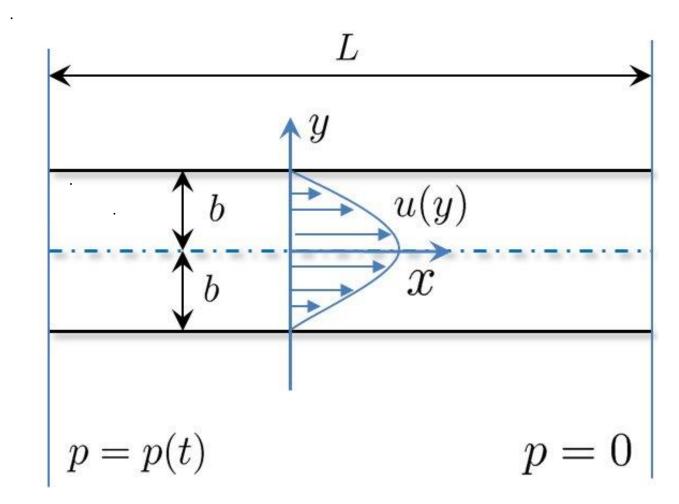
Poiseuille flow simulation via Lattice Boltzmann method

Parallel Computing in Mathematical Modeling and Data-Intensive Applications

- Optimize given sequential Lattice Boltzmann method code
- Develop the parallelized code from sequential
- Profile. Measure timing and plot speedup plot depending on number of processes/threads.
- Conclude. Is the speedup You obtained is approximately linear?

Dimensionless parameters of simulation:

Sign	Description	Value
N_{x}	Length of tube, equivalent to L	1000
$N_{\mathcal{Y}}$	Diameter of tube, equivalent to 2*b	200
$ ho_{in}$	Inlet pressure in LBM units.	1
$ ho_{out}$	Outlet pressure in LBM units	0.95
τ	Relaxation time	1
nt	Number of time steps	20000

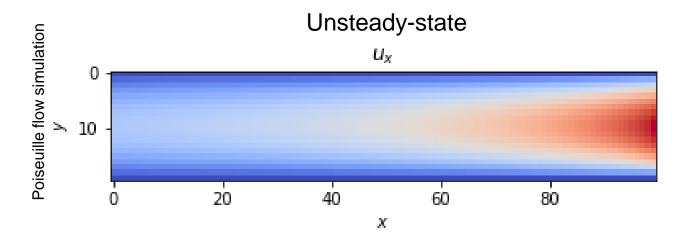


$$\widetilde{f}_i(r,t) = f_i(r + \overrightarrow{v_i}, t + 1)$$

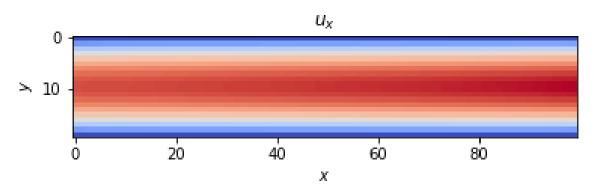
Streaming step

$$f_i(r,t) = \widetilde{f}_i(r,t) - \frac{\widetilde{f}_i - f_i^{eq}}{\tau}$$

Collision step



Steady-state



Poiseuille velocity profile:

$$u(y) = -\frac{\Delta p}{4\rho \nu L} (b^2 - y^2)$$

Lattice units:

$$\tau = \frac{\nu}{c_s^2} + 0.5$$

$$\nu - \text{kinematic}$$
viscosity of fluid

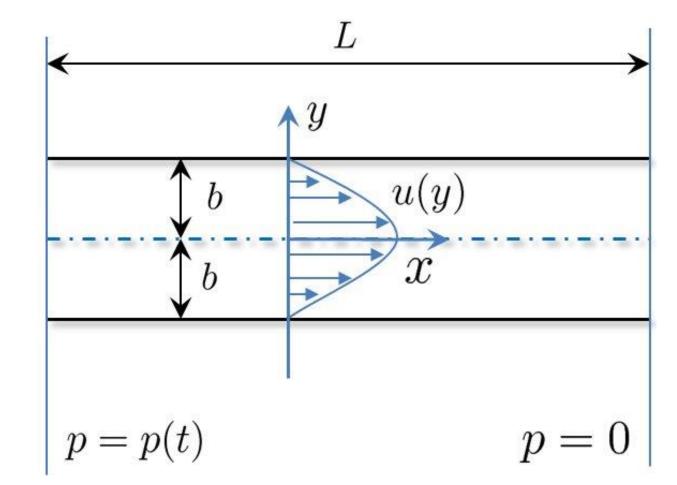
$$\rho_{phys} \neq \rho_{LB}$$

$$p_{LB} = c_s^2 \rho_{LB}$$

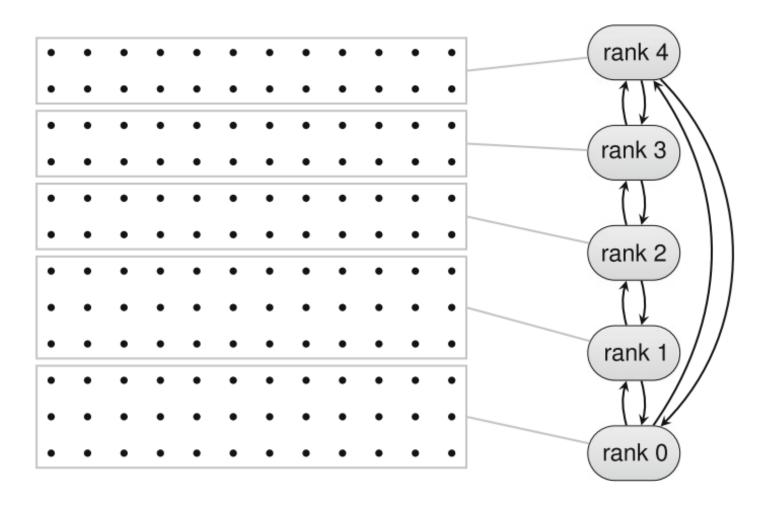
Skoltech

Several Sequential codes in shared folder:

/gpfs/gpfs0/ParallelComputingShared/Lattice _Boltzmann_2021



```
If (rank < NX % nprocs): //ranks that are less than reminder rank_nx = NX/nprocs+1; rank_xstart = rank*rank*nx; else: rank_nx = NX/procs; rank_xstart = NX-(nprocs-rank)*rank_nx print("Rank %d: %d nodes from y = %d to y = %d\n", % (rank, rank_nx, rank_xstart, rank_xstart+rank_nx-1))
```



Blocking realization

```
if (rank % 2 == 0) // even ranks send then receive
  // send below, i.e. rank-1
 MPI Send(send buffer, count, MPI DOUBLE,
           rankm1, tag, MPI COMM WORLD);
  // receive from above, i.e. rank+1
 MPI Recv(recv buffer, count, MPI DOUBLE,
           rankp1, tag, MPI COMM WORLD,
           status);
else // odd ranks receive then send
 // receive from above, i.e. rank+1
 MPI Recv (recv buffer, count, MPI DOUBLE,
          rankp1, tag, MPI COMM WORLD,
          status);
 // send below, i.e. rank-1
 MPI Send (send buffer, count, MPI DOUBLE,
          rankm1, tag, MPI COMM WORLD);
```

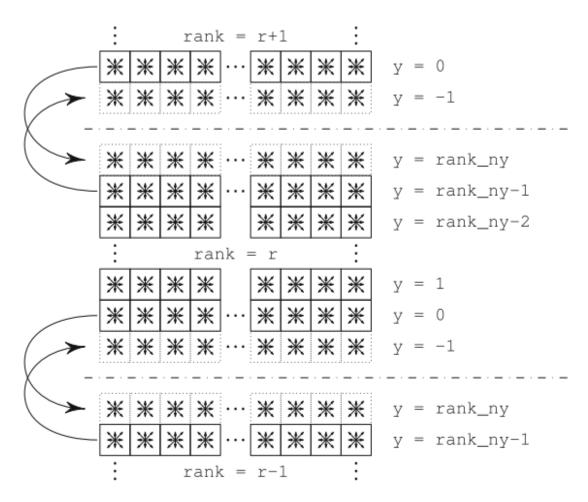


Fig. 13.10 Diagram showing the rows of data that need to be transferred across subdomain boundaries between processes. *Boxes with solid outlines* denote the nodes of the subdomain updated by each rank, while *boxes with dotted outlines* denote the extra rows used to store data from adjacent subdomains that are handled by different ranks

Blocking realization

```
size_t transfer_doubles = (ndir-1) *NX;
```

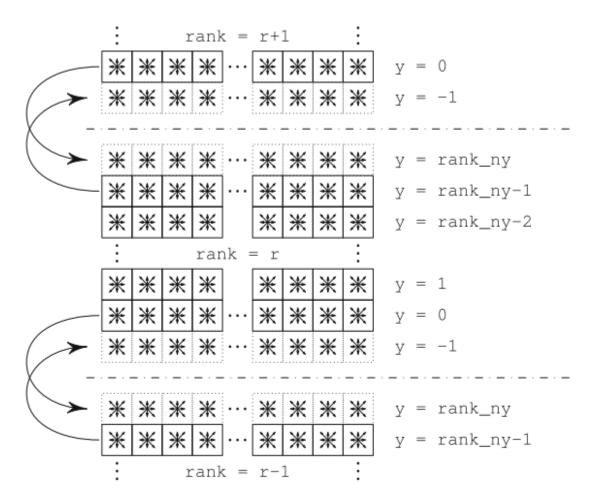


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Non-Blocking realization

```
MPI_Request reqs[4];
MPI_Status stats[4];
```

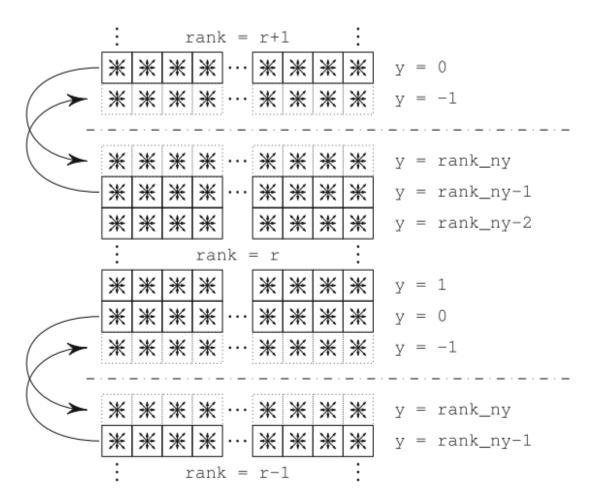


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Non-Blocking realization

```
MPI Request reqs[4];
   MPI Status stats[4];
  MPI Isend(&f1[fieldn index(0,rank ny-1,1)],
            transfer doubles, MPI DOUBLE,
            rankp1, rank, MPI COMM WORLD, &regs[0]
  MPI Irecv(&f1[fieldn index(0,-1,1)],
            transfer doubles, MPI DOUBLE,
            rankm1, rankm1,
            MPI COMM WORLD, &regs[1]);
  MPI Isend(&f1[fieldn index(0,0,1)],
            transfer doubles, MPI DOUBLE,
            rankm1, rank,
            MPI COMM WORLD, &regs[2]);
  MPI Irecv(&f1[fieldn index(0,rank ny,1)],
            transfer doubles, MPI DOUBLE,
            rankp1, rankp1,
            MPI COMM WORLD, &regs[3]);
  int MPI Waitall (int count,
                  MPI_Request *array_of_requests,
                  MPI_Status *array_of_statuses)
int MPI Testall (int count,
                MPI Request *array of requests,
                int *flag,
                MPI Status *array of statuses)
```

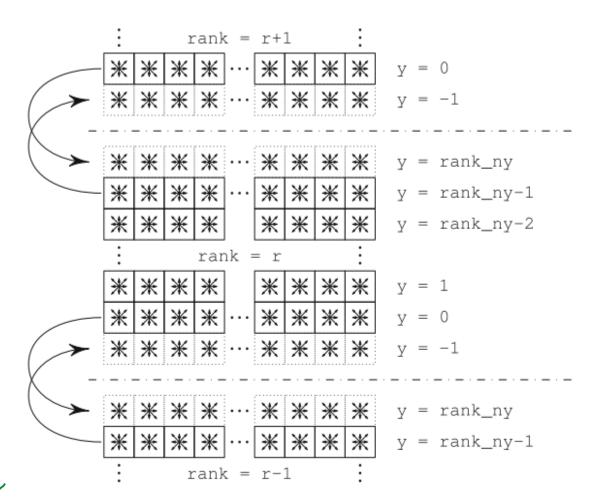


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Non-Blocking realization

Profiling tools for Python

CPU time profiling tools:

- 1. timeit
- 2. cProfile
- 3. line_profiler

Memory profiling tools:

- 1. memory_profiler
- 2. heapy

Contacts for questions:

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Email: Andrey. Olhin @skoltech.ru

MPI commands for Python:

/gpfs/gpfs0/ParallelComputingShared/Lattice_B oltzmann_2021 MPI_commands/

Lattice Boltzmann book

Principles of parallelization of LBM – Chapter 13.4

