《普通物理学 I （H）》课程期中考查答案

I. MULTIPLE CHOICE [15 points]

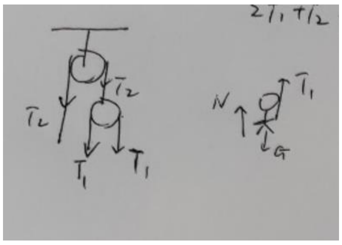
CCBDC

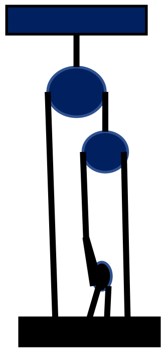
II. BLANK FILLING [20 points]

1. $\frac{4v^3 sin^2\alpha cos\alpha}{g^2}$ 2. $3\sqrt{2}m/s$ 西北 3. $22470N$ 4. $\sqrt{\frac{\rho_0}{\rho_0-\rho}\frac{d}{L}}$ 5. $18m/s$

III. CALCULATION AND ANALYSIS

1. (10points) A person with m_p=60 kg stands on a base plate with m_b=30 kg. They are connected by a rope and pulley as shown in the figure below. Assuming that the mass of the pulley and the rope can be neglected and the friction at the pulley can be disregarded, and the rope is inextensible, if the person and the base plate are to rise with an acceleration of 1 m/s², what is the force F_n exerted by the person on the rope? What is the force exerted by the person on the base plate? (Take g = 10 m/s².)





对人和平台整体受力平衡：(3points)

$$2T_1 + T_2 - (m_p + m_b)g = (m_p + m_b)a$$

对人受力平衡：(3points)

$$T_1 + N - m_p g = m_p a$$

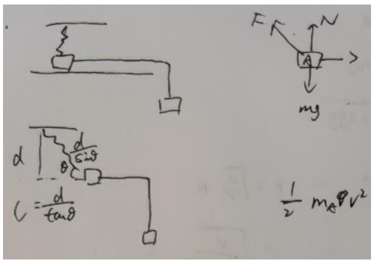
由轻滑轮，有：(2points)

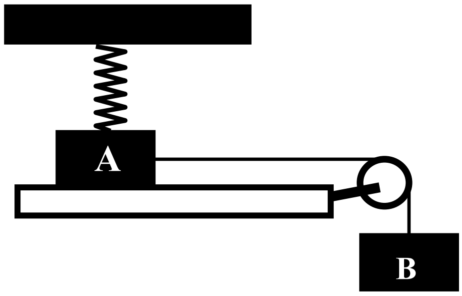
$$2T_1 = T_2$$

带入数据解方程组得：(2points) $T_1 = 247.5N$， $N = 412.5N$

2. (15points) Two blocks A and B have the same mass of 320g connected by a light

string passing over a smooth light pulley. The block A can slide smoothly on the horizontal surface. The block A is also attached to a spring with the spring constant 40N/m whose another end is fixed to the ceiling with 40cm above the horizontal surface. Initially, the spring is vertical and unstretched when the system is released to move. Find the velocity of the block A at the instant it breaks off the horizontal surface. Take g=10m/s^2





设即将离开平面时弹簧与平面夹角θ，此时支持力 N=0(2points)

胡克定律，弹力：(3points)

$$F = k\Delta x = kd\left(\frac{1}{sin\theta} - 1\right)$$

竖直方向受力平衡：(3points+2points)

$$\mathrm{Fsin\theta} = \mathrm{mg} \Rightarrow sin\theta = 0.8, \mathrm{l} = \frac{d}{tan\theta} = 0.3\mathrm{m}, \Delta x = 0.1\mathrm{m}$$

整个系统机械能守恒：(3points)

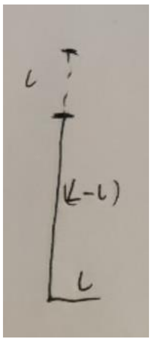
$$\frac{1}{2}m_A v^2 + \frac{1}{2}m_B v^2 = m_b gl - \frac{1}{2}k(\Delta x)^2$$

解得：(2points)

$$v = \sqrt{\frac{19}{8}} = 1.54m/s$$

3. (10points) If you first hold the upper end of a nonuniform soft chain with the length

L while the lower end just touches the floor. After you release, how large force will the chain impose on the floor? Assuming the mass density of the chain linearly increases from μ at the lower end to 2μ at the upper end.



写出链子的线密度分布(从下到上)：(2points)

$$\lambda = \mu\left(1 + \frac{l}{L}\right)$$

当下降长度 l 的时候，对地上的末端链子有：(2points)

$$m_1 = \int_0^l \lambda \, \mathrm{d}l = \mu l\left(1 + \frac{l}{2L}\right)$$

此时速度为:(2points) $v = \sqrt{2gl}$

对 dt 时间落地的 dm 微元：$dm = \lambda v\, dt$

对地面的冲力：(2points)

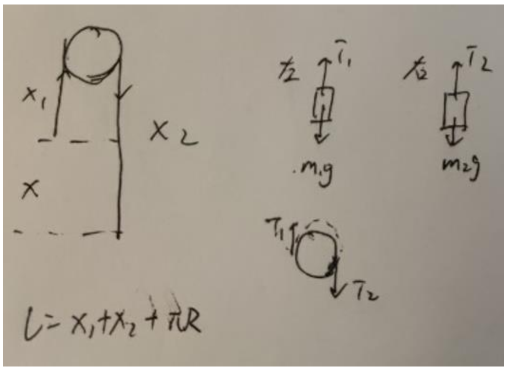
$$F_2 = \frac{dp}{dt} = \frac{dm \cdot v}{dt} = \lambda v^2$$

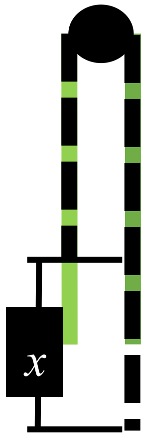
总的力：(2points)

$$F = F_1 + F_2 = m_1 g + \lambda v^2 = \mu l\left(1 + \frac{l}{2L}\right)g + \mu\left(1 + \frac{l}{L}\right)\cdot 2gl = \mu g l\left(3 + \frac{5l}{2L}\right)$$

4. (15points) A uniform flexible chain of the length l with the uniform density $\boldsymbol{\lambda = m/l}$

passes over a pulley with the radius R and the mass M. If there is no relative motion between the chain and the pulley. Initially, there are the same lengths of the chain in two sides of the pulley. If we give a very small downward drag force of one end of the chain, calculate its acceleration speed when the difference of two ends of the chain is x.





详见 quiz 6

$$T_1 - \lambda x_1 g = \lambda x_1 a$$
$$\lambda x_2 g - T_2 = \lambda x_2 a$$
$$(T_2 - T_1)R = I\alpha = \left(\frac{1}{2}MR^2 + \pi R\lambda R^2\right)\alpha$$
$$a = R\alpha$$
$$l = x_1 + x_2 + \pi R$$

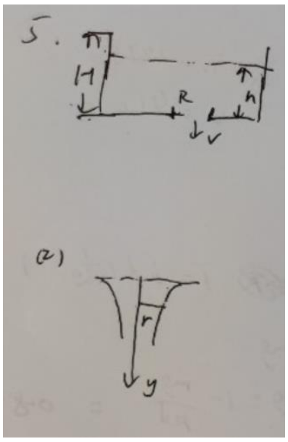
解得:

$$a = \frac{xmg}{\left(\frac{1}{2}M + m\right)l}$$

5. (15points) A cylindrical bucket with the cross-section A and the height H is filled fully with water. At the bottom of the bucket has a circular hole with radius R *as shown* in Figure below.

(1) How much time is needed when all water flows out of the bucket?

(2) The cross-section of the vertical stream of water decreases when it falls. Calculate the radius r of the stream as a function of the vertical drops y.



(1) 当桶中液面高度为 h 时，从洞中流出水的速度为：(2points)

$$v_1 = \sqrt{2gh}$$

dt 时间内下降的水柱体积等于从孔中流出体积：(3points)

$$-Adh = vdt \cdot \pi R^2$$

积分得：(2points)

$$t = \sqrt{\frac{2H}{g}}\frac{A}{\pi R^2}$$

(2) 由伯努利方程：

$$p + \rho gh + \frac{1}{2}\rho v^2 = const$$

设下落距离为 y 处流速v_2，有：(3points)

$$\rho g\, y + \frac{1}{2}\rho {v_1}^2 = \frac{1}{2}\rho {v_2}^2$$

解得: (2points)

$$v_2 = \sqrt{v_1^2 + 2gy}$$

由通量守恒即：(3points)

$$v \cdot S = const\lambda \Rightarrow v_1 R^2 = v_2 r^2 \Rightarrow r = \sqrt{\frac{v_1}{v_2}}R = \left(\frac{h}{h+y}\right)^{\frac{1}{4}} R$$