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Señales y sistemas Grupo 541

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Encuentre la transformada de Laplace

a) $2e^{4t} \to 2\mathcal{L}\{e^{4t}\} \Rightarrow \frac{2}{s-4}$

b) $3e^{-2t} \to 3\mathcal{L}\{e^{-2t}\} \Rightarrow \frac{3}{s+2}$

c) $5t-3 \to 5\mathcal{L}\{t\} - \mathcal{L}\{3\} \Rightarrow \frac{5}{s^2} - \frac{3}{s}$

d) $2t^2 - e^{-t} \to 2\mathcal{L}\{t^2\} - \mathcal{L}\{e^{-t}\} \Rightarrow \frac{4}{s^3} - \frac{1}{s+1}$

e) $3\cos 5t \to 3\mathcal{L}\{\cos 5t\} \Rightarrow \frac{3s}{s^2+25}$

f) $10 \operatorname{sen} 6t \to 10\mathcal{L}\{\operatorname{sen} 6t\} \Rightarrow \frac{60}{s^2+36}$

g) $6 \operatorname{sen} 2t - 5\cos 2t \to 6\mathcal{L}\{\operatorname{sen} 2t\} - 5\mathcal{L}\{\cos 2t\} \Rightarrow \frac{12}{s^2+4} - \frac{5s}{s^2+4}$

h) $(t^2+1)^2 \to \mathcal{L}\{t^4 + 2t^2 + 1\} \Rightarrow \frac{24}{s^5} + \frac{4}{s^3} + \frac{1}{s}$

i) $(\operatorname{sen} t - \cos t)^2 \to \mathcal{L}\{\underbrace{\cancel{\operatorname{sen}^2 t + \cos^2 t}}_{1} - \underbrace{\cancel{2\operatorname{sen} t \cos t}}_{\operatorname{sen} 2t}\}$

$\mathcal{L}\{1\} - \mathcal{L}\{\operatorname{sen} 2t\} \Rightarrow \frac{1}{s} - \frac{1}{s^2+4}$

j) $3\cosh 5t - 4\operatorname{senh} 5t \Rightarrow 3\mathcal{L}\{\cosh 5t\} - 4\mathcal{L}\{\operatorname{senh} 5t\}$

$\frac{3s}{s^2-5^2} - \frac{20}{s^2-5^2}$

Evalúe las sig. transformadas

a) $\mathcal{L}(5e^2-3)^2$

$\frac{(5e^2-3)^2}{s}$

b) $\mathcal{L}\{4\cos^2 2t\} \Rightarrow 4\mathcal{L}\{\frac{1}{2} + \frac{\cos 4t}{2}\}$

$\mathcal{L}\{2\} + 2\mathcal{L}\{\cos 4t\} \Rightarrow \frac{2}{s} + \frac{2s}{s^2+4}$

Evalue cada una de las sig. expresiones

a) $\mathcal{L}\{t^3 e^{-3t}\}$ $\frac{6}{s^4}$ donde $s = s+3$ $\Rightarrow \frac{6}{(s+3)^4}$

b) $\mathcal{L}\{e^{-t}\cos 2t\} \Rightarrow \frac{s}{s^2+2^2}$ donde $s = s+1$ $\Rightarrow \frac{s+1}{(s+1)^2+2^2}$

c) $\mathcal{L}\{2e^{3t}\operatorname{sen} 4t\} \Rightarrow \frac{8}{s^2+4^2}$ donde $s = s-3$ $\Rightarrow \frac{8}{(s-3)^2+4^2}$

d) $\mathcal{L}\{(t+2)^2 e^{t}\} \Rightarrow \mathcal{L}\{e^{t}(t^2+4t+4)\} \Rightarrow \frac{2}{(s-1)^3} + \frac{4}{(s-1)} + \frac{4}{s-1}$

e) $\mathcal{L}\{e^{2t}(3\operatorname{sen} 4t - 4\cos 4t)\}$

$3\mathcal{L}\{e^{2t}\operatorname{sen} 4t\} - 4\mathcal{L}\{e^{2t}\cos 4t\}$

$\frac{12}{(s-2)^2+4^2} - \frac{3s}{(s-2)^2+4^2}$

f) $\mathcal{L}\{e^{-4t}\cosh 2t\} \Rightarrow \frac{s+4}{(s+4)^2-4}$

g) $\mathcal{L}\{e^{-t}(\operatorname{senh} 2t - 5\cosh 2t)\}$

$\mathcal{L}\{e^{-t}\operatorname{senh} 2t\} - 5\mathcal{L}\{e^{-t}\cosh 2t\}$

$\frac{2}{(s+1)^2-2^2} - \frac{5(s+1)}{(s+1)^2-2^2}$

Propiedades de linealidad, translación y cambio de escala

Encuentre

$\mathcal{L}\{3t^4 - 2t^3 + 4e^{-3t} - 2\operatorname{sen} 5t + 3\cos 2t\}$

$3\mathcal{L}\{t^4\} - 2\mathcal{L}\{t^3\} + 4\mathcal{L}\{e^{-3t}\} - 2\mathcal{L}\{\operatorname{sen} 5t\} + 3\mathcal{L}\{\cos 2t\}$

$\frac{72}{s^5} - \frac{12}{s^4} + \frac{4}{s+3} - \frac{10}{s^2+25} + \frac{3s}{s^2+4}$

Encuentre $\mathcal{L}\{F(t)\}$ si:

a) $F(t)\begin{cases} 0 & t < 2 \\ 4 & t > 2 \end{cases}$

$4\mathcal{L}\{u(t-2)\} \Rightarrow \frac{4e^{-2s}}{s}$

b) $F(t)\begin{cases} 2t & 0 < t \leq 5 \\ 0 & t > 5 \end{cases}$

$2t\,u(t) - 2(t-5)u(t-5) - 10\,u(t-5) = F(t)$

$2\mathcal{L}\{t\,u(t)\} - 2\mathcal{L}\{(t-5)u(t-5)\} - 10\mathcal{L}\{u(t-5)\}$

$\frac{2}{s^2} - \frac{2e^{-5s}}{s^2} - \frac{10e^{-5s}}{s} \Rightarrow \frac{2e^{-5s}(-5s + e^{5s} - 1)}{s^2}$

Encuentre

a) $\mathcal{L}\{e^{-t}\,\text{sen}^2 t\} \rightarrow \mathcal{L}\{e^{-t}(\frac{1-\cos 2t}{2})\} \rightarrow \frac{1}{2}\{e^{-t}(1-\cos 2t)\}$

$\frac{1}{2}\mathcal{L}\{e^{-t}\} - \frac{1}{2}\mathcal{L}\{e^{-t}\cos 2t\} \Rightarrow \frac{1}{2(s+1)} - \frac{s+1}{2(s+1)^2+8}$

b) $\mathcal{L}\{(1+te^{-t})^3\} \rightarrow \mathcal{L}\{1 + 3te^{-t} + 3t^2e^{-2t} + t^3e^{-3t}\}$

$\mathcal{L}\{1\} + 3\mathcal{L}\{te^{-t}\} + 3\mathcal{L}\{t^2e^{-2t}\} + \mathcal{L}\{t^3e^{-3t}\}$

$\frac{1}{s} + \frac{3}{(s+1)^2} + \frac{6}{(s+2)^3} + \frac{12}{(s+3)^4}$

Si $\mathcal{L}\{F(t)\} = \frac{s^2-s+1}{(2s+1)^2(s-1)}$ encuentre $\mathcal{L}\{(F(2t))\} = \frac{1}{2}\mathcal{L}\{F(t)\}$

$\frac{s^2-s+1}{2(2s+1)^2(s-1)}$

$\frac{e^{-\frac{1}{s+1}}}{3(s+1)}$

Si $\mathcal{L}\{F(t)\} = \frac{e^{-1/s}}{s}$ encuentre $\mathcal{L}\{e^{-t}F(3t)\} = \frac{1}{3}\mathcal{L}\,e^{-t}F(t)\}$

Encuentre $\mathcal{L}\{t(3\,\text{sen}\,2t - 2\cos t)\}$

$3\mathcal{L}\{t\,\text{sen}\,2t\} - 2\mathcal{L}\{t\cos 2t\}$

$\frac{6}{(s^2+4)^2} - \frac{2(s^2-4)}{(s^2+4)^2}$

Muestre que $\mathcal{L}\{t^2 \operatorname{sen} t\} = \frac{6s^2-2}{(s^2+1)^3}$

$\mathcal{L}\{t^2 f(t)\} \rightarrow \frac{d^2 f(s)}{ds^2}$

$f(s) \Rightarrow \mathcal{L}\{\operatorname{sen} t\} \rightarrow \frac{1}{s^2+1} \approx (s^2+1)^{-1}$

$f'(s) \Rightarrow -2s(s^2+1)^{-2} \approx -\frac{2s}{(s^2+1)^2}$

$f''(s) \Rightarrow \frac{2(s^2+1)^2 - 2(s^2+1)(2s)(2s)}{((s^2+1)^2)^2} = \frac{2\cancel{(s^2+1)^2}^{(s^2+1)} - 8s^2\cancel{(s^2+1)}}{(s^2+1)^{\cancel{4}\,3}}$

$f''(s) \Rightarrow \frac{2s^2+2-8s^2}{(s^2+1)^3} \Rightarrow \frac{6s^2-2}{(s^2+1)^3}$

Evalúe $\mathcal{L}\{t \cosh 3t\}$ y $\mathcal{L}\{t \operatorname{senh} 2t\}$

$\mathcal{L}\{t \cosh 3t\}$

$\mathcal{L}\{t\left(\frac{e^{3t}+e^{-3t}}{2}\right)\}$

$\frac{1}{2}\mathcal{L}\{te^{3t}\} + \frac{1}{2}\mathcal{L}\{te^{-3t}\}$

$\frac{1}{2(s-3)^2} + \frac{1}{2(s+3)^2}$

$\mathcal{L}\{t \operatorname{senh} 2t\}$

$\mathcal{L}\{t\left(\frac{e^{2t}-e^{-2t}}{2}\right)\}$

$\frac{1}{2}\mathcal{L}\{te^{2t}\} - \frac{1}{2}\{\mathcal{L}\,te^{-2t}\}$

$\frac{1}{2(s-2)^2} - \frac{1}{2(s+2)^2}$

Encuentre $\mathcal{L}\{t^2 \cos t\}$ y $\mathcal{L}\{(t^2 - 3t + 2) \operatorname{sen} 3t\}$

$$\mathcal{L}\{t^2 \cos t\} = \frac{d^2 f(x)}{ds^2} \qquad f(s) = \mathcal{L}\{\cos t\} = \frac{s}{s^2+1}$$

$$f'(s) = \frac{-s^2+1}{(s^2+1)^2}$$

$$f''(s) = \frac{(-2s)(s^2+1)^2 - 2(s^2+1)(2s)(-s^2+1)}{(s^2+1)^4} = \frac{-2s}{(s^2+1)^2} + \frac{4s^3}{(s^2+1)^3} - \frac{4s}{(s^2+1)^3}$$

$$\mathcal{L}\{(t^2 - 3t + 2) \operatorname{sen} 3t\} = \mathcal{L}\{t^2 \operatorname{sen} 3t\} - 3\mathcal{L}\{t \operatorname{sen} 3t\} + 2\mathcal{L}\{\operatorname{sen} 3t\}$$

$$f(s) = \frac{3}{s^2+9} \qquad f''(s) = \frac{18s^2-54}{(s^2+9)^3}$$

$$f'(s) = \frac{-6s}{(s^2+9)^2} \qquad \mathcal{L}\{F(t)\} = \frac{18s^2-54}{(s^2+9)^3} - \frac{18s}{(s^2+9)^2} + \frac{6}{s^2+9}$$

Encuentre $\mathcal{L}\{t^3 \cos t\}$

$$\text{si } \mathcal{L}\{t^2 \cos t\}, \quad \mathcal{L}\{t^3 \cos t\} = -\frac{d\mathcal{L}\{t^2 \cos t\}}{ds}$$

$$\frac{-2(s^2+1)^2 - 2(s^2+1)(2s)(2s)}{(s^2+1)^4} = \frac{6s^2-2}{(s^2+1)^3} + 4\left(\frac{2s(s^2+1) - 3(s^2+1)^2(2s)s^2}{(s^2+1)^6}\right)$$

$$= \left(-\frac{16s^3+8}{(s^2+1)^4}\right) - 4\left(\frac{(s^2+1)^3 - 3(s^2+1)^2(2s)s}{(s^2+1)^6}\right) \Rightarrow$$

$$-\frac{6s^2-2}{(s^2+1)^3} + \frac{16s^3-8}{(s^2+1)^4} - \frac{20s^2-4}{(s^2+1)^4}$$

Dado $F(t)\begin{cases} 0 & 0 \leq t \leq 1 \\ t & t > 1 \end{cases}$ encuentre $\mathcal{L}\{F(t)\}$ y $\mathcal{L}\{F'(t)\}$

$\mathcal{L}\{F(t)\} = \mathcal{L}\{u(t-1)\} + \mathcal{L}\{(t-1)u(t-1)$

$\frac{e^{-s}}{s} + \frac{e^{-s}}{s^2} = \mathcal{L}\{F(t)\}$

$\mathcal{L}\{F'(t)\} = \mathcal{L}\{u(t-1)\} + \mathcal{L}\{\delta(t-1)\}$

$\frac{e^{-s}}{s} + e^{-s} = \mathcal{L}\{F'(t)\}$

Dado $F(t)\begin{cases} T^2 & 0 < t < 1 \\ 0 & t > 1 \end{cases}$ encuentre

$\mathcal{L}\{F(t)\} = \mathcal{L}\{T^2u(t) - T^2u(t-1)\}$

$F(t) = T^2(u(t) - u(t-1))$

$F'(t) = T^2(\delta(t) - \delta(t-1))$

$\mathcal{L}\{F'(t)\} = T^2\mathcal{L}\{\delta(t) - \delta(t-1)\} = T^2 - T^2e^{-s}$

$\mathcal{L}\{F''(t)\} \Rightarrow s\mathcal{L}\{F'(t)\} - F'(0)$

$\mathcal{L}\{F''(t)\} = s(T^2 - T^2e^{-s}) - T^2$

$\mathcal{L}\{F''(t)\} = sT^2 - sT^2e^{-s} - T^2 \Rightarrow T^2(s - se^{-s} - 1)$

$\mathcal{L}\{F''(t)\} = T^2(s - se^{-s} - 1)$