1. $\int \sin\left(\frac{t}{2}\right)\cos\left(\frac{t}{2}\right) dt$

let $\frac{t}{2} = \tau \Rightarrow t = 2\tau \Rightarrow dt = 2d\tau$

$$\int \sin(\tau)\cos(\tau)\, 2d\tau = \int \sin(2\tau)\, d\tau$$

$$= -\frac{1}{2}\cos(2\tau)$$

$$= -\frac{1}{2}\cos(t)$$

Subsitution Rule

2. $\int 18x^2 \sqrt[4]{6x^3+5}\, dx$

$$= \int 6\sqrt[4]{6x^3+5}\, dx^3$$

$$= \int 6\sqrt[4]{6t+5}\, dt \Big|_{t=x}$$

$$= \frac{4}{5}(6x^3+5)^{\frac{5}{4}}$$

3. $\int \left(1-\frac{1}{w}\right)\cos(w-\ln w)\, dw$

let $w - \ln w = u \qquad \left(1-\frac{1}{w}\right)dw = du$

$$\int \cos(u)\, du = \sin(u) = \sin(w-\ln w)$$

4. $$\int \frac{x}{\sqrt{1-4x^2}}\,dx$$

$\sqrt{1-4x^2} = u \Rightarrow 4x^2 + u^2 = 1 \Rightarrow (2x)^2 + u^2 = 1$

let $\begin{cases} 2x = \cos\theta \\ u = \sin\theta \end{cases} \Rightarrow x = \frac{1}{2}\cos\theta \Rightarrow dx = -\frac{1}{2}\sin\theta$

$$\int \frac{\frac{1}{2}\cos\theta}{\sin\theta}\left(-\frac{1}{2}\right)\sin\theta\, d\theta = -\frac{1}{4}\int \cos\theta$$

$$= -\frac{1}{4}\sin\theta$$

$$= -\frac{1}{4}\sqrt{1-4x^2}$$

5. $$\int \sec^2(4t)\,(3-\tan(4t))^3\,dt$$

$\boxed{\frac{d}{dt}\tan(t) = \sec^2 t}$

$$= \int \frac{1}{4}\cdot 4\sec^2(4t)\,(3-\tan(4t))^3\,dt$$

$$= 4\int [3-\tan(4t)]^3\, d(\tan 4t)$$

$$= 4\int (3-u)^3\,du = -\frac{1}{16}[3-\tan(4t)]^4$$

6. $\int \frac{3y}{(5y^2+4)^2} dy = \frac{3}{2}\int \frac{1}{(5y^2+4)^2} dy^2$

$= \frac{3}{2}\int \frac{1}{(5u+4)^2} du \Big|_{u=y^2}$

$= \frac{3}{2}\cdot\left(-\frac{1}{5}\right)\cdot \frac{1}{5y^2+4} = -\frac{3}{10}\cdot\frac{1}{5y^2+4}$

7. $\int \frac{3}{5y^2+4} dy$

$\arctan'(x) = \frac{1}{x^2+1}$

$\int \frac{1}{x^2+1} dx = \arctan(x) + C$

$= \frac{3}{4}\int \frac{1}{\frac{5}{4}y^2+1} dy$

$= \frac{3}{4}\int \frac{1}{\left(\frac{\sqrt{5}}{2}y\right)^2+1} dy \xrightarrow[dy=\frac{2}{\sqrt{5}}du]{u=\frac{\sqrt{5}}{2}y} \frac{3}{4}\cdot\frac{2}{\sqrt{5}}\int\frac{du}{u^2+1}$

$= \frac{3}{2\sqrt{5}}\arctan\left(\frac{\sqrt{5}}{2}y\right)$

8. $\int \frac{2t^3+1}{(t^4+2t)^3}\,dt$

$u = t^4+2t \Rightarrow \quad (2t^3+1)\,dt = \frac{1}{2}du$

$= \frac{1}{2}\int \frac{1}{u^3}\,du = -\frac{1}{4}(t^4+2t)^{-2}$

9. $\int \frac{2t^3+1}{t^4+2t}\,dt \quad = \quad \frac{1}{2}\ln|t^4+2t|$

Mind the absolute value!

$\int \frac{1}{u} = \ln|u| + C$

10. $\int \frac{x}{\sqrt{1-4x^2}}\,dx \xrightarrow{u=1-4x^2} -\frac{1}{8}\int u^{-\frac{1}{2}}\,du$

$= -\frac{1}{4}\sqrt{1-4x^2}$

11. $\int \frac{1}{\sqrt{1-4x^2}}dx$ $\qquad \sqrt{1-4x^2} = u$

$\Rightarrow (2x)^2 + u^2 = 1$

$\begin{cases} 2x = \sin\theta \\ u = \cos\theta \end{cases} \rightarrow dx = \frac{1}{2}\cos\theta\, d\theta$

$= \int \frac{1}{\cos\theta} \frac{1}{2}\cos\theta\, d\theta = \frac{1}{2}\theta = \frac{1}{2}\arcsin(2x)$

12. $\int \tan(x)\, dx$

$= \int \frac{\sin x}{\cos x} dx = \int \frac{-d\cos x}{\cos x} = -\ln|\cos(x)|$

13. $\int \frac{10x+3}{x^2+16} = \int \frac{10x}{x^2+16} dx + \frac{1}{16}\int \frac{3}{\frac{x^2}{16}+1} dx$

$\begin{cases} u = x^2+16 \\ v = \frac{x}{4} \end{cases}$ $\quad = 5\int \frac{1}{u} du + \frac{3}{4}\int \frac{1}{v^2+1} dv$

$= 5\ln|x^2+16| + \frac{3}{4}\arctan(\frac{x}{4})$

14. $\int \sec y \, dy =$

$$= \int \frac{\sec y \, (\sec y + \tan y)}{\sec(y) + \tan y)} \, dy$$

$$= \int \frac{\sec^2(y) + \sec y) \tan(y)}{\tan(y) + \sec(y)} \, dy$$

$u = \tan(y) + \sec(y) \longrightarrow \frac{du}{dy} = \sec^2(y) + \sec(y) \tan(y)$

$$= \int \frac{1}{u} \, du = \ln| \tan(y) + \sec(y)|$$

15. $\int \frac{\cos(\sqrt{x})}{\sqrt{x}} \, dx$ $\quad u = \sqrt{x} \Leftrightarrow u^2 = x$

$dx = 2u \, du$

$$= \int \frac{\cos(u)}{u} \cdot 2u \, du = 2\sin(\sqrt{x})$$

16. $\int e^{t+e^t}\, dt$ $\qquad u = e^t \Leftrightarrow du = e^t dt$

$\qquad\qquad\qquad\qquad\qquad\qquad du = u\,dt$

$$= \int e^t \cdot e^{e^t}\, dt = \int u \cdot e^u \cdot \frac{1}{u}\, du$$

$$= \int e^u\, du$$

$$= e^u$$

$$= e^{e^t}$$

17. $\int \frac{\arcsin(x)}{\sqrt{1-x^2}}\, dx$

$$\frac{d}{dx}\arcsin(x) = \frac{1}{\sqrt{1-x^2}}$$

$$= \int u\, du \Big|_{u=\arcsin(x)}$$

$$= \frac{1}{2}[\arcsin(x)]^2$$

Integration by Parts

18. $\int (4x^3 - 9x^2 + 7x + 3) e^{-x} \, dx$

Table:

differentiate	integrate	
$4x^3 - 9x^2 + 7x + 3$	e^{-x}	$+$
$12x^2 - 18x + 7$	$-e^{-x}$	$-$
$24x - 18$	e^{-x}	$+$
24	$-e^{-x}$	$-$
0	e^{-x}	$+$

$$\int (4x^3 - 9x^2 + 7x + 3) e^{-x} \, dx$$

$$= (4x^3 - 9x^2 + 7x + 3)(-e^{-x}) - (12x^2 - 18x + 7) e^{-x}$$

$$+ (24x - 18)(-e^{-x}) - 24 e^{-x}$$

$$= -e^{-x}(4x^3 + 3x^2 + 13x + 16)$$

19. $\int x^2 \cos(4x)\,dx$

x^2	$\cos(4x)$	$+$
$2x$	$\frac{1}{4}\sin(4x)$	$-$
2	$-\frac{1}{16}\cos(4x)$	$+$
0	$-\frac{1}{64}\sin(4x)$	$-$

$= \frac{1}{4}x^2\sin(4x)$

$-(-\frac{1}{8})x\cos(4x)$

$-\frac{1}{32}\sin(4x)$

$= (\frac{1}{4}x^2 - \frac{1}{32})\sin(4x) + \frac{1}{8}x\cos(4x)$

20. $\int 6\arctan(\frac{8}{w})\,dw$

$\arctan(\frac{8}{w}) = y \Rightarrow \frac{1}{1+(\frac{8}{w})^2}(-\frac{8}{w^2})\,dw = dy$

$dy = \frac{-8}{w^2+64}\,dw$ integration by parts

$$\boxed{= 6w\arctan(\frac{8}{w}) - \int 6w \cdot \frac{-8}{w^2+64}\,dw}$$

$= 6w\arctan(\frac{8}{w}) + 48\int \frac{w}{w^2+64}\,dw$

$= 6w\arctan(\frac{8}{w}) + 24\ln|w^2+64|$