Answer to General PhysicsI(H) Midterm

April 17, 2024

1 Dynamics of a Block-wedge System

1. From the conservation of the linear momentum, we have $m(v\cos\theta - u) = Mu$,in which u is the velocity of the wedge relative to the table and v is the velocity of the block relative to the wedge.
   From the conservation of the energy, we have $mgh = \frac{1}{2}m(v^2 + u^2 - 2uv\cos\theta) + \frac{1}{2}Mu^2$.
   Solving the two equations, we get $u = \sqrt{\frac{2m^2gh\cos^2\theta}{(M+m)(M+m\sin^2\theta)}}$, and its direction is to the left.

2. Displacement is the integral of velocity, we apply this to the conservation of the linear momentum and get $m(h\cot\theta - d) = Md$, in which d is the displacement of the wedge relative to the table and $h\cot\theta$ is the displacement of the block relative to the wedge.
   So we can get $d = \frac{m}{M+m}h\cot\theta$.

3. We consider the problem in the wedge coordinate. The acceleration of the two items is $\frac{F}{M+m}$, and thus, the inertial force of the block is $\frac{m}{M+m}F$.
   The block is stationary in the wedge coordinate, so $\frac{m}{M+m}F = mg\tan\theta$.
   Thus, $F = (M+m)g\tan\theta$, and its direction is to the right.

2 Walking on a Turntable

1. We can get that $I = \frac{1}{2}MR^2 = \frac{1}{2}\rho\pi R^4 = 400kg\cdot m^2$.

2. We can get that $\omega = \alpha t = \frac{M}{I}t = \frac{FR}{I}t = 0.1rad\cdot s^{-1}$ and is counterclockwise.

3. We can get that $v = at = \frac{F}{m}t = 0.4m\cdot s^{-1}$.

4. We can get that $P = F\cdot v_r = F\cdot(\omega R + v) = 12W$, in which v_r is the velocity of the woman relative to the turntable.

3 Simple Harmonic Motion in a 1D Potential

1. For each equilibrium position, $U^{'}(x)/\lambda = \frac{x^3}{2} - \frac{x^2}{2} - 2x + 2 = \frac{1}{2}(x-1)(x+2)(x-2) = 0$.
   Thus, the equilibrium positions are $x = -2, 1, 2$.
   For $x = -2$, $U^{''}(x)/\lambda > 0$, so it is a stable equilibrium.
   For $x = 1$, $U^{''}(x)/\lambda < 0$, so it is an unstable equilibrium.
   For $x = 2$, $U^{''}(x)/\lambda > 0$, so it is a stable equilibrium.

2. We can get that $x_0 = -2$ for it has the lowest potential energy, and $x = x_0 + s = s - 2$.
   Apply this relation and omit the high-order items, we can get that $U(x) = \lambda(3s^2 - \frac{8}{3})$ and $F(x) = -U^{'}(x) = -6\lambda s$.

3. The force $F(x)$ is a linear restoring force, so the motion is simple harmonic.
   $k = 6\lambda$, so $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{6\lambda}{m}}$.
   At $t = 0$, $s(0) = A\cos\phi = s_0$, $v(0) = s^{'}(0) = -\omega A \sin\phi = -v_0$.
   So we can get that $\phi = \arctan\frac{v_0}{s_0\omega} = \frac{v_0}{s_0}\sqrt{\frac{m}{6\lambda}}$ and $A = \sqrt{s_0^2 + \frac{m}{6\lambda}v_0^2}$.

4 Infinite One-dimensional Monoatomic Chain Including Next-nearest-neighbour Interactions

1. We have $M\ddot{u_n} = -\frac{d}{du_n}(k_1(u_n - u_{n+1})^2 + k_1(u_n - u_{n-1})^2 + k_2(u_n - u_{n+2})^2 + k_2(u_n - u_{n-2})^2) = k_1(u_{n-1} + u_{n+1} - 2u_n) + k_2(u_{n-2} + u_{n+2} - 2u_n)$.

2. From 1 we can get that $M\omega^2 A_0 e^{i(kna-\omega t)} = k_1 A_0 e^{i(kna-\omega t)}(e^{ika} + e - ika - 2) + k_2 A_0 e^{i(kna-\omega t)}(e^{2ika} + e^{-2ika} - 2)$.
   Simplify the equation using $e^{i\theta} + e^{-i\theta} = 2\cos\theta = 2 + 4\sin^2\frac{\theta}{2}$, we have $M\omega^2 = 4k_1\sin^2\frac{ka}{2} + 4k_2\sin^2 ka$.
   So, $\omega = 2\sqrt{\frac{1}{M}(\sin^2 ka + \sin^2\frac{ka}{2})}$.

3. In the long-wavelength limit, $k \cdot a \to 0$, so $\sin ka \to ka$ and $\sin\frac{ka}{2} \to \frac{ka}{2}$.
   Use this approximation, we get that $\omega = ka\sqrt{\frac{1}{M}(k_1 + 4k_2)}$.