1. What are the differences between the ARCH model and the GARCH model?

En el modelo ARCH, la estimación de la varianza solo depende de las innovaciones pasadas de rendimientos y la varianza a largo plazo. Por otro lado, en el modelo GARCH incorpora también las varianzas anteriores.

2. What are the differences between the EWMA model and the GARCH (1, 1) model?

La diferencia clave entre estos modelos es que el modelo GARCH incorpora el parámetro de varianza a largo plazo en su estructura, volviendo EWMA un modelo más simplista y menos confiable a largo plazo.

3. Analyze the case of the EWMA and ARCH models to determine if these models have any characteristics like the GARCH for estimating future volatilities to a period n, when only a period t is known. If they have it, determine which one it is, if they do not have it explain why. To resolve this exercise, help yourself with the previous development of future volatility in the GARCH model and follow the same steps, but with the respective models.

   Remember that the EWMA model is given by

$$\sigma_t^2 = (1-\lambda)R_{t-1}^2 + \lambda\hat{\sigma}_{t-1}^2$$

   And that the ARCH (1) model is given by

$$\sigma_t^2 = \omega + \alpha_1 R_{t-1}^2$$

EWMA: Reescribimos el modelo EWMA:

$$\sigma^2_{t+n} = (1-\lambda) R^2_{t+n-1} + \lambda \sigma^2_{t+n-1}$$

Si consideramos que $E\left[R^2_{t+n-1}\right] = \sigma^2_{t+n-1}$, entonces:

$$E\left[\sigma^2_{t+n}\right] = (1-\cancel{\lambda}) E\left[\sigma^2_{t+n-1}\right] + \cancel{\lambda} E\left[\sigma^2_{t+n-1}\right]$$

$$E\left[\sigma^2_{t+n}\right] = \underline{E\left[\sigma^2_{t+n-1}\right]}$$

ARCH (1): $\sigma^2_{t+n} = (1-\alpha) V_L + \alpha R^2_{t+n-1}$

Si consideramos que $E\left[R^2_{t+n-1}\right] = \sigma^2_{t+n-1}$, entonces:

$$E\left[\sigma^2_{t+n}\right] = (1-\alpha) V_L + \alpha E\left[\sigma^2_{t+n-1}\right]$$

$$E[\sigma^2_{t+n}] = V_L + \alpha\left(E[\sigma^2_{t+n-1}] - V_L\right)$$

Aplicando lags sucesivos hasta el n.vo lag:

$$E[\sigma^2_{t+n}] = V_L + \alpha^n\left(E[\sigma^2_{t}] - V_L\right)$$

4. Assume that S&P 500 at close of trading yesterday was 1,040 and the daily volatility of the index was estimated as 1% per day at that time. The parameters in a GARCH (1,1) model are ω=0.000002, α=0.06, and β=0.92. If the level of the index at close of trading today is 1,060, what is the new volatility estimate?

$$R_{t-1} = \frac{P_t}{P_{t-1}} - 1 = \frac{1}{52}$$

$$\sigma^2_t = \omega + \alpha R^2_{t-1} + \beta \sigma^2_{t-1}$$

$$\sigma^2_t = 0.000002 + 0.06\left(\frac{1}{52}\right)^2 + 0.92\,(0.01)^2$$

$$\sigma_t = \sqrt{}$$

$$= 0.010779$$

5. The parameters of a GARCH (1,1) model are estimated as ω=0.000004, α=0.05, and β=0.92. What is the long-run average volatility and what is the equation describing the way that the variance rate reverts to its long-run average? If the current volatility is 20%per year, what is the expected volatility in 20 days?

a. $$\omega = \gamma V_L \rightarrow V_L = \frac{\omega}{\gamma} = \frac{\omega}{1-\alpha-\beta} = \frac{0.000004}{1-0.05-0.92} = 0.0001\overline{3} \qquad 1/7500$$

b. $$E[\sigma^2_{t+n}] = V_L + (\alpha+\beta)^n\left(\sigma^2_t - V_L\right)$$

c. $$E[\sigma_{t+n}] = \sqrt{\frac{1}{7500} + (0.05+0.92)^{20}\left[\left(0.2/\sqrt{252}\right)^2 - \frac{1}{7500}\right]}$$

$$= 0.01213\sqrt{252} = 0.1925 = 19.25\ \%\ \text{anual}$$