Vv285 Recitation Class 11
Extrema of Multi-Variable Functions

Yuxiang Chen

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Outline

1. Procedure of calculating extrema
2. Reference

Calculating Extrema

Suppose $f : \Omega \to \mathbb{R},\ \Omega \subseteq \mathbb{R}^n$, the procedures of calculating extrema of f are listed as follows:

1. Calculate the critical point: $\frac{\partial f}{\partial x_1} = 0, ..., \frac{\partial f}{\partial x_n} = 0$. Solve for vectors such that $\nabla f(x) = 0$. Suppose found $\{P\}_i$, a set of critical points.
2. Calculate the Hessian at each P_i and determine whether it's positive definite, negative definite or other. Determine local maximum and local minimum.
3. Special check at $\partial\Omega$. Compare to value in step 2 and conclude on global maximum and global minimum.

Practice - Analyse multivariable functions

TASK

Find and classify all the critical point of $f(x,y) = 4 + x^3 + y^3 - 3xy$

① critical points

$\frac{\partial f}{\partial x} = f_x : 3x^2 - 3y = 0 \Rightarrow y = x^2$

$f_y : 3y^2 - 3x = 0 \Rightarrow x^4 = x$

$x = 0$ or 1

$P_1(0,0)$, $P_2(1,1)$

$\text{Hess } f(x) = \begin{pmatrix} 6x & -3 \\ -3 & 6y \end{pmatrix}$

$\det \text{Hess} = 36xy - 9$. sign $= 6x$

② verify

$P_1(0,0)$

sign $= 0$

$\times$

$P_2(1,1)$

sign $= 6 > 0$

$\det \text{Hes} = 36 - 9 = 27 > 0$

$\Rightarrow$ local minima

③ let y be $f(x)$

$f(x) : 4 + x^3 - (3y) \cdot x + y^3$

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Practice - Analyse multivariable functions

TASK

Find and classify all the critical point of $f(x, y) = 4 + x^3 + y^3 - 3xy$

TASK

Find the absolute minimum and absolute maximum of $f(x, y) = 2x^2 - y^2 + 6y$ *on the disk of radius 4,* $x^2 + y^2 \leq 16$.

① critical points

$f_x = 4x = 0 \Rightarrow x = 0$

$f_y = -2y + 6 \Rightarrow y = 3$

$f(0, 3) = -9 + 18 = 9$,

$\text{Hess } f(x) = \begin{pmatrix} 4 & 0 \\ 0 & -2 \end{pmatrix}$

$\det \text{Hess} = -8 < 0$ sign $= 4 > 0$

NO EXTREMA!

② Check boundary

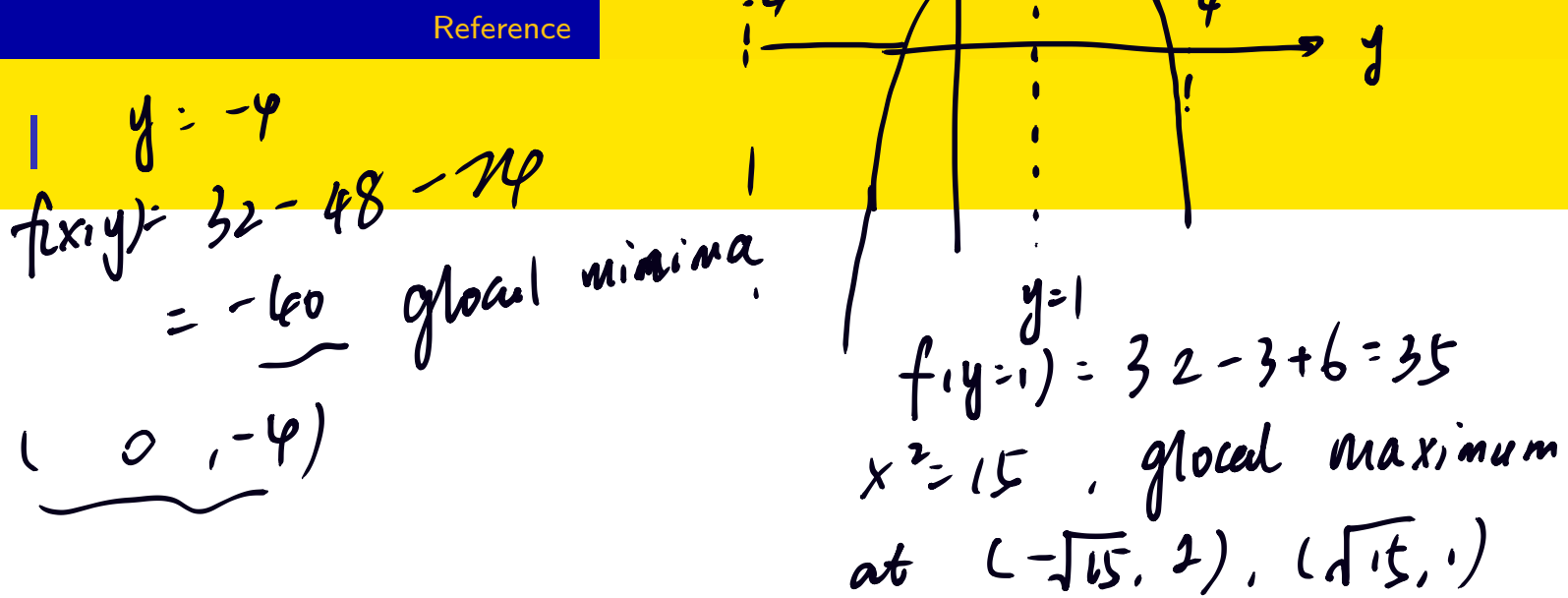
$x^2 + y^2 = 16$

$x^2 = 16 - y^2$

$f(x, y) = 32 - 3y^2 + 6y$

$f \nearrow \; : \; \searrow$

References I



- VV285 slides from Horst Hohberger
- Paul's online note
  `https://tutorial.math.lamar.edu/Classes/CalcIII/AbsoluteExtrema.aspx`