The Currency Hegemon's Optimal Inflation Rate

Anne Laski*
Brandeis University

December 26, 2021

Abstract

Given that US currency is used heavily abroad, how do changes in inflation impact the welfare of domestic and foreign consumers? I formulate a two-country model in which agents face idiosyncratic income risk and thereby hold fiat currency for consumption smoothing. All transactions and savings are done using a single dominant fiat currency. Higher inflation increases the level of seigniorage the domestic country collects from abroad but increases the cost of holding currency for both domestic and foreign agents. Hence, for the domestic currency issuing country, neither a zero-inflation regime nor one targeting Friedman's rule is optimal. The results demonstrate the presence of a seigniorage channel, which can be used to inform the hegemon's inflation target.

Key words: Dominant currency, inflation, incomplete markets

1 Introduction

In this paper I identify one factor—the seigniorage channel—that informs the monetary policy of a fictional currency hegemon. This seigniorage channel biases the optimal inflation target upward. I use two key assumptions to demonstrate that, due to this channel, the hegemon's optimal rate of inflation is an increasing function of foreign demand.

*alaski@brandeis.edu, Brandeis International Business School, 415 South Street, Waltham, MA 02453, 857-210-8548. I am indebted to my chair, Professor George Hall and committee members Professor Maya Eden and Professor Blake LeBaron. I am also grateful for comments from Stephen Cecchetti, Judy Dean, Jean-Paul l'Huillier, Carol Osler, Ben Shiller, Raphael Schoenle, and Cliff Zhu. Finally, I am hugely thankful for the support from Brandeis library staff.

First, I assume that markets are incomplete and that agents consequently hold a fiat currency as a means to smooth consumption across time. Secondly, I assume that global demand for a fiat currency is supplied by a single country. Concave preferences and idiosyncratic income shocks induce money demand for self-insurance. This money demand creates a basis for the central bank to extract an inflation tax, or seigniorage revenue. When lump-sum transfers of new cash are delivered only to residents of one country, these agents can use their advantage in trade to extract seigniorage revenue from abroad. These seigniorage gains subsidize domestic consumption and represent the primary source of welfare gains to the currency producer.[1] However, inflation discourages savings, implying that seigniorage revenue has the same functional form as a Laffer curve and that identifying the optimal inflation target involves a trade-off. Following İmrohoroğlu [1992], I focus on a stationary equilibrium with a constant rate of inflation.

The long-observed global demand for U.S. dollars motivates this analysis. For example, approximately 60% of official reserves are held in U.S. dollars and, according to S.W.I.F.T. payment data, roughly 40% of global payments are transacted in dollars.[2] The exclusive control the U.S. exercises over the printed supply and relative cost of holding these dollars has accorded it an —also long-observed— unique advantage in its interactions with the rest of the world. When the U.S. emerged as the world's currency hegemon after World War II, Valéry Giscard d'Estaing famously coined the term, "exorbitant privilege," to describe the role played by this advantage in global debt markets. Since then, the large and increasing influence of the dollar has prompted a growing movement to decouple international markets from U.S. monetary policy. In this paper, I illustrate the common sense of why a hegemon like the U.S. would be reluctant to alter the prevailing market structure. To do so, I frame the question in terms of rents accumulated by a monopolist that supplies a savings device to households who are meanwhile subjected to income shocks. In steady state equilibrium, these rents motivate a higher optimal inflation target for the currency hegemon.

Conventional monetary theory predicts that when agents are unable to perfectly share risk, the best available policy is a Friedman rule—setting inflation to offset agents' rate of time preference. However, the incomplete markets literature has since demonstrated first that equilibrium does not always exist under a Friedman rule and further that a Friedman rule can be dominated by positive inflation. My analysis provides a parable to explain why a positive inflation rate may be optimal in the context of a dominant currency. Importantly,

[1] Engel [2016] similarly presents a two-country economy wherein the home government chooses a consumption subsidy (more precisely, a negative tax on labor) in order to maximize welfare in that country. Under incomplete markets, the optimal choice is to use such a subsidy to manipulate the terms of trade. In contrast, my hegemon's instrument is a printing rule, which ultimately feeds into that country's trade balance.

[2] See Gourinchas et al. [2019].

in this parable, any welfare gains to the hegemon come at a cost to the rest of the world.[3]

In the incomplete markets model I construct below, agents are accorded unequal access to a single savings instrument in a two-country world. Restricting agents from borrowing or independently devising substitute savings instruments are indeed severe assumptions, but they enable us to isolate the seigniorage channel, while reflecting some of the constraints experienced under what is sometimes termed the “TINA" regime of the dollar: “there is no alternative." Since supplanting the pound sterling during Europe's post-war reconstruction, the U.S. has benefitted from a quasi-monopoly in producing “securities that are always in high demand by the rest of the world." Goldberg and Tille [2008] and Gopinath and Stein [2021] study invoicing choices, particularly the out-sized share of invoicing conducted in U.S. dollars and transactions outside of the U.S. The empirical evidence of the former and the theoretical framework of the latter form a narrative in which competition and network effects reinforce patterns of currency use. Gourinchas et al. [2019] catalogue the self-reinforcing nature of currency hegemony as well as the broad literature surrounding the dynamics and fragility that accompany a center-periphery system of international markets. In the words of Gourinchas [2021]: “The simple reality is that we live in a dollar world." Hyperbolizing Gourinchas' observation to be literal highlights the seigniorage channel and thereby offers a contribution to the academic debate around the optimal rate of inflation.

So long as the global supply of cash does not completely erode savings demand, trading cash balances overseas allows residents of the currency producer to run a current account deficit. Though the model abstracts away from a multitude of factors characterizing international financial markets that may influence monetary policy, the divergence in domestic and foreign savings patterns is reminiscent of the United States' somewhat notorious recent history of very low rates of consumer savings, relative to other advanced markets. In this model, the financial account—a function of foreign demand—is what drives the current account result.

I calibrate this hypothetical economy using existing estimates of global demand for U.S. hard currency, or outside money. Judson [2017] estimates that approximately 60% of all U.S. currency and roughly 75% of $100 bills are held overseas, although the location of any physical currency is of course, imperfectly observed. This obscurity has incited calls to drastically reduce the quantity of cash issued by the Federal Reserve, on the grounds that much of overseas dollars holdings are used for illicit transactions and the finance of criminal activities.[4] If this is the case, my results imply that positive inflation rates effectively tax such activities, to the profit of domestic citizens.

Setting an optimal inflation target encompasses many factors and channels from the

[3] This model assumes that the countries do not cooperate. For work exploring competitive and cooperative games, see Bodenstein et al. [2020], Egorov and Mukhin [2020], and Engel [2016].

[4] See Rogoff [2017].

domestic and international economy. My analysis demonstrates one channel to consider. The model aims to frame dollar hegemony and the associated optimal policy in terms of first principles. Doing so provides a baseline for future studies measuring the impact of substitutes in the context of monetary policy and aggregate welfare.

The paper is organized as follows. Section 2 describes a two-country incomplete markets model. Section 3 discusses the calibration and numerical solution of this model. Section 4 presents the model's results and section 5 explains the key mechanisms and intuition. Section 6 performs some sensitivity analyses. Section 7 concludes.

2 Monetary Economy with Heterogeneity

Agents live in a world with income shocks, a consumption good, and a fiat currency with which they can partially self-insure. They are heterogeneous with respect to their income and money-holdings positions. This heterogeneity follows the tradition and requires the fixed point numerical techniques of the Bewley class of models. İmrohoroğlu made early contributions to the incomplete markets literature by applying Bewley's heterogeneous agent framework to study the consequences of monetary policy. I extend her closed economy model of money demand to a two-country world.

The residents of these two countries are infinitely-lived and identical in all respects except their access to the economy's savings device, or equivalently, the resources they have to self-insure. The domestic economy is Country A; the foreign economy is Country B. The continuum of agents in the world is mass 1. The population of each country, or the likelihood of an agent being a resident of either Country $(P(i))$ is defined by the modeler. Agents are assigned their Country i at birth and never migrate.

For simplicity, agents in the two countries receive identical endowment values that are subject to the same transition probabilities. Finally, I assume that agents in the domestic and foreign countries have identical preferences. They make choices over consumption and money holdings in order to maximize:

$$E\sum_{t=0}^{\infty}\beta^t U(c_t^i), \tag{1}$$

where $0 < \beta < 1$ is their subjective time discount factor and c_t^i is their consumption of the nonstorable good. The utility function is twice continuously differentiable, increasing and concave in c_t^i, and has the following form:

$$U(c_t^i) = \frac{(c_t^i)^{1-\gamma} - 1}{1-\gamma}, \quad \gamma > 0. \tag{2}$$

In addition, agents face uncertainty in their endowment streams, determined by exogenous transition probabilities, given by:

$$\chi = (s, s') = Pr(s_{t=1} = s', s_t = s) \quad \text{for} \quad s, s' \in S = s_1, s_2. \tag{3}$$

When an agent is in state $s = s_1$, the high income state, he receives y units of the consumption good. In state $s = s_2$, he instead receives θy units, where $0 < \theta < 1$.

Because of these fluctuations, agents allocate their endowment across the savings device and the consumption good, trading with other agents in order to maximize their lifetime utility. There are no barriers to this trade. Agents begin each time period of their life with the present value of whatever fiat money they chose to save during the previous period. Time is discrete and each period lasts six weeks. Finally, because all agents use the same currency for savings, any inflation due to τ is fully passed through during all transactions. Equivalently, I assume that the quantity theory of money holds in this economy and inflation affects all agents simultaneously.

Country A agents receive a lump-sum transfer equal to $\frac{\tau}{P(i=A)}\tilde{M}_{t-1}$, where $\tilde{M}_{t-1}$ is the nominal global per-capita money supply in time $t-1$ and τ is an exogenously assigned printing parameter assigned by the central bank in Country A. $P(i = A)$ represents the global population share of Country A agents. The rules of this global economy allow Country A agents to both save money and receive government transfers, while Country B agents can only save. The money supply follows the law of motion

$$\tilde{M}_t = \Big(1 + \frac{\tau}{P(i = A)}\Big)\tilde{M}_{t-1}. \tag{4}$$

The economy's prices, p_t, are defined as the price of money in terms of the consumption good at time t. Country A agents have budget constraints written as

$$c_t^A + p_t\tilde{m}_t^A = p_t\tilde{m}_{t-1}^A + y_t^A + \frac{\tau}{P(i = A)}p_t\tilde{M}_{t-1}^{A+B} \tag{5}$$

and Country B agents have budget constraints written as

$$c_t^B + p_t\tilde{m}_t^B = p_t\tilde{m}_{t-1}^B + y_t^B, \tag{6}$$

where

$$y = \begin{cases} y \text{ if } s = s_1, \\ \theta y \text{ if } s = s_2, 0 < \theta < 1. \end{cases} \tag{7}$$

To rewrite these equations in real terms (in terms of the consumption good), we define $p_t\tilde{m}_t = m_t$ and $\frac{1}{1+\pi_t} = \frac{p_t}{p_{t-1}}$:

$$c_t^A + m_t^A = \frac{1}{1 + \pi_t}m_{t-1}^A + y_t^A + \frac{\tau}{(1 + \pi_t)P(i = A)}M_{t-1}^{A+B} \tag{8}$$

and

$$c_t^B + m_t^B = \frac{1}{1+\pi_t} m_{t-1}^B + y_t^B. \tag{9}$$

I assume that Walras' law always holds and that all agents are utility maximizing. Therefore, for a given printing parameter, τ, the exchange rate between goods and money adjusts until markets clear and the individual Euler equations for all agents hold. Because I assume that the quantity theory of money holds, it follows that prices immediately adjust when the money supply changes, so that $\tau = \pi$.

There is very limited role for government in this model and the entirety of its actions are performed by the central bank in Country A. The central bank collects seigniorage by printing money; its only expenditure consists of the provision of lump-sum transfers to the Country A agents. Following Friedman, I assume there are no additional costs to printing new currency. The optimal inflation rate is that which corresponds to maximized aggregate utility in Country A.

Define the government's (of Country A) nominal budget constraint as:

$$\tilde{M}_t - \tilde{M}_{t-1} = \tilde{G}_t. \tag{10}$$

Under the model's definition of prices and law of motion for money growth in equation (4), the corresponding real constraint for the central bank in Country A is then:

$$\frac{\tau}{(1+\tau)P(i=A)} M_{t-1} = G_t. \tag{11}$$

Diverse agents make individual consumption and savings decisions to solve a dynamic programming program, which is implemented through a Bellman equation. The state variables in this problem are agents' quantities of real balances m and the endowment state s they draw from the stochastic process. The corresponding choice variables are present consumption c^i and the quantity of savings to take into the future, $m^{i\prime}$. Agents' Bellman equation is thus:

$$V(m^i, s^i) = \max_{m,c} \left\{ U(c^i) + \beta \sum_{s'} \chi(s^i, s^{i\prime}) V(m^{i\prime}, s^{i\prime}) \right\}, \tag{12}$$

where agents' maximization problems are subject to the budget constraints specified in equations (5) and (6). The solution criteria and algorithm are formally discussed in the next sections. Equilibrium in this context involves iteratively computing the distributions of agents' individual savings-endowment positions, defined as $\lambda(m, s, i)$.

In the two-country context, I find these distributions for each country as well as for the global economy. Computing the average measure of real balances requires taking the cross product of the distribution of agents and the vector of agents' (conditionally) optimal choices of savings, $m^{i\prime}$ to carry into the next period. Average endowment resources and consumption can be similarly computed. (Note that in the endowment economy, the average endowment is a constant.) The initial assumptions that no aggregate shocks exist and that agents never migrate permit this approach to aggregation; with these assumptions, the cross products represent an expected value which should consistently occur in a stationary equilibrium by applying the law of large numbers.

There are no barriers to trade, so deriving the law of motion for the economy's global money supply requires writing agents' respective country-specific budget constraints, equations (5) and (6), in terms of their average "trade balances." Aggregating across countries can be done by weighting each country's average consumption, endowment, and real balances by its relative share of the global population, because we have prohibited any migration and there is no aggregate uncertainty. Thus,

$$\begin{aligned} &P(i=A)\Big[\sum_{s}\sum_{m\in\Omega(m',s)}\lambda_A(m,s)y_t^A - \sum_{s}\sum_{m\in\Omega(m',s)}\lambda_A(m,s)c_t^A\Big] \\ &= P(i=A)p_t\Big[\Delta\sum_{s}\sum_{m\in\Omega(m',s)}\lambda_A(m,s)\tilde{m}_t^A\Big] - p_t\tau\tilde{M}_{t-1}^{A+B}. \end{aligned} \tag{13}$$

$$\begin{aligned} &P(i=B)\Big[\sum_{s}\sum_{m\in\Omega(m',s)}\lambda_B(m,s)y_t^B - \sum_{s}\sum_{m\in\Omega(m',s)}\lambda_B(m,s)c_t^B\Big] \\ &= P(i=B)p_t\Big[\Delta\sum_{s}\sum_{m\in\Omega(m',s)}\lambda_B(m,s)\tilde{m}_t^B\Big], \end{aligned} \tag{14}$$

where Δ represents the change in agents' stock of money holdings. Equations (12) and (13) represent estimates of the average per-capita quantities of each country.

Although this analysis is restricted to that of the stationary equilibrium, equations (13) and (14) suggest some narrative of the different costs and benefits imposed on Country A and Country B agents when inflation is increased. Equation (13) is a variation of İmrohoroğlu's (and indeed Friedman's) original story: agents in Country A receive lump-sum transfers of cash and observe a corresponding change in the relative quantities of currency and goods, which is immediately reflected in the price level. Those agents in Country B only experience a change in prices, reminiscent of the small open economy tradition.

Combining equations (13) and (14) gives us an expression for the global economy:

$$
\begin{aligned}
&P(i=A)\Big[\sum_s \lambda_A(m,s) y_t^A - \sum_s \sum_{m\in\Omega(m',s)} \lambda_A(m,s) c_t^A\Big] \\
&+ P(i=B)\Big[\sum_s \lambda_B(m,s) y_t^B - \sum_s \sum_{m\in\Omega(m',s)} \lambda_B(m,s) c_t^B\Big] \\
&= p_t P(i=A) \sum_s \sum_{m\in\Omega(m',s)} \lambda_A(m,s) \tilde{m}_t^A + p_t P(i=B) \sum_s \sum_{m\in\Omega(m',s)} \lambda_B(m,s) \tilde{m}_t^B \\
&- p_t P(i=A) \sum_s \sum_{m\in\Omega(m',s)} \lambda_A(m,s) \tilde{m}_{t-1}^A - p_t P(i=B) \sum_s \sum_{m\in\Omega(m',s)} \lambda_B(m,s) \tilde{m}_{t-1}^B \\
&- \tau p_t \tilde{M}_{t-1}^{A+B}.
\end{aligned} \tag{15}
$$

As suggested by the term "aggregate trade balances," equation (15) describes each of Country A and B's balance of payments, scaled by the countries' global population share. It follows that the left hand side of equation (15) represents the sum of the current accounts or trade balances of A and B, while the right hand side represents the corresponding changes in their respective financial accounts. It is immediately evident that so long as agents from Country B are importing the fiat currency and such trade is subsidized through the lump-sum transfers to Country A agents, Country A will run a current account deficit.

In the world economy described in equation (15), the left hand side must zero out. When those terms drop out of the equation, so can the price terms. Define the equilibrium quantity of fiat currency as:

$$
\tilde{M}_t = P(i=A) \sum_s \sum_{m\in\Omega(m',s)} \lambda_A(m,s) \tilde{m}_t^A + P(i=B) \sum_s \sum_{m\in\Omega(m',s)} \lambda_B(m,s) \tilde{m}_t^B. \tag{16}
$$

Equation (16) provides the $t-1$ basis for the law of motion for the world money supply described in equation (4). When this expression of the total global demand for money holdings is rearranged, one obtains an expression for what informs the transfer to Country A agents in equation (5):

$$
\frac{1}{P(i=A)} \tilde{M}_t = \sum_s \sum_{m\in\Omega(m',s)} \lambda_A(m,s) \tilde{m}_t^A + \frac{P(i=B)}{P(i=A)} \sum_s \sum_{m\in\Omega(m',s)} \lambda_B(m,s) \tilde{m}_t^B. \tag{17}
$$

Equation (17) illustrates the advantage built in when only the Country A agents receive the transfer: these agents receive both the per-capita demand by Country A agents as well as a weighted share of the Country B demand. As already discussed, receiving any lump-sum transfer works to the advantage of Country A agents. So long as there exists demand for Country A's currency, these agents can use seigniorage revenue, a subsidy, to increase their average level of consumption.

2.1 Stationary Competitive Equilibrium

A stationary competitive equilibrium with agents in two countries $i = A, B$, is defined as a sequence of prices in terms of the consumption good $\{p_t\}$, a set of decision rules $m'(m,s,i), c(m,s,i)$ for each agents in each country, and an invariant distribution $\lambda(m,s,i)$ across countries that is a measure of agents of position (m,s,i), such that given these prices, agents optimize; that is, agents' decision rules solve their Bellman equations while prices (defined as the relative quantity of goods to fiat money) adjust to facilitate the following equilibria:

1. in the goods market. That is:

$$\sum_i \sum_{m,s} \lambda(m,s,i) c(m,s,i) = \sum_i \sum_{m,s} \lambda(m,s,i) y(s) \quad \text{or} \tag{18}$$

$$P(i = A) \sum_{m,s} \lambda_A(m,s)(y^A(s) - c^A(m,s)) + P(i = B) \sum_{m,s} \lambda_B(m,s)(y^B(s) - c^B(m,s)) = 0.$$

2. in the money market. That is:

$$\sum_i \sum_{m,s} \lambda(m,s,i) m(m,s,i) = M, \tag{19}$$

(where M is the total real balances available for all agents), and

$$P(i = A) \sum_{m,s} \lambda_A(m,s) \left[\tilde{m}_t^A - \tilde{m}_{t-1}^A\right] - P(i = B) \sum_{m,s} \lambda_B(m,s) \left[\tilde{m}_t^B - \tilde{m}_{t-1}^B\right] = \tau \tilde{M}_{t-1}^{A+B}.$$

3. Finally it must be that the aggregate $\lambda(m,s,i)$ is a stationary distribution such that:

$$\lambda(m',s',i) = \sum_{i \in I} \sum_s \sum_{m \in \Omega(m',s)} \chi(s,s') \lambda(m,s,i) \tag{20}$$

where $i = \{A, B\}$ and $\Omega(m',s) = \{m : m' = m'(m,s)\}$.

3 Recursive Competitive Equilibrium

To solve for the stationary equilibrium described above, I apply İmrohoroğlu's nested fixed-point algorithm. I use a discrete grid search method, and using two grids with 4000 intervals each, I solve equation (12) at each grid point for a series of uniformly spaced m. Each of the two money holdings grids corresponds to one of the two endowment states s, so that each position in the set of 8002 gridpoints represents a discrete savings-endowment state or position in of the countries, (m,s,i).

The choice of real balances that maximizes (11)—that is, the level of savings that maximizes utility, conditional on agents' present holdings and income draw— is carried forward as $m^{i\prime}$ and the value function is computed again and again, until the value function and corresponding decision rules converge. These steps are implemented separately for Country A and Country B agents. Next, during each iteration, global money demand is computed according to equation (16); lump-sum transfers to agents in Country A are updated accordingly. Finally, I compute the cross-product of the equilibrium distributions of money holdings and endowment draws produced by these iterations to solve for average measures of consumption,[5] money demand, and utility. Although agents may change their relative position in within the distribution, the law of large numbers implies that these values represent the average decision quantities and utilities realized by any agent of either country or in the general economy.

3.1 Calibration

Agents' preferences are calibrated using İmrohoroğlu's original choices. In section 6, I discuss how adjusting agents' risk aversion affects the model's main result. The endowment value y is normalized to 1.

β	γ	θ	y	$P(i = A)$
0.995	1.5	0.25	1	0.4

Fully calibrating this model requires assigning values to the agents' transition probability matrix; these values determine the endowment draws of any agents and thereby incorporate the idiosyncratic risk that motivates precautionary savings and characterizes this family of economic models. I assume that the Country B agents are subject to the same income dynamics as those in Country A. I update İmrohoroğlu's original transition probabilities from 1990 (12 weeks) to correspond to the unemployment duration in the U.S. prior to the Covid-19 pandemic.

In 2019, unemployment duration had nearly doubled from 1990 to approximately 21.6 weeks, according to the Bureau of Labor Statistics; 21.6 weeks corresponds to 3.6 model periods. The average duration of any agent's time in either of the two (high and low) endowment states can be calculated as $(1 - \chi(s,s))^{-1}$. For calibration purposes, an unemployment duration of 3.6 implies $\chi(u,u) = 1 - 1/D_u = 0.73$, and since the matrix rows sum to one, this is sufficient to also specify the transitions from the high income state to the low income state. I calibrate the row describing transition to the high state (from the high state and from the low state) using the same steady state of time spent in the

[5] Consumption choices are computed using the budget constraints in equations (5) and (6) and agents' decision rules for real balances. In practice, there is a vector of consumption values for agents in the low income state and a second vector of consumption values for agents in the high income state.

high income state, $N = 0.92$ as İmrohoroğlu.[6] This implies an unemployment rate of 8%, which is slightly higher than the average unemployment rate from 1990-2019. The model's transition matrix is then

$$\chi = \begin{bmatrix} 0.9765 & 0.0235 \\ 0.27 & 0.73 \end{bmatrix}. \tag{21}$$

To calibrate the population shares of Country A and Country B, I use estimates provided by Judson [2017]. Judson estimates that approximately 60% of U.S. hard currency is held outside of the U.S. Rather than assigning the country sizes based on observed population shares, I do so based on observed demand for U.S. cash. Doing so is consistent with my assumption that agents in the model are otherwise identical and that differences in aggregate money demand are entirely attributable to Country A agents' unique access to money. Admittedly, a 40-60 split is reasonable but not perfect, since in this model, countries' shares of global money demand do not necessarily match their population shares. Nevertheless, the model's key result—that seigniorage revenue funds a current account deficit and raises aggregate welfare in the currency produces—is robust to a number of specifications, discussed in section 6. Using Judson's measures, the global population share of Country A is 40%, or $P(i = A) = 0.4$.

From the global market clearing conditions in equations (15)-(16), one can observe the advantages of increased foreign demand for the currency. As mentioned, the cash transfers distributed to Country A agents increase with the relative population size of Country B. Consider the calibration choice of $P(i = A) = P(i = B)$ versus a calibration where $P(i = A) < P(i = B)$. Holding τ fixed, the increase in Country B's contribution to global demand exceeds the increase in $\frac{1}{P(i=A)}\tilde{M}$, the basis of the per capita supply of money (in nominal terms). This difference implies higher growth of external demand than growth in the per capita money supply, raising the price of money in terms of the consumption good, a windfall for Country A agents in possession of extra savings. Consequently, I observe that in this setting, lower values of $P(i = A)$ are associated with higher levels of average utilities and that these utility values in turn correspond to higher rates of optimal inflation.

4 Results Summary

I approximate the optimal rate of inflation for a currency producer in a two-country world with idiosyncratic income shocks and imperfect self-insurance.[7] I estimate a sample of market equilibria and determine the compensating variation required to maintain the

[6] That is, the top two matrix elements are selected so that the average time in the high state is 0.92.

[7] When computing the optimal inflation rate, I consider only the average welfare of agents in the currency producing economy.

average utility agents in each country would experience under three baseline cases. Using compensating variation to measure changes in welfare was demonstrated by İmrohoroğlu to be more appropriate than the Harberger triangles that had been computed in previous studies by Lucas [1981] and Fischer [1981]. Using the 2019 calibration and the assumption that $P(i = A) = 0.4$, I find that aggregate utility in Country A reaches its maximum under an inflation rate of 6.8%. Computing the corresponding compensating variation suggests that the optimal rate of inflation lies between 5% and 6.8%.[8]

My results illustrate the utility losses incurred when seigniorage revenues are left uncollected or transferred abroad—for example, under a Friedman rule—which immediately implies an interior solution for the optimal rate of inflation. The model's results are intuitive when we consider agents' incentives to hold real balances. Friedman's initial logic—under the assumption that money is costless to print—sets the inflation rate to balance the marginal benefits of cash balances with the cost of holding them. Agents are compensated at the rate at which they discount the future: in a closed economy, this condition requires paying interest on balances. Under incomplete markets, fiat currency is a store of value, and Friedman's rule ensures that value is constant.

However, Friedman's result breaks down when the marginal benefit of holding cash can instead increase within some positive range of inflation. Such is the case when a currency producer can extract seigniorage revenue from abroad. Friedman [1969] originally induces money demand by offsetting the cost of real balances. Bewley [1979] and Townsend [1980] motivate money demand for the purpose of consumption smoothing, a critical driver behind my results.

Because they are prevented from perfectly sharing risk, agents face a trade-off between maximizing the level of lifetime consumption and minimizing the volatility of this consumption. Jensen's inequality predicts that the uncertainty around future consumption induces agents to divert present consumption to savings. The third derivative of equation (2) establishes such precautionary savings for this model. İmrohoroğlu's original results demonstrate how the cost of inflation erodes this mechanism. Consistent with Friedman and İmrohoroğlu, on a global basis (A with B), average utility is highest when deflationary monetary policy best facilitates self-insurance; a constant value of money holdings drives up money demand and correspondingly, the equilibrium price of the consumption good.[9,10] As mentioned above, this outcome is not in the interest of Country A: in a two-country

[8] This inexact measure is due to rounding that arises in the numerical computations.

[9] Bewley [1983] demonstrated the infinite money demand that results from a Friedman rule within an incomplete markets structure and the associated non-existence of any equilibrium. Consequently, the equilibria approximated here begin just inside a Friedman rule.

[10] Such an argument could be consistent with recent arguments for monetary policies in advanced economies being accommodating to the interests of lower income countries, but I reserve the exploration of such implications for future research.

world with a Friedman rule, the higher global utility is driven by the utility gains to Country B, which come at the cost of agents in Country A.

My results are consistent with closed economy models that find positive optimal inflation rates. What I contribute to this literature on positive optimal inflation rates is the open economy dimension. Shortly after Bewley [1983] demonstrated the nonexistence of equilibrium at a Friedman rule, Mehrling [1995] demonstrated the welfare costs of the lump-sum inflation taxation needed to support positive interest necessary to generate money demand:

> "Because the agent is liable for the tax whether or not he holds any money, whatever insurance against income fluctuation he enjoys when he holds large money balances is gained at the expense of greater vulnerability to income fluctuations when he holds low balances."

More recently, Akyol [2004] builds onto Bewley's intuition and extends the analysis to a choice set of money holdings and interest bearing illiquid assets. Akyol finds that inflation plays a role in household welfare even after controlling for real interest rates and ultimately pins down the optimal rate of inflation at 10%. In their overlapping generations economy with incomplete markets and aggregate shocks, Kryvtsov et al. [2011] demonstrate a positive optimal inflation rate and that it can be implemented through inflation targeting rules.

5 Demand, Seigniorage Revenue, and Compensating Variation

As in prior studies, precautionary savings in a dynamic program implies a mean-variance trade-off for all agents in the model. That is, agents choose between minimizing the volatility of lifetime consumption (consumption smoothing) and maximizing the lifetime expected value of consumption. When recursively solving their dynamic problem for a given level of real balances and with knowledge of the probability of transitioning into either endowment state, agents form decision rules that determine present and future consumption simultaneously. Because consumption is a function of existing real balances, low present savings feed into the Euler equation by raising the marginal utility of future consumption. It follows that present consumption adjusts accordingly. Assigning too much priority to consumption over savings implies the possibility of very low levels of consumption, should the agent with low savings experience an endowment shock. Because utility is a function of consumption, rising levels of inflation increase this impulse to prioritize present consumption over future consumption streams, holding discount factors constant.

The optimal inflation target for the currency producer then depends on three interlinked aggregate measures: money demand in Country B, the average consumption in Country A, and the standard deviation of consumption in Country A. The marginal cost of an increase in inflation is increased consumption volatility, which Country A agents are willing to tolerate if there is an equal or greater increase in average consumption.

By assigning one country a monopoly on currency production, the model explicitly engineers a comparative advantage for some agents, while concave preferences preserve savings demand among all agents. Thus, average consumption in Country A increases with the seigniorage revenue it is able to collect from Country B (an income effect). Country B's demand for real balances forms the tax base from which any extra inflation tax or seigniorage revenue is collected. Aggregate money demand falls with higher levels of inflation, reflecting the higher cost of real balances (a wealth effect). Consequently, increases in inflation eventually result in a diminishing tax base, so that seigniorage revenue, as a function of inflation, looks like a Laffer curve.

The peak of this Laffer curve corresponds to an upper bound on expected consumption in Country A and depends on the relative size of money demand in Country B.[11] An inflation policy set according a Friedman rule represents the lower bound on the Country A agent's consumption volatility and in an environment of imperfect risk-sharing, is the policy that most closely facilitates full insurance. At the same time, the assumption of full pass-through implies that all agents face the same cost of consumption in terms of real balances, or equivalently, that all money holdings depreciate at the same rate. The universal cost of inflation makes it a blunt policy instrument and creates the mean-variance trade-off for agents in Country A. This trade-off can be seen in Country A's hump-shaped utility curve illustrated in Figure 1.

Because agents are limited to one means of self-insurance, money demand is rather robust to higher levels of inflation. So long as there are some Country B agents who can still afford to acquire new money holdings, precautionary savings motives guarantee that any excess money holdings can be exchanged abroad for the consumption good. Meanwhile, without any lump-sum transfer, Country B agents only experience inflation as a cost and will therefore always prefer the full-insurance option. Indeed, a deflationary monetary policy in this context implies seigniorage transfers abroad (via the terms of trade channel), so naturally such policies become utility-maximizing for agents in Country B. Such a dynamic captures a point argued by Aiyagari:[12]

> "If the inflation tax were eliminated, resources would implicitly be transferred from U.S. citizens, who don't use U.S. currency much, to citizens of other

[11] In standard theory, the countervailing forces of demand and price cause any Laffer curve to peak when demand elasticity equals one.

[12] See Aiyagari et al. [1990].

countries, who do. This clearly implies some welfare loss to U.S. citizens."

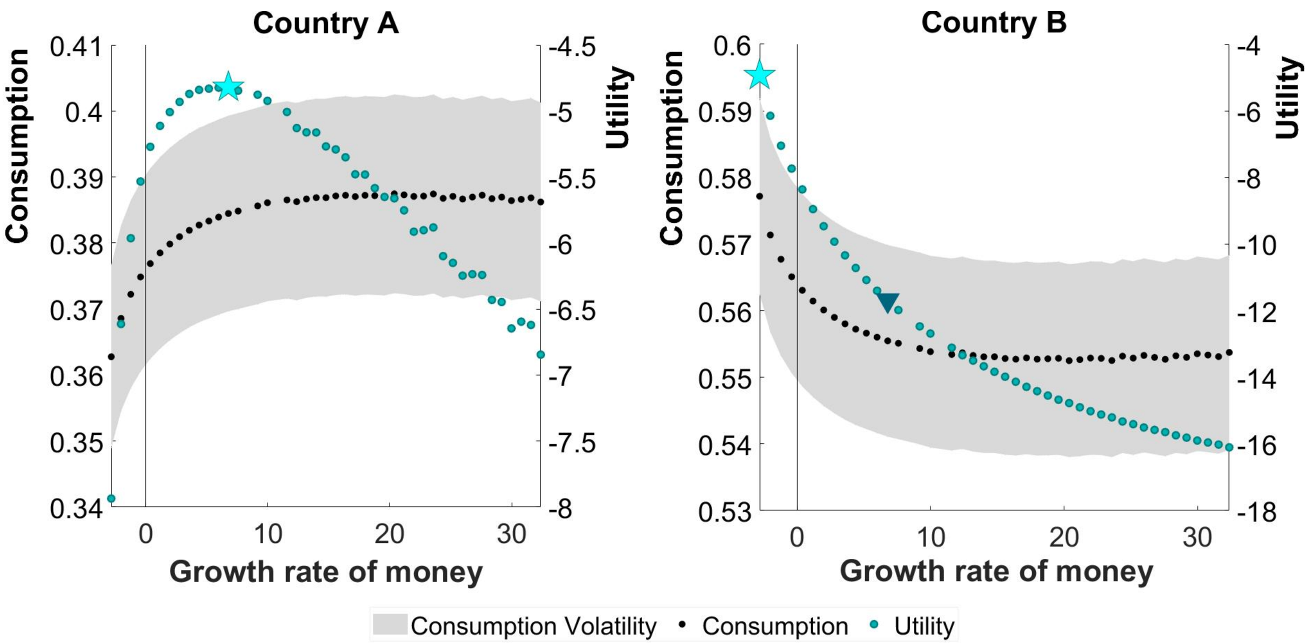
Implementing comparative advantage reduces to assigning the (fixed) probability that agents are born residents of Country A or B, or more to the point, the likelihood that they receive a lump-sum transfer each period. While global utility increases as the system approaches full insurance, varying this additional probability alters the division of global utility across the countries. Because the government's only expenditure is the lump-sum transfers to the Country A agents and this analysis concentrates on the economy's stationary distribution, seigniorage revenue can be computed using equation (12). The real value of equation (12) naturally represents the present value of transfers to the Country A agents, and importantly, the lump-sum cash transfers to Country A agents—included in the Country A budget constraint, equation (5)—reflect average global demand, where average demand is computed using the stationary distribution, $\lambda(m, s, i)$, described in equation (20).

İmrohoroğlu's closed economy nests within the two-country setting as the case when $P(i = A) = 1$. In the closed economy, all agents are affected by the changes in the inflation rate identically; although there are direct implications for the real return on savings, all agents are equally subsidized by the transfers and bear the costs of inflation equally and fully, eliminating any role for comparative advantage in agents' demand functions. The seigniorage gains in the closed economy are much less pronounced than in the open economy that includes the international trade described above. Intuitively, in the open economy, the expanded seigniorage gains are entirely attributable to foreign demand.

When average consumption and utility in Country A depend on demand and imports from Country B, optimal inflation policy reflects not only the mean-variance trade-off of domestic agents but also that of foreign agents. Because these elements all enter the third derivative of equation (2), the relationship between inflation and the tolerance for consumption volatility cannot be solved analytically. However equation (18), the expression for the nominal lump-sum transfer to domestic agents, provides some illustration of how the relative size of the overseas market for currency shapes the mean-variance trade-off of Country A agents.

The weight $\frac{P(i=B)}{P(i=A)}$ implies that seigniorage from abroad is amplified once $P(i = A) < P(i = B)$. Equation (18) and the money market clearing condition in equation (19) imply that, for a given money printing rule, the additional seigniorage (or imports exchanged for seigniorage) gained by Country A agents is sufficient to offset the corresponding level of consumption volatility in steady state. Without aggregate shocks, the share of Country A's population in any savings-endowment state also represents the average time spent in that state for an individual. Equilibria located to the right of the optimal inflation rate are points where marginal increases consumption volatility dominate increases in agents' expected consumption.

The optimizing decision rule of the Country B agents, or average money demand in



This figure plots the long-run equilibrium values of aggregate utility, consumption, and consumption volatility in Countries A and B for a sample range of printing parameters, τ. The star markers represent the location of either country's maximum utility. The triangle marks aggregate utility in Country B at the value of τ associated with Country A's optimal inflation target.

Figure 1: Average Consumption and Utility in Countries A and B

Country B, is represented by the term $\sum_s \sum_{m \in \Omega(m',s)} \lambda_B(m,s) \tilde{m}_t^B$, which naturally falls in response increases in τ. The relative size of Country B, $P(i = B)$, is reflected in the Country A agents' decisions through the evolution of the equilibrium price. Because the endowment supply is fixed and money supply reflects the size of the domestic market for currency, the larger the foreign market relative to the domestic market, the greater the scarcity of fiat money relative to the consumption good. It follows that when $P(i = B) > P(i = A)$, the price of money in terms of the consumption good will be relatively high, so that Country A agents can import a greater quantity of the good per unit of currency exported. Hence, greater global population shares of Country A, or greater values of $P(i = A)$, correspond to optimal inflation rates near a Friedman rule, while lower values of $P(i = A)$ correspond to optimal inflation rates near the upper bound, or the inflation rate associated with maximum seigniorage.

5.1 Autarky

Given the costs imposed on agents in the country that does not produce currency, it seems fair to ask whether money demand for these agents would not collapse to zero for any positive value of inflation. The answer here comes from the concavity of equation (2) and

Jensen's inequality: agents are assigned precautionary savings motives for consumption smoothing. Without access to the currency produced by country A, country B agents' consumption patterns entirely reflect the movement of their endowments, which is less than ideal for their preferences.

5.2 Friedman Rule

Without a social planner's intervention to offset idiosyncratic endowment shocks, the closest the economy can approach complete markets or perfect insurance is to implement inflation according to a Friedman or Chicago style rule. A β of 0.995 corresponds to a rate of time preference $\mu = \frac{1}{\beta} - 1 \approx 0.005$. An annualized basis applies β^8, and implies a μ approximately equal to 0.04 or an inflation rate of -4%. According to Friedman [1969]'s logic, setting the rate of inflation to the negative of the rate of time preference should eliminate the private cost of holding money; Friedman's result holds in İmrohoroğlu's closed economy.

In the open-economy framework, a Friedman rule does not minimize average consumption volatility in Country B. Rather, minimum consumption volatility is determined by the prevailing price of money and the Country B agent's own mean-variance calculation. At low values of inflation, savings demand in Country B and the favorable terms of trade imply higher exports of the consumption good sent to Country A, which result in initially falling average levels of consumption in Country B. In other words, for low levels of inflation, agents with high endowment draws trade more of their goods away in exchange for money and demonstrate consumption levels closer to their fellow citizens in the low endowment state. In a sense, at these levels of inflation, wealth in Country B is measured more in money holdings rather than in consumption. Ultimately, increases in the inflation rate cause this pattern to reverse, and consumption volatility in Country B begins to increase with τ as a diminishing number of agents in the high income state can afford to hold money.

In contrast, average consumption volatility in Country A initially increases, because rising seigniorage allows an increasing share of the population to consume greater quantities of the consumption good; eventually, however, the high cost of the consumption good combined with insufficient savings press these agents into a hand-to-mouth pattern of consumption. This trade-off and the corresponding incentive of the hegemon to expand its financial account reflect results produced by Farhi and Maggiori [2018] in their model of hegemonic currency issuance and by Bodenstein et al. [2020] in their two-country model with incomplete markets.[13]

[13] Bodenstein et al. [2020] use a two stage game rather than heterogeneous agents to model optimal policies. Egorov and Mukhin [2020] construct a New Keynesian model wherein the U.S. is a Stackelberg leader that chooses its best policy response to partners' strategies.

Figure 1 plots the equilibrium results produced by the model for an assumption that the global population share of Country A comprises 40% of the world, $P(i = A) = 0.4$, and a discrete set of values for the inflation parameter, τ, beginning with a Friedman rule (or something close to it) and including the τ associated with maximum seigniorage revenue. The black markers in the figure plot out the average utility scores for economies A and B under this parameterization. The shaded area around the equilibrium utility markers illustrates the increasing level of consumption volatility associated with increases in τ. The corresponding average utility is plotted in green markers, with the maximum average utility values marked by stars. The triangle in the right hand side panel corresponds to the Country B average utility that results from the currency producer's optimal inflation rate.

Besides the non-monotonic relationship between inflation and Country A utility, the y-axis range of figure 1 illustrates that Country B agents are much more sensitive to changes in inflation than are Country A agents. Specifically, foreign utility covers a much wider set of outcomes and a higher standard deviation of consumption (see tables below). This sensitivity to τ increases when the global population share of Country B, of $P(i = B)$, is increased.

To evaluate the welfare costs of inflation in the two-country model, I compute the compensating variation compared to the baseline utilities associated with autarky and a Friedman rule. In addition, I compute the Gini coefficients that correspond to each inflation rate. In either country, the Gini coefficients initially rise as inflation increases above a Friedman rule (an increase in inequality), before the costs of inflation cause average consumption to converge at a lower level.

Value of τ	**Corresponds to**
-2.8%	-μ or Friedman rule proxy*
0%	İmrohoroğlu welfare baseline
5%	İmrohoroğlu analysis
6.8%	Approximate maximum utility in A when $P(i = A) = 0.4$
10%	İmrohoroğlu analysis

*-2.8% represents the value closest to μ that produces real values of aggregate utility.

Table 1: Selected τ calibrations

Table 1 describes a sample of inflation rates and Table 2 displays the corresponding equilibrium values. Specifically, Table 2 presents the standard deviations of consumption in each country, the consumption as a share of endowment observed in Country A, each county's average utility, and each country's Gini coefficient. Table 3 reports the compensating variation (as a share of each country's GDP) required by each country to maintain the welfare associated with the baseline cases described above.

τ	σ_{C_A}	σ_{C_B}	$\frac{C}{Y}_A$	$U(C)_A$	$U(C)_B$	**Gini**$_A$	**Gini**$_B$
-2.8%	0.4466	0.4742	0.9653	-7.8804	-4.9605	0.6487	0.4745
0%	0.4645	0.4665	1	-5.3601	-8.0402	0.6515	0.4773
5%	0.4759	0.4643	1.0192	-4.8323	-11.0211	0.6441	0.4678
6.8%	0.4782	0.4643	1.0226	-4.8317	-11.7003	0.6424	0.4646
10%	0.4813	0.4644	1.0269	-4.9185	-12.6895	0.6393	0.4614

From left to right: selected values of the printing parameter, τ, the standard deviation of consumption, $\sigma_{C_A}, \sigma_{C_B}$, Country A's consumption as a share of its endowment, $\frac{C}{Y}_A$, aggregate utility for each country, $U(C)_A, U(C)_B$, and the Gini coefficients for each country.

Table 2: Selected model estimates when $P(i = A) = 0.4$

Perhaps unsurprisingly, the steady state for $\tau = 0$ confers no economic advantage to the Country A agents, because this inflation rate kills any of their comparative advantage. For advocates of U.S. polices less punitive to foreign economies engaged in dollar-denominated activities, a policy of zero inflation appears to work in this direction, although such a policy comes at the expense of the currency producer's agents, in line with Aiyagari's argument. As mentioned, these model results are elements of a descriptive parable. At the approximated optimal level of inflation, seigniorage revenue is just over 9% of Country A's GDP. For comparison, Rogoff [2017] reports an average seigniorage revenue of 0.4% of GDP from 2006 to 2015.

The persistent trade deficits maintained by Country A under positive inflation are presented in the last three rows of Table 2 wherein Country A's average consumption to endowment ratios remain above 1. Their ability to run such deficits declines as the level of inflation rises and only ceases when annual inflation exceeds 200%.

6 Sensitivity to Calibration

I have presented a highly stylized simple model. The assumptions of incomplete markets and asymmetric access to the savings device drive the key result of the current account deficit and optimality inside of the Friedman rule. The model's key contribution is the counter-intuitive result that higher inflation may be welfare-maximizing under these assumptions.

However, it is clear that the magnitude of the optimal inflation rate computed here reflects the model's calibration. Indeed, stylized as this model is, its results should be interpreted as a parable of factors to consider rather than as a robust or precise policy recommendation. Nevertheless, we can gain some insight on the key mechanisms at work

Baseline

τ	Autarky $\frac{CV}{Y}_A$	$\frac{CV}{Y}_B$	Friedman Rule $\frac{CV}{Y}_A$	$\frac{CV}{Y}_B$
0	-63.12%	-27.05%	-17.57%	9.58%
5%	-62.295%	-22.46%	-19.35%	13.11%
6.8%	-62.37%	-21.40%	-19.31%	13.84%
10%	-61.18%	-19.89%	-18.90%	14.93%

These values measure compensating variation as a share of either country's aggregated endowment $\frac{CV}{Y}$. For a given value of τ and its corresponding equilibrium money supply (or per-capita cash transfer), $\lambda(m, s, i)$ and each country's aggregate utility are recomputed based on budget constraints that include guesses of CV_A and CV_B. This operation is repeated until the value functions from the parameterized computation converge with the value functions associated with the baseline utility values. Using the value functions, or agents' discounted streams of lifetime utility, gives a fuller picture of the role of consumption smoothing in agents' decision-making and welfare.

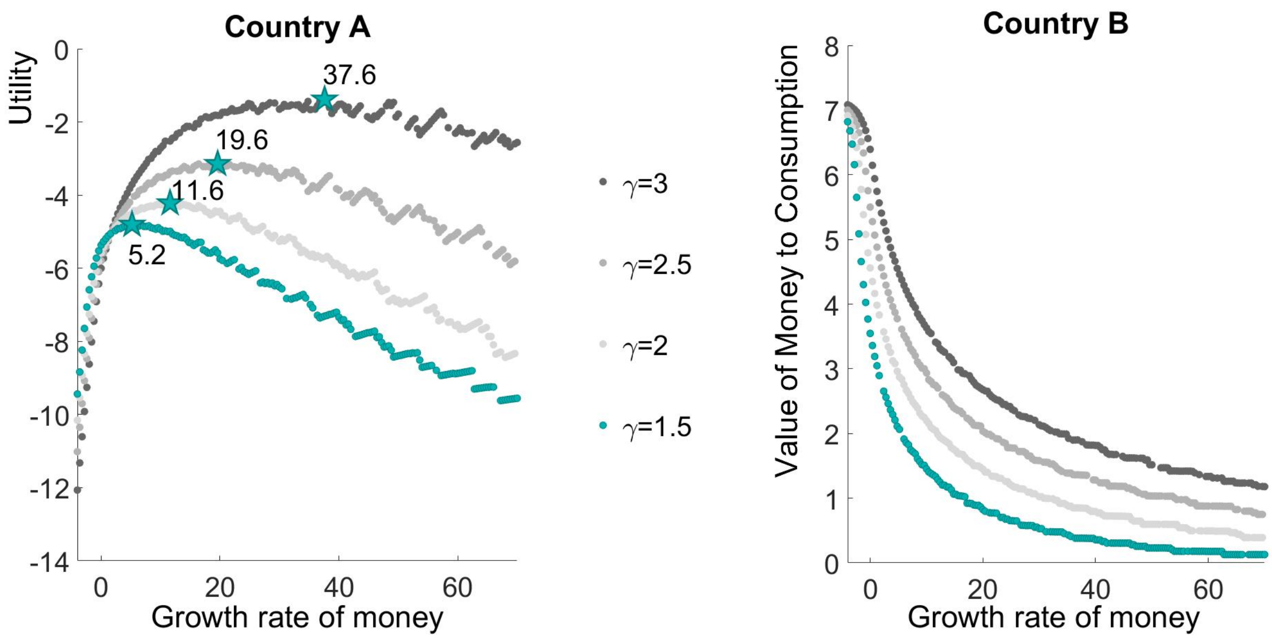
Table 3: Compensating Variation Needed for Aggregate Utility to Maintain Baseline Utility Values

by adjusting the parameterization.

6.1 Risk Aversion Parameter

The interest rate in this model is effectively negative, since under positive inflation rates, savings only lose value. Adjusting the risk aversion parameter in equation (2) makes agents more or less sensitive to increases or decreases in inflation. A higher γ implies greater curvature in the utility function, or a higher sensitivity to changes in the consumption argument. In other words, higher risk aversion implies a high commitment to consumption smoothing, analytically represented by a low elasticity of intertermporal substitution (the inverse of the risk aversion coefficient). Because the central bank in the model is motivated to maximize aggregate utility in Country A, lower intertemporal elasticities of substitution translate into higher levels of optimal inflation.

This result again illustrates the model's key trade-off, as seen in Figure 2. If preferences of all agents are identical, higher levels of inflation imply that Country A can extract higher levels of seigniorage revenue from Country B. This is entirely due to agents' desire to consumption smooth, even if such smoothing necessitates lower levels of consumption throughout their lifetime. Thus under higher risk aversion parameters, Country B is willing to trade away higher levels of the consumption good for money, all else fixed. While households in Country A also bear the cost of inflation, the marginal gain from additional



The left hand size illustrates Country A aggregate utility for a selected set of τ values under various parameterizations of γ, the coefficient of risk aversion. The right hand side plots the relative quantities of real balances and consumption in Country B in equilibrium. The values plotted here correspond to an asset gridspace of 300 intervals.

Figure 2: Risk Aversion and Intertemporal Elasticity of Substitution: Increased Foreign Money Demand Increases Optimal Inflation Target

seigniorage is much greater than the cost from inflation.

6.2 Asset State Space

İmrohoroğlu's original analysis discretized the money-holdings state space using a grid spanning values 0 to 8 with 300 gridpoints. When this setup is used in the two-country extension, the optimal inflation rate for Country A is approximately 5.2%. Computing demand and utility at a Friedman rule proxy (inflation just inside of -μ) required expanding the grid space from 8 to 100, due to the high demand for money at negative inflation rates. I then refine the gridpoints to 4000, so that the grid is constructed in intervals of 0.025.

Under this finer, longer grid, the highest aggregate consumption in Country A corresponds to an inflation rate of about 6.8%, suggesting that further refining the grid would produce higher optimal inflation rates but these rates approach a limit as the interval space approaches zero. By separating out some of the money demand grouped together under the rougher grid, the finer grid catches a higher aggregate money demand and a higher value of seigniorage revenues. With a fixed endowment of the consumption good, higher equilibrium quantities of real balances cause relative prices to adjust enough that there is no change in the standard deviation of consumption.

7 Conclusion

My analysis shows a seigniorage channel that biases the optimal inflation target of a currency hegemon upward. A monopoly on currency production permits the hegemon to run a persistent current account deficit, which leads to higher average welfare. These results are robust to various calibrations and are attributable to agents' motives to self-insure against income shocks and smooth consumption. As international financial markets and trade increasingly rely on a small number of currencies—the dollar in particular—the surrounding literature has responded by incorporating the effects of foreign demand for dominant currencies into macroeconomic analysis. To this end, I contribute a baseline model with which to analyze demand for an absolutely dominant currency.

In this setting, when calibrated to 2019 U.S. employment data, the positive inflation rate that is welfare maximizing for the currency producer is consistent with prior studies that illustrate the suboptimality of a Friedman rule. The mechanism that makes positive inflation optimal is a mean-variance trade-off for agents: balancing the marginal seigniorage gains with marginal increases in consumption volatility drives an internal solution. The magnitude of this rate is a function of the relative size of foreign demand.

The natural next step for this line of inquiry is to permit a substitute currency or store of value. Algan and Ragot [2010] build a model that includes a second asset. Doing so, they induce heterogeneous responses of money demand to inflation and thus non-neutrality in the long-run. In the dominant currency literature, Gopinath and Stein [2021] demonstrate how, all else equal, granting one currency a small advantage in trade invoicing leads to currency dominance in a world characterized by representative agents.

Standard theory indicates that substitutes should be welfare-enhancing for Country B in my model. With the gains from currency monopoly reduced, the effect of the seigniorage channel will be muted, decreasing the optimal inflation target. However, if both countries are extended money-printing technology in the two-country world discussed here, inflation rates will necessarily converge. Thus studying the welfare gains due to market structure requires preserving the incomplete markets features of this model as well as building in new frictions to motivate differential returns and portfolio diversification. I save this challenge for future work.

References

S Rao Aiyagari et al. Deflating the Case for Zero Inflation. *Federal Reserve Bank of Minneapolis Quarterly Review*, 14(3):2–11, 1990.

Ahmet Akyol. Optimal Monetary Policy in an Economy with Incomplete Markets and Idiosyncratic Risk. *Journal of Monetary Economics*, 51(6):1245–1269, 2004.

Yann Algan and Xavier Ragot. Monetary Policy with Heterogeneous Agents and Borrowing Constraints. *Review of Economic Dynamics*, 13(2):295–316, 2010.

Truman Bewley. The Optimum Quantity of Money. Technical report, Discussion Paper, 1979.

Truman Bewley. A Difficulty with the Optimum Quantity of Money. *Econometrica: Journal of the Econometric Society*, pages 1485–1504, 1983.

Martin Bodenstein, Giancarlo Corsetti, and Luca Guerrieri. The Elusive Gains from Nationally-Oriented Monetary Policy. *FRB International Finance Discussion Paper*, (1271), 2020.

Konstantin Egorov and Dmitry Mukhin. Optimal Policy under Dollar Pricing. *Available at SSRN 3404660*, 2020.

Charles Engel. Policy Cooperation, Incomplete Markets, and Risk Sharing. *IMF Economic Review*, 64(1):103–133, 2016.

Emmanuel Farhi and Matteo Maggiori. A Model of the International Monetary System. *The Quarterly Journal of Economics*, 133(1):295–355, 2018.

Stanley Fischer. Towards an Understanding of the Costs of Inflation: II. In *Carnegie-Rochester Conference Series on Public Policy*, volume 15, pages 5–41. Elsevier, 1981.

Milton Friedman. The Optimum Quantity of Money, and Other Essays. Technical report, 1969.

Linda S Goldberg and Cédric Tille. Vehicle Currency Use in International Trade. *Journal of international Economics*, 76(2):177–192, 2008.

Gita Gopinath and Jeremy C Stein. Banking, Trade, and the Making of a Dominant Currency. *The Quarterly Journal of Economics*, 136(2):783–830, 2021.

Pierre-Olivier Gourinchas. The Dollar Hegemon? Evidence and Implications for Policymakers. In *The Asian Monetary Policy Forum: Insights for Central Banking*, pages 264–300. World Scientific, 2021.

Pierre-Olivier Gourinchas, Hélène Rey, and Maxime Sauzet. The International Monetary and Financial System. *Annual Review of Economics*, 11:859–893, 2019.

Ayşe İmrohoroğlu. The Welfare Cost of Inflation under Imperfect Insurance. *Journal of Economic Dynamics and Control*, 16(1):79–91, 1992.

Ruth Judson. The Death of Cash? not So Fast: Demand for us Currency at Home and Abroad, 1990-2016. 2017.

Oleksiy Kryvtsov, Malik Shukayev, and Alexander Ueberfeldt. Optimal Monetary Policy under Incomplete Markets and Aggregate Uncertainty: A Long-Run Perspective. *Journal of Economic Dynamics and Control*, 35(7):1045–1060, 2011.

Robert Lucas. Comments on: Towards an Understanding of the Costs of Inflation. In *Carnegie-Rochester Conference on Public Police, K. Brunner and A. Meltzer, eds. Autumn*, volume 15, pages 43–52, 1981.

Perry Mehrling. A Note on the Optimum Quantity of Money. *Journal of Mathematical Economics*, 24(3):249–258, 1995.

Kenneth S Rogoff. *The Curse of Cash*. Princeton University Press, 2017.

Robert M Townsend. Models of Money with Spatially Separated Agents. *Models of Monetary Economies*, pages 265–303, 1980.