

ALGORITHMS AND DATA STRUCTURES

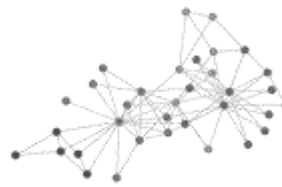
LECTURE 1 - RECURSION

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RECURSION OVERVIEW

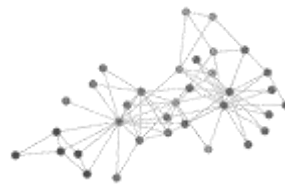
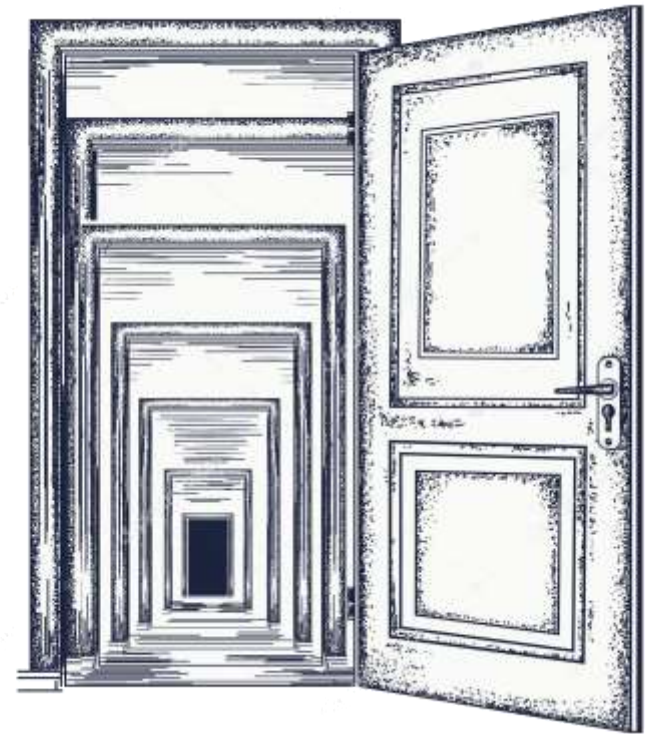
Recursion is the process of repeating items in a self-similar way

A way to design solutions by Divide-and-Conquer

- Reduce a problem to simpler versions of the same problem

A programming technique where a function calls itself

- Must have at least 1 **base case**
- **Base case** means that there exist one or more inputs for which the function produces a result trivially (without recurring)
- Must solve the same problem on some other input with the goal of simplifying the larger problem input



SIMPLE EXAMPLE

$$N! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot \dots \cdot N$$

fact(N)

$$N! = \underbrace{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot \dots \cdot (N-1)}_{\text{fact(N-1)}} \cdot N$$

fact(N-1) * N

$$N! = \underbrace{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot \dots \cdot (N-2)}_{\text{fact(N-2)}} \cdot (N-1) \cdot N$$

fact(N-2) * (N-1) * N

$$N! = \underbrace{1}_{\text{Base case!}} \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot \dots \cdot N$$

fact(1) * 2 * 3 * ... * N

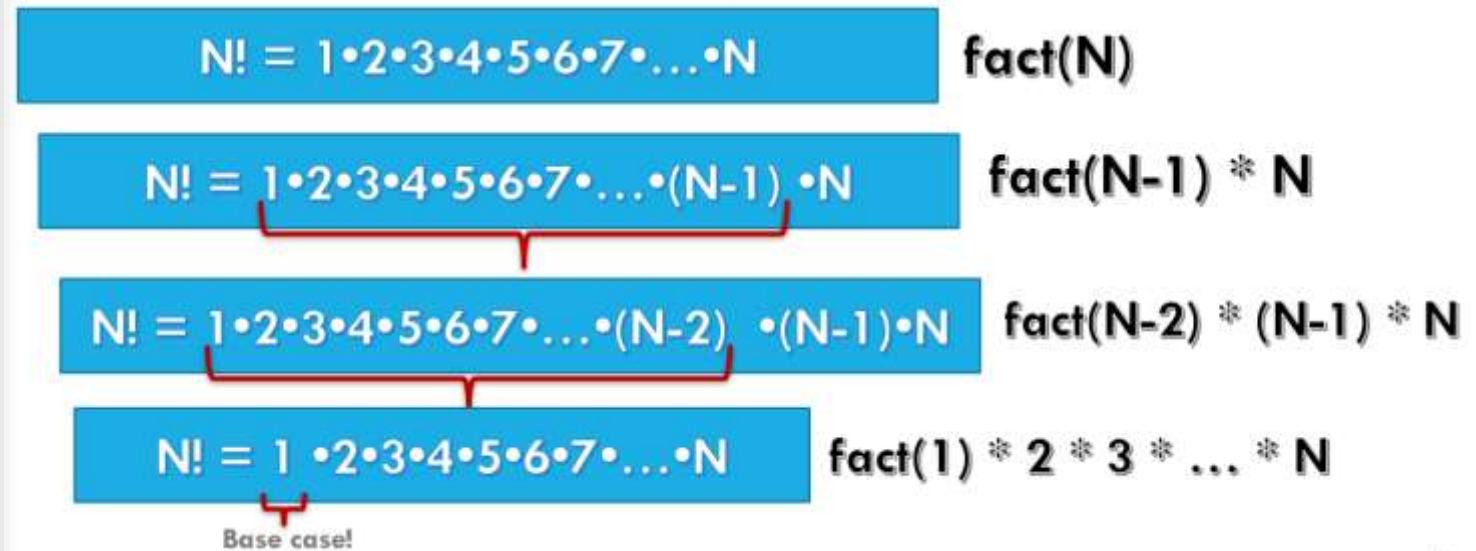
Base case!



SIMPLE EXAMPLE

```
public static int factorial(int N) {
    if (N <= 1) return 1; // base case
    return factorial(N - 1) * N;
}

public static void main(String[] args) {
    Scanner sc = new Scanner(System.in);
    int n = sc.nextInt();
    int result = factorial(n);
    System.out.println(result);
}
```



HOW IT WORKS?

Recursion is no different than a function call

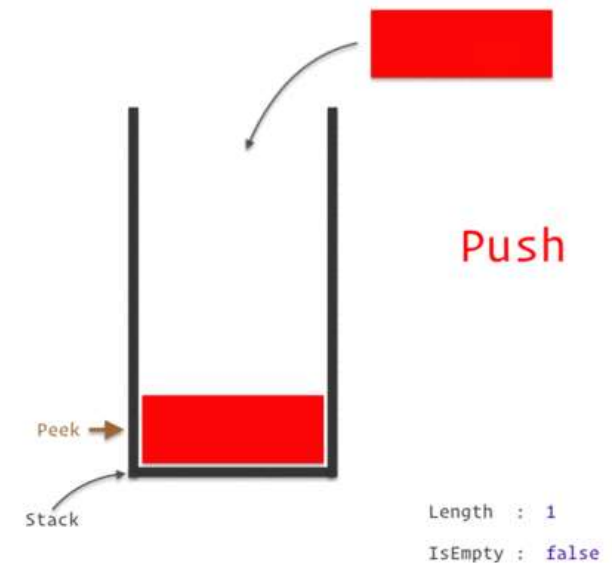
Every function call creates a new frame (block) inside the stack

The system keeps track of the sequence of method calls that have been started but not finished yet (active calls)

- order matters

Recursion pitfalls

- miss base-case (infinite recursion, stack overflow)
- no convergence (solve recursively a problem that is not simpler than the original one)



Stack

FUNCTION CALL AND STACK

When you run a program, the computer creates a **stack** for you

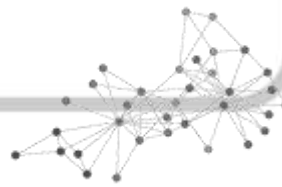
Each time you invoke a method, the method is placed to the stack

A stack is a **last-in/first-out** memory structure. The first item referenced or removed from a stack is always the last item entered into the stack

If some function call has produced an excessively long chain of recursive calls, it can lead to **stack overflow**

```
int factorial(int N) {  
    if (N <= 1) return 1; // base case  
    return factorial(N - 1) * N;  
}  
  
void main() {  
    int result = factorial(N:3);  
    System.out.println(result);  
}
```

```
void main() {  
    > int result = factorial(N:3);  
    System.out.println(result);  
}
```



ITERATION VS RECURSION

Iteration

- Uses repetition structures (for, while or do...while)
- Repetition through explicitly use of repetition structure
- Terminates when loop-continuation condition fails
- Controls repetition by using a counter

```
public static int factorial(int N) {  
    int product = 1;  
    for (int i = 1; i <= N; i++) {  
        product *= i;  
    }  
  
    return product;  
}
```

Recursion

- Uses selection structures (if, if...else or switch)
- Repetition through repeated method calls
- Terminates when base case is satisfied
- Controls repetition by dividing problem into simpler one

```
public static int factorial(int N) {  
    if (N <= 1) return 1; // base case  
  
    return factorial(N - 1) * N;  
}
```



ITERATION VS RECURSION

Repetition

- Iteration: explicit loop
- Recursion: repeated function calls

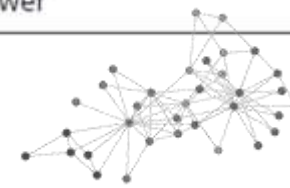
Termination

- Iteration: loop condition fails
- Recursion: base case recognized

Both can have infinite loops

Balance between performance (iteration) and good software engineering (recursion)

Criteria	Iteration	Recursion
Mode of implementation	Implemented using loops	Function calls itself
State	Defined by the control variable's value	Defined by the parameter values stored in stack
Progression	The value of control variable moves towards the value in condition	The function state converges towards the base case
Termination	Loop ends when control variable's value satisfies the condition	Recursion ends when base case becomes true
Code Size	Iterative code tends to be bigger in size	Recursion decreases the size of code
No Termination State	Infinite Loops uses CPU Cycles	Infinite Recursion may cause Stack Overflow error or it might crash the system
Execution	Execution is faster	Execution is slower



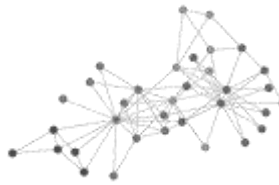
HOW TO CREATE A RECURSIVE ALGORITHM?

1. Think about a problem at a high level of abstraction

2. Figure out the **base case** for the program

3. Redefine the answer in terms of a simpler sub-problem

4. Combine the results in the formulation of the answer



FIBONACCI SOLUTION

Fibonacci sequence

- 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55 ...
- Each element is the sum of previous two
- Starts from 0 and 1

Task: Find the Fibonacci number at the given position

Example:

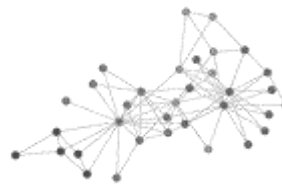
- 3rd element is 5
- 6th element is 8

Solution:

$$\text{fib}(n) = \text{fib}(n-2) + \text{fib}(n-1)$$

$\text{fib}(0) = 0$ and $\text{fib}(1) = 1$ // this is a base case

```
public static int fib(int n) {  
    if (n <= 1) return n; // base case  
    // no need to write "else", since the  
    // previous one will return  
    return fib(n-2) + fib(n-1);  
}
```



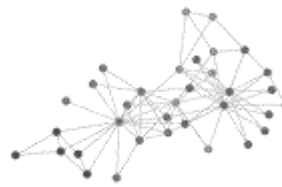
LITERATURE

Algorithms, 4th Edition, by Robert Sedgewick and Kevin Wayne, Addison-Wesley

- Chapter 1.1

Grokking Algorithms, by Aditya Y. Bhargava, Manning

- Chapter 3



GOOD LUCK!

