CMPE 185 Autonomous Mobile Robots

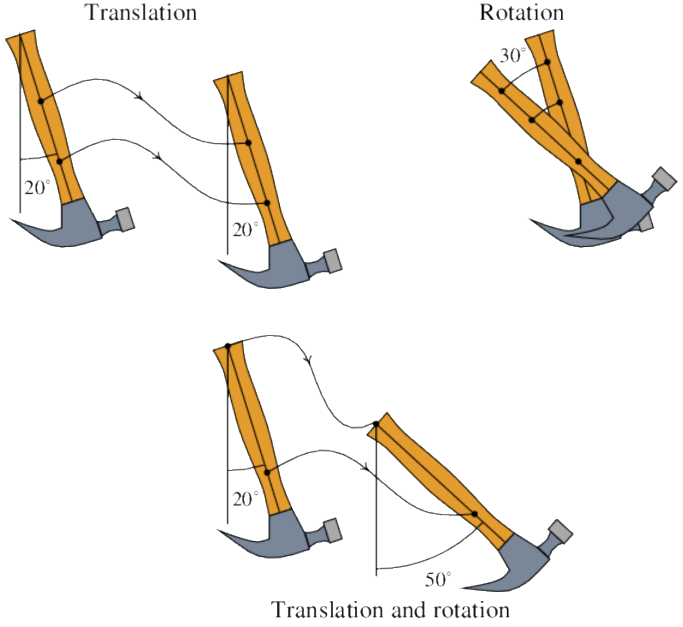
Mobile Robot Kinematics

Dr. Wencen Wu

Computer Engineering Department

Kinematics

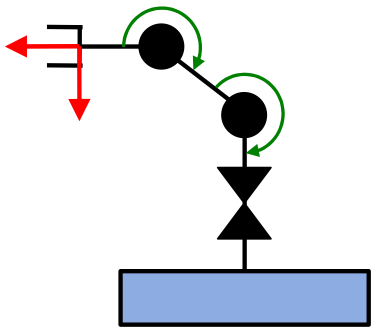
- Definition of kinematics
  - "Description of the motion of points, bodies or systems of bodies"
  - …without consideration of the causes of motion (=> dynamics)
  - Required for kinematic simulation and control



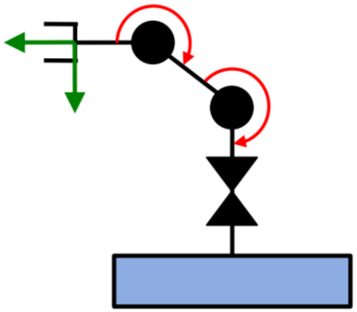
- Types of motion of single bodies
  - Translation
  - Rotation
  - Combined motion

Kinematics

- **Forward kinematics**
  - Given a set of actuator positions, determine the corresponding pose



- **Inverse kinematics**
  - Given a desired pose, determine the corresponding actuator positions



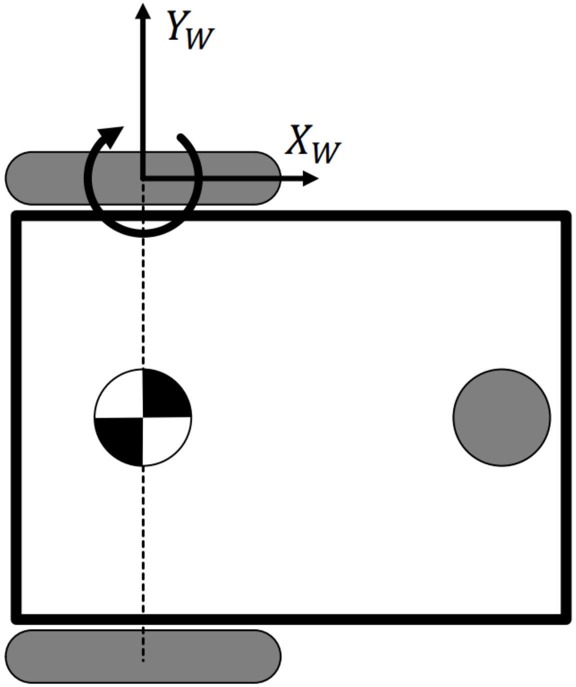
Wheeled Kinematics

- Not all degrees of freedom of a wheel can be actuated or have encoders
- Wheels can impose **differential constraints** that complicate the computation of kinematics

rolling constraint

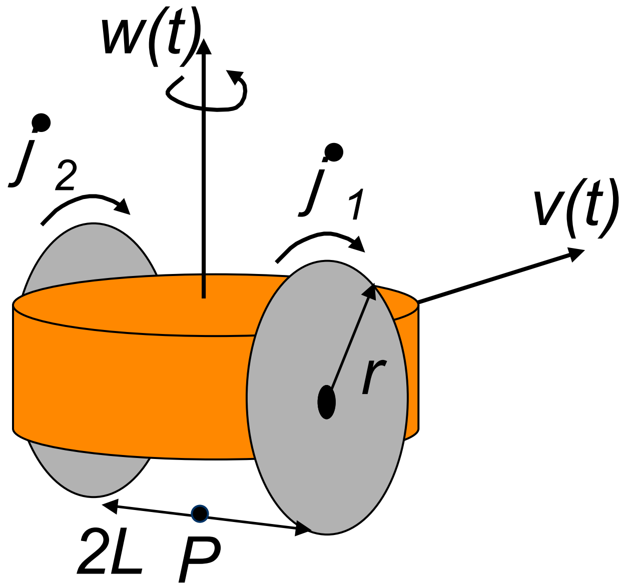
$$\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = \begin{bmatrix}\dot{\varphi} r\\ 0\end{bmatrix}$$

no-sliding constraint



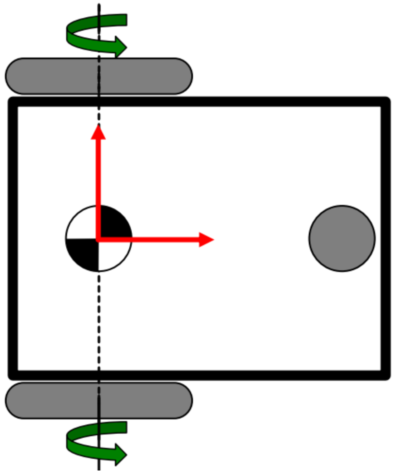
Wheeled Kinematics

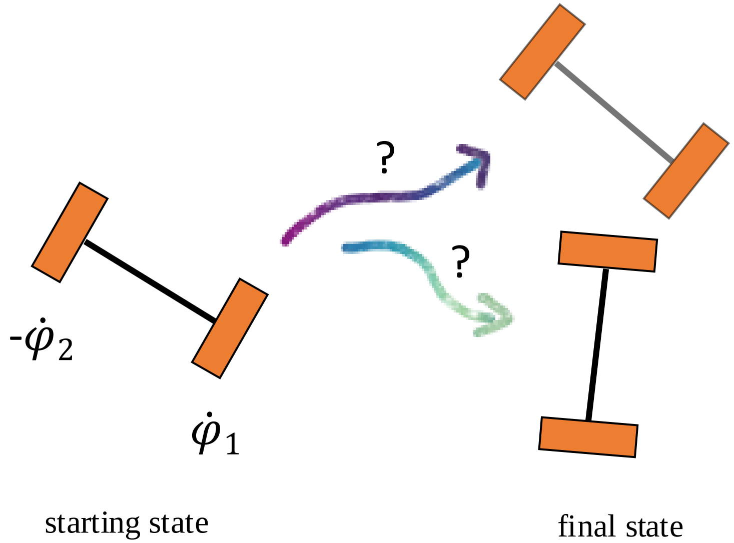
- *P*: center of the robot
- *r*: radius of the wheel
- *2L*: length of the axles
- *v*: linear velocity of the robot
- *w*: angular (rotational) velocity of the robot
- $\dot{\varphi}_1$: rotational speed of the right wheel
- $\dot{\varphi}_2$: rotational speed of the left wheel



Differential Kinematics

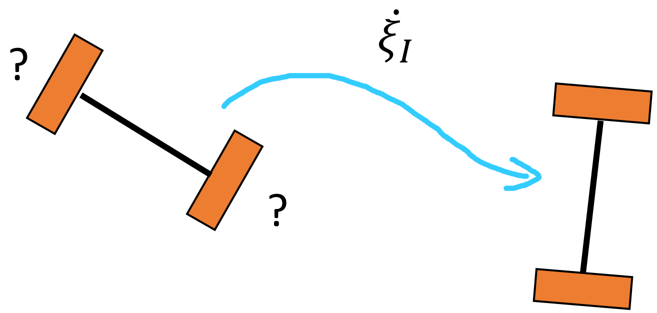
- Differential forward kinematics
  - Given the wheels' speed inputs - $\dot{\varphi}_1$ and $\dot{\varphi}_2$, determine the robot's velocity $\dot{\xi}_I = [\dot{x}, \dot{y}, \dot{\theta}]$ or state (x, y, θ) in the global frame





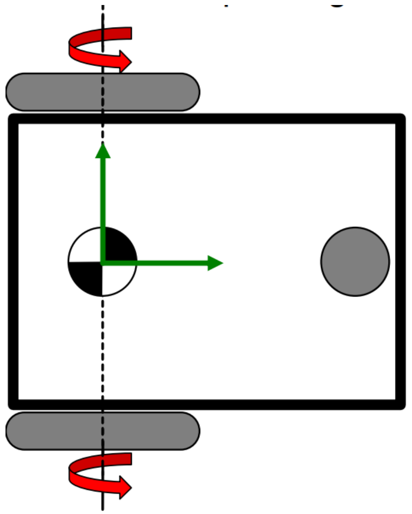
Differential Kinematics

- Differential inverse kinematics
  - Given the desired $\dot{\xi}_I = [\dot{x}, \dot{y}, \dot{\theta}]_{\text{desired}}$ or state $(x, y, \theta)_{\text{desired}}$ of the robot in the global frame, determine the corresponding wheels' speed input $(\dot{\varphi}_1, \dot{\varphi}_2)$



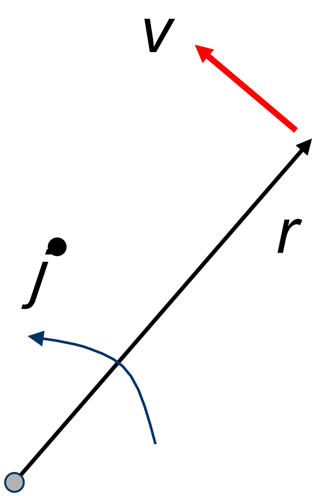
starting state

desired final state



Differential Forward Kinematics

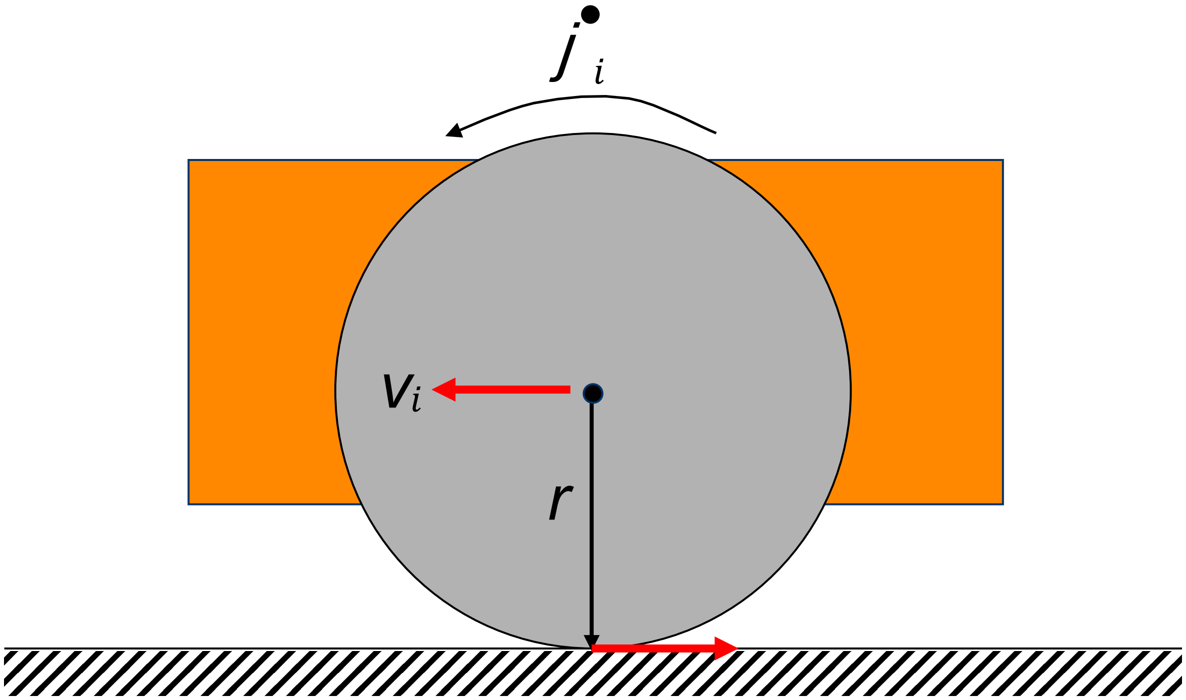
- Before we continue, we need to understand the relation between angular velocity and linear velocity



$$r\dot{\varphi} = v$$

Differential Forward Kinematics

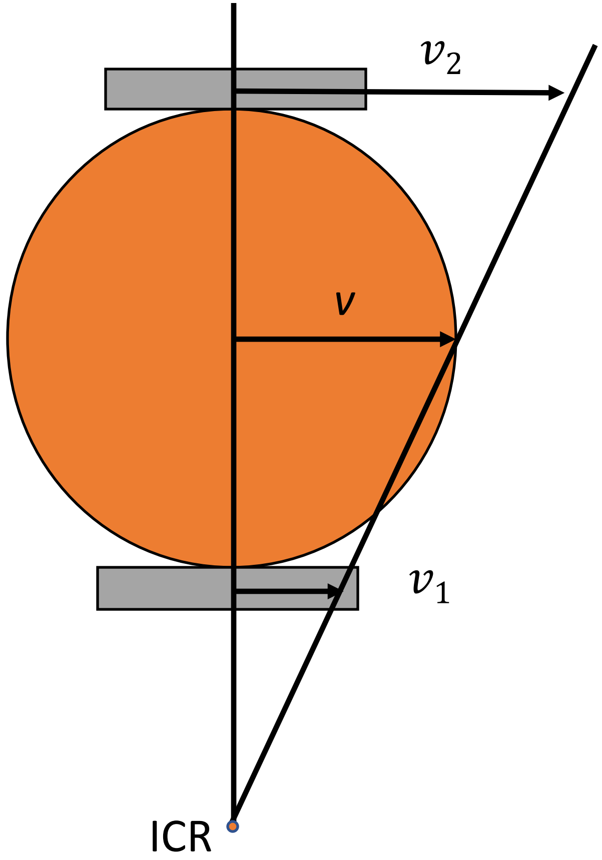
- Apply this to a wheel on the robot
- Kinematic constraint: $r\dot{\varphi}_i = v_i$



Two-Wheeled Robot Kinematic Model

- Linear velocity of the robot is the average of the two wheel velocities

$$v = \frac{v_1 + v_2}{2} = \frac{r\dot{\varphi}_1 + r\dot{\varphi}_2}{2}$$

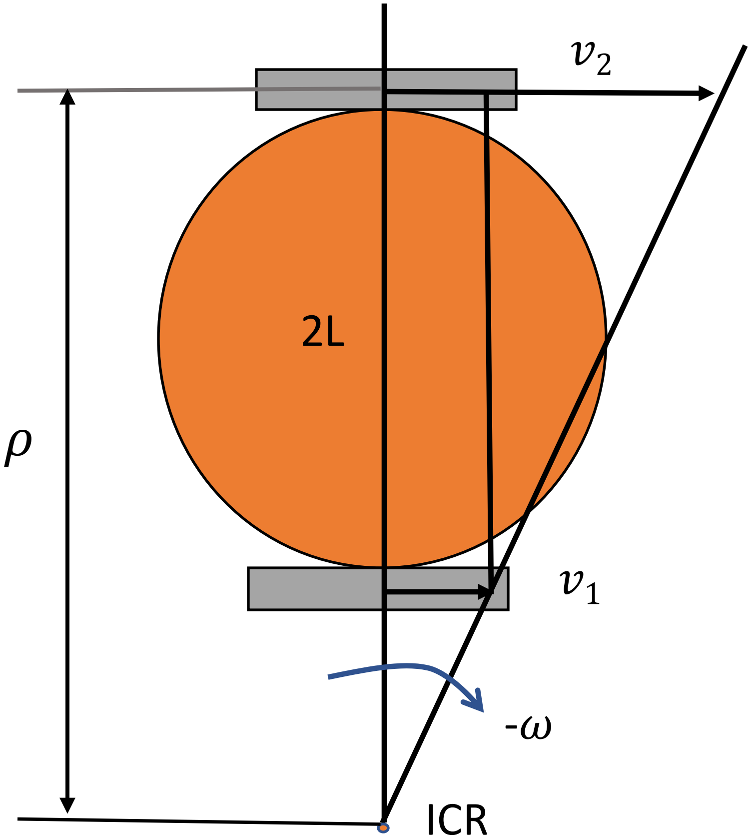


Two-Wheeled Robot Kinematic Model

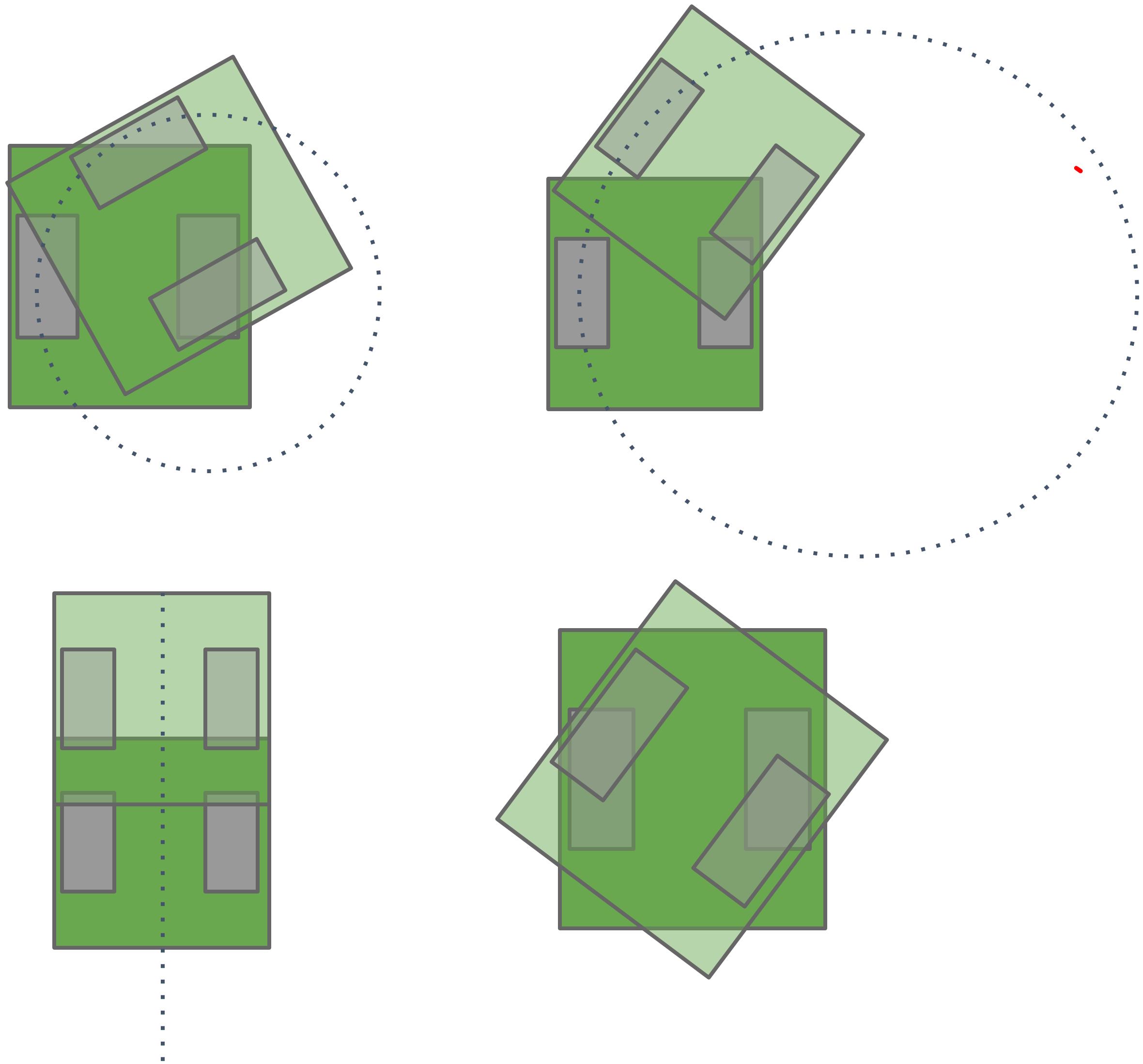
- Use the instantaneous center of rotation (ICR)
- Similar triangles give the angular rate of rotation

$$\omega = \frac{-v_2}{\rho} = \frac{-(v_2 - v_1)}{2L}$$

$$\omega = \frac{r\dot{\varphi}_1 - r\dot{\varphi}_2}{2L}$$



Fun Time



$v_L = +k, v_R = 0$

$v_L = +k, v_R = +k$

$v_L = +k, v_R = +k'$

$v_L = +k, v_R = -k$

Forward Kinematics

- We now know how to calculate how wheel speeds affect the robot velocities (linear velocity v and angular velocity w) in the global coordinate frame
- This will be useful when we want to control the robot to track points in the global coordinate frame by controlling wheel speeds

$$\dot{x}_I = v \cdot \cos\theta$$

$$\dot{y}_I = v \cdot \sin\theta$$

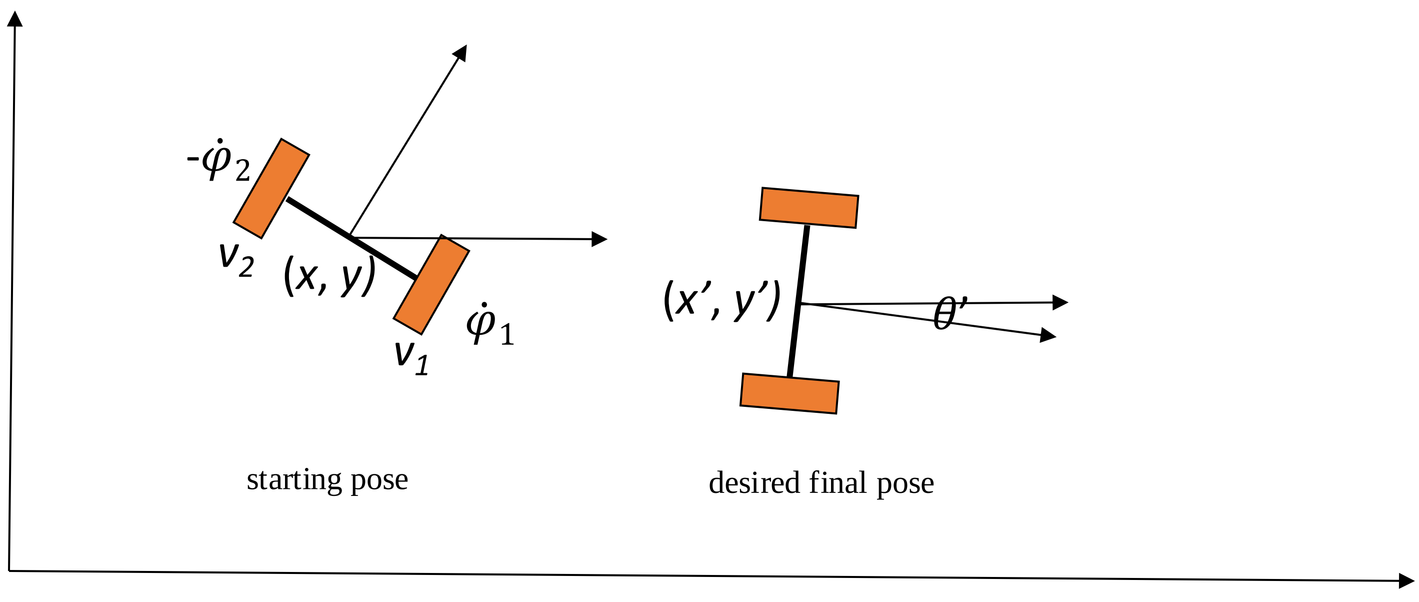
$$\dot{\theta}_I = \dot{\theta}_R = w$$

Inverse Kinematics

- **How to determine the speed of the wheels to obtain the desired velocities of the robot?**

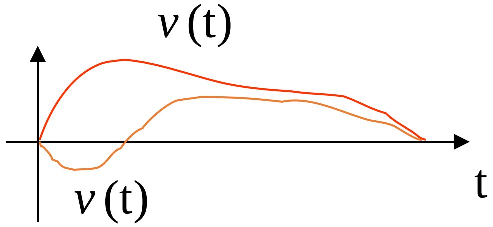
Differential Inverse Kinematics

- Given the desired velocity of the robot, determine the corresponding wheel's speed, or
- Standing in pose (x, y, θ) at time t, determine the control parameters, i.e., (v, w) or $(\dot{\varphi}_1, \dot{\varphi}_2)$, such that the pose at time $t + \boldsymbol{\delta}t$ is (x', y', θ')



Differential Inverse Kinematics

- Finding some solutions are not hard, but finding the "best" solution is very difficult
- "Best" in the sense of
  - Quickest time
  - Most energy efficient
  - Smoothest velocity profiles
  - Etc…

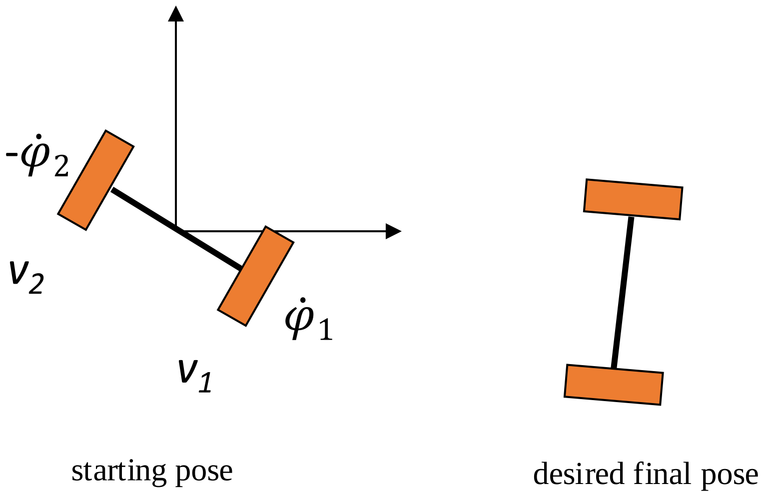


- Some solutions are not feasible for robots because of constraints – nonholonomic robots

Differential Inverse Kinematics – Decomposition

- One simple working approach: decompose the problem into two operations:
  - Move in a straight line

    $v_1 = v_2$, $\omega\delta t = 0$
  - Rotate in place about center
    - $v_1 = -v_2$
    - $\theta' = \theta + \omega\delta t$



Differential Inverse Kinematics – Decomposition

- Step 1: turn so that the wheels are parallel to the line between the original and final position of the robot origin.

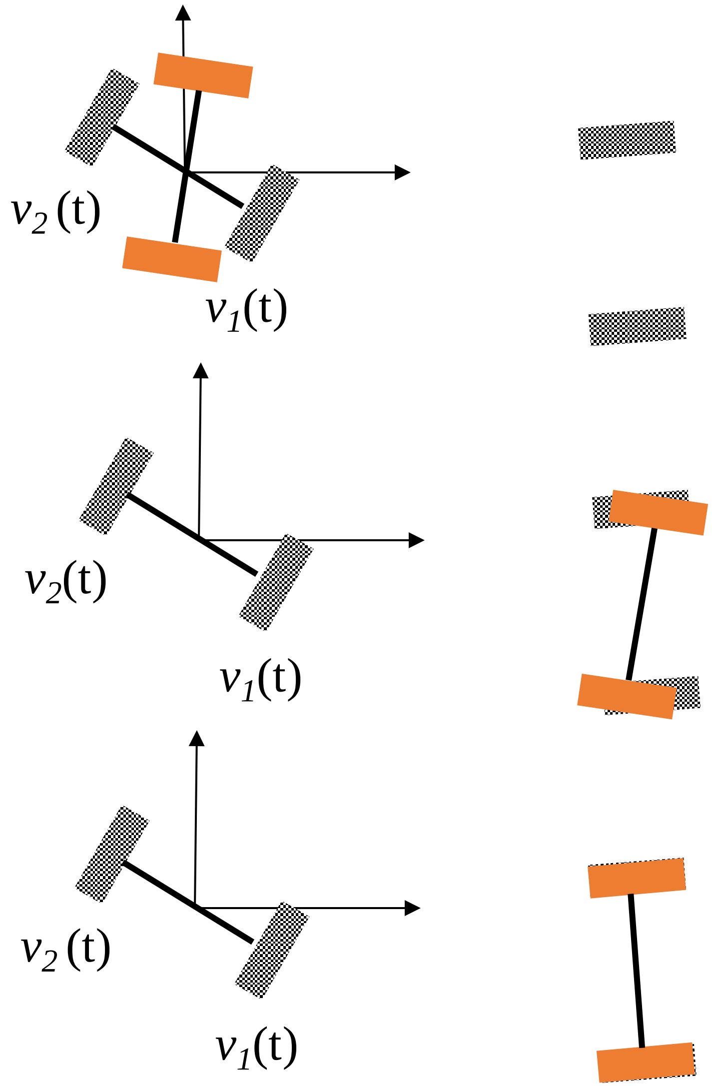
$$v_1 (t) = -v_2(t) = v_{\max}$$

- Step 2: drive straight until the robot's origin coincides with the destination

$$v_1 (t) = v_2 (t) = v_{\max}$$

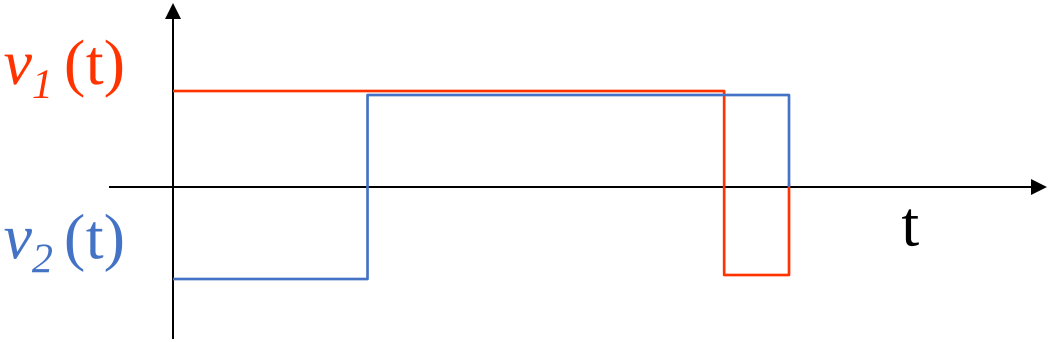
- Step 3: rotate again in order to achieve the desired final orientation

$$-v_1 (t) = v_2 (t) = v_{\max}$$



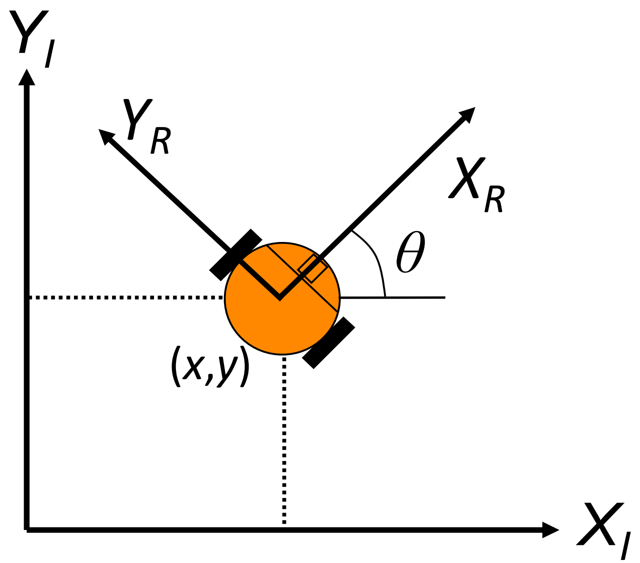
Differential Inverse Kinematics – Decomposition

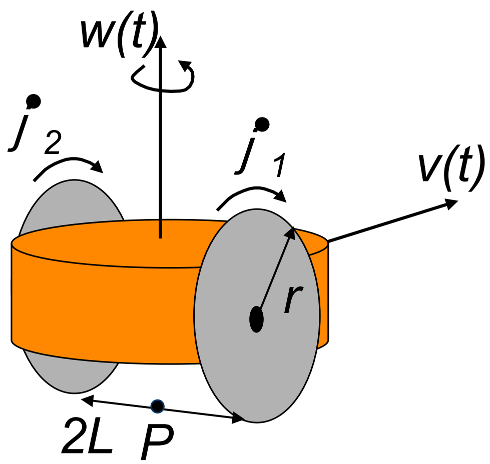
- Velocity profile



Kinematic Models of a Simple 2D Robot in Practice

- Two models





$$\begin{cases} \dot{x} = v \cos \theta \\ \dot{y} = v \sin \theta \\ \dot{\theta} = w \end{cases}$$

Design for this model!

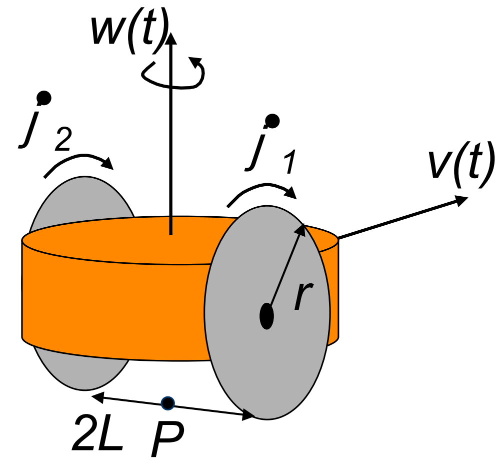
$$\begin{cases} \dot{x} = \dfrac{r}{2}(\dot{\varphi}_1 + \dot{\varphi}_2) \cos \theta \\ \dot{y} = \dfrac{r}{2}(\dot{\varphi}_1 + \dot{\varphi}_2) \sin \theta \\ \dot{\theta} = \dfrac{r}{2L}(\dot{\varphi}_2 - \dot{\varphi}_1) \end{cases}$$

Implement this model

Kinematic Model of a Simple 2D Robot

- Continuous time model:

$$\begin{cases} \dot{x} = \frac{r}{2}(\dot{\varphi}_1 + \dot{\varphi}_2)\cos\theta \\ \dot{y} = \frac{r}{2}(\dot{\varphi}_1 + \dot{\varphi}_2)\sin\theta \\ \dot{\theta} = \frac{r}{2L}(\dot{\varphi}_2 - \dot{\varphi}_1) \end{cases}$$

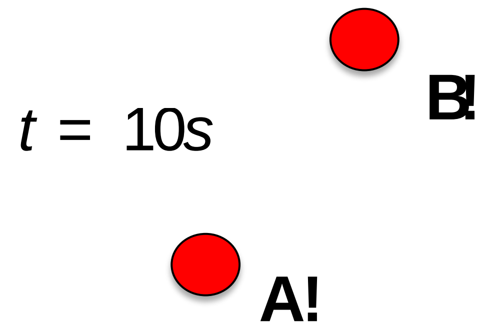


- Discrete time model

$$\begin{cases} x_{k+1} = x_k + \frac{r}{2}(\dot{\varphi}_{1,k} + \dot{\varphi}_{2,k})\cos\theta_k \Delta t \\ y_{k+1} = y_k + \frac{r}{2}(\dot{\varphi}_{1,k} + \dot{\varphi}_{2,k})\sin\theta_k \Delta t \\ \theta_{k+1} = \theta_k + \frac{r}{2L}(\dot{\varphi}_{2,k} - \dot{\varphi}_{1,k})\Delta t \end{cases}$$

From One Model to Another

- A simple task: move from A to B in 10s
- High level task!
- Control design: *v* and ω
- Commands sent to the robots: $\dot{\varphi}_1$ and $\dot{\varphi}_2$



$$v(t) = \frac{r}{2}(\dot{\varphi}_1 + \dot{\varphi}_2) \quad \Rightarrow \quad \frac{2v}{r} = \dot{\varphi}_1 + \dot{\varphi}_2$$

$$w(t) = \frac{r}{2L}(\dot{\varphi}_2 - \dot{\varphi}_1) \quad \Rightarrow \quad \frac{wL}{r} = \dot{\varphi}_2 - \dot{\varphi}_1$$

$$\dot{\varphi}_1 = \frac{2v + wL}{2r}$$

$$\dot{\varphi}_2 = \frac{2v - wL}{2r}$$

From One Model to Another

- An intuitive example
- For inputs $v = 0$, $\omega = C$ (a constant), find the corresponding angular wheels velocities $\dot{\varphi}_1$ and $\dot{\varphi}_2$

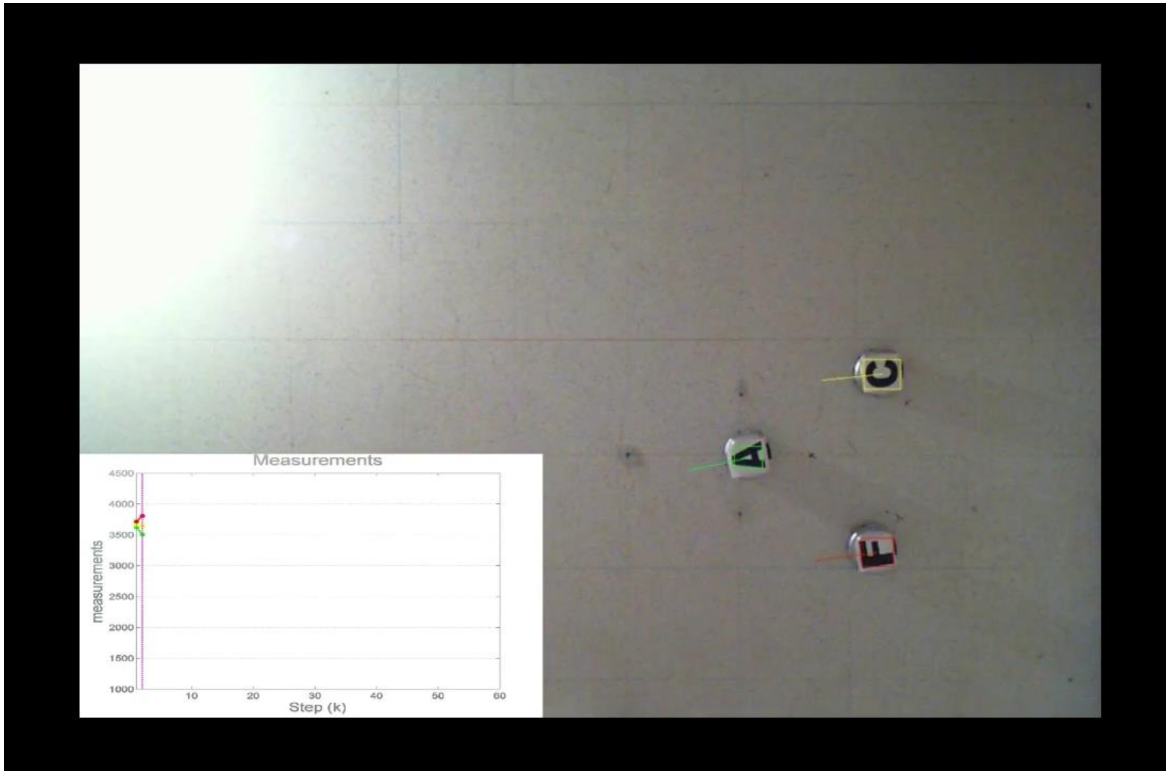
$$\dot{\varphi}_1 = \frac{2v + wL}{2r}$$

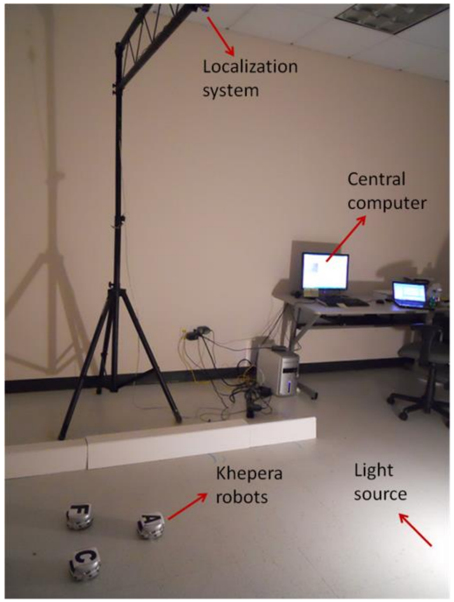
$$\dot{\varphi}_2 = \frac{2v - wL}{2r}$$

$$\dot{\varphi}_1 = \frac{CL}{2r}$$

$$\dot{\varphi}_2 = -\frac{CL}{2r}$$

Experiment: Multi-Robot Source Seeking

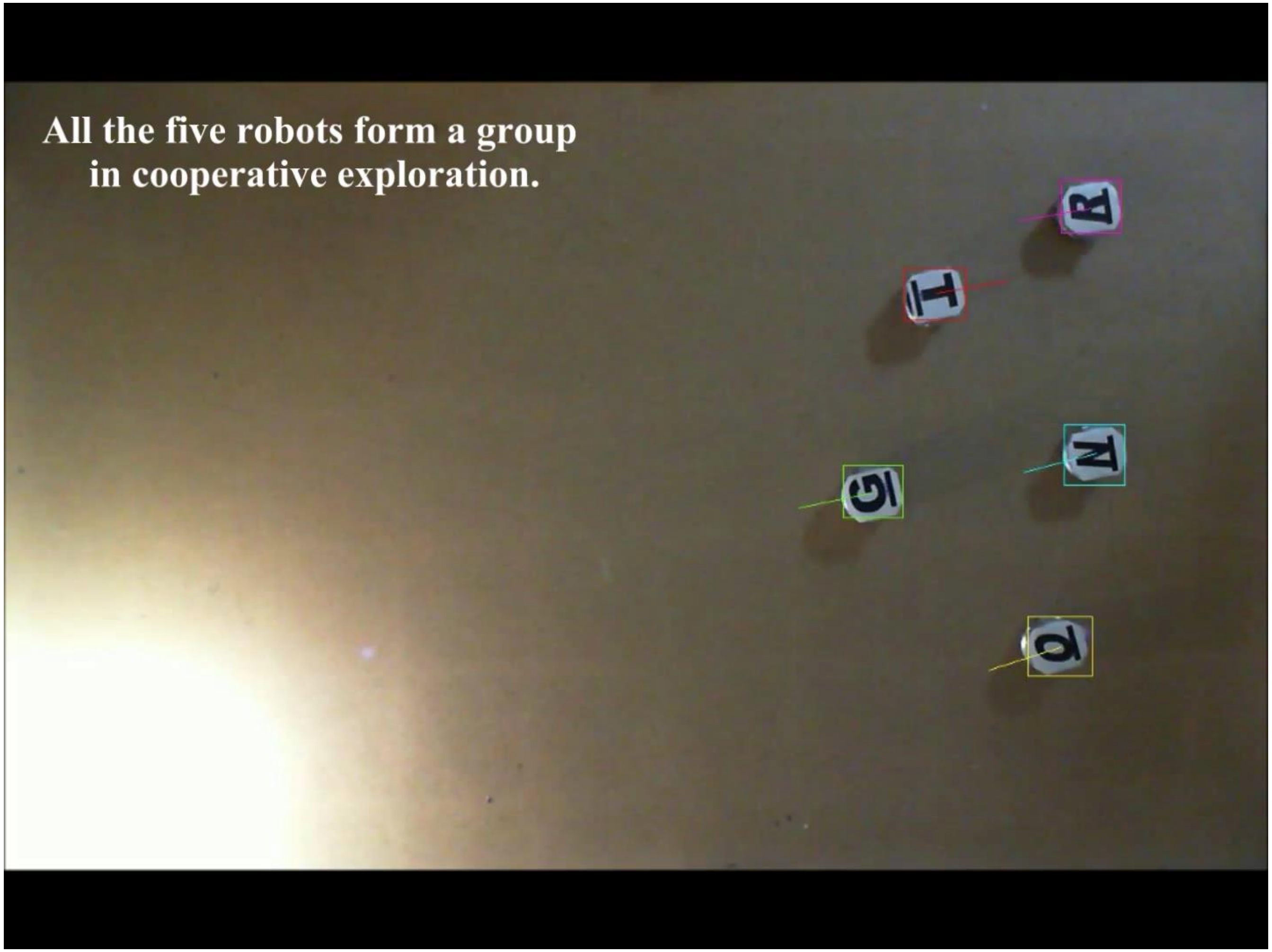




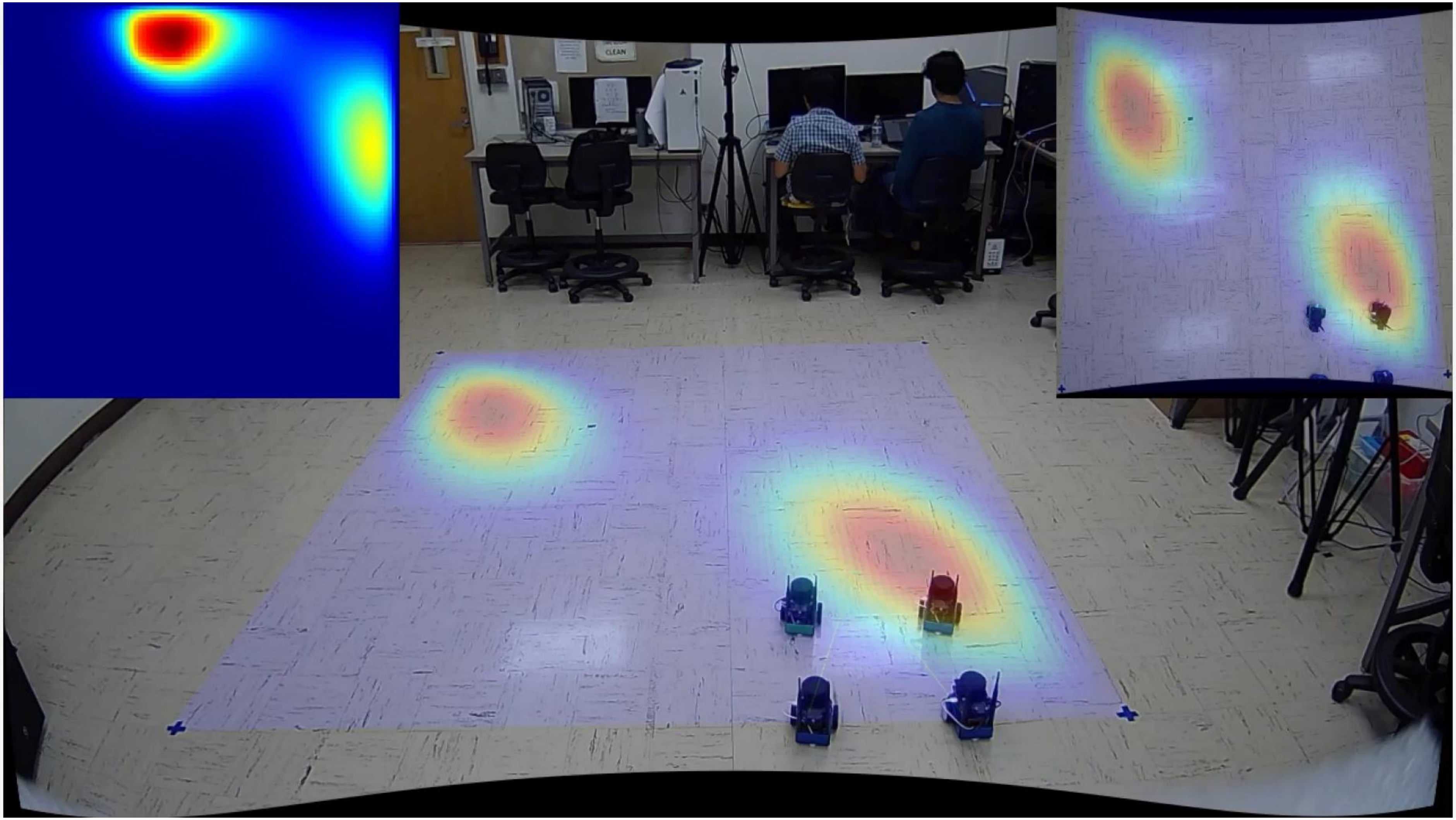
- Khepera III robots
- Infrared sensors

W. Wu and F. Zhang, "Robust Cooperative Exploration with a Switching Strategy," *IEEE Transactions on Robotics*, 28(4), 828-839, 2012.

Self-Organizing Swarming Robots



Multi-robot Dynamic Field Mapping



- Thank you!