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Grupo 541

Señales y sistemas

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Encuentre la transformada de Laplace

$$a) 2e^{4t} \rightarrow 2\mathcal{L}\{e^{4t}\} \Rightarrow \frac{2}{s-4}$$

$$b) 3e^{-2t} \rightarrow 3\mathcal{L}\{e^{-2t}\} \Rightarrow \frac{3}{s+2}$$

$$c) 5t-3 \rightarrow 5\mathcal{L}\{t\} - \mathcal{L}\{3\} \Rightarrow \frac{5}{s^2} - \frac{3}{s}$$

$$d) 2t^2 - e^{-t} \rightarrow 2\mathcal{L}\{t^2\} - \mathcal{L}\{e^{-t}\} \Rightarrow \frac{4}{s^3} - \frac{1}{s+1}$$

$$e) 3\cos 5t \rightarrow 3\mathcal{L}\{\cos 5t\} \Rightarrow \frac{3s}{s^2+25}$$

$$f) 10\sin 6t \rightarrow 10\mathcal{L}\{\sin 6t\} \Rightarrow \frac{60}{s^2+36}$$

$$g) 6\sin 2t - 5\cos 2t \rightarrow 6\mathcal{L}\{\sin 2t\} - 5\mathcal{L}\{\cos 2t\} \Rightarrow \frac{12}{s^2+4} - \frac{5s}{s^2+4}$$

$$h) (t^2+1)^2 \rightarrow \mathcal{L}\{t^4 + 2t^2 + 1\} \Rightarrow \frac{24}{s^5} + \frac{4}{s^3} + \frac{1}{s}$$

$$i) (\sin t - \cos t)^2 \rightarrow \mathcal{L}\{\sin^2 t + \cos^2 t - 2\sin t \cos t\}$$

$$\mathcal{L}\{1\} - \mathcal{L}\{\sin 2t\} \Rightarrow \frac{1}{s} - \frac{1}{s^2+4}$$

$$j) 3\cosh 5t - 4\sinh 5t \rightarrow 3\mathcal{L}\{\cosh 5t\} - 4\mathcal{L}\{\sinh 5t\}$$

$$\frac{3s}{s^2-25} - \frac{20}{s^2-25}$$

Evalúe las sig. transformadas

$$a) \mathcal{L}(5e^2 - 3t^2)$$

$$b) \mathcal{L}\{4\cos^2 2t\} \Rightarrow 4\mathcal{L}\left\{\frac{1}{2} + \frac{\cos 4t}{2}\right\}$$

$$\frac{(5e^2 - 3)^2}{s}$$

$$\mathcal{L}\{2\} + 2\mathcal{L}\{\cos 4t\} \Rightarrow \frac{2}{s} + \frac{2s}{s^2+16}$$

Evalúe cada una de las sig. expresiones

a) $\mathcal{L}\{t^3 e^{-3t}\} \Rightarrow \frac{6}{s^4} \text{ donde } s \Rightarrow s+3 \Rightarrow \frac{6}{(s+3)^4}$

b) $\mathcal{L}\{e^{-t} \cos 2t\} \Rightarrow \frac{s}{s^2+2^2} \text{ donde } s \Rightarrow s+1 \Rightarrow \frac{s+1}{(s+1)^2+2^2}$

c) $\mathcal{L}\{2e^{3t} \sin 4t\} \Rightarrow \frac{8}{s^2+4^2} \text{ donde } s \Rightarrow s-3 \Rightarrow \frac{8}{(s-3)^2+4^2}$

d) $\mathcal{L}\{(t+2)^2 e^t\} \Rightarrow \mathcal{L}\{e^t(t^2+4t+4)\} \Rightarrow \frac{2}{(s-1)^3} + \frac{4}{(s-1)} + \frac{4}{s-1}$

e) $\mathcal{L}\{e^{2t} (3 \sin 4t - 4 \cos 4t)\}$
 $3 \mathcal{L}\{e^{2t} \sin 4t\} - 4 \mathcal{L}\{e^{2t} \cos 4t\}$
 $\frac{12}{(s-2)^2+4^2} - \frac{3s}{(s-2)^2+4^2}$

f) $\mathcal{L}\{e^{-4t} \cosh 2t\} \Rightarrow \frac{s+4}{(s+4)^2-4}$

g) $\mathcal{L}\{e^{-t} (\sinh 2t - 5 \cosh 2t)\}$
 $\mathcal{L}\{e^{-t} \sinh 2t\} - 5 \mathcal{L}\{e^{-t} \cosh 2t\}$
 $\frac{2}{(s+1)^2-2^2} - \frac{5(s+1)}{(s+1)^2-2^2}$

Propiedades de linealidad, traslación y cambio de escala
 Encuentre

$\mathcal{L}\{3t^4 - 2t^3 + 4e^{-3t} - 2 \sin 5t + 3 \cos 2t\}$

$3 \mathcal{L}\{t^4\} - 2 \mathcal{L}\{t^3\} + 4 \mathcal{L}\{e^{-3t}\} - 2 \mathcal{L}\{\sin 5t\} + 3 \mathcal{L}\{\cos 2t\}$

$\frac{72}{s^5} - \frac{12}{s^4} + \frac{4}{s+3} - \frac{10}{s^2+25} + \frac{3s}{s^2+4}$

Encuentre $\mathcal{L}\{F(t)\}$ si:

a) $F(t) = \begin{cases} 0 & 0 < t < 2 \\ 4 & t > 2 \end{cases}$

$4\mathcal{L}\{u(t-2)\} \Rightarrow \frac{4e^{-2s}}{s}$

b) $F(t) = \begin{cases} 2t & 0 < t \leq 5 \\ 0 & t > 5 \end{cases}$

$2t u(t) - 2(4-s)u(t-5) - 10u(t-5) = F(t)$
 $2\mathcal{L}\{t u(t)\} - 2\mathcal{L}\{(4-s)u(t-5)\} - 10\mathcal{L}\{u(t-5)\}$
 $\frac{2}{s^2} - \frac{2e^{-5s}}{s^2} - \frac{10e^{-5s}}{s} \Rightarrow \frac{2e^{-5s}(-s+4+5)}{s^2}$

Encuentre

a) $\mathcal{L}\{e^{-t} \sin^2 t\} \Rightarrow \mathcal{L}\{e^{-t} (\frac{1-\cos 2t}{2})\} \Rightarrow \frac{1}{2} \mathcal{L}\{e^{-t}(1-\cos 2t)\}$

$\frac{1}{2} \mathcal{L}\{e^{-t}\} - \frac{1}{2} \mathcal{L}\{e^{-t} \cos 2t\} \Rightarrow \frac{1}{2(s+1)} - \frac{s+1}{2(s+1)^2 + 4}$

b) $\mathcal{L}\{(1+te^{-t})^3\} \Rightarrow \mathcal{L}\{1 + 3te^{-t} + 3t^2e^{-2t} + t^3e^{-3t}\}$

$\mathcal{L}\{1\} + 3\mathcal{L}\{te^{-t}\} + 3\mathcal{L}\{t^2e^{-2t}\} + \mathcal{L}\{t^3e^{-3t}\}$

$\frac{1}{s} + \frac{3}{(s+1)^2} + \frac{6}{(s+2)^3} + \frac{12}{(s+3)^4}$

$S_0 \mathcal{L}\{F(t)\} = \frac{s^2 - s + 1}{(s+1)^2(s-1)}$ encuentre $\mathcal{L}\{F(2t)\} = \frac{1}{2} \mathcal{L}\{F(t)\}$

$\frac{s^2 - s + 1}{2(s+1)^2(s-1)}$

$\frac{e^{-\frac{1}{2s+1}}}{3(s+1)}$

si $\mathcal{L}\{F(t)\} = \frac{e^{-1/s}}{s}$ encuentre $\mathcal{L}\{e^{-t} F(3t)\} = \frac{1}{3} \mathcal{L}\{e^{-t} F(t)\}$

Encuentre $\mathcal{L}\{t(3 \sin 2t - 2 \cos 2t)\}$

$3\mathcal{L}\{t \sin 2t\} - 2\mathcal{L}\{t \cos 2t\}$

$\frac{6}{(s^2+4)^2} - \frac{2(s^2-4)}{(s^2+4)^2}$

Muestre que $\mathcal{L}\{t^2 \sin t\} = \frac{6s^2 - 2}{(s^2 + 1)^3}$

$$\mathcal{L}\{t^2 f(t)\} \rightarrow \frac{d^2 f(s)}{ds^2}$$

$$f(s) \Rightarrow \mathcal{L}\{\sin t\} \rightarrow \frac{1}{s^2 + 1} \approx (s^2 + 1)^{-1}$$

$$f'(s) \Rightarrow -2s(s^2 + 1)^{-2} \approx -\frac{2s}{(s^2 + 1)^2}$$

$$f''(s) \Rightarrow \frac{2(s^2 + 1)^2 - 2(s^2 + 1)(2s)(2s)}{(s^2 + 1)^4} = \frac{2(s^2 + 1)^2 - 8s^2(s^2 + 1)}{(s^2 + 1)^4}$$

$$f''(s) \Rightarrow \frac{2s^2 + 2 - 8s^2}{(s^2 + 1)^3} \Rightarrow \frac{6s^2 - 2}{(s^2 + 1)^3}$$

Evalúe $\mathcal{L}\{t \cosh 3t\}$ y $\mathcal{L}\{t \sinh 2t\}$

$$\mathcal{L}\{t \cosh 3t\}$$

$$\mathcal{L}\left\{t \left(\frac{e^{3t} + e^{-3t}}{2}\right)\right\}$$

$$\frac{1}{2} \mathcal{L}\{t e^{3t}\} + \frac{1}{2} \mathcal{L}\{t e^{-3t}\}$$

$$\frac{1}{2(s-3)^2} + \frac{1}{2(s+3)^2}$$

$$\mathcal{L}\{t \sinh 2t\}$$

$$\mathcal{L}\left\{t \left(\frac{e^{2t} - e^{-2t}}{2}\right)\right\}$$

$$\frac{1}{2} \mathcal{L}\{t e^{2t}\} - \frac{1}{2} \mathcal{L}\{t e^{-2t}\}$$

$$\frac{1}{2(s-2)^2} - \frac{1}{2(s+2)^2}$$

Encuentre $\mathcal{L}\{t^2 \cos t\}$ y $\mathcal{L}\{t^2 - 3t + 2\} \sin 3t$

$$\mathcal{L}\{t^2 \cos t\} = \frac{d^2 f(s)}{ds^2} \quad f(s) = \mathcal{L}\{\cos t\} = \frac{s}{s^2 + 1}$$

$$f'(s) = \frac{-s^2 + 1}{(s^2 + 1)^2}$$

$$f''(s) = \frac{(-2s)(s^2 + 1)^2 - 2(s^2 + 1)(2s)(-s^2 + 1)}{(s^2 + 1)^4} = \frac{-2s}{(s^2 + 1)^2} + \frac{4s^3}{(s^2 + 1)^3} - \frac{4s}{(s^2 + 1)^3}$$

$$\mathcal{L}\{t^2 - 3t + 2\} \sin 3t = \mathcal{L}\{t^2 \sin 3t\} - 3\mathcal{L}\{t \sin 3t\} + 2\mathcal{L}\{\sin 3t\}$$

$$f(s) = \frac{3}{s^2 + 9} \quad f'(s) = \frac{18s^2 - 54}{(s^2 + 9)^2}$$

$$f''(s) = \frac{-6s}{(s^2 + 9)^2} \quad \mathcal{L}\{f(t)\} = \frac{18s^2 - 54}{(s^2 + 9)^3} - \frac{18s}{(s^2 + 9)^2} + \frac{6}{s^2 + 9}$$

Encuentre $\mathcal{L}\{t^3 \cos t\}$

$$s \cdot \mathcal{L}\{t^2 \cos t\}, \mathcal{L}\{t^3 \cos t\} = -\frac{d}{ds} \mathcal{L}\{t^2 \cos t\}$$

$$= -\frac{2(s^2 + 1)^2 - 2(s^2 + 1)(2s)(2s)}{(s^2 + 1)^4} = \frac{6s^2 - 2}{(s^2 + 1)^3} + 4 \left(\frac{2s(s^2 + 1) - 3(s^2 + 1)^2(2s)}{(s^2 + 1)^6} \right)$$

$$= \left(\frac{-16s^3 + 8}{(s^2 + 1)^4} \right) - 4 \left(\frac{(s^2 + 1)^3 - 3(s^2 + 1)^2(2s)s}{(s^2 + 1)^4} \right)$$

$$= \frac{6s^2 - 2}{(s^2 + 1)^3} + \frac{16s^2 - 8}{(s^2 + 1)^4} = \frac{20s^2 - 4}{(s^2 + 1)^4}$$

Dado $F(t) = \begin{cases} 0 & 0 \leq t \leq 1 \\ t & t > 1 \end{cases}$ encuentre $\mathcal{L}\{F(t)\} + \mathcal{L}\{F'(t)\}$

$$\mathcal{L}\{F(t)\} = \mathcal{L}\{u(t-1)\} + \mathcal{L}\{(t-1)u(t-1)\}$$

$$\frac{e^{-s}}{s} + \frac{e^{-s}}{s^2} = \mathcal{L}\{F(t)\}$$

$$\mathcal{L}\{F'(t)\} = \mathcal{L}\{u(t-1)\} + \mathcal{L}\{\delta(t-1)\}$$

$$\frac{e^{-s}}{s} + e^{-s} = \mathcal{L}\{F'(t)\}$$

Dado $F(t) = \begin{cases} T^2 & 0 \leq t < 1 \\ 0 & t > 1 \end{cases}$ encuentre

$$\mathcal{L}\{F(t)\} = \mathcal{L}\{T^2 u(t) - T^2 u(t-1)\}$$

$$F(t) = T^2 (u(t) - u(t-1))$$

$$F'(t) = T^2 (\delta(t) - \delta(t-1))$$

$$\mathcal{L}\{F'(t)\} = T^2 \mathcal{L}\{\delta(t) - \delta(t-1)\} = T^2 - T^2 e^{-s}$$

$$\mathcal{L}\{F''(t)\} = s \mathcal{L}\{F'(t)\} - F'(0)$$

$$\mathcal{L}\{F''(t)\} = s(T^2 - T^2 e^{-s}) - T^2$$

$$\mathcal{L}\{F''(t)\} = sT^2 - sT^2 e^{-s} - T^2 = T^2 (s - se^{-s} - 1)$$

$$\mathcal{L}\{F''(t)\} = T^2 (s - se^{-s} - 1)$$