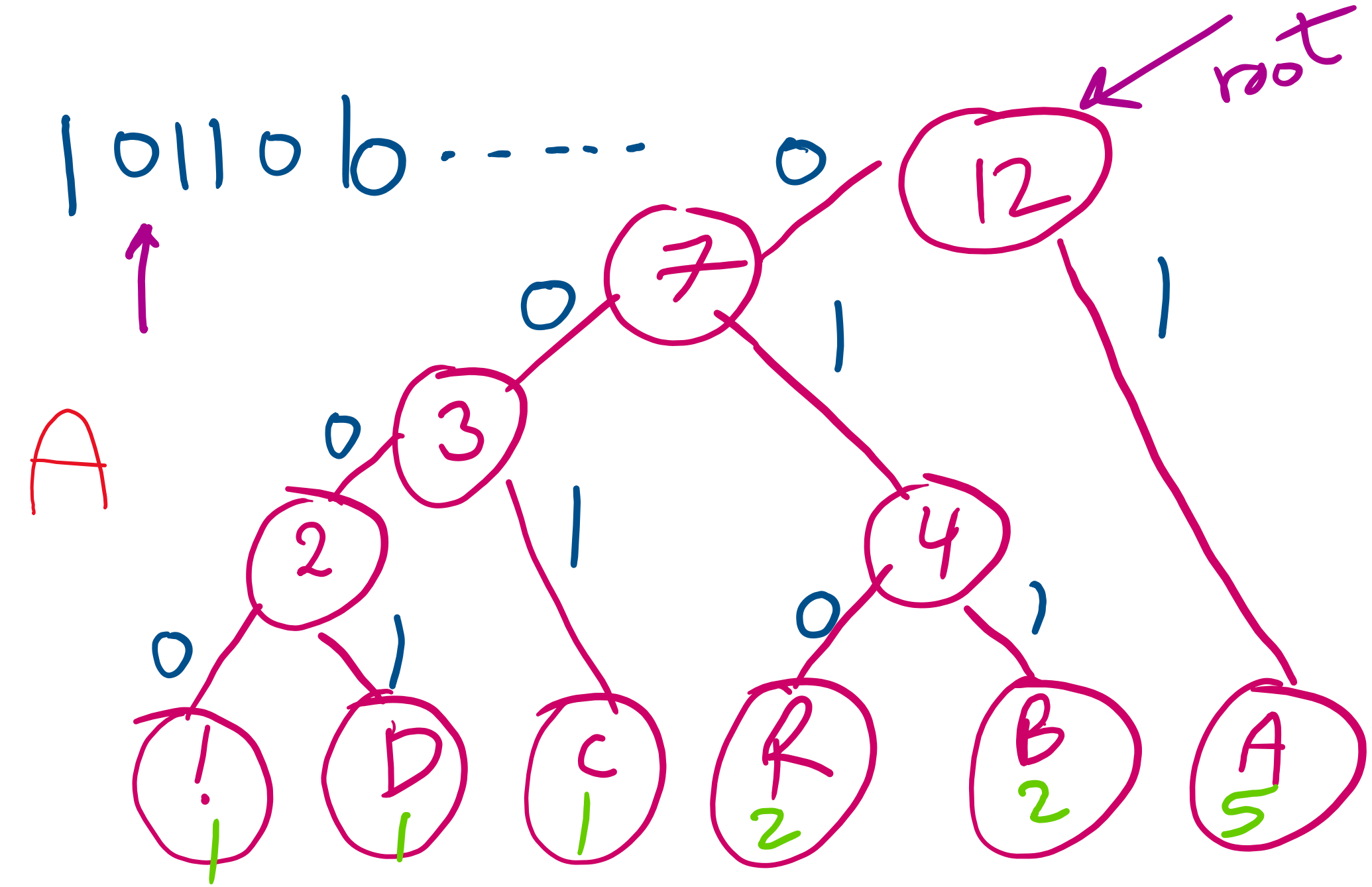
Huffman Compression: Decoding



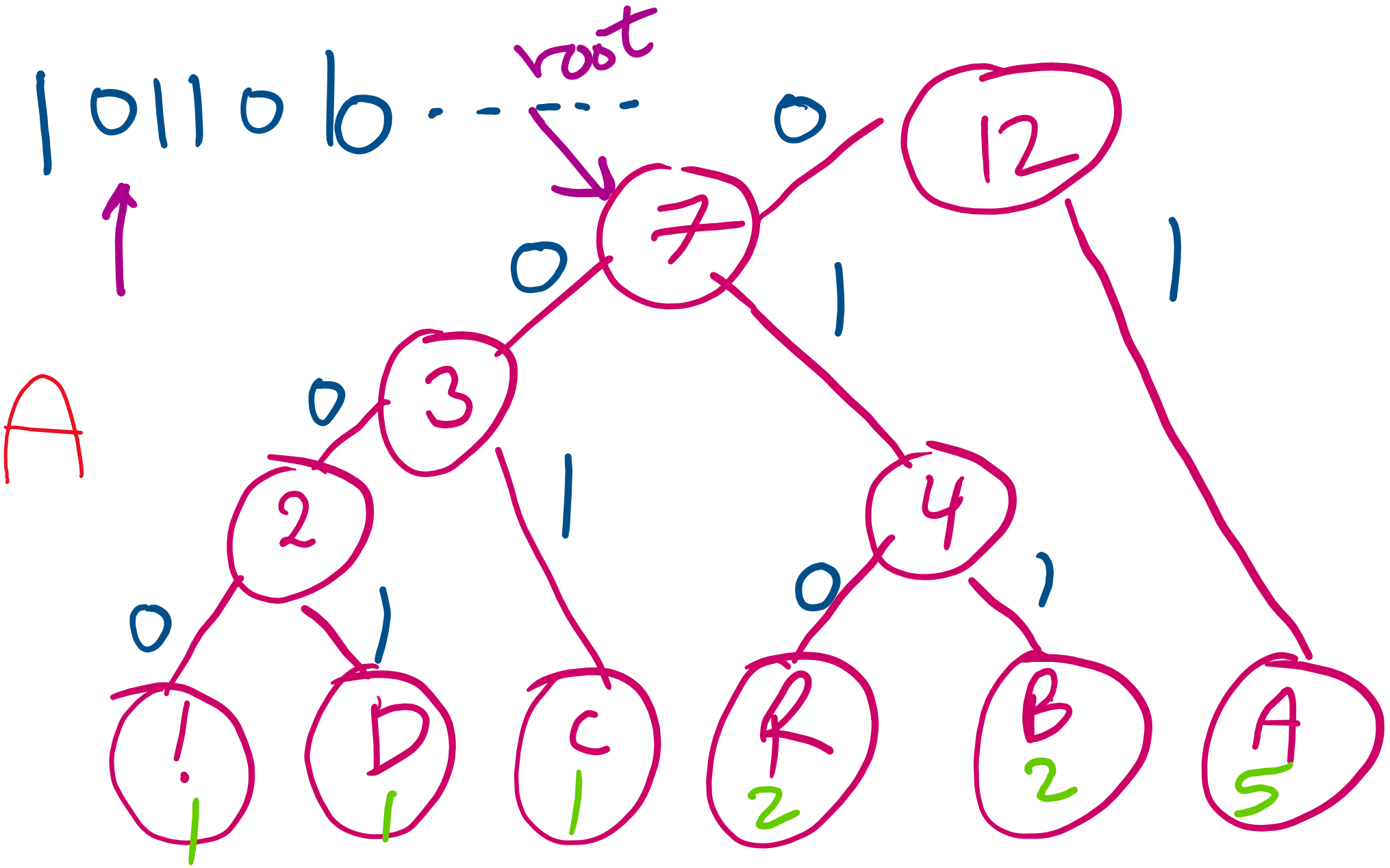
Huffman Compression: Decoding



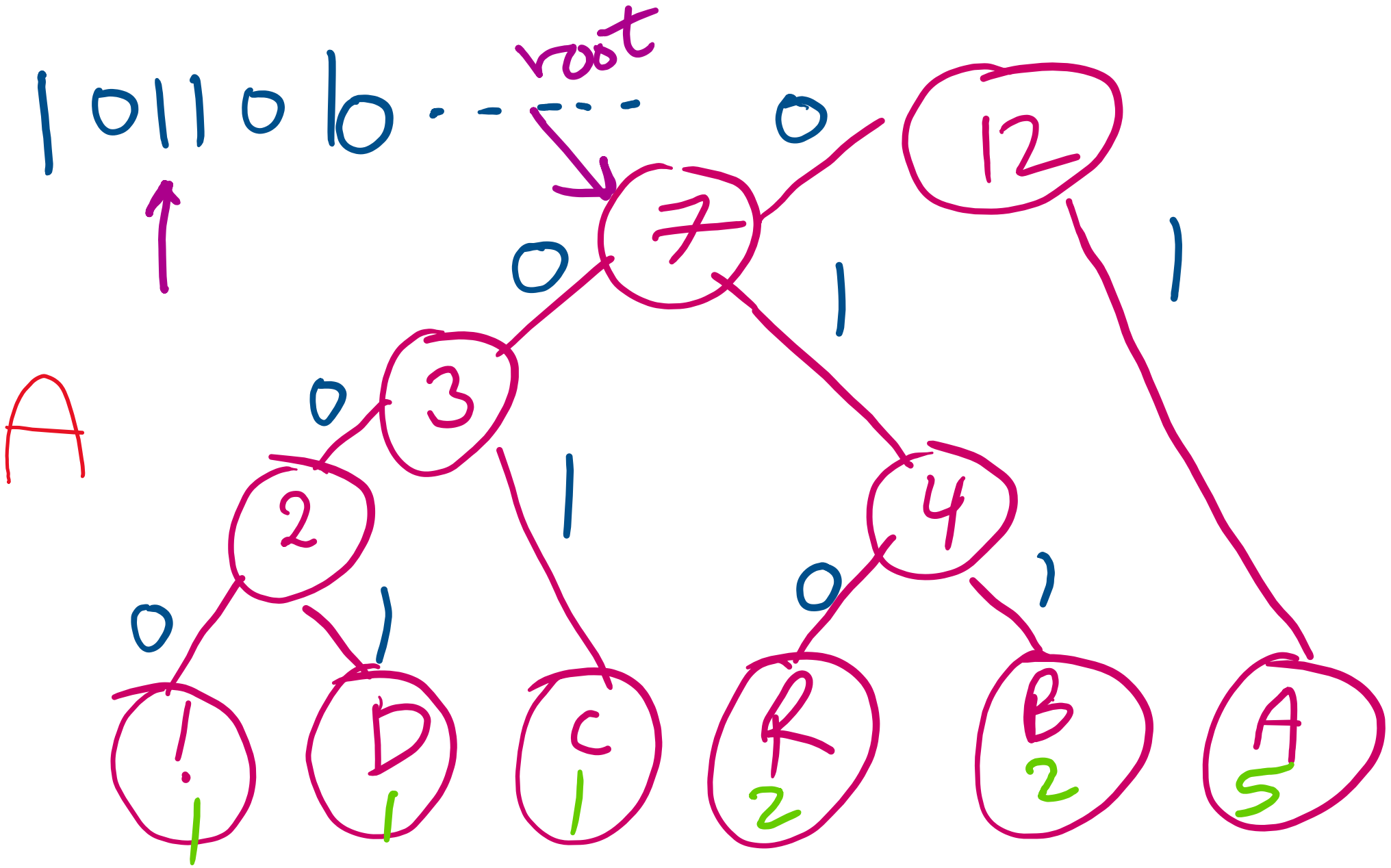
Huffman Compression: Decoding



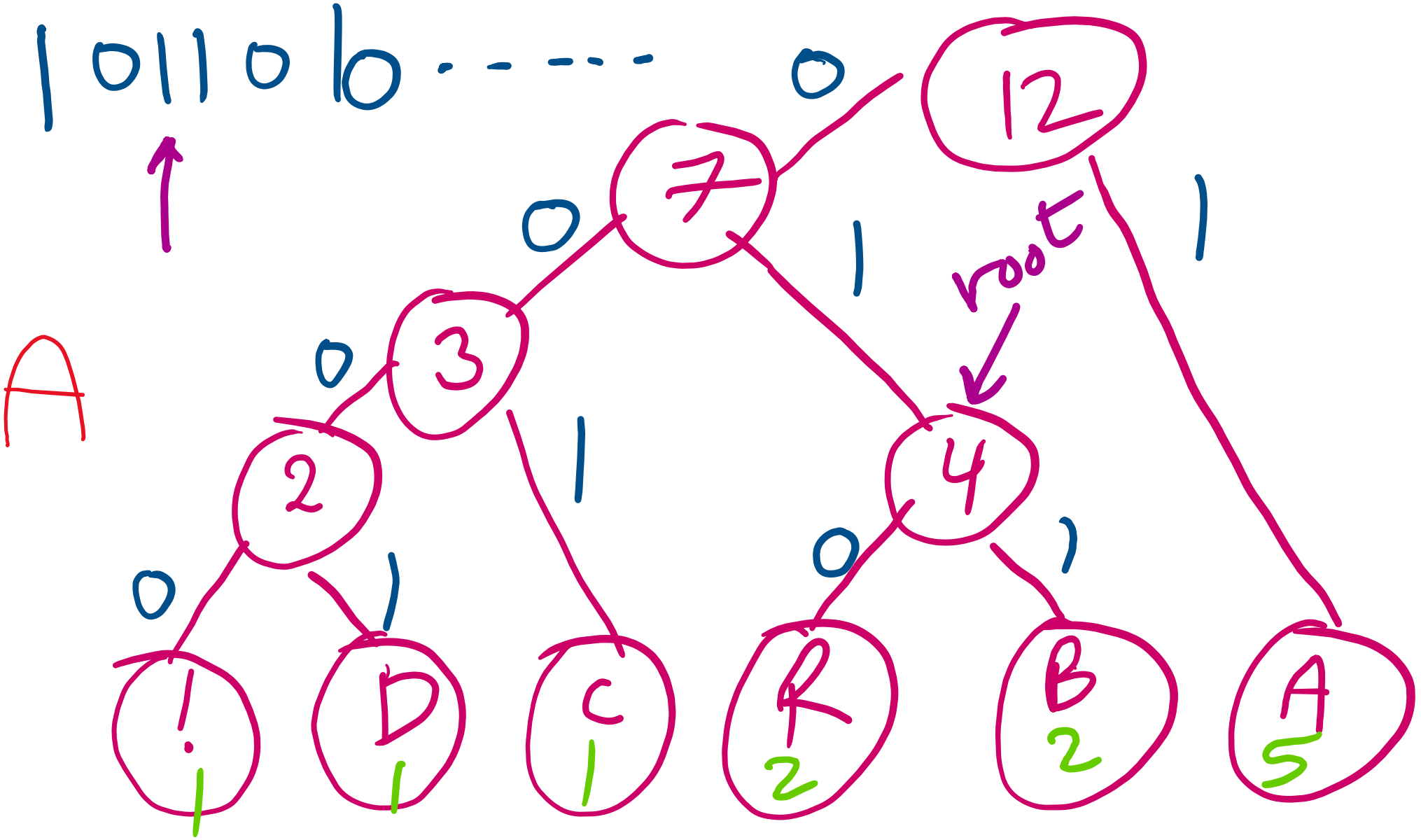
Huffman Compression: Decoding



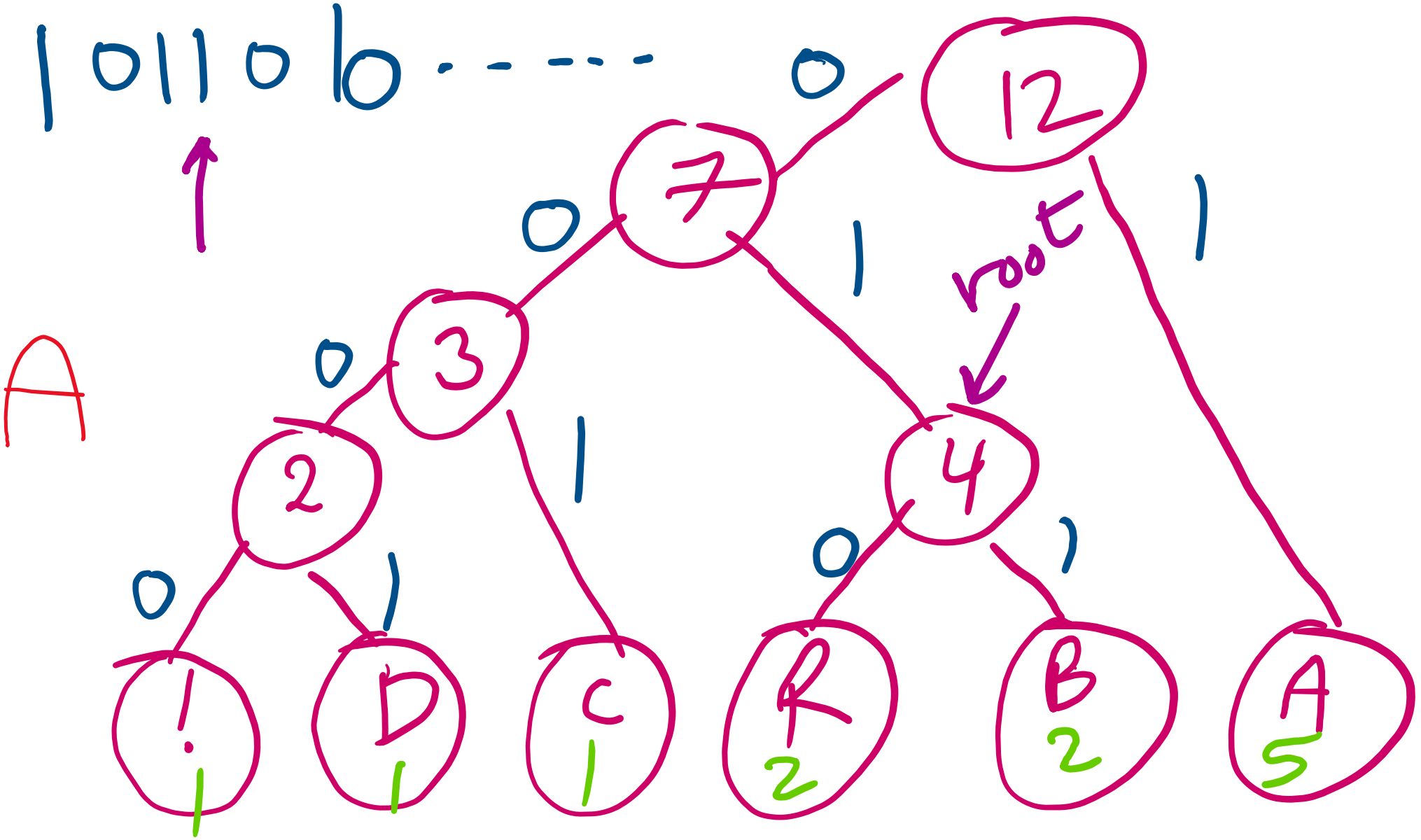
Huffman Compression: Decoding



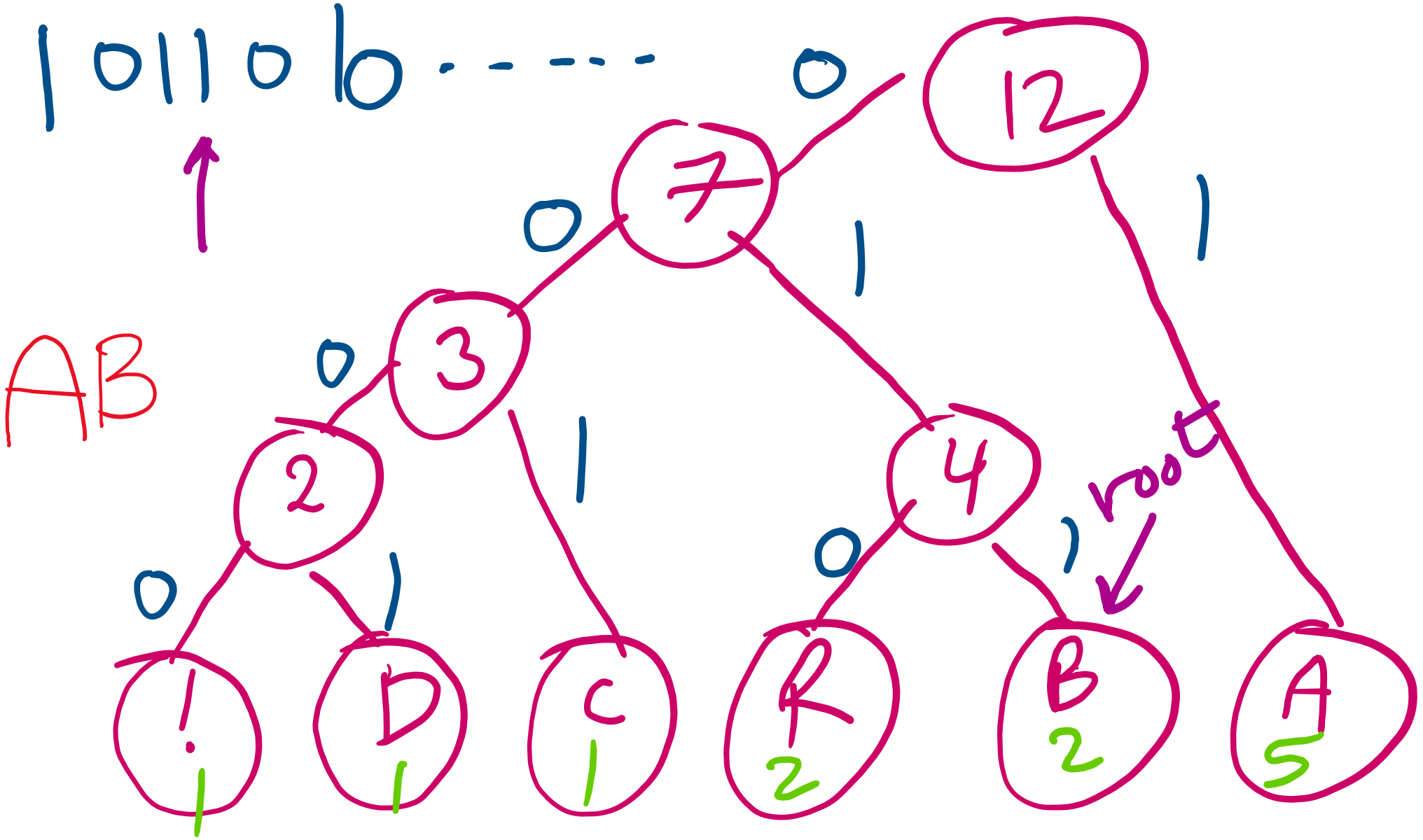
Huffman Compression: Decoding



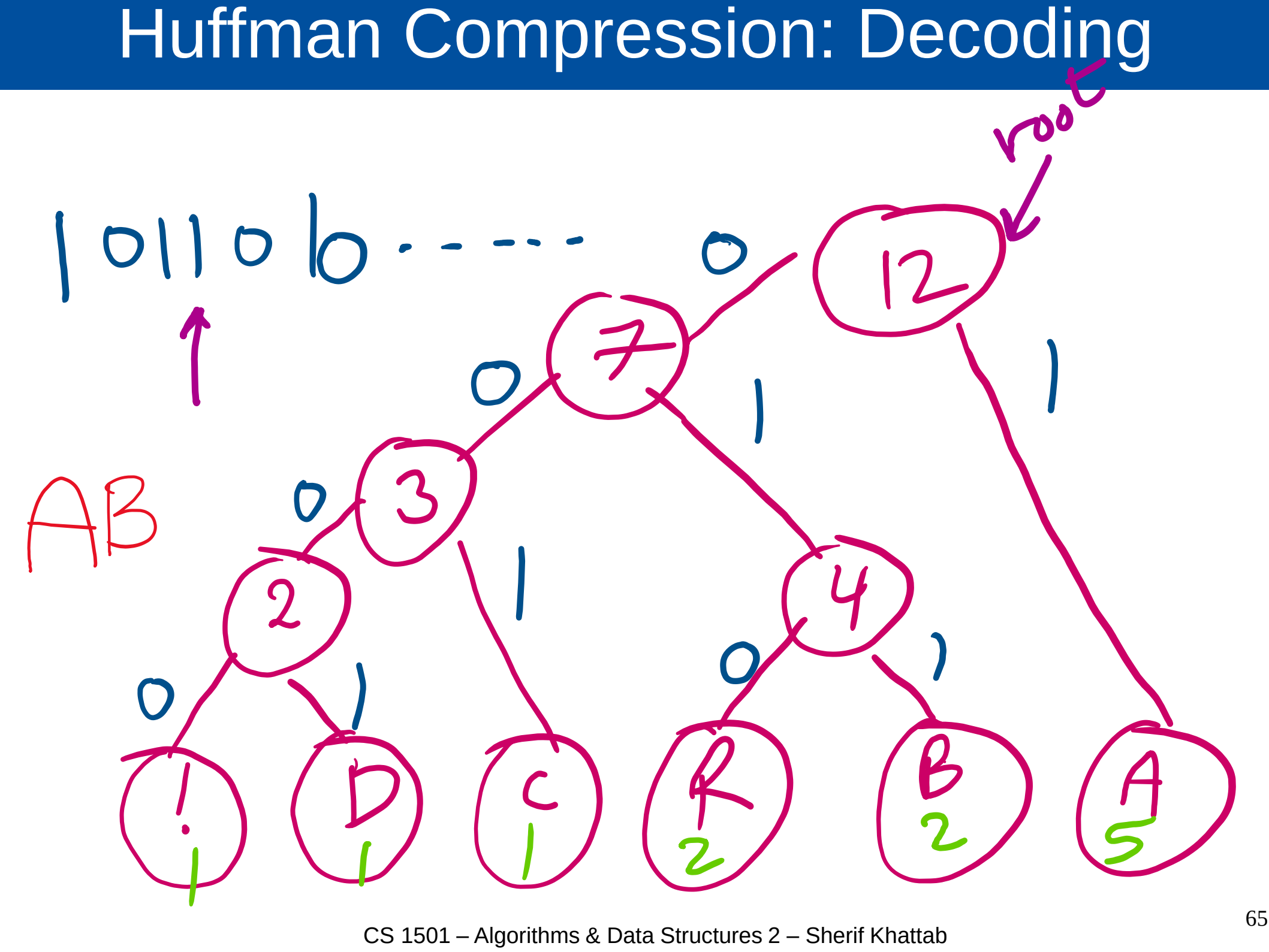
Huffman Compression: Decoding



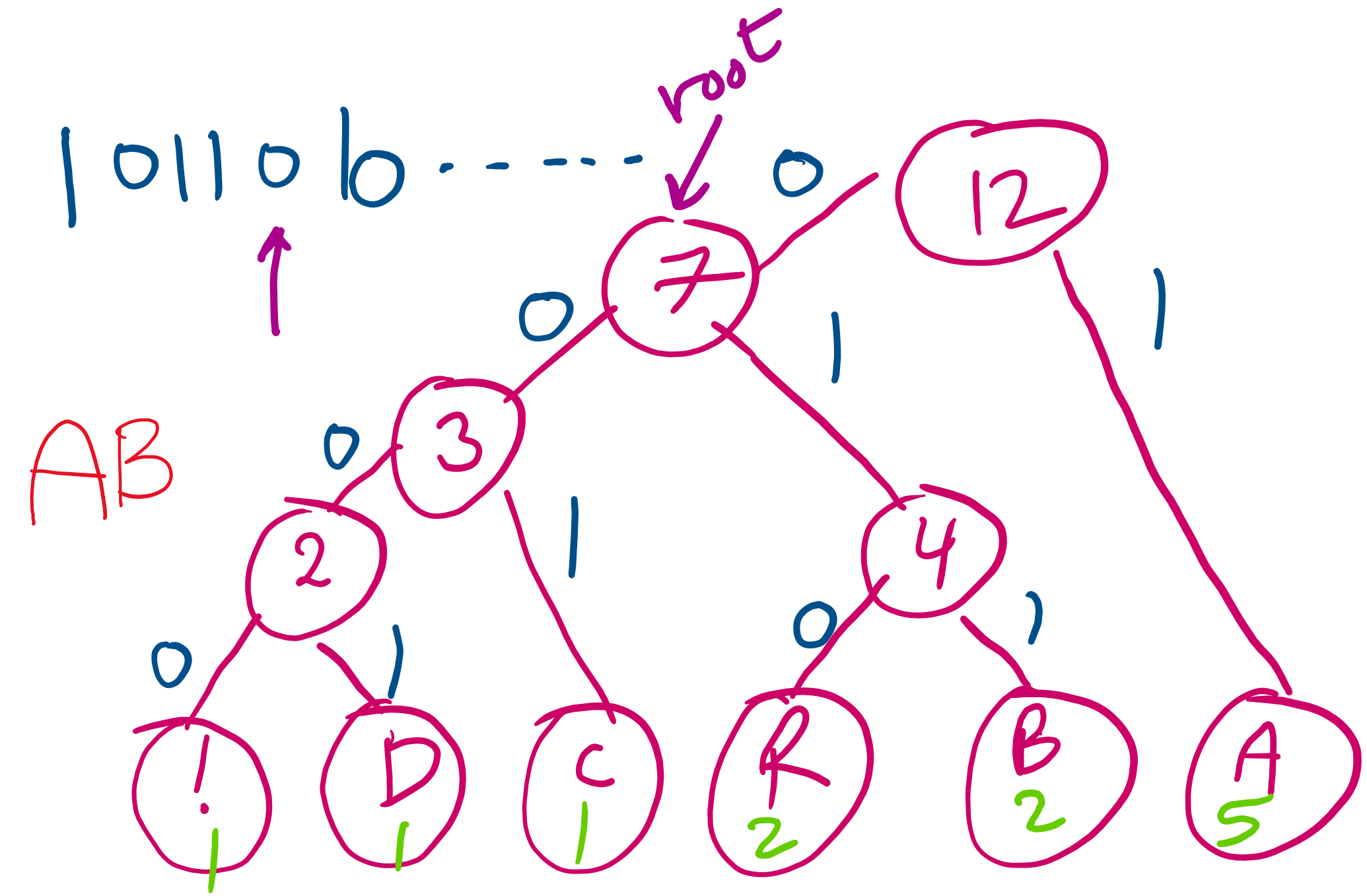
Huffman Compression: Decoding



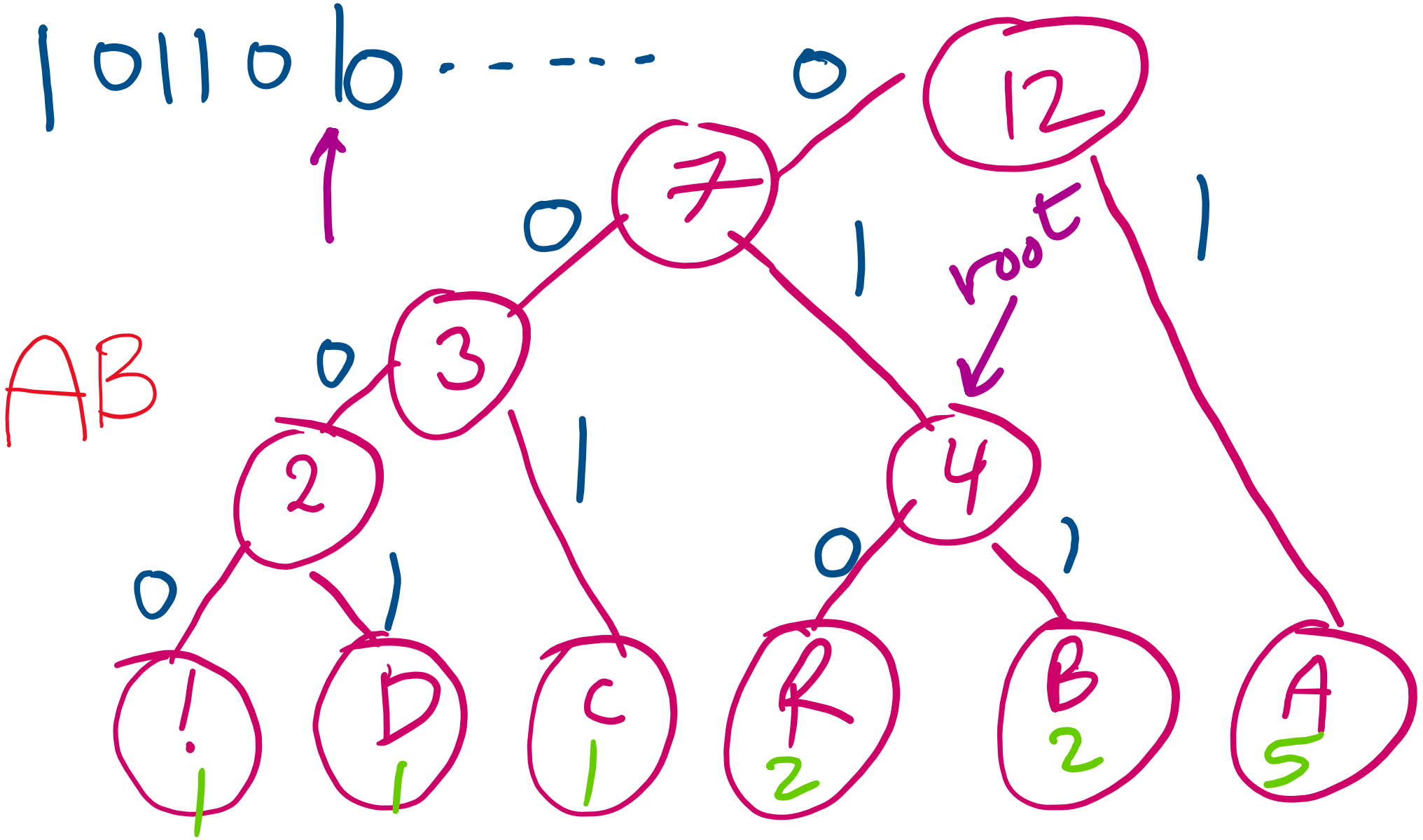
Huffman Compression: Decoding



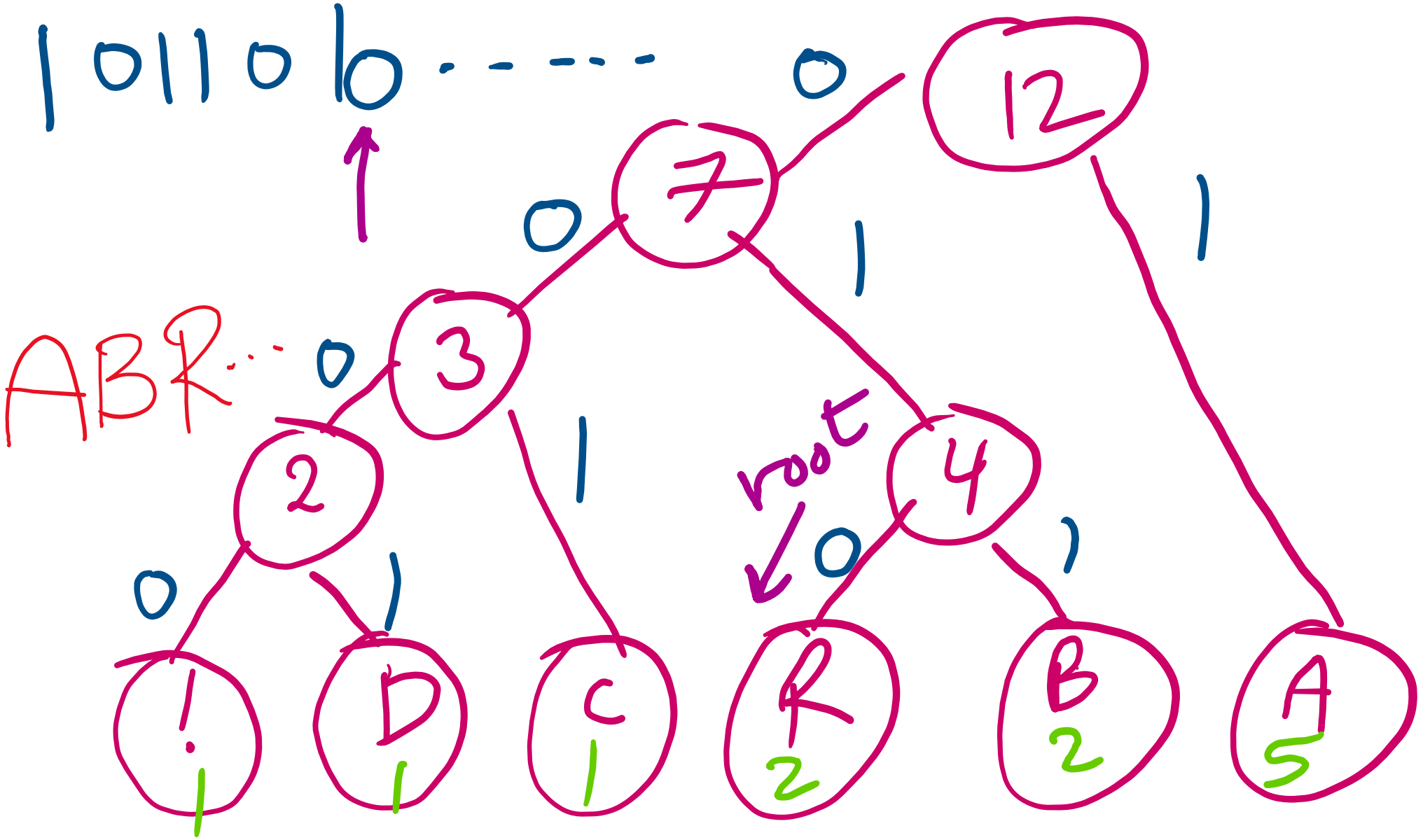
Huffman Compression: Decoding



Huffman Compression: Decoding



Huffman Compression: Decoding



How do we determine character frequencies?

- Option 1: Preprocess the file to be compressed
  - Upside: Ensure that Huffman's algorithm will produce the best output for the given file
  - Downsides:
    - Requires two passes over the input, one to analyze frequencies/build the trie/build the code lookup table, and another to compress the file
    - Trie must be stored with the compressed file, reducing the quality of the compression
      - This especially hurts small files
      - Generally, large files are more amenable to Huffman compression
        - Just because a file is large, however, does not mean that it will compress well!

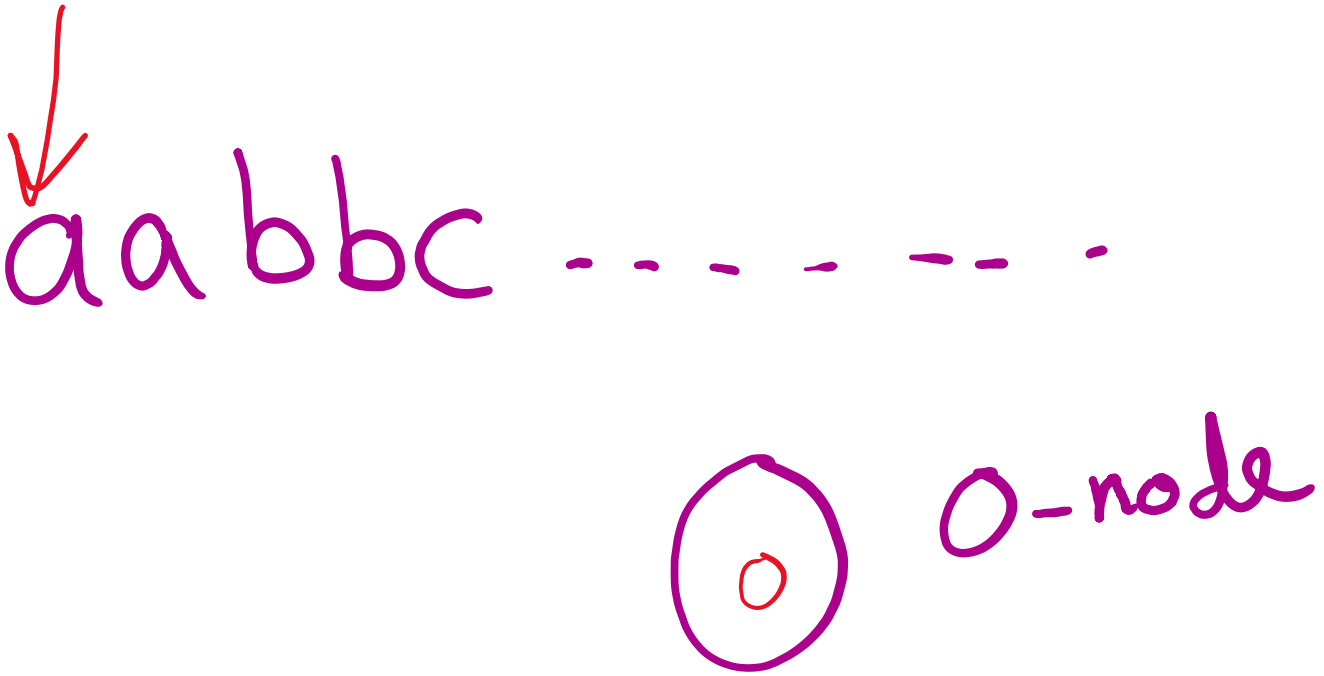
How do we determine character frequencies?

- Option 2: Use a static trie
  - Analyze multiple sample files, build a single tree that will be used for all compressions/expansions
  - Saves on trie storage overhead…
  - But in general not a very good approach
    - Different character frequency characteristics of different files means that a code set/trie that works well for one file could work very poorly for another
      - Could even cause an increase in file size after “compression”!

How do we determine character frequencies?

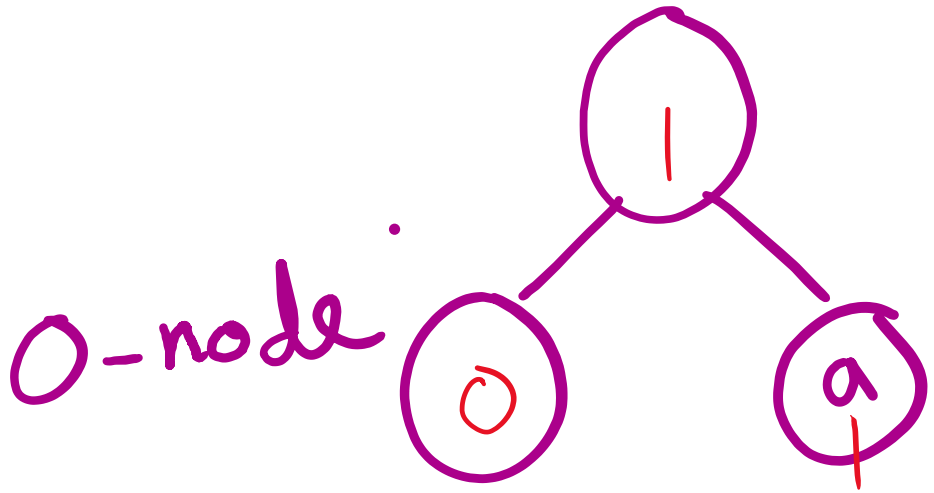
- Option 3: Adaptive Huffman coding
  - Single pass over the data to construct the codes and compress a file with no background knowledge of the source distribution
  - Not going to really focus on adaptive Huffman in the class, just pointing out that it exists...

Adaptive Huffman



Adaptive Huffman

aabbc.....

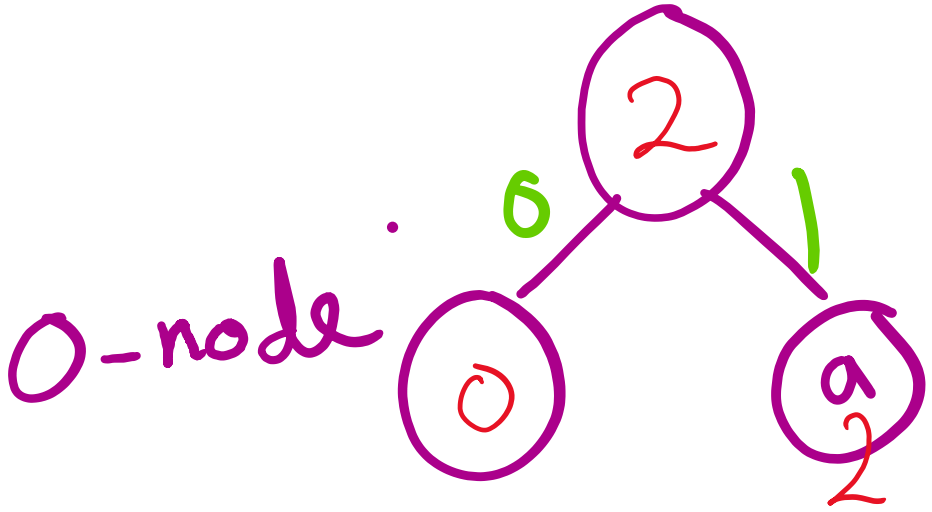


0-node

Ascii of a

Adaptive Huffman

aabbc.....

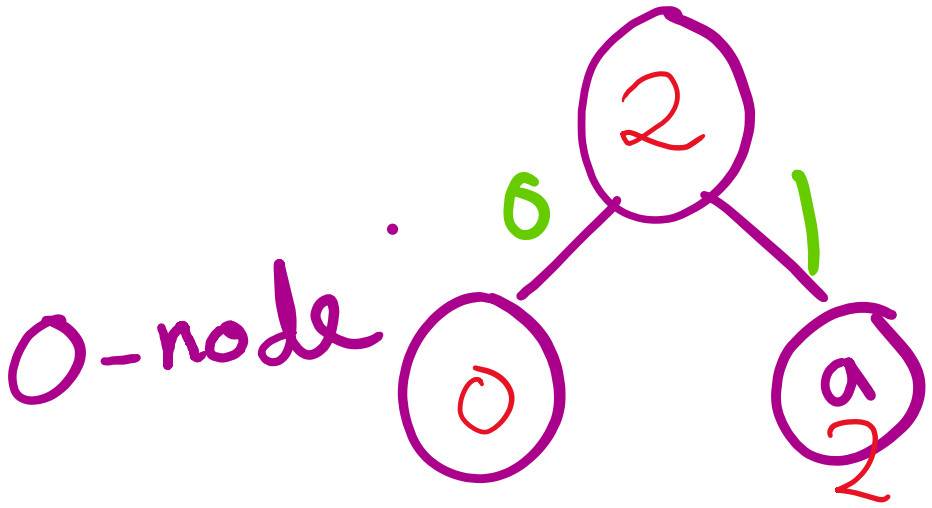


0-node

Ascii of a 1

Adaptive Huffman

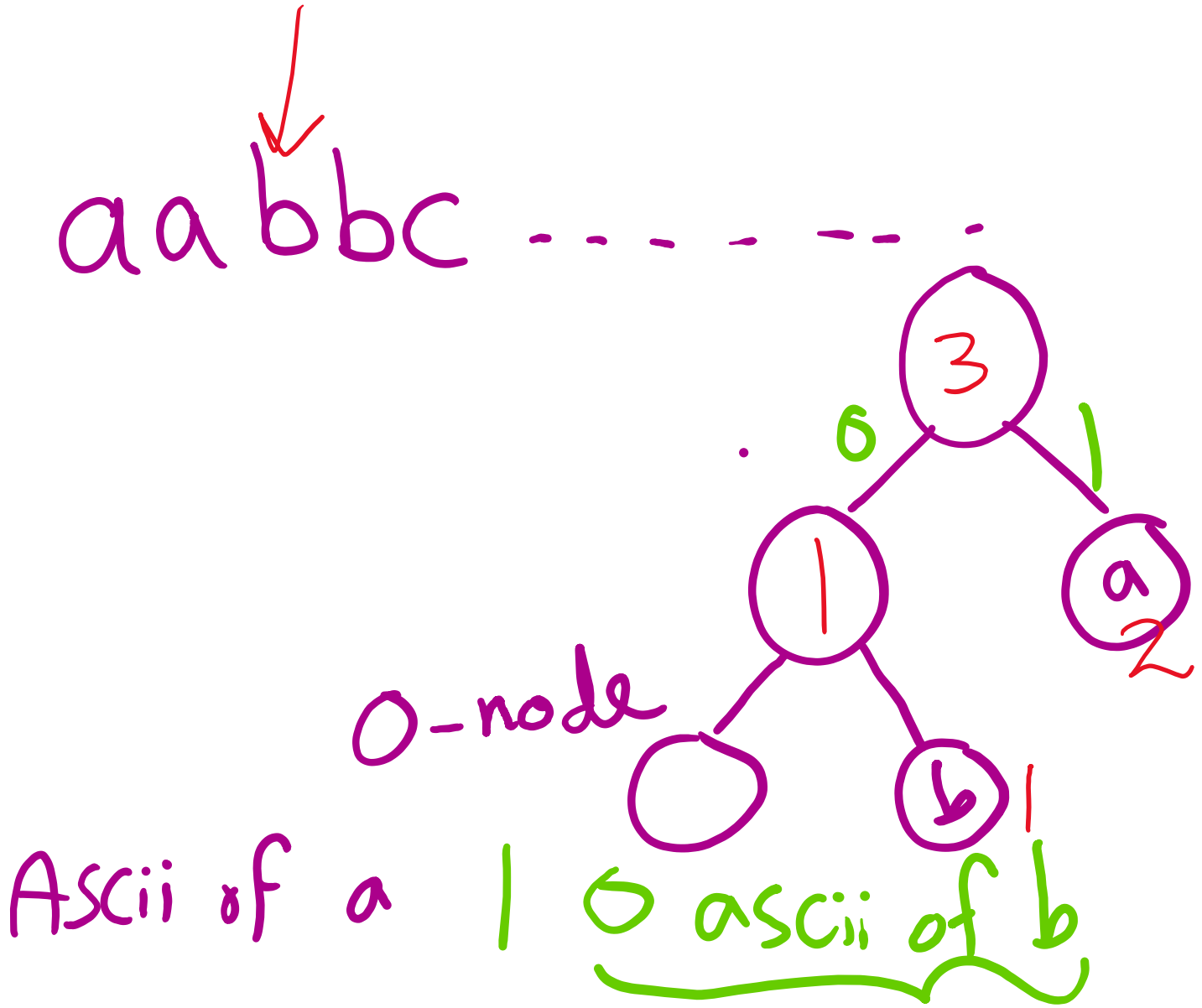
aabbc.....



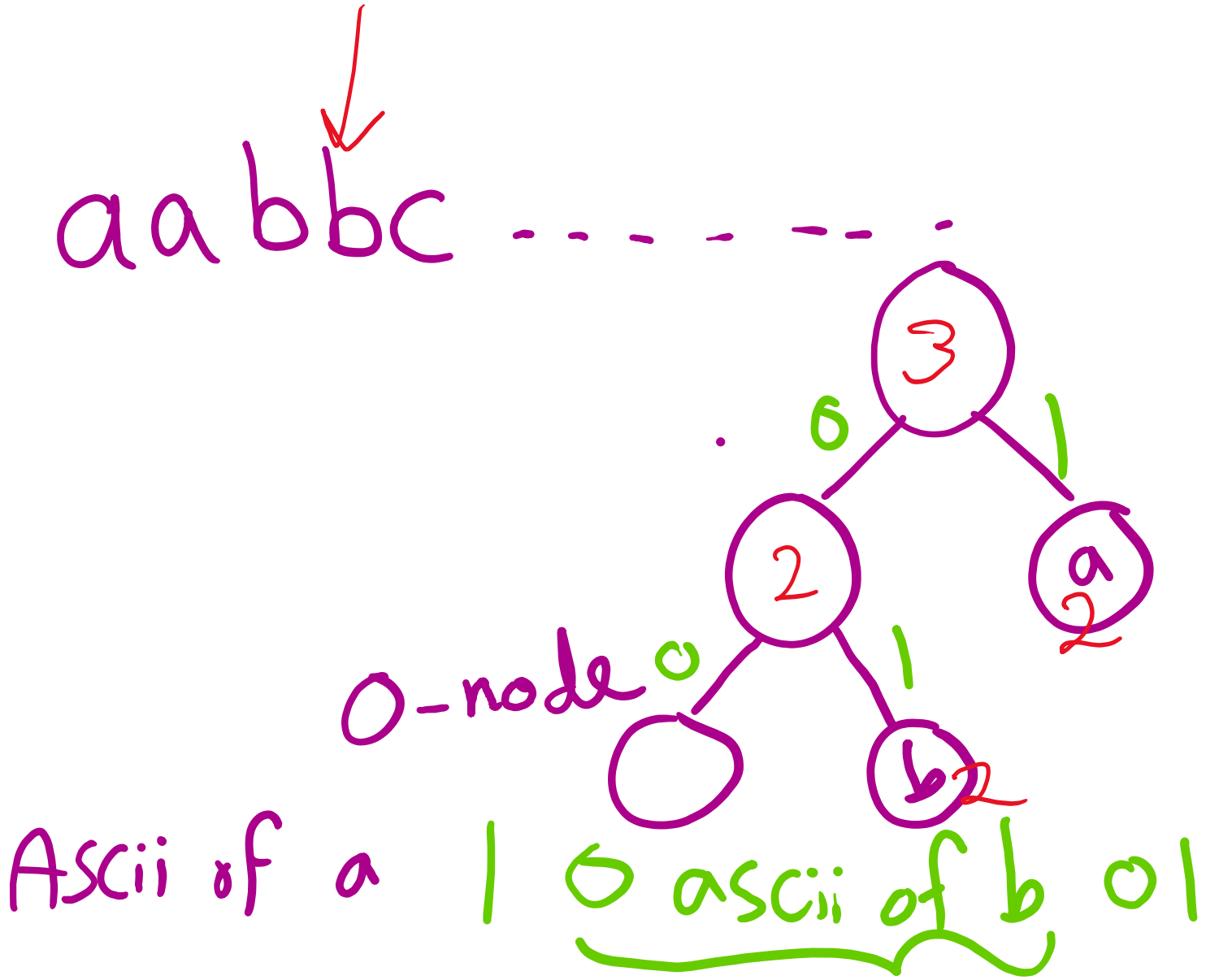
0-node

Ascii of a | 0 ascii of b

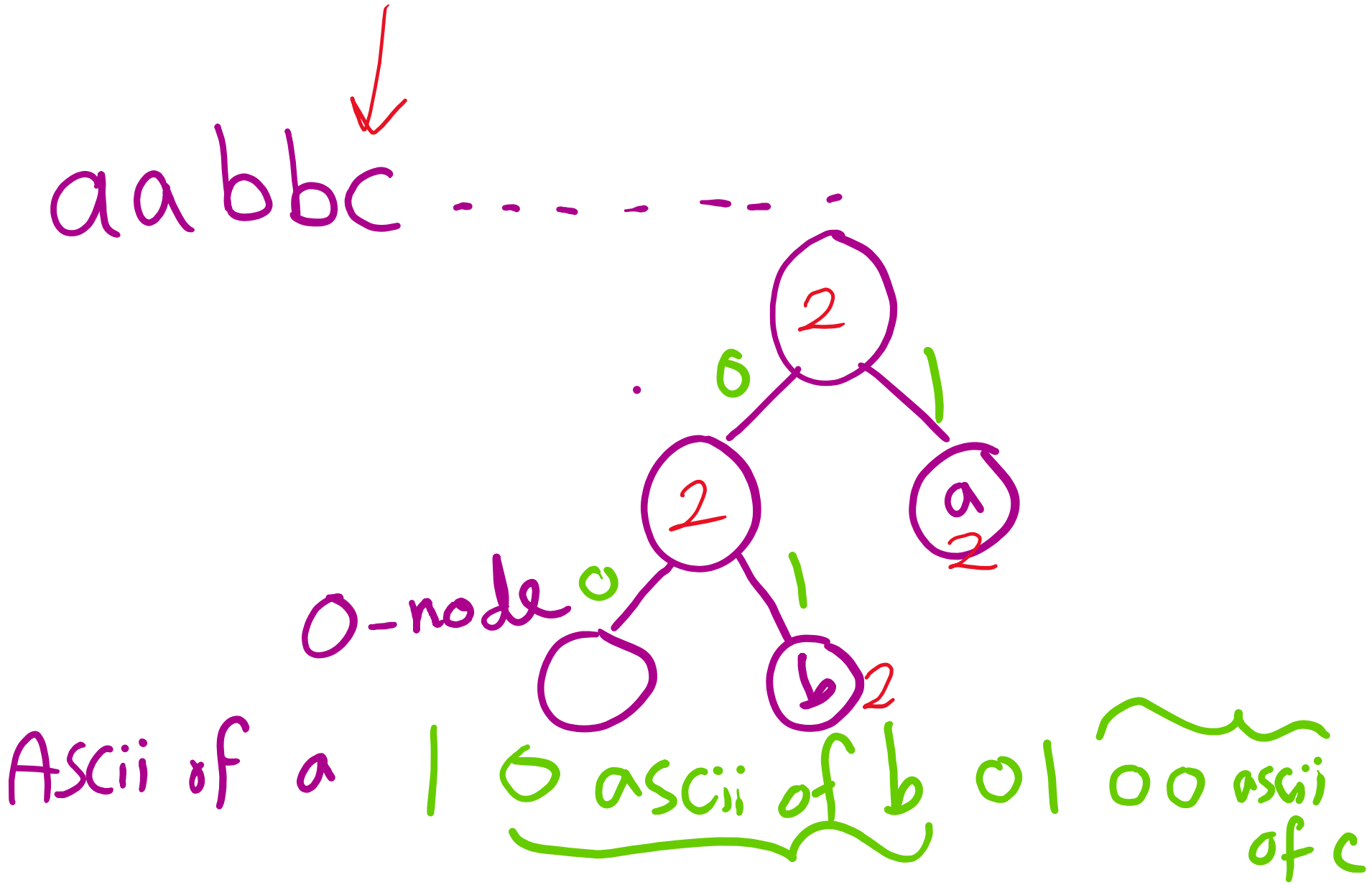
Adaptive Huffman



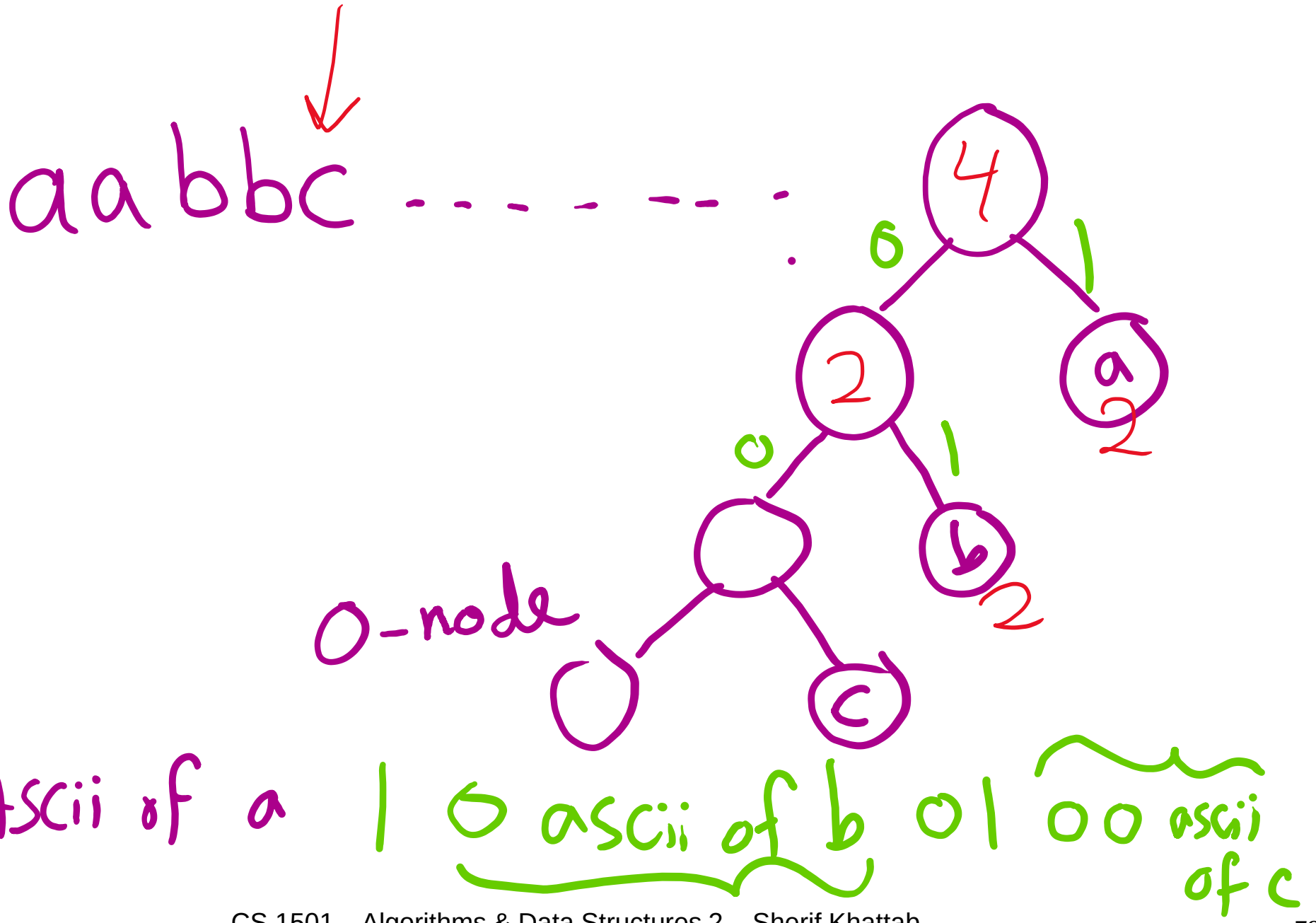
Adaptive Huffman



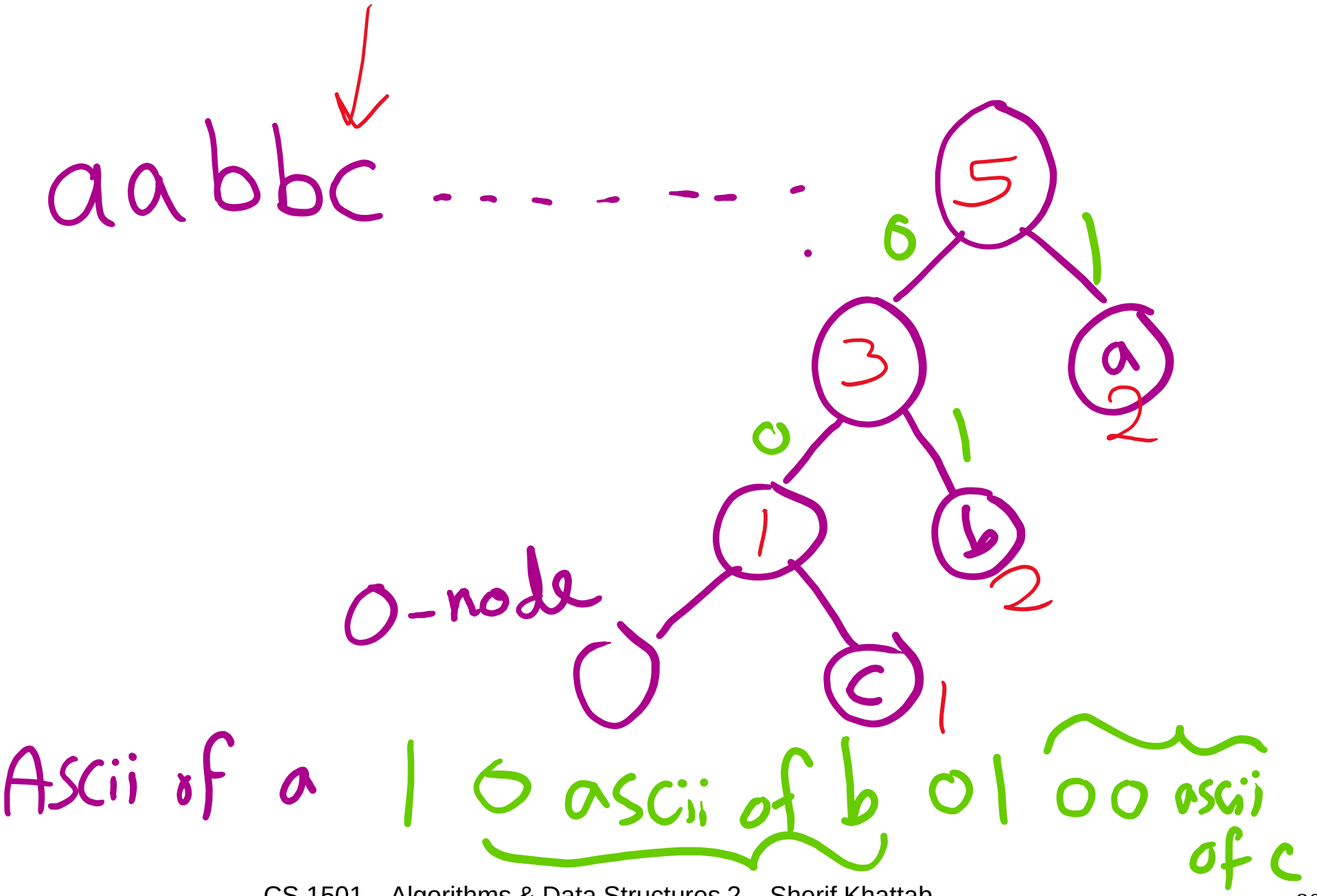
Adaptive Huffman



Adaptive Huffman

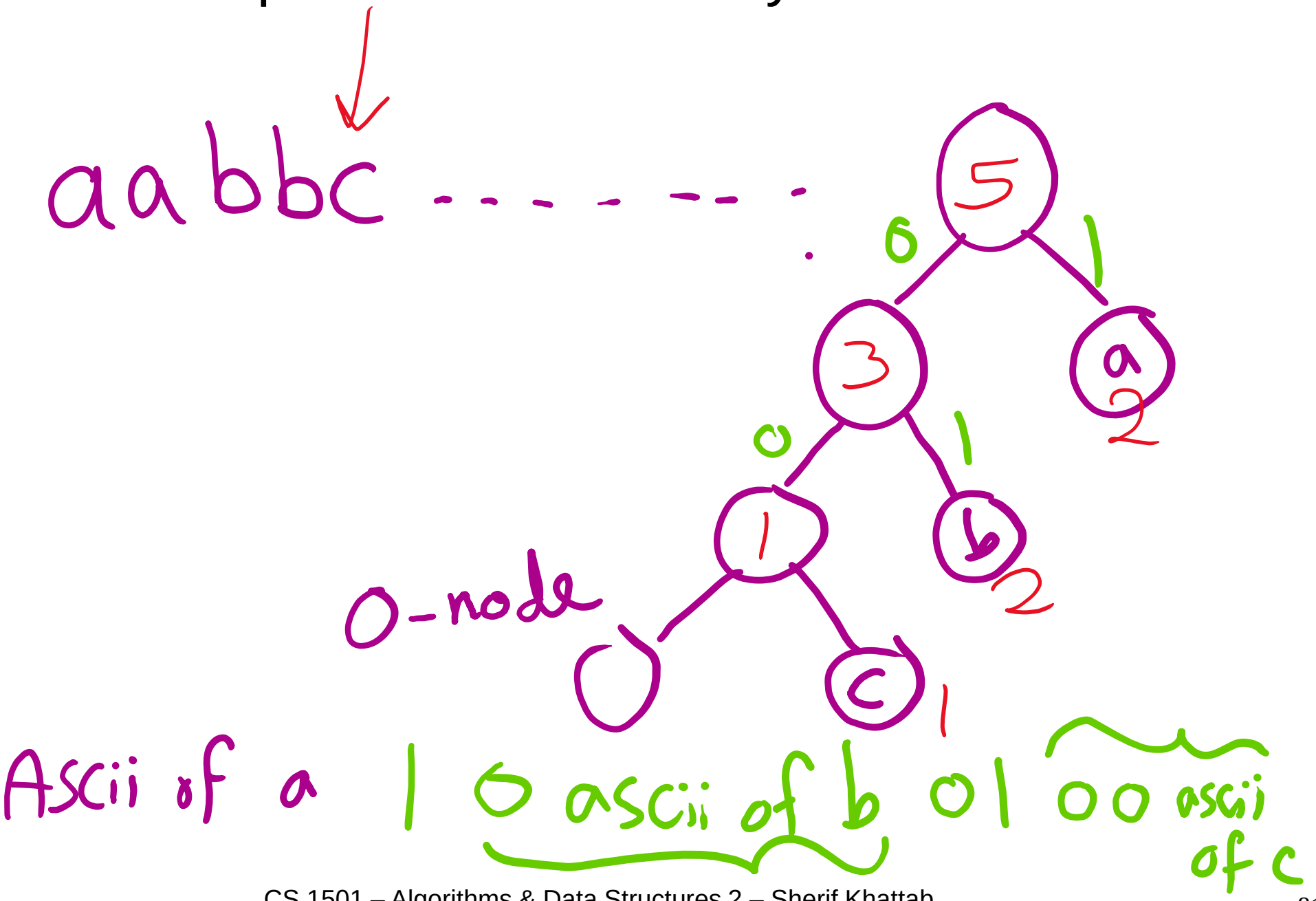


Adaptive Huffman



Adaptive Huffman

- need to swap nodes if necessary



Ok, so how good is Huffman compression

- ASCII requires 8n bits to store n characters
- For a file containing c different characters
  - Given Huffman codes $\{h_0, h_1, h_2, \ldots, h_{(c-1)}\}$
  - And frequencies $\{f_0, f_1, f_2, \ldots, f_{(c-1)}\}$
  - Sum from 0 to c-1: $|h_i| * f_i$
- Total storage depends on the differences in frequencies
  - The bigger the differences, the better the potential for compression
- Huffman is optimal for character-by-character prefix-free encodings
  - Proof in Propositions T and U of Section 5.5 of the text
- If all characters in the alphabet have the same usage frequency, we can't beat block code (fixed-size codewords)
  - Huffman reduces to fixed-size codewords
  - On a character by character basis…

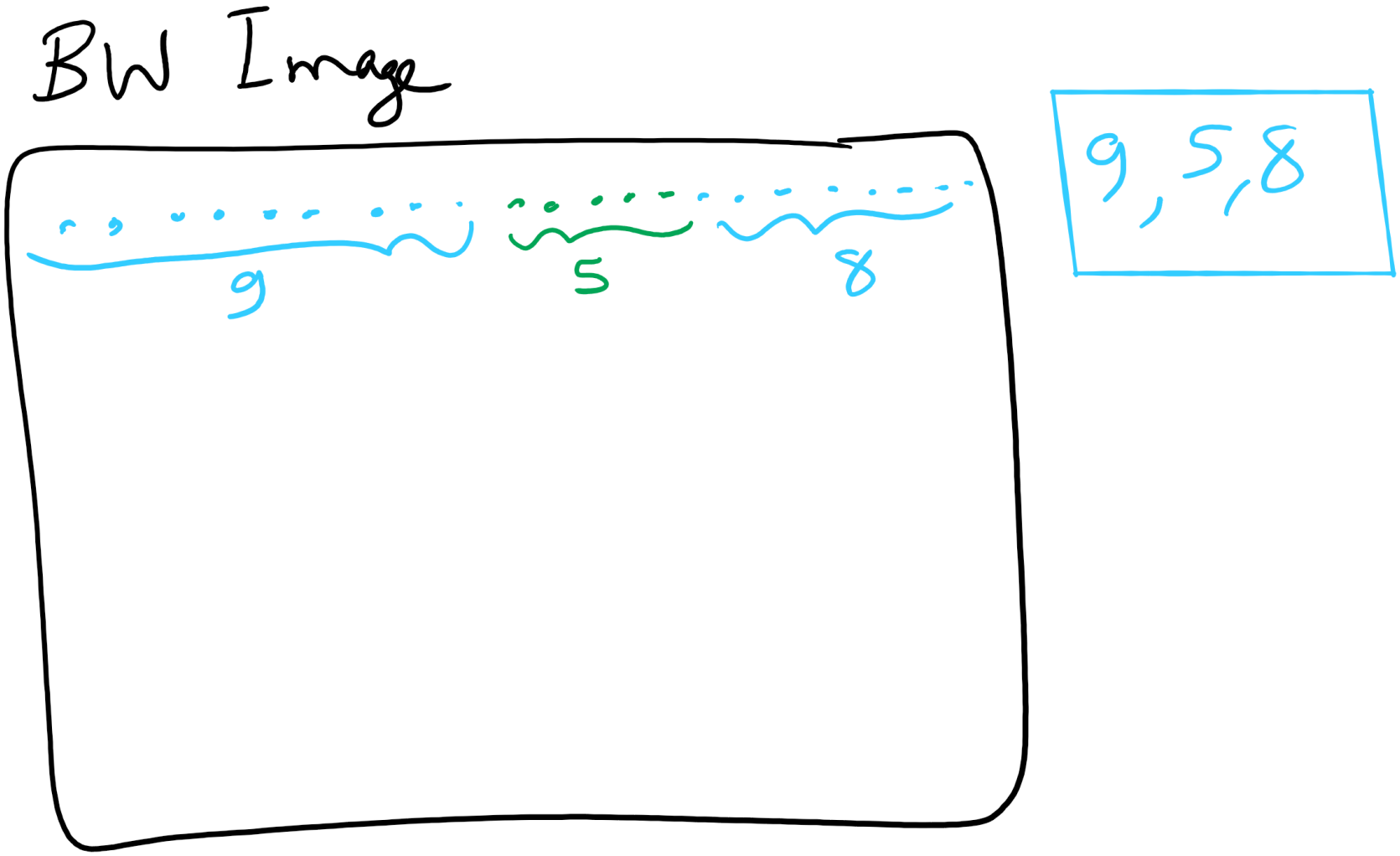
That seems like a bit of a caveat...

- Where does Huffman fall short?
  - What about repeated patterns of multiple characters?
    - Consider a file containing:
      - 1000 A's
      - 1000 B's
      - ...
      - 1000 of every ASCII character
    - Will this compress at all with Huffman encoding?
      - Nope!
    - But it seems like it should be compressible...

Run length encoding

- Could represent the previously mentioned string as:
  - 1000A1000B1000C, etc.
    - Assuming we use 10 bits to represent the number of repeats, and 8 bits to represent the character…
      - 4608 bits needed to store run length encoded file
      - vs. 2048000 bits for input file
      - Huge savings!
- Note that this incredible compression performance is based on a very specific scenario…
  - Run length encoding is not generally effective for most files, as they often lack long runs of repeated characters

Run-length Encoding



What else can we do to compress files?

Patterns are compressible, need a general approach

- Huffman used variable-length codewords to represent fixed-length portions of the input…
  - Let's try another approach that uses fixed-length codewords to represent variable-length portions of the input
- Idea: the more characters can be represented by a single codeword, the better the compression
  - Consider "the": 24 bits in ASCII
  - Representing "the" with a single 12 bit codeword cuts the used space in half
    - Similarly, representing longer strings with a 12 bit codeword would mean even better savings!

How do we know that “the” will be in our file?

- Need to avoid the same problems as the use of a static trie for Huffman encoding…
- So use an adaptive algorithm and build up our patterns and codewords **as we go** through the file

LZW compression

- Initialize codebook to all single characters
  - e.g., character maps to its ASCII value
  - codewords 0 to 255 are filled now
- While !EOF: (EOF is End Of File)
  - Find the longest match in codebook
  - Output codeword of that match
  - Take this longest match + the next character in the file
    - add that to the codebook with the next available codeword value
  - Start from the character right after the match

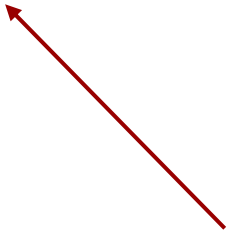
LZW compression example

- Compress, using 12 bit codewords:
  - TOBEORNOTTOBEORTOBEORNOT

Cur	Output	Add
T	84	TO:256
O	79	OB:257
B	66	BE:258
E	69	EO:259
O	79	OR:260
R	82	RN:261
N	78	NO:262
O	79	OT:263
T	84	TT:264
TO	256	TOB:265
BE	258	BEO:266
OR	260	ORT:267
TOB	265	TOBE:268
EO	259	EOR:269
RN	261	RNO:270
OT	263	--

LZW expansion

- Initialize codebook to all single characters
  - e.g., ASCII value maps to its character
- While !EOF:
  - Read next codeword from file
  - Lookup corresponding pattern in the codebook
  - Output that pattern
  - Add the previous pattern + the first character of the current pattern to the codebook



Note this means no codebook addition after first pattern output!

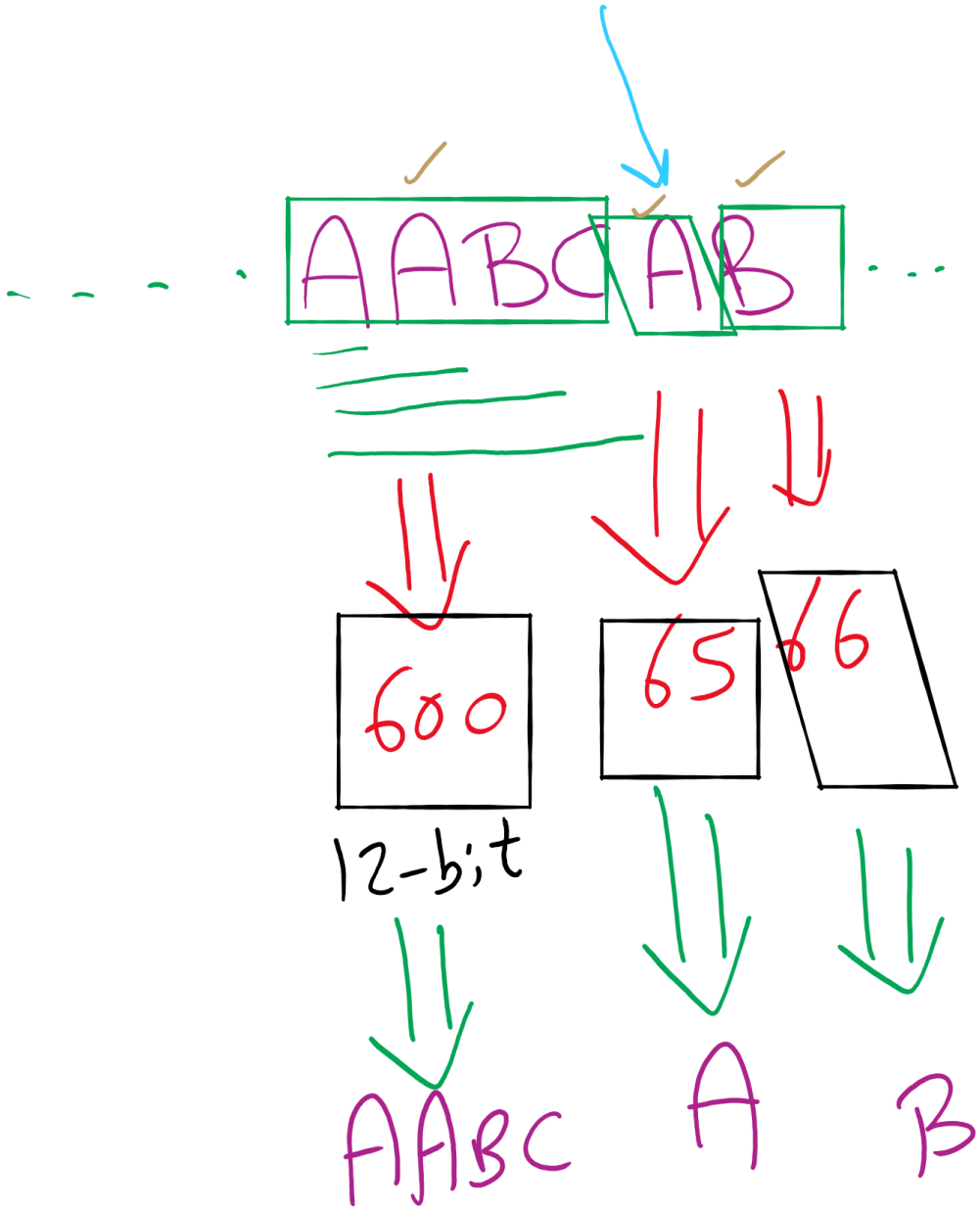
LZW expansion example

Cur	Output	Add
84	T	--
79	O	256:TO
66	B	257:OB
69	E	258:BE
79	O	259:EO
82	R	260:OR
78	N	261:RN
79	O	262:NO
84	T	263:OT
256	TO	264:TT
258	BE	265:TOB
260	OR	266:BEO
265	TOB	267:ORT
259	EO	268:TOBE
261	RN	269:EOR
263	OT	270:RNO

How does this work out?

- Both compression and expansion construct the same codebook!
  - Compression stores character string → codeword
  - Expansion stores codeword → character string
  - They contain the same pairs in the same order
    - Hence, the codebook doesn't need to be stored with the compressed file, saving space

LZW Example



A	65
B	66
AA	300
AAB	400
AABB	500
AABC	600
AABCA	601
AB	602

Just one tiny little issue to sort out...

- Expansion can sometimes be a step ahead of compression…
  - If, during compression, the (pattern, codeword) that was just added to the dictionary is immediately used in the next step, the decompression algorithm will not yet know the codeword.
  - This is easily detected and dealt with, however

LZW corner case example

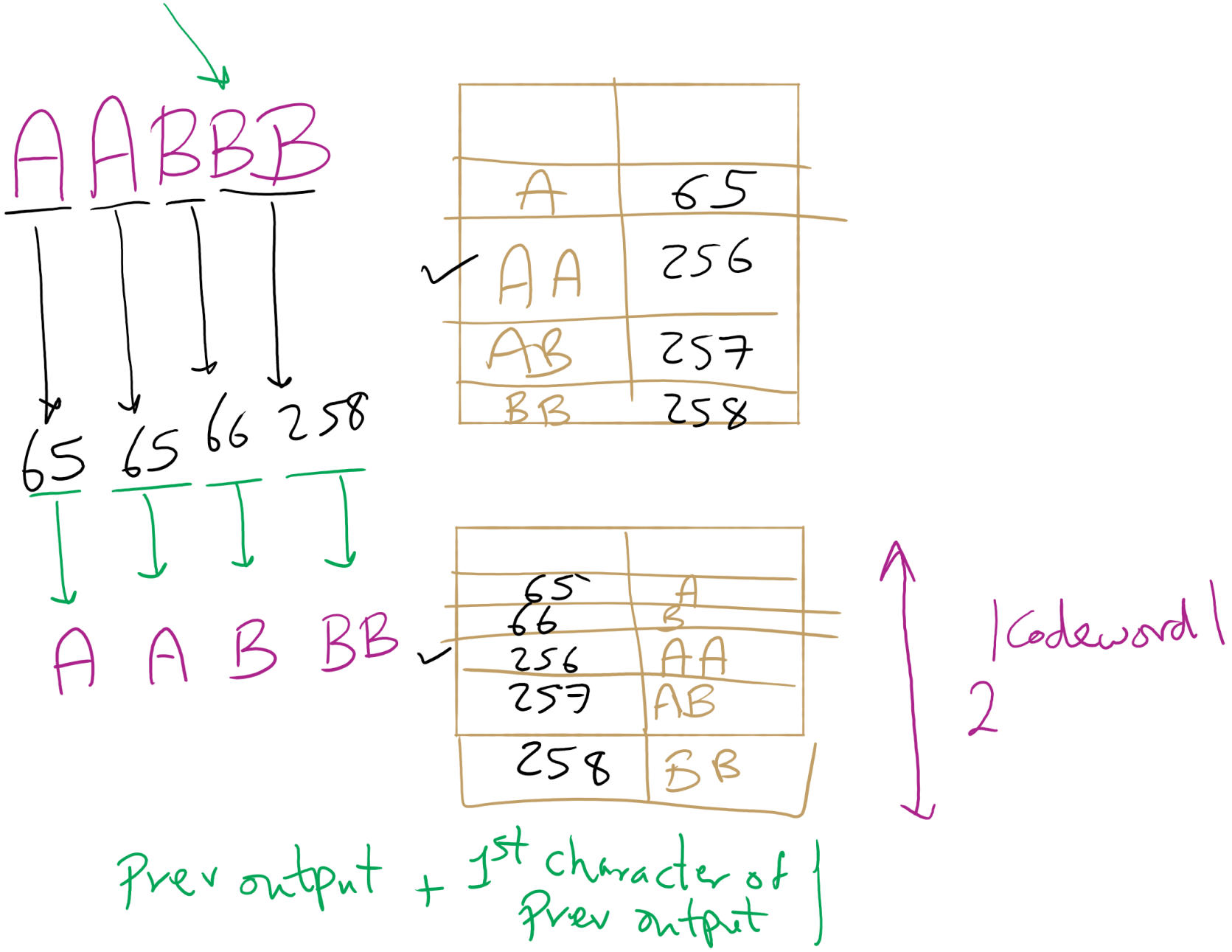
- Compress, using 12 bit codewords: AAAAAA

Cur	Output	Add
A	65	AA:256
AA	256	AAA:257
AAA	257	--

- Expansion:

Cur	Output	Add
65	A	--
256	AA	256:AA
257	AAA	257:AAA

LZW Corner Case



LZW implementation concerns: codebook

- How to represent/store during:
  - Compression
  - Expansion
- Considerations:
  - What operations are needed?
  - How many of these operations are going to be performed?
- Discuss

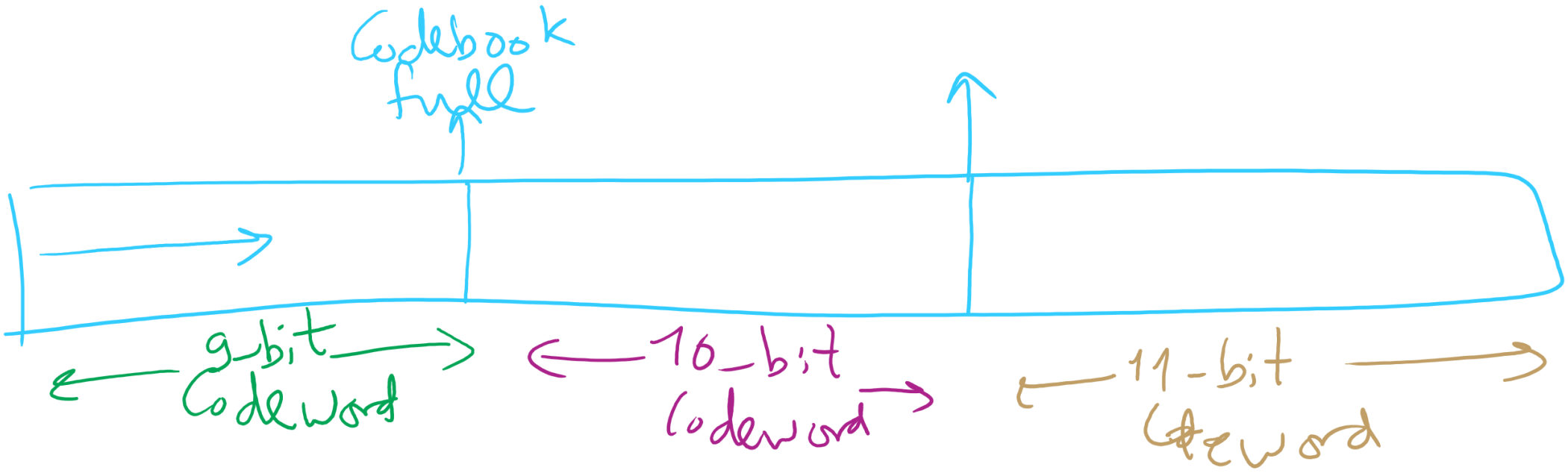
Further implementation issues: codeword size

- How long should codewords be?
  - Use fewer bits:
    - Gives better compression earlier on
    - But, leaves fewer codewords available, which will hamper compression later on
  - Use more bits:
    - Delays actual compression until longer patterns are found due to large codeword size
    - More codewords available means that greater compression gains can be made later on in the process

Variable width codewords

- This sounds eerily like variable length codewords…
  - Exactly what we set out to avoid!
- Here, we're talking about a different technique
- Example:
  - Start out using 9 bit codewords
  - When codeword 512 is inserted into the codebook, switch to outputting/grabbing 10 bit codewords
  - When codeword 1024 is inserted into the codebook, switch to outputting/grabbing 11 bit codewords…
  - Etc.

Adaptive Codeword Size



Even further implementation issues: codebook size

- What happens when we run out of codewords?
  - Only 2^n possible codewords for n bit codes
  - Even using variable width codewords, they can't grow arbitrarily large…
- Two primary options:
  - Stop adding new keywords, use the codebook as it stands
    - Maintains long already established patterns
    - But if the file changes, it will not be compressed as effectively
  - Throw out the codebook and start over from single characters
    - Allows new patterns to be compressed
    - Until new patterns are built up, though, compression will be minimal