

Universidad Autónoma de Baja California
Facultad de Ciencias Químicas e Ingeniería



SEÑALES Y SISTEMAS
Aplicaciones de la Transformada Z

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① Considere un sistema discreto cuya $H(z)$ es

$$H(z) = \frac{z}{z - 1/2}$$

$$y(n) = H(n) * x(n)$$

$$y(n) = z^{-1} \{ H(z) \cdot x(z) \}$$

a) Encuentre la respuesta al escalón $u[n]$

$$Z\{u[n]\} = X(z) \Rightarrow \frac{z}{z-1}$$

$$\frac{z}{z-1} \cdot \frac{z}{z-1/2} = \frac{z^2}{(z-1)(z-1/2)} = X(z)$$

$$y(n) = z^{-1} \{ X(z) \} \Rightarrow z^{-1} \left\{ \frac{z^2}{z^2 - \frac{3}{2}z + \frac{1}{2}} \right\} \quad \left. \begin{array}{l} \text{Fracción Parcial} \\ z^2 = A(z-1/2) + B(z-1) \\ z=1 \quad 1 = \frac{1}{2}A \quad A=2 \\ z=1/2 \quad \frac{1}{2} = \frac{1}{2}B \quad B=1 \end{array} \right\} \quad z^{-1} \left\{ \frac{2}{z-1} - \frac{1/2}{z-1/2} \right\} = 2u[n]$$

b) la salida $y[n]$ cuando $x[n] = nu[n]$

$$Z\{nu[n]\} = X(z) \Rightarrow \frac{z}{(z-1)^2}$$

$$y(n) = x(n) * h(n)$$

$$y(z) = x(z) \cdot h(z)$$

$$y(n) = z^{-1} \{ y(z) \}$$

$$y(z) = \frac{z}{(z-1)^2} \cdot \frac{z}{z-1/2}$$

$$\frac{y(z)}{z} = \frac{1}{(z-1)^2(z-1/2)}$$

$$\frac{y(z)}{z} = \frac{2}{z-1/2} - \frac{2}{z-1} + \frac{2}{z}$$

$$\frac{z^0}{(z-1)^2} \Big| = 2, \quad \frac{z}{(z-1/2)} \Big| = 2, \quad y(z) = \frac{2z}{z-1/2} + \frac{2z}{z-1} + \frac{2}{z}$$

$$y(n) = 2\left(\frac{1}{2}\right)^n u[n] + 2u[n] + 2\delta[n]$$

$$\frac{2-2}{(2z-1)^2} = -2$$

$$y(n) = 2u[n]\left(\frac{1}{2}^n - 1 + n\right)$$

$$y(n) = 2u[n](2^{-n} + n - 1)$$

II Considere un sistema discreto relacionado por
 $y[n] - \frac{5}{6}y[n-1] + \frac{1}{6}y[n-2] = x[n]$

① Encuentre la Función de Transferencia

$$y(z) - \frac{5}{6} [z^{-1}y(z) + y(-1)] + \frac{1}{6} [z^{-2}y(z) + y(-2) + z^{-1}y(-1)] = x(z)$$

$$y(z) - \frac{5}{6} z^{-1}y(z) + \frac{1}{6} z^{-2}y(z) = x(z)$$

$$y(z) [1 - \frac{5}{6} z^{-1} + \frac{1}{6} z^{-2}] = x(z)$$

$$\frac{y(z)}{x(z)} = \frac{1}{1 - \frac{5}{6} z^{-1} + \frac{1}{6} z^{-2}} = \text{F.T. ó } h(z)$$

② Encuentre la respuesta al impulso

$$x(n) = \delta(n) \rightarrow x(z) = 1$$

$$y(z) = \frac{1}{1 - \frac{5}{6} z^{-1} + \frac{1}{6} z^{-2}} \rightarrow \frac{y(z)}{x(z)} = \frac{6z}{36z^2 - 30z + 6} = \frac{6z}{(z - \frac{1}{2})(z - \frac{1}{3})}$$

$$y(z) = 1 - \frac{5}{6} z^{-1} + \frac{1}{6} z^{-2}$$

$$A = \frac{6z}{z - \frac{1}{2}} = 12, \quad B = \frac{6z}{z - \frac{1}{3}} = 12$$

$$y(z) = \frac{6z^2(6z-5) + 6z}{6z^3}$$

$$\frac{12}{z - \frac{1}{2}} - \frac{12}{z - \frac{1}{3}} = y(z)$$

$$y(z) = \frac{36z^3 - 30z^2 + 6z}{6z^3}$$

$$y(n) = 12 \cdot 2^{-n} - 12 \cdot 3^{-n}$$

$$y(z) = \frac{6z^3}{z(16z^3 - 30z^2 + 6z)}$$

$$y(n) = 9 \cdot 2^{-n+1} - 4 \cdot 3^{-n+1}$$

II) Resuelve las ec. dif. siguientes

a) $y[n] - 3y[n-1] = x[n]$ $x[n] = 4u[n]$ $y[-1] = 1$

$$y(z) - 3[z^{-1}y(z) + y[-1]] = 4u(z)$$

$$y(z) - 3z^{-1}y(z) + 3 = 4\left[\frac{z}{z-1}\right]$$

$$y(z)[1 - 3z^{-1}] = \frac{4z}{z-1} + 3$$

$$y(z)[1 - 3z^{-1}] = \frac{4z + 3(z-1)}{z-1}$$

$$y(z) = \frac{4z + 3(z-1)}{z-1}$$

$$1 - 3/z$$

$$\frac{z(7z-3)}{(z-1)(z-3)}$$

$$\frac{y(z)}{z} = \frac{7z-3}{(z-1)(z-3)}$$

Fracciones Parciales

$$y(z) = \frac{9z}{z-3} - \frac{2z}{z-1}$$

$$\frac{7z-3}{z-3} \Big|_1 = 2, \quad \frac{7z-3}{z-1} \Big|_3 = 9$$

$$y(n) = 9 \cdot 3^n u(n-1) - 2^n u(n-1)$$

$$y(n) = 3^{n+1} u(n-1) - 2^n u(n-1)$$

b) $y[n] - 5y[n-1] + 6y[n-2] = x[n]$ $x[n] = nu[n]$

$$y[-1] = 3$$

$$y[-2] = 1$$

$$y(z) - 5[z^{-1}y(z) + y[-1]] + 6[z^{-2}y(z) + y[-2] + z^{-1}y[-1]] = nu(z)$$

$$y(z) - 5z^{-1}y(z) + 15 + 6z^{-2}y(z) + 6 + 18z^{-1} = z/(z-1)^2$$

$$y(z) - 5z^{-1}y(z) + 6z^{-2}y(z) = \frac{z}{(z-1)^2} - 21 + 18z^{-1}$$

$$y(z)[1 - 5z^{-1} + 6z^{-2}] = \frac{9z^3 - 18z^2 + 45z - 18}{z(z-1)^2}$$

$$y(z) = \frac{9z^3 - 18z^2 + 45z - 18}{z(z-1)^2}$$

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$$y(z) = \frac{9z^3 - 18z^2 + 45z - 18}{z(z-1)^2}$$

$$z(z-1)^2$$

$$= \frac{9z^3 - 18z^2 - 45z^3 - 18z^2}{z(z-1)^2(z-3)(z-2)}$$

$$y(z) = \frac{z(9z^3 - 35z^2 + 45z - 18)}{(z-3)(z-2)(z-1)^2}$$

IV Encuentre las $y(n)$ de las expresiones

$$a) Y(z) = \frac{2z(z - \frac{3}{2})}{(z - \frac{1}{2})(z - \frac{1}{3})}$$

$$A = \frac{2(z - \frac{3}{2})}{z - \frac{1}{3}} \Big|_{\frac{1}{3}} = 1$$

$$\frac{Y(z)}{z} = \frac{2(z - \frac{3}{2})}{(z - \frac{1}{2})(z - \frac{1}{3})}$$

$$B = \frac{2(z - \frac{3}{2})}{z - \frac{1}{2}} \Big|_{\frac{1}{2}} = 1$$

$$Y(z) = \frac{z}{z - \frac{1}{2}} + \frac{z}{z - \frac{1}{3}} \Rightarrow Y(n) = 2^n u(n) + 3^n u(n)$$

$$b) Y(z) = \frac{z}{2z^2 - 3z + 1}$$

$$A = \frac{1}{z - \frac{1}{2}} \Big|_{\frac{1}{2}} = 2$$

$$\frac{Y(z)}{z} = \frac{1}{(z - 1)(z - \frac{1}{2})}$$

$$B = \frac{1}{z - 1} \Big|_{1} = -2$$

$$Y(z) = \frac{2z}{z - 1} - \frac{2z}{z - \frac{1}{2}} \Rightarrow Y(n) = 2u(n) - 2 \cdot 2^{-n} u(n)$$

$$Y(n) = 2u(n) - 2^{n+1} u(n)$$

$$c) Y(z) = \frac{6z^3 + 2z^2 - z}{z^3 - z^2 - z + 1}$$

$$\frac{Y(z)}{z} = \frac{6z^2 + 2z - 1}{(z - 1)^2(z + 1)} = \frac{A}{z + 1} + \frac{\lambda_1}{z - 1} + \frac{\lambda_2}{(z - 1)^2}$$

$$p_{aux} = \frac{6z^2 + 2z - 1}{z + 1}$$

$$\lambda_1 = \frac{d(p_{aux})}{dz} \Big|_{-1} = \frac{21}{4}$$

$$A = \frac{6z^2 + 2z - 1}{(z - 1)^2} \Big|_{-1} = \frac{3}{4} \quad \lambda_2 = p_{aux} \Big|_{-1} = \frac{7}{2}$$

$$Y(z) = \frac{\frac{3}{4}z}{z + 1} + \frac{\frac{21}{4}z}{z - 1} + \frac{\frac{7}{2}z}{(z - 1)^2} \Rightarrow \frac{3}{4}(-1)^n u(n) + \frac{21}{4}u(n) + \frac{7}{2}n u(n)$$

$$d) Y(z) = \frac{z+1}{(z-0.5)(z^2-0.5z+0.75)}$$

$$Y(z) = \frac{z+1}{(z-1/2)(z^2-1/2z+1/4)} \rightarrow \frac{1 + \frac{1}{z}}{(z-1/2)(z-(\frac{1+\sqrt{3}i}{4}))(z-(\frac{1-\sqrt{3}i}{4}))} = \frac{Y(z)}{z}$$

Fracciones Parciales

$$A = \frac{1 + \frac{1}{z}}{(z - (\frac{1+\sqrt{3}i}{4}))(z - (\frac{1-\sqrt{3}i}{4}))} \Big|_{z=1/2} = 12 \quad B = \frac{1 + \frac{1}{z}}{(z-1/2)(z - (\frac{1-\sqrt{3}i}{4}))} \Big|_{z=\frac{1+\sqrt{3}i}{4}} = \frac{-2 + 10\sqrt{3}i}{3}$$

$$C = \frac{1 + \frac{1}{z}}{(z-1/2)(z - (\frac{1+\sqrt{3}i}{4}))} \Big|_{z=\frac{1-\sqrt{3}i}{4}} = \frac{-2 - 10\sqrt{3}i}{3}$$

$$\frac{Y(z)}{z} = \frac{A}{z-1/2} + \frac{B}{(z - (\frac{1+\sqrt{3}i}{4}))} + \frac{C}{(z - (\frac{1-\sqrt{3}i}{4}))}$$

$$Y(z) = \frac{12z}{z-1/2} + \frac{-2 + \frac{10\sqrt{3}i}{3}}{(z - (\frac{1+\sqrt{3}i}{4}))} - \frac{2 + \frac{10\sqrt{3}i}{3}}{(z - (\frac{1-\sqrt{3}i}{4}))}$$

$$12 \cdot 2^{n+1} u(n) + \left(-2 + \frac{10\sqrt{3}i}{3} \right) \left(\frac{1+\sqrt{3}i}{4} \right)^n u(n) + \left(-2 - \frac{10\sqrt{3}i}{3} \right) \left(\frac{1-\sqrt{3}i}{4} \right)^n u(n)$$

\downarrow
 $3 \cdot 2^{n+1} u(n) +$