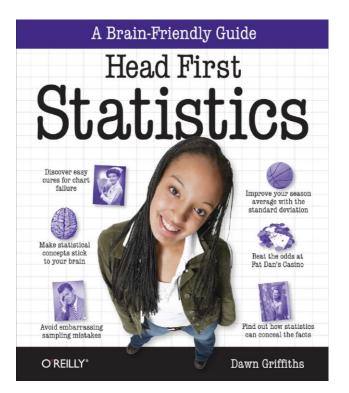
# **Statistics**

### **Textbook and References**

- You can find the textbook on the web
- Other references will be guided when being used



### Course Schedule (Section 001)

#### week.1

- Course introduction
- Visualizing Information:

First Impression

week.2 ~ 3

- Measuring Central Tendency
- Measuring Spread
- Calculating probability

week.4 ~ 5

- week. 6 ~ 7
- Permutations and Combinations Normal distribution
- Discrete Probability Distribution Mid-term examination

#### "Head First Statistics" Dawn Griffiths, O'Reilly

#### week.8 - 9

- Estimating Your Populations
- Constructing Confidence
   Intervals

week.10 ~ 11

- Correlation and Regression
- Multi-Regression

week.12 ~ 13

- Regression Project
- Regression Project

Week.14 ~ 15

- Project Presentation
- Final examination

Class: Monday 1pm ~ 4pm, Office Hour: over Kakao talk (010-6799-6636)



## Each class will consist of

- → 10 min Re-cap of the previous week (occasionally with Quiz)
- → 40 min Lecture on key concepts
- → 40 min Breakout session
- → 40 min team presentation
- → 20 min Q&A and wrap-up

## **Evaluation Criteria**

- 20% Attendance
- 40% Mid/Final Term Exam
- 20% Participation

(In-class Exercise, Assignment, Quiz)

20% Final Project

## **Grade Guideline**

- Relative evaluation
- Fail for those who haven't attended the class more than
   1/3

#### visualizing information

#### **First Impressions**

#### Can't tell your facts from your figures?

Statistics help you make sense of confusing sets of data. They make the complex simple. And when you've found out what's really going on, you need a way of visualizing it and telling everyone else. So if you want to pick the best chart for the job, grab your coat, pack your best slide rule, and join us on a ride to Statsville.

Company Profit per Month
2.5
8 2.0
25 6 20 13 10 10 10 10 10 10 10 10 10 10 10 10 10
1.0
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Jul Aug Sep Oct Nov Dec
Month
See what I mean, the
profit's about the same each month.
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No, this
profit's amazing. Look at it soarl
Cook at It south

Statistics are everywhere	2
But why learn statistics?	3
A tale of two charts	4
The humble pie chart	8
Bar charts can allow for more accuracy	10
Vertical bar charts	10
Horizontal bar charts	11
It's a matter of scale	12
Using frequency scales	13
Dealing with multiple sets of data	14
Categories vs. numbers	18
Dealing with grouped data	19
Make a histogram	20
Step 1: Find the bar widths	26
Step 2: Find the bar heights	27
Step 3: Draw your chart	28
Introducing cumulative frequency	34
Drawing the cumulative frequency graph	35
Choosing the right chart	39



#### measuring central tendency

#### The Middle Way

2

Sometimes you just need to get to the heart of the matter. It can be difficult to see patterns and trends in a big pile of figures, and finding the average is often the first step towards seeing the bigger picture. With averages at your disposal, you'll be able to quickly find the most representative values in your data and draw important conclusions. In this chapter, we'll look at several ways to calculate one of the most important statistics in town—mean, median, and mode—and you'll start to see how to effectively summarize data as concisely and usefully as possible.



Welcome to the Health Club	4
A common measure of average is the mean	4
Mean math	4
Dealing with unknowns	4
Back to the mean	
Back to the Health Club	
Everybody was Kung Fu fighting	
Our data has outliers	5
The outliers did it	5
Watercooler conversation	6
Finding the median	6
How to find the median in three steps:	6
Business is booming	(
The Little Ducklings swimming class	(
What went wrong with the mean and median?	6
What should we do for data like this?	6
The Mean Exposed	7
Introducing the mode	7
Three steps for finding the mode	7







#### measuring variability and spread

#### Power Ranges

#### Not everything's reliable, but how can you tell?

Averages do a great job of giving you a typical value in your data set, but they don't tell you the full story. OK, so you know where the center of your data is, but often the mean, median, and mode alone aren't enough information to go on when you're summarizing a data set. In this chapter, we'll show you how to take your data skills to the next level as we begin to analyze ranges and variation.





Wanted: one player	84
We need to compare player scores	85
Use the range to differentiate between data sets	86
The problem with outliers	89
We need to get away from outliers	91
Quartiles come to the rescue	92
The interquartile range excludes outliers	93
Quartile anatomy	94
We're not just limited to quartiles	98
So what are percentiles?	99
Box and whisker plots let you visualize ranges	100
Variability is more than just spread	104
Calculating average distances	105
We can calculate variation with the variance	106
but standard deviation is a more intuitive measure	107
Standard Deviation Exposed	108
A quicker calculation for variance	113
What if we need a baseline for comparison?	118
Use standard scores to compare values across data sets	119
Interpreting standard scores	120
Statsville All Stars win the league!	125

#### calculating probabilities

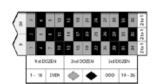
#### **Taking Chances**

#### Life is full of uncertainty.

Sometimes it can be impossible to say what will happen from one minute to the next. But certain events are more likely to occur than others, and that's where probability theory comes into play. Probability lets you predict the future by assessing how likely outcomes are, and knowing what could happen helps you make informed decisions. In this chapter, you'll find out more about probability and learn how to take control of the future!







Fat Dan's Grand Slam	128
Roll up for roulette!	129
What are the chances?	132
Find roulette probabilities	135
You can visualize probabilities with a Venn diagram	136
You can also add probabilities	142
Exclusive events and intersecting events	147
Problems at the intersection	148
Some more notation	149
Another unlucky spin	155
Conditions apply	156
Find conditional probabilities	157
Trees also help you calculate conditional probabilities	159
Handy hints for working with trees	161
Step 1: Finding P(Black ∩ Even)	167
Step 2: Finding P(Even)	169
Step 3: Finding P(Black 1 Even)	170
Use the Law of Total Probability to find P(B)	172
Introducing Bayes' Theorem	173
If events affect each other, they are dependent	181
If events do not affect each other, they are independent	182
More on calculating probability for independent events	183



#### using discrete probability distributions

#### Manage Your Expectations

#### Unlikely events happen, but what are the consequences?

So far we've looked at how probabilities tell you how likely certain events are. What probability doesn't tell you is the overall impact of these events, and what it means to you. Sure, you'll sometimes make it big on the roulette table, but is it really worth it with all the money you lose in the meantime? In this chapter, we'll show you how you can use probability to predict long-term outcomes, and also measure the certainty of these predictions.



Back at Fat Dan's Casino	198
We can compose a probability distribution for the slot machine	201
Expectation gives you a prediction of the results	204
and variance tells you about the spread of the results	205
Variances and probability distributions	206
Let's calculate the slot machine's variance	207
Fat Dan changed his prices	212
There's a linear relationship between E(X) and E(Y)	217
Slot machine transformations	218
General formulas for linear transforms	219
Every pull of the lever is an independent observation	222
Observation shortcuts	223
New slot machine on the block	229
Add $E(X)$ and $E(Y)$ to get $E(X + Y)$	230
and subtract E(X) and E(Y) to get E(X - Y)	231
You can also add and subtract linear transformations	232
Jackpot!	238

#### permutations and combinations

#### **Making Arrangements**

#### Sometimes, order is important.

Counting all the possible ways in which you can order things is time consuming, but the trouble is, this sort of information is crucial for calculating some probabilities. In this chapter, we'll show you a quick way of deriving this sort of information without you having to figure out what all of the possible outcomes are. Come with us and we'll show you how to count the possibilities.



The Statsville Derby	24
It's a three-horse race	24
How many ways can they cross the finish line?	24
Calculate the number of arrangements	24
Going round in circles	24
It's time for the novelty race	25
Arranging by individuals is different than arranging by type	25
We need to arrange animals by type	25
Generalize a formula for arranging duplicates	25
It's time for the twenty-horse race	25
How many ways can we fill the top three positions?	25
Examining permutations	25
What if horse order doesn't matter	26
Examining combinations	26
Combination Exposed	26
Does order really matter?	26
It's the end of the race	26



geometric, binomial, and poisson distributions

#### **Keeping Things Discrete**

Calculating probability distributions takes time.

So far we've looked at how to calculate and use probability distributions, but wouldn't it be nice to have something easier to work with, or just quicker to calculate? In this chapter, we'll show you some special probability distributions that follow very definite patterns. Once you know these patterns, you'll be able to use them to calculate probabilities, expectations, and variances in record time. Read on, and we'll introduce you to the geometric, binomial and Poisson distributions.

We need to find Chad's probability distribution

There's a pattern to this probability distribution

Your quick guide to the Poisson distribution

		The probability distribution can be represented algebraically	277
		The geometric distribution also works with inequalities	279
		The pattern of expectations for the geometric distribution	280
		Expectation is 1/p	281
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Peorn machine	Drinks machine	A quick guide to the geometric distribution	284
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	(6)	Should you play, or walk away?	291
B HAMMER		Generalizing the probability for three questions	293
0-0-0-0		Let's generalize the probability further	296
		What's the expectation and variance?	298
		Binomial expectation and variance	301
		Your quick guide to the binomial distribution	302
		Expectation and variance for the Poisson distribution	308
	Ouch! Rock! Ouch!	So what's the probability distribution?	312
	Flagi Ouchi Treel	Combine Poisson variables	313
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#### **Being Normal**

Discrete probability distributions can't handle every situation.

So far we've looked at probability distributions where we've been able to specify exact values, but this isn't the case for every set of data. Some types of data just don't fit the probability distributions we've encountered so far. In this chapter, we'll take a look at how continuous probability distributions work, and introduce you to one of the most important probability distributions in town—the normal distribution.



273 274

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#### using the normal distribution ii

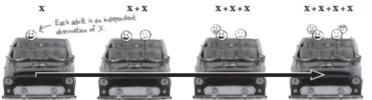
#### **Beyond Normal**

If only all probability distributions were normal.

Life can be so much *simpler* with the normal distribution. Why spend all your time working out individual probabilities when you can look up entire ranges in one swoop, and still leave time for game play? In this chapter, you'll see how to solve more complex problems in the blink of an eye, and you'll also find out how to bring some of that normal goodness to other probability distributions.



All aboard the Love Train	363
Normal bride + normal groom	364
It's still just weight	365
How's the combined weight distributed?	367
Finding probabilities	370
More people want the Love Train	375
Linear transforms describe underlying changes in values	376
and independent observations describe how many values you have	377
Expectation and variance for independent observations	378
Should we play, or walk away?	383
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When to approximate the binomial distribution with the normal	389
Revisiting the normal approximation	394
The binomial is discrete, but the normal is continuous	395
Apply a continuity correction before calculating the approximation	396
The Normal Distribution Exposed	404
All aboard the Love Train	405
When to approximate the binomial distribution with the normal	407
A runaway success!	413



#### using statistical sampling

#### **Taking Samples**

#### Statistics deal with data, but where does it come from?

Some of the time, data's easy to collect, such as the ages of people attending a health club or the sales figures for a games company. But what about the times when data isn't so easy to collect? Sometimes the number of things we want to collect data about are so huge that it's difficult to know where to start. In this chapter, we'll take a look at how you can **effectively gather data** in the real world, in a way that's efficient, accurate, and can also save you time and money to boot. Welcome to the world of sampling.



The Mighty Gumball taste test	416
They're running out of gumballs	417
Test a gumball sample, not the whole gumball population	418
How sampling works	419
When sampling goes wrong	420
How to design a sample	422
Define your sampling frame	423
Sometimes samples can be biased	424
Sources of bias	425
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Simple random sampling	430
How to choose a simple random sample	431
There are other types of sampling	432
We can use stratified sampling	432
or we can use cluster sampling	433
or even systematic sampling	433
Mighty Gumball has a sample	439

estimating your population

#### **Making Predictions**

Wouldn't it be great if you could tell what a population was like, just by taking one sample?

Before you can claim **full sample mastery**, you need to know how to use your samples to best effect once you've collected them. This means using them to **accurately predict** what the population will be like and coming up with a way of saying how **reliable** your predictions are. In this chapter, we'll show you how knowing your sample helps you **get to know your population**, and vice versa.

So how long does flavor really last for?	443
Let's start by estimating the population mean	445
Point estimators can approximate population parameters	44
Let's estimate the population variance	448
We need a different point estimator than sample variance	449
Which formula's which?	45
It's a question of proportion	45
So how does this relate to sampling?	459
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So what's the expectation of P <sub>s</sub> ?	463
And what's the variance of P <sub>s</sub> ?	46
Find the distribution of P <sub>s</sub>	46
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The sampling distribution of the mean	475
Find the expectation for X	47
What about the the variance of X?	476
So how is X distributed?	480
If n is large, X can still be approximated by the normal distribution	48
Using the central limit theorem	483



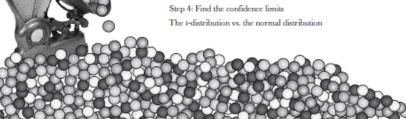
#### constructing confidence intervals

#### **Guessing with Confidence**

#### Sometimes samples don't give quite the right result.

You've seen how you can use point estimators to estimate the precise value of the population mean, variance, or proportion, but the trouble is, how can you be certain that your estimate is completely accurate? After all, your assumptions about the population rely on just one sample, and what if your sample's off? In this chapter, you'll see another way of estimating population statistics, one that allows for uncertainty. Pick up your probability tables, and we'll show you the ins and outs of confidence intervals.

Mighty Gumball is in trouble	488
The problem with precision	489
Introducing confidence intervals	490
Four steps for finding confidence intervals	491
Step 1: Choose your population statistic	492
Step 2: Find its sampling distribution	492
Step 3: Decide on the level of confidence	494
Step 4: Find the confidence limits	496
Start by finding Z	497
Rewrite the inequality in terms of m	498
Finally, find the value of X	501
You've found the confidence interval	502
Let's summarize the steps	503
Handy shortcuts for confidence intervals	504
Step 1: Choose your population statistic	508
Step 2: Find its sampling distribution	509
Step 3: Decide on the level of confidence	512
Step 4: Find the confidence limits	513
The t-distribution vs. the normal distribution	515



#### using hypothesis tests

#### Look at the Evidence

Not everything you're told is absolutely certain.

The trouble is, how do you know when what you're being told isn't right? Hypothesis tests give you a way of using samples to test whether or not statistical claims are likely to be true. They give you a way of weighing the evidence and testing whether extreme results can be explained by mere coincidence, or whether there are darker forces at work. Come with us on a ride through this chapter, and we'll show you how you can use hypothesis tests to confirm or allay your deepest suspicions.

Statsville's new miracle drug	522
Resolving the conflict from 50,000 feet	526
The six steps for hypothesis testing	527
Step 1: Decide on the hypothesis	528
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Step 3: Determine the critical region	532
Step 4: Find the p-value	535
Step 5: Is the sample result in the critical region?	537
Step 6: Make your decision	537
What if the sample size is larger?	540
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Step 2: Choose the test statistic	544
Use the normal to approximate the binomial in our test statistic	547
Step 3: Find the critical region	548
Let's start with Type I errors	556
What about Type II errors?	557
Finding errors for SnoreCull	558
We need to find the range of values	559
Find P(Type II error)	560
Introducing power	561



#### the X2 distribution

#### There's Something Going On...

Sometimes things don't turn out quite the way you expect.

When you model a situation using a particular probability distribution, you have a good idea of how things are likely to turn out long-term. But what happens if there are differences between what you expect and what you get? How can you tell whether your discrepancies come down to normal fluctuations, or whether they're a sign of an underlying problem with your probability model instead? In this chapter, we'll show you how you can use the  $\chi^2$  distribution to analyze your results and sniff out suspicious results.

There may be trouble ahead at Fat Dan's Casino	568
Let's start with the slot machines	569
The χ <sup>2</sup> test assesses difference	571
So what does the test statistic represent?	572
Two main uses of the χ <sup>2</sup> distribution	573
V represents degrees of freedom	574
What's the significance?	575
Hypothesis testing with χ <sup>2</sup>	576
You've solved the slot machine mystery	579
Fat Dan has another problem	585
The $\chi^2$ distribution can test for independence	586
You can find the expected frequencies using probability	587
So what are the frequencies?	588
We still need to calculate degrees of freedom	591
Generalizing the degrees of freedom	596
And the formula is	597
You've saved the casino	599
You've saved the casino	



#### correlation and regression

#### What's My Line?

#### Have you ever wondered how two things are connected?

So far we've looked at statistics that tell you about just one variable—like men's height, points scored by basketball players, or how long gumball flavor lasts—but there are other statistics that tell you about the connection between variables. Seeing how things are connected can give you a lot of information about the real world, information that you can use to your advantage. Stay with us while we show you the key to spotting connections: correlation and regression.



Let's analyze sunshine and attendance	607
Exploring types of data	608
Visualizing bivariate data	609
Scatter diagrams show you patterns	612
Correlation vs. causation	614
Predict values with a line of best fit	618
Your best guess is still a guess	619
We need to minimize the errors	620
Introducing the sum of squared errors	621
Find the equation for the line of best fit	622
Finding the slope for the line of best fit	623
Finding the slope for the line of best fit, continued	624
We've found b, but what about a?	625
You've made the connection	629
Let's look at some correlations	630
The correlation coefficient measures how well the line fits the data	631
There's a formula for calculating the correlation coefficient, r	632
Find r for the concert data	633
Find r for the concert data, continued	634



#### leftovers

# i

#### The Top Ten Things (we didn't cover)

Even after all that, there's a bit more. There are just a few more things we think you need to know. We wouldn't feel right about ignoring them, even though they only need a brief mention. So before you put the book down, take a read through these short but important statistics tidbits.



#1. Other ways of presenting data	64
#2. Distribution anatomy	64
#3. Experiments	64
#4. Least square regression alternate notation	64
#5. The coefficient of determination	64
#6. Non-linear relationships	65
#7. The confidence interval for the slope of a regression line	65
#8. Sampling distributions - the difference between two means	65
#9. Sampling distributions - the difference between two proportions	65
#10. E(X) and Var(X) for continuous probability distributions	65

#### statistics tables



#### **Looking Things up**

Where would you be without your trusty probability tables? Understanding your probability distributions isn't quite enough. For some of them, you need to be able to look up your probabilities in standard probability tables. In this appendix you'll find tables for the normal, t and X² distributions so you can look up probabilities to your heart's content.



tandard normal probabilities	658
distribution critical values	660
critical values	661

### **Exercise - Team**

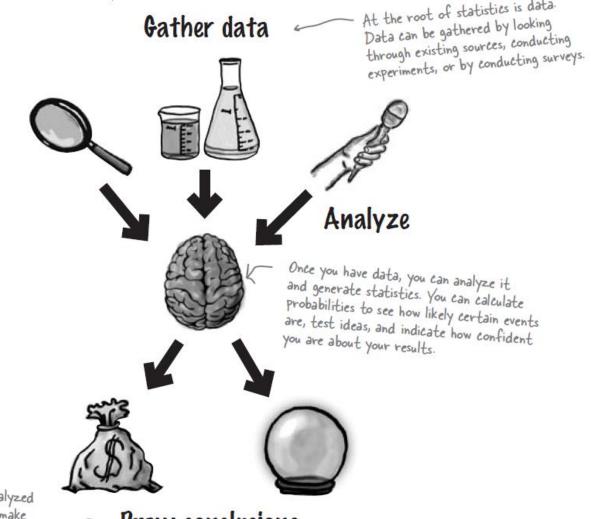
Visit the below Google Spreadsheet document

https://docs.google.com/spreadsheets/d/18c9bl0CizLxbuRnIkdflUXQYzU

I9P6NxBgN8\_5Se8OA/edit?usp=sharing

- Form a Team 3~5 people / team
- Put your team's name right to your name (C column)
  - Put a brief introduction of yourself in column D
  - ✓ For KR students, please put your English name before your Korean name.

## **Statistics**



When you've analyzed your data, you make Griffit decisions and predictions.

**Praw** conclusions

### **Statistics**

Statistics are numbers that summarize raw facts and figures in some meaningful way

## **But why learn statistics?**

Statistics can be a convenient way of summarizing key truths about data, but there's a dark side too.



## **But why learn statistics?**

Take a look at the profits made by a company in the latter half of last year

Month	Jul	Aug	Sep	Oct	Nov	Dec
Profit (millions)	2.0	2.1	2.2	2.1	2.3	2.4



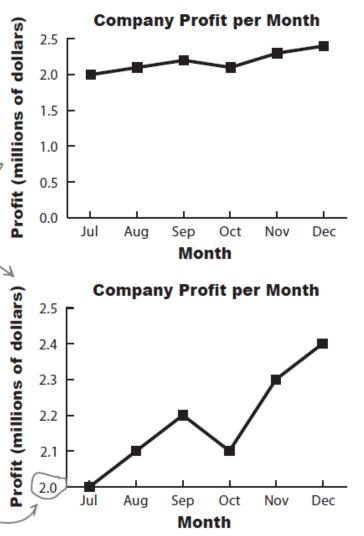
Both charts are based on the same underlying data, but they each send a different message.

The first chart shows that the profit is relatively steady. It achieves this by having the vertical axis start at O, and then plotting the profit for each month against this.

Look, the vertical axes are different on each chart.

The second chart gives a different impression by making the vertical axis start at a different place and adjusting the scale accordingly. At a glance, the profits appear to be rising dramatically each month. It's only when you look closer that you see what's really going on.

The axis for this chart starts at 2.0, not 0. No wonder the profit looks so a wesome.



Source: <Head First Statistics> Dawn Griffiths, O'Reill

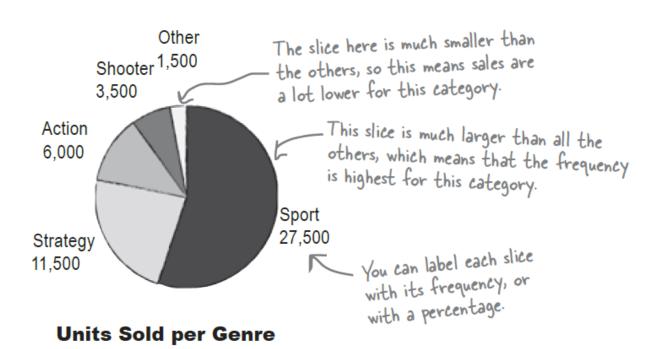
# Visualizing Information

## Manic Mango needs some charts

One company that needs some charting expertise is Manic Mango, an innovative games company that is taking the world by storm.

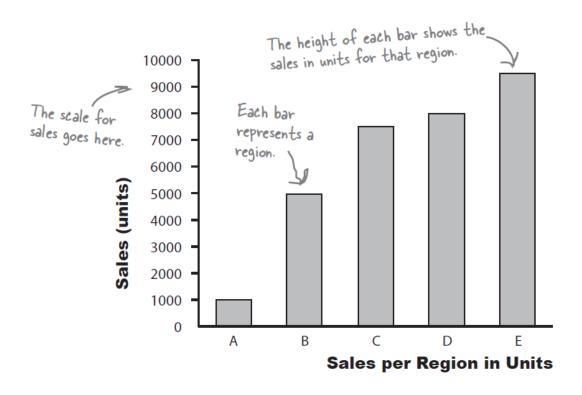
The CEO has been invited to deliver a keynote presentation at the next worldwide games expo. He needs some quick, slick ways of presenting data.

### Pie chart



Genre	Units sold
Sports	27,500
Strategy	11,500
Action	6,000
Shooter	3,500
Other	1,500

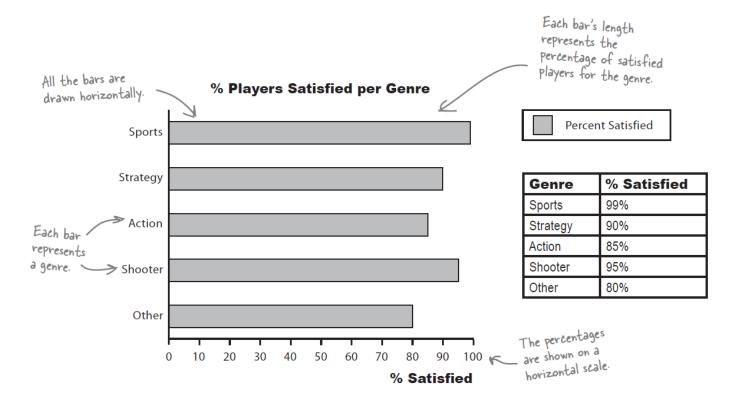
### Vertical bar charts





Region	Sales (units)
Α	1,000
В	5,000
С	7,500
D	8,000
Е	9,500

## **Horizontal bar charts**

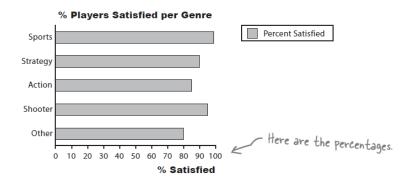


### It's a matter of scale

### You need to chart the satisfied players per game

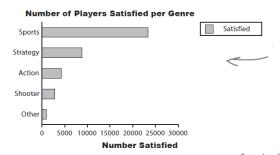
#### Using percentage scales

Let's start by taking a deeper look at the bar chart showing player satisfaction per game genre. The horizontal axis shows player satisfaction as a **percentage**, the number of people out of every hundred who are satisfied with this genre.



#### Using frequency scales

You can show frequencies on your scale instead of percentages. This makes it easy for people to see exactly what the frequencies are and compare values.



This chart reflects how many people are satisfied rather than the percentage

## Dealing with multiple sets of data

#### The split-category bar chart

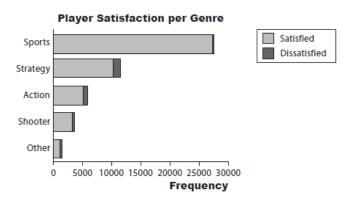
One way of tackling this is to use one bar for the frequency of satisfied players and another for those dissatisfied, for each genre. This sort of chart is useful if you want to **compare frequencies**, but it's difficult to see proportions and percentages.

#### The segmented bar chart

If you want to show frequencies *and* percentages, you can try using a segmented bar chart. For this, you use one bar for each category, but you split the bar proportionally. The overall length of the bar reflects the total frequency.

This sort of chart allows you to quickly see the total frequency of each category—in this case, the total number of players for each genre—and the frequency of player satisfaction. You can see proportions at a glance, too.

Sports
Strategy
Action
Shooter
Other
0 5000 10000 15000 20000 25000 30000
Frequency



Source: <Head First Statistics> Dawn Griffiths, O'Reilly

## Dealing with grouped data - Histogram

### How to represent the score frequency data?

The scores are numeric and grouped into intervals

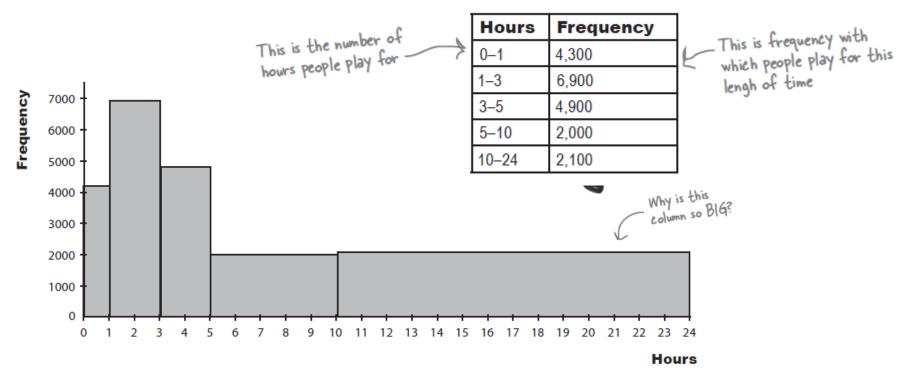
	Score	Frequency
	0-199	5
)	200-399	29
	400-599	56
	600-799	17
	800-999	3

That's easy, don't we just use a bar chart like we did before? We can treat each group as a separate category.



We could, but there's a better way

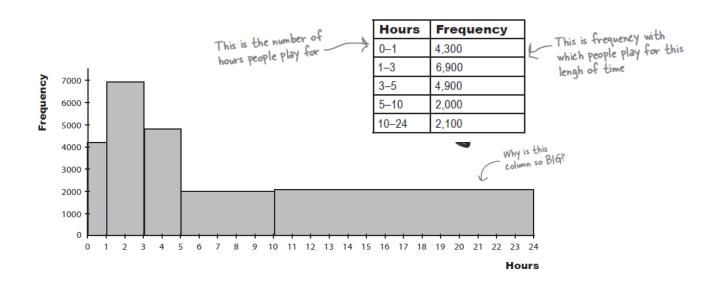
## Histogram



A histogram's bar area must be proportional to frequency Source: <Head First Statistics Dawn Griffiths, O'Reilly

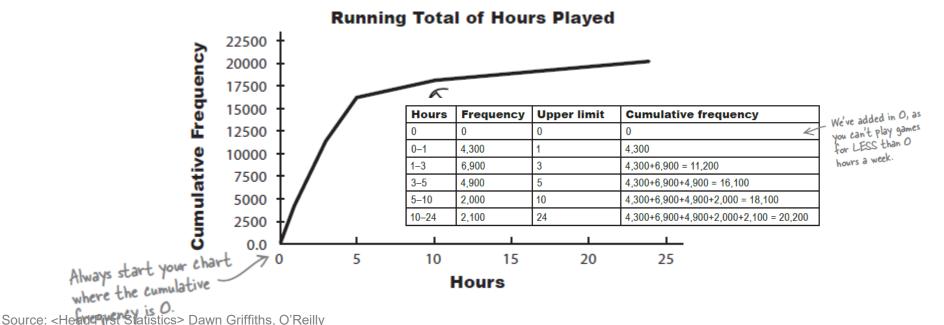
## **Exercise - Histogram**

Find out what's wrong in below histogram, and draw a correct one

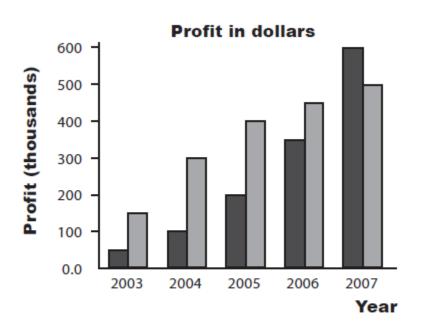


## Introducing cumulative frequency

How to represent the number of players who play less than certain hours, like 10 hours?



## Bar chats to compare the profit in each year

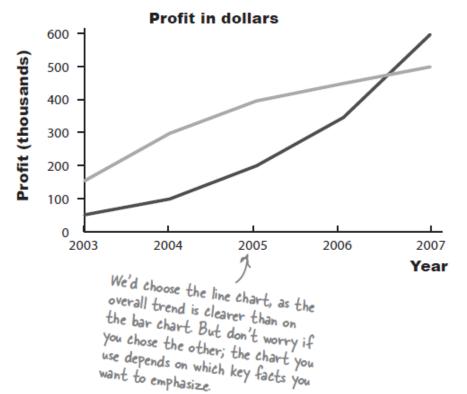




The bar chart does a good job of comparing the profit on a year-by-year basis, and it's great if you want to compare profits in an individual year. As an example, we can see that up to 2007, the competitor made a bigger profit, but in 2007 Manic Mango did.

A weaknesses of this chart is that if the CEO suddenly decided to add a third competitor, it might make the chart a bit harder to take in at a single glance.

### Line charts works better to show the trend





The line chart is better at showing a trend, the year-on-year profits for each company. The trend line for each company is well-defined, which means we easily see the pattern profits: Manic Mango profits are climbing well, where its competition is beginning to slacken off. It would also be easy to add another company without swamping the chart.

A weakness is that you can also compare year-by-year profit, but perhaps the bar chart is clearer.

## **Exercise - Visualizing Information**

- Please visit "World Bank Group"
- Download "GDP per capita" data



- Draw any 3 charts which you'd like to describe, share to explain
- We'll have a team presentation time next week

### **Next Week**

- Measuring central tendency
  - Mean, median, mode
- Measuring variability and spread
  - Range, quartile
  - Variance, standard deviation

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# End of Week 1