Vv285 Recitation Class 7
Integration over $\mathbb{R}^n$

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Outline

1. Integration over (Ordinate) Regions
2. Integration over different coordinate system
3. Change of variables
4. Reference

Determine the Pivot for the Region

Property

1. $\iint_R \lambda f(x,y) + \mu g(x,y) dA = \lambda \iint_R f(x,y) dA + \mu \iint_R g(x,y) dA.$
   Since the integration operator is linear.
2. *If region R can be split into two disjoint regions D_1, D_2 with $D_1 \cap D_2 = \varnothing$, then*

$$\iint_R f(x,y) dA = \iint_{D_1} f(x,y) dA + \iint_{D_2} f(x,y) dA$$

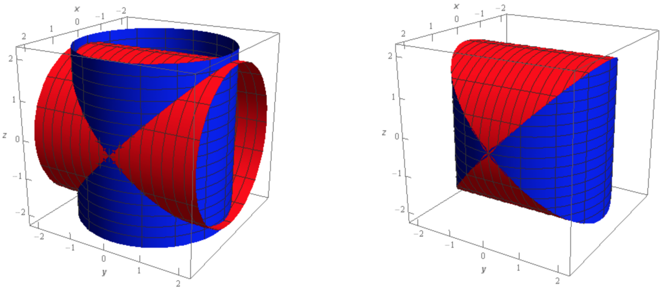
TASK

Evaluate $\iint_D 42y^2 - 12x dA$ *where*
$D = \{(x,y)\colon 0 \le x \le 4,\ (x-2)^2 \le y \le 6\}$

Visualize Geometry

TASK

Find the volume of the region which is formed by the intersection of two cylinders $x^2 + y^2 = 4$ *and* $x^2 + z^2 = 4$ *in* $\mathbb{R}^3$



(a) a

(b) b

Figure: Help your imagination

Cylindrical Coordinate

Property

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \\ \det(\varphi) = r \\ r = \sqrt{x^2+y^2} \\ \theta = \arctan\frac{y}{x} \end{cases}$$

TASK

1. *Identify the surface defined by the equation:* $r^2 - 4r\cos(\theta) = 14$
2. *Visualize the surface defined by the equation:* $z = 7 - 4r^2$

Visualization

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ContourPlot3D[z + 4 x^2 + 4 y^2 == 7, {x, -4, 4}, {y, -4, 4},
  {z, -4, 8}, PlotTheme → "Scientific"]
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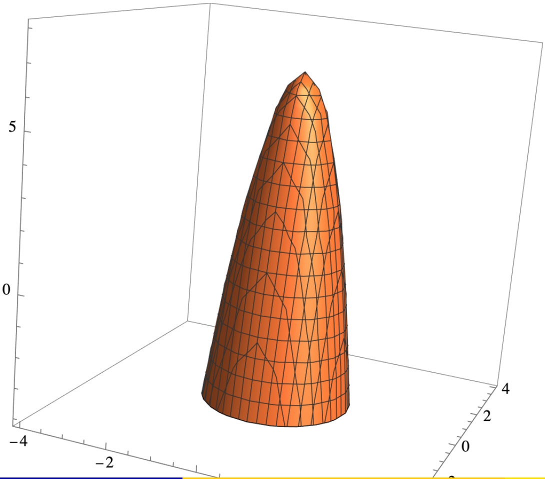

Integration Practice

TASK

Determine the volume of the solid which is inside the cylinder $x^2 + y^2 = 16$, *below* $z = 2x^2 + 2y^2$ *and above the* xy *plane.*

Integration Practice

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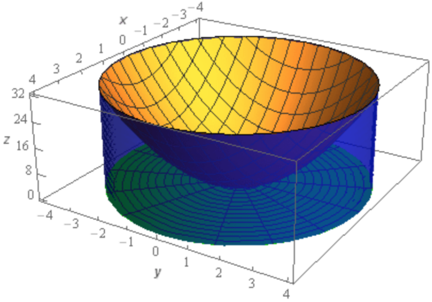

Integration Practice

TASK

Determine the volume of the solid which is **bounded** *by* $z = 8 - x^2 - y^2$ *and* $z = 3x^2 + 3y^2 - 4$

Integration Practice

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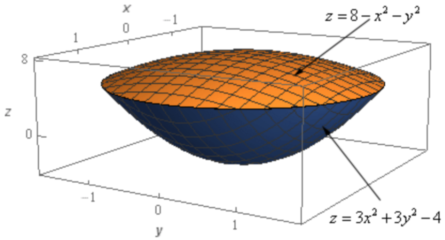

Spherical Coordinate

Property

$$x = r\cos\theta\sin\varphi, \quad y = r\sin\theta\sin\varphi, \quad z = r\cos\theta$$

Suppose f is the transformation from standard coordinate to spherical coordinate, i.e, $f:(x,y,z)\mapsto(r,\theta,\varphi)$, then $\det Df = r^2\sin\varphi$

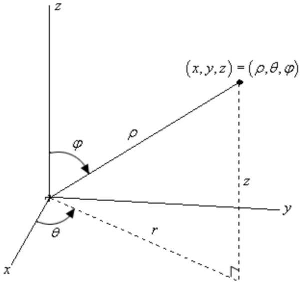

Figure: illustration of θ and φ

Integration Practice

TASK

Evaluate the following triple integral:

$$\int_{-1}^{0}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{\sqrt{6x^2+6y^2}}^{\sqrt{7-x^2-y^2}} 18y \; dzdydx$$

Change of Variables

Property

$$\iint_R f(x,y)dA = \iint_S f(g(u,v),h(u,v))|\det J_{g,h}|dA'$$

where $x = g(u,v)$ and $y = h(u,v)$.

TASK

Evaluate $\displaystyle\iint_R 6x - 3ydA$ *where R is the parallelogram with vertices $(2,0),(5,3),(6,7),(3,4)$ using transformation $x = \dfrac{1}{3}(v-u)$ and $y = \dfrac{1}{3}(4v-u)$*

References I

- VV285 slides from Horst Hohberger
- Paul's online note https://tutorial.math.lamar.edu/Classes/CalcIII/ChangeOfVariables.aspx