Universidad Autónoma de Baja California
Facultad de Ciencias Químicas e Ingeniería

SEÑALES Y SISTEMAS

Aplicaciones de la Transformada Z

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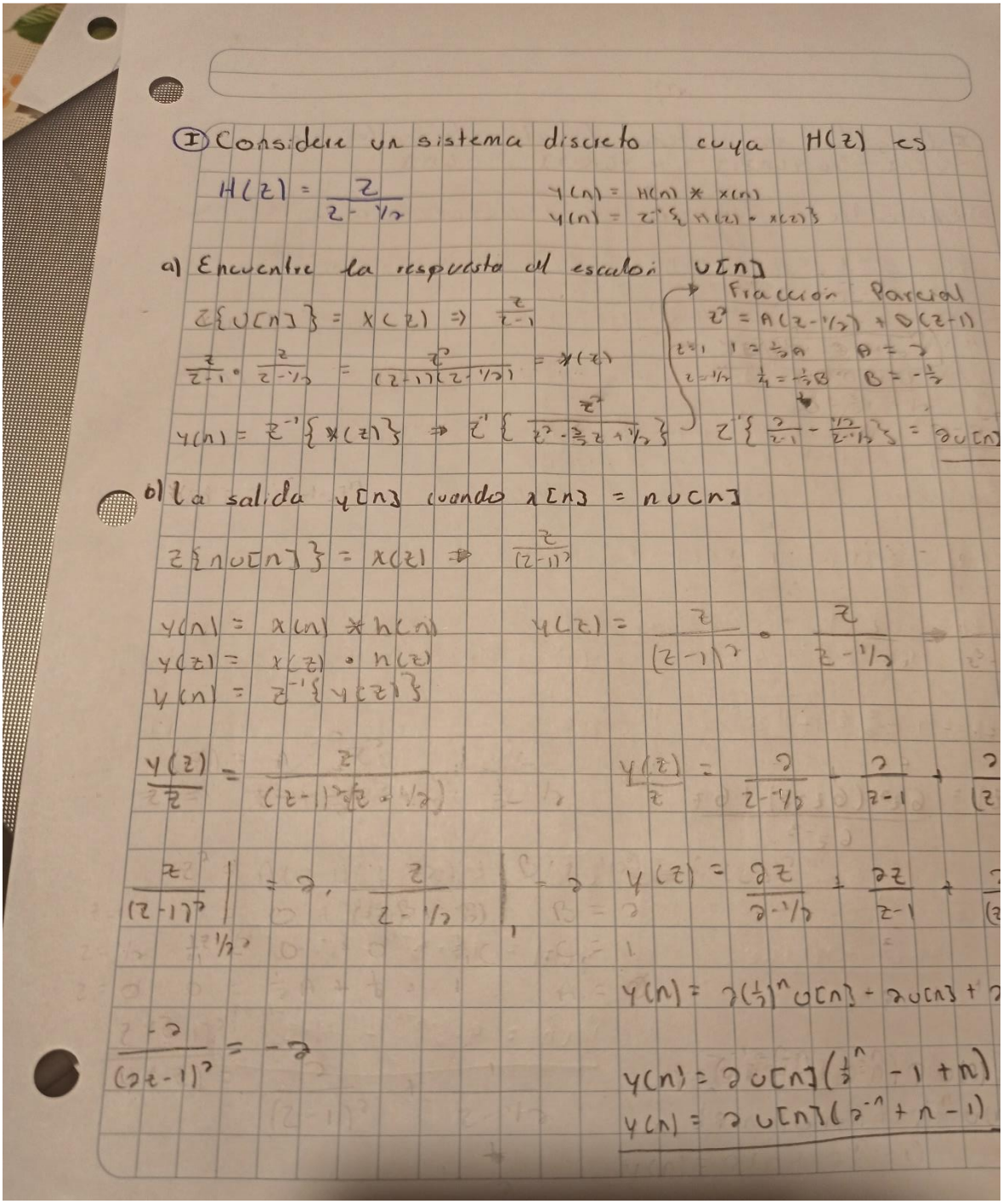

Ⓘ Considere un sistema discreto cuya $H(z)$ es

$$H(z) = \frac{z}{z - \frac{1}{4}}$$

$$y(n) = H(n) * x(n)$$
$$y(n) = z^{-1}\{H(z) \cdot X(z)\}$$

a) Encuentre la respuesta al escalón $u[n]$

$$z\{u[n]\} = X(z) \Rightarrow \frac{z}{z-1}$$

$$\frac{z}{z-\frac{1}{4}} \cdot \frac{z}{z-1} = \frac{z^2}{(z-\frac{1}{4})(z-1)} = \frac{z}{z-\frac{1}{4}} \cdot \frac{z}{z-1}$$

Fracción Parcial

$$z^2 = A(z-\tfrac{1}{4}) + B(z-1)$$

$z = 1 \quad 1 = \frac{3}{4}A \quad A = \frac{4}{3}$

$z = \frac{1}{4} \quad \frac{1}{4} = -\frac{3}{4}B \quad B = -\frac{1}{3}$

$$z^{-1}\left\{\frac{\frac{4}{3}z}{z-1} - \frac{\frac{1}{3}z}{z-\frac{1}{4}}\right\} = y[n]$$

$$y[n] = \left(\frac{4}{3} - \frac{1}{3}\left(\frac{1}{4}\right)^n\right)u[n]$$

b) La salida $y[n]$ cuando $x[n] = n\,u[n]$

$$z\{n\,u[n]\} = X(z) \Rightarrow \frac{z}{(z-1)^2}$$

$$y[n] = x[n] * h[n]$$
$$Y(z) = X(z) \cdot H(z)$$
$$y[n] = z^{-1}\{Y(z)\}$$

$$Y(z) = \frac{z}{(z-1)^2} \cdot \frac{z}{z-\frac{1}{4}}$$

$$\frac{Y(z)}{z} = \frac{z}{(z-1)^2(z-\frac{1}{4})}$$

$$\frac{Y(z)}{z} = \frac{A}{z-1} + \frac{B}{(z-1)^2} + \frac{C}{z-\frac{1}{4}}$$

$$\frac{Y(z)}{z} = \frac{z}{(z-1)^2} + \frac{z}{z-1} - \frac{z}{z-\frac{1}{4}}$$

$$y[n] = n\,u[n] + u[n] - \left(\frac{1}{4}\right)^n u[n]$$

$$y[n] = u[n]\left(n + 1 - \left(\frac{1}{4}\right)^n\right)$$

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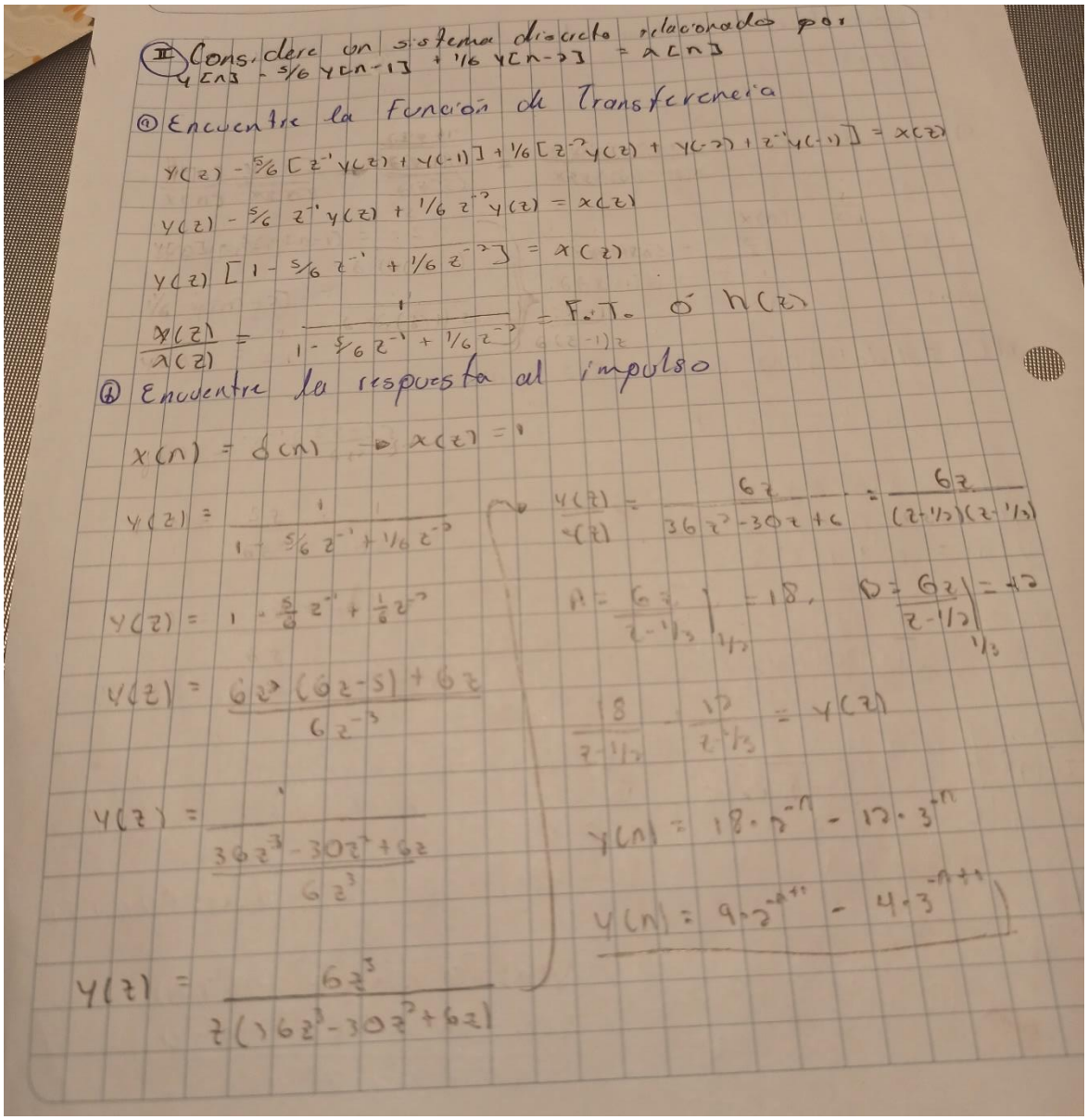
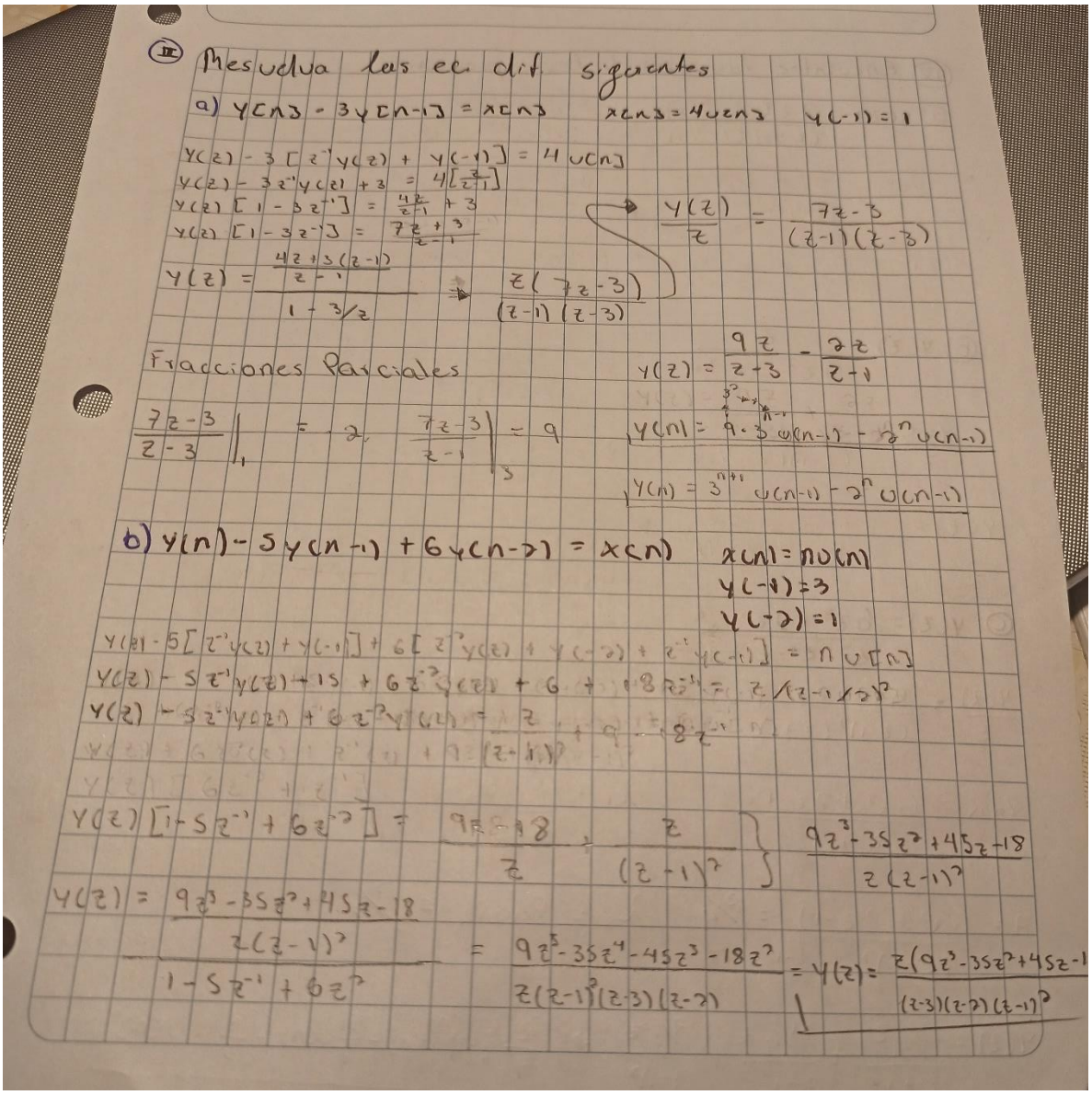
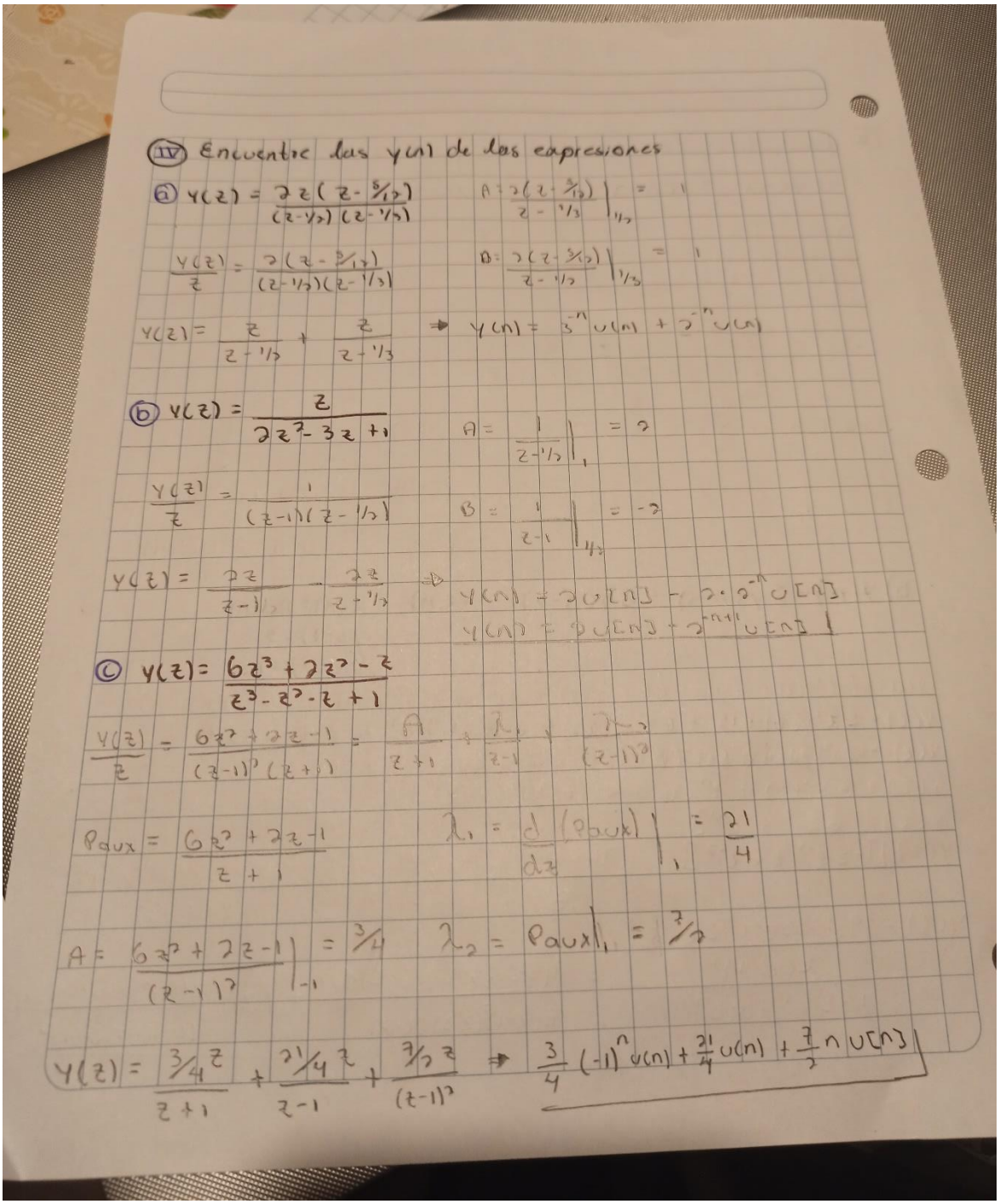

II Considere un sistema discreto relacionado por:
y[n] - 5/6 y[n-1] + 1/6 y[n-2] = x[n]
a) Encuentre la Función de Transferencia
Y(z) - 5/6 [z⁻¹Y(z) + y(-1)] + 1/6 [z⁻²Y(z) + y(-2) + z⁻¹y(-1)] = X(z)
Y(z) - 5/6 z⁻¹Y(z) + 1/6 z⁻²Y(z) = X(z)
Y(z) [1 - 5/6 z⁻¹ + 1/6 z⁻²] = X(z)
Y(z)/X(z) = 1/(1 - 5/6 z⁻¹ + 1/6 z⁻²) = F.T. ó h(z)
b) Encuentre la respuesta al impulso
x(n) = δ(n) → X(z) = 1
Y(z) = 1/(1 - 5/6 z⁻¹ + 1/6 z⁻²)
Y(z) = 1 - 5/6 z⁻¹ + 1/6 z⁻²
Y(z) = (6z³(6z-5) + 6z)/6z⁻³
Y(z) = 1/((36z³ - 30z² + 6z)/6z³)
Y(z) = 6z³/(z(36z³ - 30z² + 6z))
Y(z)/z = 6z²/(36z² - 30z + 6)
6z²/(36z² - 30z + 6) = 6z/((z-1/2)(z-1/3))
A = (6z/(z-1/3))|z=1/2 = 18
B = (6z/(z-1/2))|z=1/3 = -12
18/(z-1/2) - 12/(z-1/3) = Y(z)
y(n) = 18·(1/2)ⁿ - 12·(1/3)ⁿ
y(n) = 9·2^(-n+1) - 4·3^(-n+1)




IV Encuentre las y(n) de las expresiones
a) Y(z) = 2z(z - 5/12) / ((z - 1/4)(z - 1/3))
A = 2(z - 5/12)/(z - 1/3) |z=1/4 = 1
B = 2(z - 5/12)/(z - 1/4) |z=1/3 = 1
Y(z)/z = 2(z - 5/12)/((z - 1/4)(z - 1/3))
Y(z) = z/(z - 1/4) + z/(z - 1/3)
⇒ y(n) = 3^-n u(n) + 2^-n u(n)
b) Y(z) = z/(2z^2 - 3z + 1)
A = 1/(2z - 1) |z=1 = 2
B = 1/(2z - 1) |z=1/2 = -2
Y(z)/z = 1/((z - 1)(z - 1/2))
Y(z) = 2z/(z - 1) - 2z/(z - 1/2)
⇒ y(n) = 2u[n] - 2·2^-n u[n]
y(n) = 2u[n] - 2^(-n+1) u[n]
c) Y(z) = (6z^3 + 2z^2 - z)/(z^3 - z^2 - z + 1)
Y(z)/z = (6z^2 + 2z - 1)/((z - 1)^2 (z + 1)) = A/(z + 1) + λ1/(z - 1) + λ2/(z - 1)^2
Paux = (6z^2 + 2z - 1)/(z + 1)
λ1 = d/dz (Paux) |z=1 = 21/4
A = (6z^2 + 2z - 1)/(z - 1)^2 |z=-1 = 3/4
λ2 = Paux |z=1 = 7/2
Y(z) = 3/4 z/(z + 1) + 21/4 z/(z - 1) + 7/2 z/(z - 1)^2
⇒ 3/4 (-1)^n u(n) + 21/4 u(n) + 7/2 n u[n]