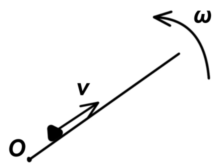
1 Quizzes

1.1 Problems

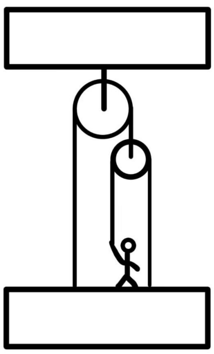
1. A rob rotates with an angular speed ω around one of the rob end O on the horizontal plane. An insect moves away from the fixed end O with the constant speed v along the rob at $t = 0$. Calculate the acceleration speed of the insect relative to the ground.



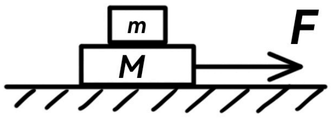
2. A person with $m_p = 60kg$ stands on a base plate with $m_b = 30kg$. They are connected by a rope and pulley as shown in the figure below. Assuming that the mass of the pulley and the rope can be neglected and the friction at the pulley can be disregarded, and the rope is inextensible, if the person and the base plate are to rise with an acceleration of $1m/s^2$. (Take $g = 10m/s^2$)

   (a)What is the force F_n exerted by the person on the rope?

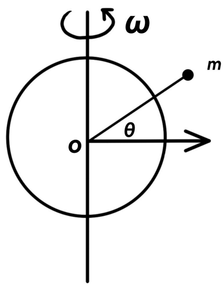
   (b)What is the pressure exerted by the person on the base plate?



3. Two rectangular blocks are stacked on a table as shown in the figure below. The masses of the upper and lower blocks are m and M, respectively.The kinetic friction coefficient between the lower block and the table is μ_1, The static friction coefficient between two blocks is μ_2. A string is attached to the lower block, and an external force F is applied horizontally, pulling on the string. What is the maximum force that can be applied to the string without having the upper block slide off?

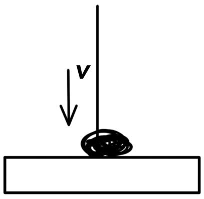


4. A ball with mass m is dropped from height $h \ll R$, at the north latitude θ. How far to the east is the ball deflected, by the time it hits the ground?

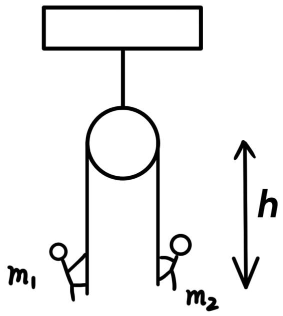


5. A uniform soft chain with length L and mass M initially lays on the table. If you hold an end of the chain and lift it upward with a constant speed v, how large is the lifting force needed at time t?

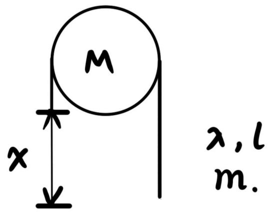
6. If you first hold the upper end of a uniform soft chain while the lower end just touches the floor. After you release, how large force will the chain impose on the floor? Assuming that the chain has the length L and the mass M.



7. There are 2 persons with masses m_1 and m_2 ($m_1 > m_2$), respectively.They each hold onto the two ends of the massless ropes hanging over the massless and frictionless pulley. If at the beginning, the distances of both persons away from the pulley are h. Find the distance of the person with the heavier mass away from the pulley when the person with the lighter mass reaches the pulley at the time t after they climb upwards.



8. A uniform flexible chain of length l with linear density $\lambda = m/l$ passes over a pulley with the radius R and the mass M. If there is no relative motion between the chain and the pulley. Initially, there are the same lengths in two sides of the pulley. If we give a very small downward drag force of one end of the chain, calculate its acceleration speed when the difference of two ends of the chain is x.

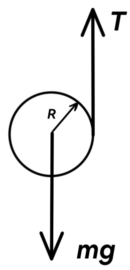


9. The problem is related with Yo-Yo which is the most favorite game for pupils. Assuming Yo-Yo is a weightless line winding a disk of the radius R and the mass m.

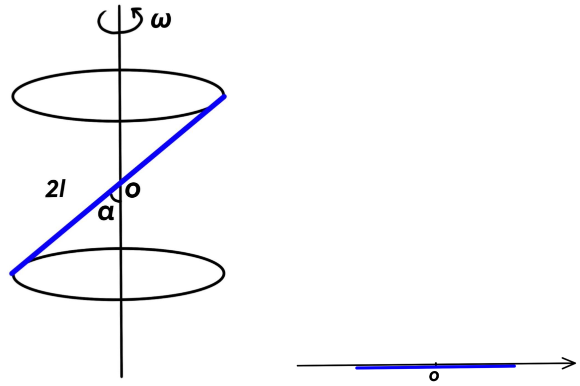
   a. If we hold the end of the line to be steady, calculate the acceleration of the disk and the force to act on the end of the line.

   b. If we want the disk to maintain a fixed height, how large lift force is needed and what is the angular acceleration?

   c. If the lift force is $2mg$, what is the acceleration of the line end that we held?

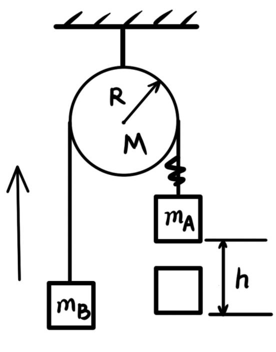


10. Consider a nonuniform rigid rod with mass density $\rho = \rho_0(1 - \frac{x^2}{4l^2})$ and length $2l$. The midpoint (point O) of the rod is fixed to a vertical axis which rotates at angular speed ω. The angle between the rod and the axis is α. Find the angular momentum of the rod about the midpoint of the rod.

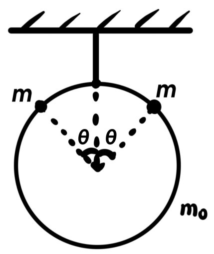


$$\rho = \rho_0(1 - \frac{x^2}{4l^2})$$

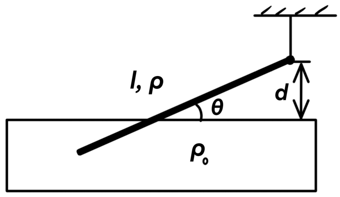
11. A massless rope is passed over a fixed pulley with mass M and the radius R. Two objects A and B with masses m_A and m_B ($m_B > m_A$) are tied to the two ends of the rope. Object B is at rest on the table. Object A is lifted to make the rope slack. When object A falls freely for a distance h, the rope becomes tight. Find the velocities of the two objects and the maximum height that object B can rise.



12. A large ring with mass m_0 is hung from the ceiling by a thin wire. Two small rings with mass m set in the large ring can move freely. If two small rings simultaneously release from the top of the large ring at rest, they will slip down oppositely. When two small rings slip to the angle θ, the large ring will start to rise up. Please calculate this angle and the minimum mass of the small rings required for rising the large ring.

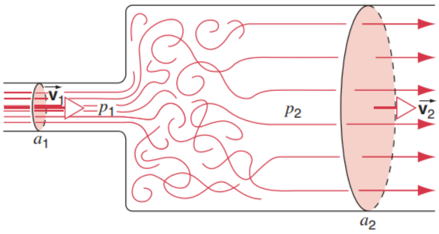


13. A uniform rod with mass density ρ and length l is partly floating on the liquid with mass density $\rho_0(> \rho)$. The cross-section of the rod is S. When one end of the rod is hanged above the liquid surface with the height d. Calculate the angle between the rod and the liquid surface.



14. Consider a stream of fluid of the density ρ with speed v_1, passing abruptly from a cylindrical pipe of the cross-sectional area A_1 into a wider cylindrical pipe of the cross-sectional area A_2. The jet will mix with the surrounding fluid and after the mixing, will flow on almost uniformly with an average speed v_2. Without referring to the details of the mixing, Calculate the difference of the pressure at the two ends.

    (Clipped from Chapter 16 Problem 5.)



15. A block of mass $4m$ is attached to a vertical spring or elastic string with rest length l_0 and spring or string constant k. The block is made by gluing two blocks of mass $3m$ and m, respectively. Initially, the block is in equilibrium and at rest (as shown in Figure 1 and 2). At $t = 0$, the part of the block with mass $3m$ falls down. Considering hanging point on ceiling as $x = 0$ and downward direction as positive x, find the x-coordinate of the remaining block with mass m as a function of time for the spring and the period of the oscillation for the string.

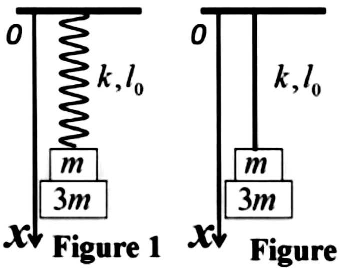


Figure 1 Figure 2

16. A wave source

$$y = 0.5 \cos 10\pi(t - 0.5x)$$

is launched at the position A. After reaching the wall B that is $10m$ away from A, the wave will be reflected. Please find the points between A and B that remain at rest.

17. Show that the total energy between the node and the antinode remains constant and there is no energy flowing through nodes and antinodes for a standing wave

$$y = y_m \cos kx \sin \omega t.$$

18. A car with a gasoline-powered internal ‘combustion engine travels with a speed of $v = 30m/s$ on a level road and uses gas at a rate of $\chi = 10L/100km$. The energy content of gasoline is $E_g = 35MJ/L$. If the engine has an efficiency of $e = 20\%$,how much power is delivered to keep the car moving at a constant speed.

1.2 Answers

$\boxed{\text{The answers below are written by DeepSeek / Prof. ZHIWEI MA.}}$

1. Answer-1:

   极坐标加速度公式：$\boxed{\mathbf{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\mathbf{e}_\theta}$

   杆以恒定角速度旋转：$\dot{\theta} = \omega$，$\ddot{\theta} = 0$；昆虫沿杆匀速运动：$\dot{r} = v$，$\ddot{r} = 0$；初始位置 $r = 0$，故 $r(t) = vt$

   径向分量：$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - (vt)\omega^2 = -vt\omega^2$

   切向分量：$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2v\omega$

   答案：$\boxed{\mathbf{a} = -vt\omega^2\,\mathbf{e}_r + 2v\omega\,\mathbf{e}_\theta}$

2. Answer-2:

   (a) To find the force F_n exerted by the person on the rope, consider the system's total mass ($m_p + m_b = 90\,\text{kg}$) and the required upward acceleration. The tension T in the rope provides the force. Using Newton's second law for the system: $4T - (m_p + m_b)g = (m_p + m_b)a$

   Solving for T: $T = \dfrac{(m_p + m_b)(g + a)}{4} = \dfrac{90 \times 11}{4} = 247.5\,\text{N}$, Thus, $F_n = \boxed{247.5\,\text{N}}$.

   (b) For the pressure (force) exerted by the person on the base plate, analyze the normal force N. Using equations for the person and base plate separately:

   $$\begin{cases} T + N = m_p(g + a) \\ 3T - N = m_b(g + a) \end{cases}$$

   Subtracting these: $4N = (3m_p - m_b)(g + a) \implies N = \frac{(3\cdot 60 - 30)(10+1)}{2} = 825\,\text{N}$. Thus, the force is $\boxed{825\,\text{N}}$.

3. Answer-3:

   To determine the maximum force F that can be applied without causing the upper block to slide off, we analyze the forces acting on both blocks:

   1. Upper Block (mass m): - The static friction f_s provides the necessary centripetal force to accelerate the block. - Maximum static friction: $f_{s_max} = \mu_2 mg$. - Newton's second law: $f_{s_max} = ma \Rightarrow a = \mu_2 g$. This is the maximum acceleration before slipping.

   2. Lower Block (mass M): - Forces: Applied force F, kinetic friction $f_k = \mu_1(M + m)g$, and static friction $f_s = ma = m\mu_2 g$. - Net force equation: $F - f_k - f_s = Ma$. - Substitute $a = \mu_2 g$, f_k, and f_s:

   $$F = \mu_1(M + m)g + \mu_2 g(M + m)$$

   Thus, the maximum force is $\boxed{(M + m)g(\mu_1 + \mu_2)}$.

4. Answer-4:

To determine the eastward deflection of a ball dropped from height h at a northern latitude θ, we consider the Coriolis effect. Here's the step-by-step solution:

1. Time of Fall: The time T taken for the ball to hit the ground is given by free fall kinematics:

$$T = \sqrt{\frac{2h}{g}}$$

2. Coriolis Acceleration: The horizontal (eastward) Coriolis acceleration is:

$$a_x = 2\omega \cos\theta \cdot v_z$$

where $v_z = -gt$ (downward velocity). Substituting $|v_z| = gt$, the acceleration becomes:

$$a_x = 2\omega \cos\theta \cdot gt$$

3. Velocity Integration: Integrate the acceleration to find the eastward velocity:

$$v_x(t) = \int_0^t a_x\, dt = \omega g \cos\theta \int_0^t 2t'\, dt' = \omega g \cos\theta \cdot t^2$$

4. Displacement Integration: Integrate the velocity to find the eastward displacement:

$$x(T) = \int_0^T v_x(t)\, dt = \omega g \cos\theta \int_0^T t^2\, dt = \frac{\omega g \cos\theta \cdot T^3}{3}$$

5. Substitute Time of Fall: Substitute $T = \sqrt{\frac{2h}{g}}$:

$$x = \frac{\omega g \cos\theta}{3} \left(\sqrt{\frac{2h}{g}}\right)^3 = \frac{2\sqrt{2}\omega \cos\theta \cdot h^{3/2}}{3\sqrt{g}}$$

Final Answer:

$$\boxed{\frac{2\sqrt{2}\omega \cos\theta\, h^{3/2}}{3\sqrt{g}}}$$

5. Answer-5:

To determine the lifting force required at time t when lifting a uniform chain with constant speed v, consider both the gravitational force on the lifted portion and the force needed to accelerate each new chain segment to speed v.

1. Gravitational Force: The length lifted by time t is $y(t) = vt$. The mass of this portion is $\frac{M}{L}vt$, leading to a gravitational force: $F_g = \left(\frac{M}{L}vt\right) g = \frac{M}{L} gvt$

2. Force Due to Momentum Transfer: Each segment of the chain is accelerated from rest to speed v. The rate of mass flow is $\frac{dm}{dt} = \frac{M}{L}v$, requiring a force: $F_p = \frac{dm}{dt}v = \frac{M}{L}v^2$

Total Lifting Force:

$$F(t) = F_g + F_p = \frac{M}{L}gvt + \frac{M}{L}v^2 = \frac{Mv}{L}(gt + v)$$

Answer: The lifting force at time t is $\boxed{\frac{Mv}{L}(gt+v)}$.

6. Answer-6:

   To determine the force the chain imposes on the floor after release, consider two components: 1. Weight of the chain already on the floor: As the chain falls, a portion $x(t)$ accumulates on the floor. Its weight is $F_{\text{weight}} = \frac{M}{L}x(t)g$. 2. Momentum transfer from falling segments: Each segment of the chain hits the floor with velocity $v = \sqrt{2gx(t)}$. The rate of mass flow onto the floor is $\frac{dm}{dt} = \frac{M}{L}\frac{dx}{dt}$, leading to a force $F_{\text{impulse}} = v\frac{dm}{dt}$.

   Using $x(t) = \frac{1}{2}gt^2$ and $\frac{dx}{dt} = gt$: - $F_{\text{weight}} = \frac{M}{L} \cdot \frac{1}{2}gt^2 \cdot g = \frac{Mg^2t^2}{2L}$, - $F_{\text{impulse}} = \frac{M}{L}gt \cdot gt = \frac{Mg^2t^2}{L}$.

   Total force:

   $$F_{\text{total}} = \frac{Mg^2t^2}{2L} + \frac{Mg^2t^2}{L} = \frac{3Mg^2t^2}{2L}.$$

   When the entire chain has fallen ($t = \sqrt{2L/g}$), substitute $t^2 = \frac{2L}{g}$:

   $$F_{\text{total}} = \frac{3Mg^2 \cdot \frac{2L}{g}}{2L} = 3Mg.$$

   However, once the chain is fully on the floor, the force should stabilize at Mg. This discrepancy arises because the derived force applies **during the fall**. Standard physics problems interpret such scenarios as requiring the maximum force during the fall, which is three times the chain's weight.

   Answer: The force imposed on the floor is $\boxed{3Mg}$.

7. Answer 7:

   $$\begin{cases} m_1\ddot{x} = T - m_1 g \\ m_2\ddot{y} = T - m_2 g \end{cases}$$

   上式减下式得：$m_1\ddot{x} - m_2\ddot{y} = (m_2 - m_1)g$.

   对 t 积分两次并与 x,y 关于 h 的函数式联立得

   $$\begin{cases} m_1 x - m_2 y = \frac{1}{2}(m_2 - m_1)gt^2 \\ y = h \\ x = h - x_0 \end{cases}$$

   解得 $x_0 = \dfrac{m_1 - m_2}{m_1}(h + \dfrac{gt^2}{2})$.

8. Answer 8:

   1. Key Observations: - The chain has linear density $\lambda = m/l$, where m is the total mass of the chain. - The pulley has mass M and radius R, with moment of inertia $I = \frac{1}{2}MR^2$. - When the length difference between the two sides is x, the masses on the left and right sides are $m_1 = \lambda\frac{l+x}{2}$ and $m_2 = \lambda\frac{l-x}{2}$, respectively.

2. Force Analysis: - Net gravitational force difference: $\Delta F = (m_1 - m_2)g = \lambda xg$. - Newton's laws for the two sides: - Left side: $m_1 g - T_1 = m_1 a \quad T_1 = m_1(g-a)$. - Right side: $T_2 - m_2 g = m_2 a$ $T_2 = m_2(g+a)$.

3. Torque and Rotational Motion: - Tension difference creates torque: $\tau = (T_1 - T_2)R$. - Rotational equation: $\tau = I\alpha = \frac{Ia}{R}$. - Substituting T_1 and T_2:

$$(m_1(g-a) - m_2(g+a))R = \frac{1}{2}MR^2 \cdot \frac{a}{R}.$$

- Simplify using $m_1 - m_2 = \lambda x$ and $m_1 + m_2 = \lambda l$:

$$\lambda xgR^2 - \lambda laR^2 = \frac{1}{2}MR^2 a.$$

- Cancel R^2 and solve for a:

$$a = \frac{\lambda xg}{\lambda l + \frac{M}{2}} = \frac{mxg}{l(m + \frac{M}{2})}.$$

4. Final Answer: The acceleration when the length difference is x is:

$$\boxed{a = \frac{mgx}{l\left(m + \frac{M}{2}\right)}}$$

9. Answer 9:

a.

$$\begin{cases} mg - T = ma \\ TR = I\alpha = \frac{1}{2}mR^2\alpha \end{cases}$$

$$\implies \alpha = \frac{2g}{3R}, a_c = R\alpha = \frac{2g}{3}.$$

$$\text{So, } T = \frac{1}{3}mg.$$

b.

$$\begin{cases} mg - T = 0 \\ TR = I\alpha = \frac{1}{2}mR^2\alpha \end{cases}$$

$$\text{So, } \alpha = \frac{2g}{R}.$$

c.

$$\begin{cases} T - mg = ma_c \\ TR = \frac{1}{2}mR^2\alpha \end{cases}$$

$$\implies \alpha = \frac{4g}{R}, a = \alpha R + a_c = 5g.$$

10. Answer 10:

我们考虑一个微元 dx，其质量为 $\mathrm{d}m = \rho \mathrm{d}x$，离 O 点的距离是 x，我们有 $|p| = \mathrm{d}m(\omega x \sin\alpha)$，故

$$|\mathrm{d}\vec{L}| = |\vec{r} \times \vec{p}| = x \cdot \mathrm{d}m \cdot (\omega x \sin\alpha) = (\omega x^2 \sin\alpha)\mathrm{d}m = (\omega x^2 \sin\alpha)\rho \mathrm{d}x$$

注意到我们利用了 $\vec{r} = x\hat{r}$，这样总的角动量是:

$$|\vec{L}| = \int_{-l}^{l} |\mathrm{d}\vec{L}| = \rho_0 \int_{-l}^{l} x^2 \omega \sin\alpha (1 - \frac{x^2}{4l^2}) \mathrm{d}x = \boxed{\frac{17}{30}\rho_0 l^3 \omega \sin\alpha}$$

11. Answer 11:

1. Velocities After the Rope Tightens - Free fall of A: When A falls distance h, its velocity is $v_A = \sqrt{2gh}$. - Conservation of angular momentum (about the pulley's axis) at the moment the rope tightens:

$$m_A v_A R = \left(\frac{1}{2}MR^2 + m_A R^2 + m_B R^2\right)\frac{v}{R}$$

Simplifying:

$$v = \frac{m_A\sqrt{2gh}}{\frac{M}{2} + m_A + m_B}$$

Result: Both A and B move with velocity v (downward for A, upward for B).

2. Maximum Height of B

- Energy conservation after the rope tightens: Initial kinetic energy converts to gravitational potential energy as B rises:

$$\frac{1}{2}\left(m_A + m_B + \frac{M}{2}\right)v^2 = (m_B - m_A)gH$$

Substituting v and solving for H:

$$H = \frac{2m_A^2 h}{(M + 2m_A + 2m_B)(m_B - m_A)}$$

Final Answers:

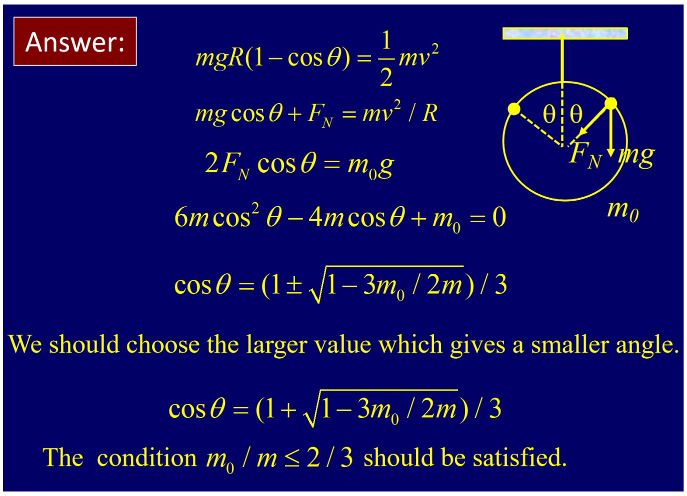
1. Velocities:

$$v = \boxed{\frac{m_A\sqrt{2gh}}{\frac{M}{2} + m_A + m_B}}$$

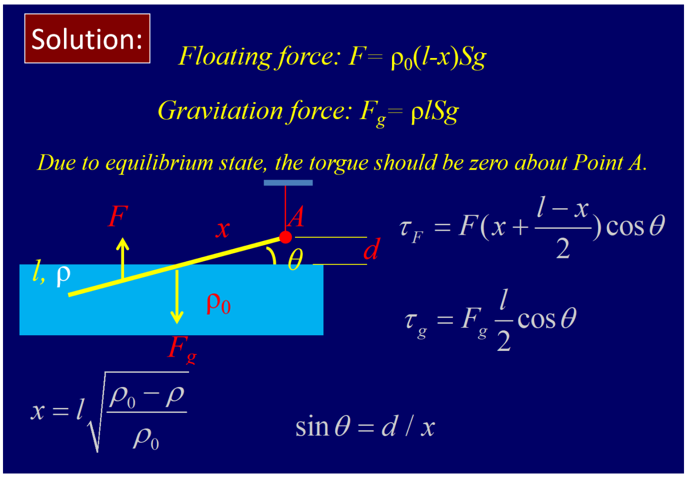
2. Maximum Height:

$$H = \boxed{\frac{2m_A^2 h}{(M + 2m_A + 2m_B)(m_B - m_A)}}$$

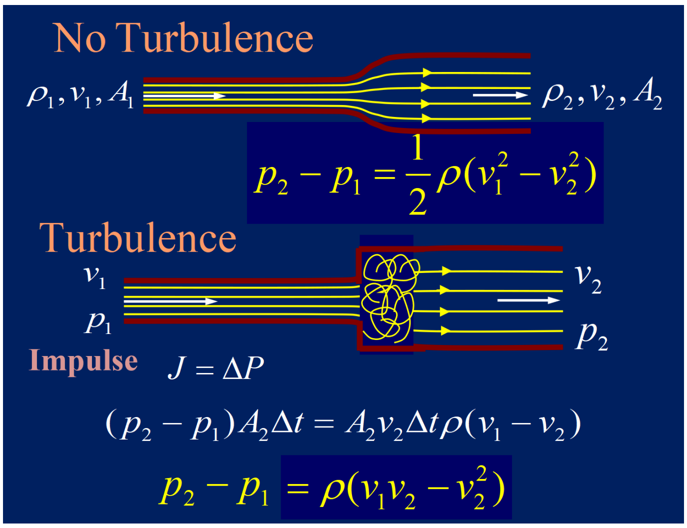
12. Answer 12:



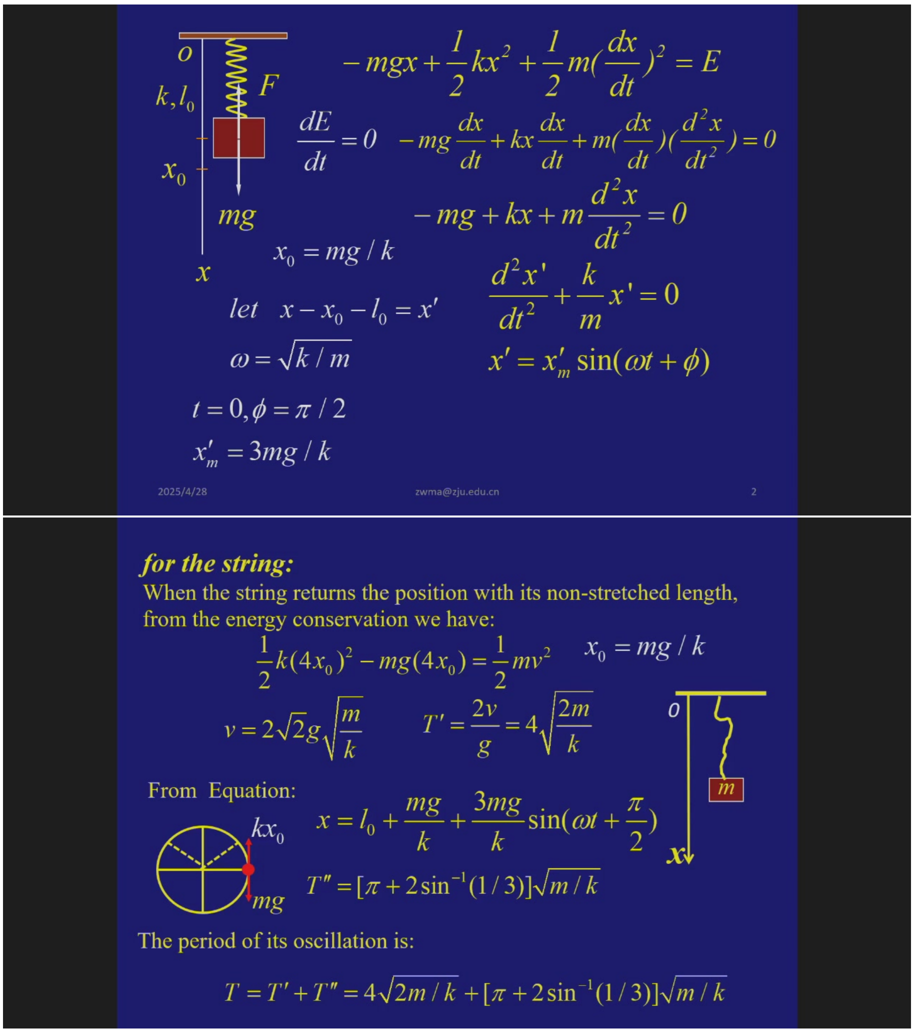
13. Answer 13:



14. Answer 14:



15. Answer 15:



16. Answer 16:

Solution:

The reflection wave is

$$y' = 0.5\cos[10\pi(t+\frac{x}{2}-\frac{20}{2})-\pi]$$
$$= -0.5\cos 10\pi(t+0.5x)$$

$$y+y' = 0.5\cos 10\pi(t-0.5x) - 0.5\cos 10\pi(t+0.5x)$$

$$= \sin 10\pi t \sin 5\pi x$$

$$5\pi x = n\pi \qquad x=\frac{n}{5} \qquad x = 0, 0.2, 0.4, 0.6\ldots$$

17. Answer 17:

Solution:

Kinetic energy: $dK = \frac{1}{2}(\mu dx)(\frac{dy}{dt})^2$ $\qquad \frac{dy}{dt} = y_m \omega \cos kx \cos \omega t$

$$= \frac{1}{2}(\mu dx) y_m{}^2 \omega^2 \cos^2 kx \cos^2 \omega t$$

Potential energy: $dU = \frac{F}{2}(\frac{dy}{dx})^2 dx$ $\qquad \frac{dy}{dx} = -ky_m \sin kx \sin \omega t$

$$= \frac{Fdx}{2}(y_m k \sin kx \sin \omega t)^2$$

$$= \frac{1}{2} dx \mu y_m{}^2 \omega^2 \sin^2 x \sin^2 \omega t \qquad F = u^2 \mu = \frac{\omega^2}{k^2}\mu$$

$$dE = dK + dU = \frac{1}{2} dx \mu y_m{}^2 \omega^2 [\sin^2 kx \sin^2 \omega t + \cos^2 kx \cos^2 \omega t]$$

$$E = \frac{1}{2}\mu y_m{}^2 \omega^2 \int_0^{\lambda/4} [\sin^2 kx \sin^2 \omega t + \cos^2 kx \cos^2 \omega t] dx$$

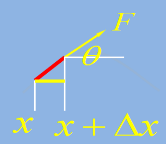
$$= \frac{1}{2}\mu y_m{}^2 \omega^2 [\frac{1}{2k}\frac{\pi}{2}\sin^2 \omega t + \frac{1}{2k}\frac{\pi}{2}\cos^2 \omega t]$$

$$E = \frac{\mu y_m{}^2 \omega^2 \lambda}{16}$$

Total energy remains constant between N and AN.

Instantaneous power transferred from left part to right part is P_i

$$P_i = \vec{F}\cdot\vec{v} = -F\sin\theta \frac{dy}{dt} = -F(\frac{dy}{dx})(\frac{dy}{dt})$$



$$P_i = -F(-y_m k \sin kx \sin \omega t)(y_m \omega \cos kx \cos \omega t)$$

$$= F y_m{}^2 k\omega \sin \omega t \cos \omega t \sin kx \cos kx$$

$P_i = 0$ At a node or an antinode.

There is no power transmitted through a node or an antinode.

18. Answer 18:

The burn rate of gasoline by the car $\gamma_g = \chi v$

The power per unit time $p = \gamma_g E_g$

The efficiency by the definition

$$e = W/Q_H = (W/t)/(Q_H/t) = P_{delivered}/P$$

So

$$P_{delivered} = eP = e\chi v E_g = 0.2 \times \frac{10}{100 \cdot 10^3} \times 30 \times 35 \cdot 10^6 = 2.1 \times 10^4 W$$