Vv285 Recitation Class 4
Practice Questions on Determinant

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Outline

Judge the Followings

[1]Suppose $A \in \mathrm{Mat}(n \times n; \mathbb{C})$ is a matrix with $\det A \neq 0$. Judge T or F:

- $\det A \in \mathbb{R}$.
- A is invertible.
- The row rank of A is n
- $\ker A = \{0\}$
- For any $y \in \mathbb{C}^n$, $x_0 = A^{-1}y$ is the only solution of $Ax = y$
- $\det (A^{-1})^T = \det A$
- $\det c \cdot A = c \cdot \det A$, where $c \in \mathbb{R}$

[1]This question is provided by Leyang Zhang. Vielen Dank!

Determinant of Vandermonde Matrix

Prove that the determinant of Vandermonde matrix $V_n \in \mathrm{Mat}(n \times n; \mathbb{R})$, which is defined by $V_1 = 1$ and when $n \leq 2$:

$$V_n = \begin{pmatrix} 1 & x_1 & x_1^2 & \dots & x_1^{n-2} & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \dots & x_2^{n-2} & x_2^{n-1} \\ . & . & . & \dots & . & . \\ . & . & . & \dots & . & . \\ . & . & . & \dots & . & . \\ 1 & x_n & x_n^2 & \dots & x_n^{n-2} & x_n^{n-1} \end{pmatrix}$$

is $\det V_n = \Pi_{1 \leq i < j \leq n}(x_j - x_i)$

Linear Transform on Parallelogram

Suppose $P \subseteq \mathbb{R}^2$ is a paralellogram spanned by two vectors $p, q \in \mathbb{R}^2$. Let $L\colon \mathbb{R}^2 \to \mathbb{R}^2$ be a linear map and the denoted the set $L(P)$ as

$$L(P) = \{y \in \mathbb{R}^2 \colon \exists x \in P,\ y = Lx\}$$

1. Prove that $L(P)$ is a parallelogram spanned by $L(p)$ and $L(q)$.
2. Prove that $S_{L(P)} = (\det L) \cdot S_P$
3. Prove that (2)'s result also works in $\mathbb{R}^3$

Determinant of block matrices

[2]Calculate the determinant of D_1 and D_2:

$$D_1 = \begin{pmatrix} A & B \\ 0 & C \end{pmatrix} \quad D_2 = \begin{pmatrix} G_1 & * & \dots & * & * \\ 0 & G_2 & \dots & * & * \\ 0 & 0 & G_3 & \dots & * \\ 0 & 0 & 0 & \dots & G_n \end{pmatrix}$$

where A, B, C are $m \times m$ matrices and $A_1, ..., A_n$ are $n \times n$ matrices

[2]This exercise is from Leyang Zhang. Vielen Dank!

References I

- VV285 sample exam from Horst Hohberger
- Practice questions from Leyang Zhang