1 A falling string (25 points)

A string of length L was initially hung above the ground, where the top end was held by hand, with the bottom end almost touching the ground (as shown in Fig.1).At t=0 the top end was released from rest,and the string begins to fall freely. Asume the mass of the string is λ per unit length. We also assume the tension in the string is zero in other words, the part of the string that has been lying on the ground does not affect the remaining part of the string tha is falling freely.
(a) What is the speed v of the top end when it has fallen a distance h?
(b) Two forces are relevant for the string,namely, the gravitational force and the supportive force from the ground.For each force, state whether it is a conservative force,and explain why.
(c) Give an expression for the supportive force acting on the string by the ground when the string has fallen a distance h.

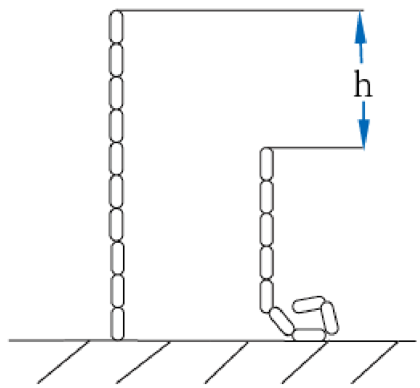

Figure 1: falling string

2 Rod hinged to a massive pulley (25 points)

Consider the configuration displayed in Figure 2.At the particular moment shown,the uniform rod of length L=0.8m and mass M_R=6.5kg is oriented horizontally and is rotating with an angular velocity ω=3.5 rad s^{-1}.One end of the rod is fixed, and the other end of the rod(point B)is attached to a massless string.The string wraps around a massive pulley of mass M_P=10.2kg and radius R_P=0.2m without slipping, and attached to other end of the string is a block of mass M_A=12.1kg. Treat the pulley as a uniform disc that rotates about the axis through P without friction,and take the acceleration due to gravity to be g=9.8m/s^2.
(a)Show that the moment of inertia I_P of the pulley about an axis through its center(point P),perpendicular to the circular cross section,is $I_P = \frac{1}{2}M_P{R_P}^2$.
(b)Show that the moment of inertia I_R of the rod about an axis at one end of the rod is $I_R = \frac{1}{3}M_R L^2$.
(c)Find the tangential component of the accerleration $a_T$of the end of the rod at point B and its direction.
(d)Find the radial component of the acceleration $a_R$of the end of the rod at point B and its direction.

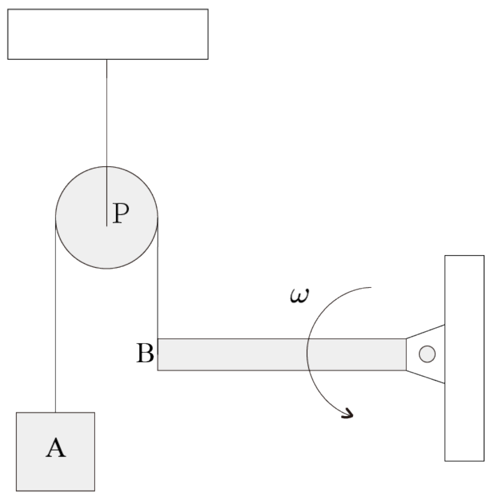

Figure 2: The hinged pulley system

3 Coupled harmonic oscillators (25 points)

Two blocks with mass m_1 and m_2,respectively,are connected by one massless spring with spring constant k and natural length l(as shown in Fig.3).Their movements are limited to the one-dimensional line in the x direction.At the beginning,they are still on the frictionless ground and the spring is at its natural length.
(a)When the system is disturbed,the dispalcements of m_1 and m_2 are x_1 and x_2,respectively,whose positive direction is to the right.Write down the equations of motion for each block $m_1$and$m_2$in the form of $F_i = m_i \frac{d^2 x_i}{dt^2}$;
(b)Suppose the solutions for x_1 and x_2 are $x_i = x_{i0} cos(\omega t + \phi)$.Find the equations for x_{10} and x_{20} ,which are the amplitudes of oscillation for m_1 and m_2.Solve the eigenfrequency ω and draw a schematic illustration of x_{10} and x_{20} for each normal mode.
(c)Suppose a third block with the same mass m_1 and an initial velocity $v_0$moving from the left undergoes an elastic collision with the block $m_1$at time t=0.After the collision, the system of $m_1$and$m_2$will move as a function of time t,following a superposition of the above normal modes with initial conditions $x_1 = 0, x_2 = 0; v_1 = v_0, v_2 = 0$. Determine the respective oscillation amplitude $x_{i0}$of each block in terms of $v_0$and ω,as well as the phase ϕ.

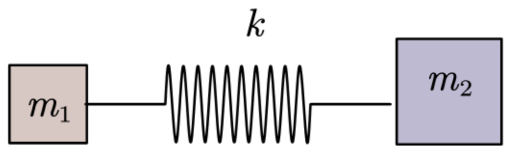

Figure 3: The coupled harmonic oscillators

4 Transverse waves on a rope (25 points)

Consider a uniform rope whose mass per unit length is μ.
(a)Initially the rope is straightened horizontally along the x-axis, as shown in Fig.4(a). The origin O of the xy coordinate system is on the rope. Suppose the right end of the rope is fixed on a wall at x=x_0. Then a transverse wave progating to the right is generated by shaking the left end of the rope vertically. We assume that the rope does not deviate too much from the x-axis,i.e.,the amplitude of the wave is small. The tension T in the rope is constant.Derive the wave equation for the transverse wave (neglect the gravity of the rope).
(b)Suppose the teansverse wave in (a)is a sinusoidal wave with wave function $y(x,t) = Asin(kx - wt)$.At time $t = t_0$, the wave just reaches the right end of the rope at $x = x_0$,as shown in Fig.4(b).The reflected wave function is $y'(x,t) = Asin(kx + \omega t + \phi)$. Find the phase ϕ according to the boundary condition at the right end of the rope.
Hint:The right end of the rope has zero displacement at all times.
$sin\alpha + sin\beta = 2sin\frac{\alpha+\beta}{2}cos\frac{\alpha-\beta}{2}; cos\alpha + cos\beta = 2cos\frac{\alpha+\beta}{2}cos\frac{\alpha-\beta}{2}$
(c)Consider a different situation from (a) and (b): the rope initially hangs vertically from the ceiling along the y-axis, as shown in Fig.4(c).The origin O of the y-axis is at the lower end of the rope. The tension in the rope solely results from its gravity. If the rope does not deviate too much from the y-axis when a transverse wave is propagating on it, derive the wave equation for the transverse wave.Is a sinusoidal wave still a solution of the wave equation in this case?

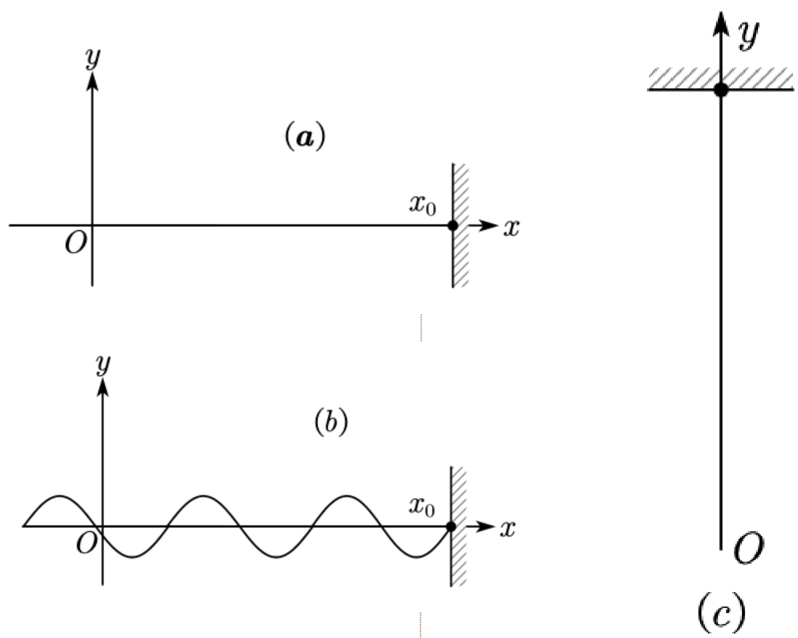

Figure 4: The transverse waves on a rope