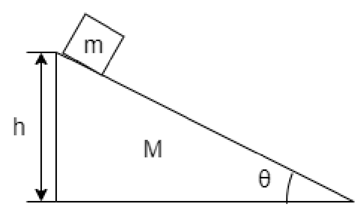
General PhysicsI(H) Midterm

April 16, 2024

1 Dynamics of a Block-wedge system

A right triangular wedge of mass M, height h, angle θ, supports a cubic block of mass m on its side. Initially the wedge rest on a horizontal table, and the block is put at the highest point on the wedge side. Then the block is released from rest. Neglect the size of the block, the friction between the block and the wedge, and the friction between the wedge and the table.



1. After the block is released, it will slide down the wedge. When the block reaches the table, what is the velocity of the wedge relative to the table.

2. Following 1, find the distance that the wedge has moved when the block reaches the table.

3. Now we do NOT want the block to slide down the wedge as in 1 and 2. To achieve this, we can apply a suitable addtional horizontal force F to the wedge at the moment of releasing the block, such that the block will keep stationary relative to the wedge. Find the magnitude and the direction of this force.

2 Walking on a Turntable

A woman with mass $m = 50kg$ stand at the rim of a horizontal turntable with area density $\rho = \frac{50}{\pi} kg \cdot m^{-2}$ and radius $R = 2.0m$. The turntable is initially at rest and is free to rotate about a frictionless, vertical axle through its center. The woman then starts walking around the rim clockwise (as viewed from above the system) by continuously exerting the force $F = 20N$ on the turntable along its tangentialdirection. In the following problems, give the values in SI-units.

1. Calculate the moment of inertia I of the turntable about the axle.
2. In what direction and with what angular speed ω does the turntable rotate at the $t = 1.0s$ after the woman started walking.
3. Determine the woman's speed v relative to the ground at that moment.
4. How the power P is done by the woman at that moment.

3 Simple Harmonic Motion in a 1D Potential

A potential of mass m is situated in a one-dimensional potential $U(x)$ given by $U(x) = \lambda(\frac{x^4}{8} - \frac{x^3}{6} - x^2 + 2x)$ where $\lambda > 0$.

1. Show that the particle is in equilibrium at $x = -2$, $x = 1$ and $x = 2$. For each equilibrium position, is it a stable of unstable equilibrium? [You may use $(x-1)(x+2)(x-2) = x^3 - x^2 - 4x + 4$.]
2. Write an approximate expression for the potential $U(s)$ and force $F(s)$ for small s where $s = x - x_0$, and x_0 is the stable equilibrium position with the lowest potential energy.
3. At $t = 0$, the particle is released with an inital velocity $-v_0$ at the distance $s = s_0$. Write the equation of motion for the particle and show that it undergoes simple harmonic motion. Show that $s(t) = A\cos(\omega t + \phi)$ is a solution, and find expressions for the frequency ω, the amplitude A, and the phase ϕ.

4 Infinite One-dimensional Monoatomic Chain Including Next-nearest-neighbour Interactions

In the lecture, we demonstrated the example of an one-dimensional monoatomic chain with nearest-neighbour interactions only, where we get the linear wave equation in the long-wavelength limit. With the condition of only nearest-neighbour interactions, the interatomic potential is $U^{ham} = \frac{1}{2}k_1\Sigma_n(x_{n+1} - x_n - a)^2 = \frac{1}{2}k_1\Sigma_n(u_{n+1} - u_n)^2$, where x_n is the position of the n^{th} atom and u_n is the displacement of the n^{th} atom from its equilibrium position $n \cdot a$, if we would include the next-nearest-neighbour interactions, it will be $U^{ham} = \frac{1}{2}k_1\Sigma_n(u_{n+1} - u_n)^2 + \frac{1}{2}k_2\Sigma_n(u_{n+2} - u_n)^2$.

1. Using the relation $M\ddot{u_n} = -fracdU^{ham}du_n$, write down the equation of motion for the n^{th} atom incluing next-nearest-neighbour interactions.

2. Consider normal modes in the form of $u_n = A_0 e^{i(kna - \omega t)}$, where k is the wave number, ω is the angular frequency, and A_0 is the amplitude of the oscillation for the n^{th} atom. Derive an expression for ω as a function of k.

3. In the long-wavelength limit, $k \cdot a \to 0$, derive the linear relation between ω and k.