HW 08

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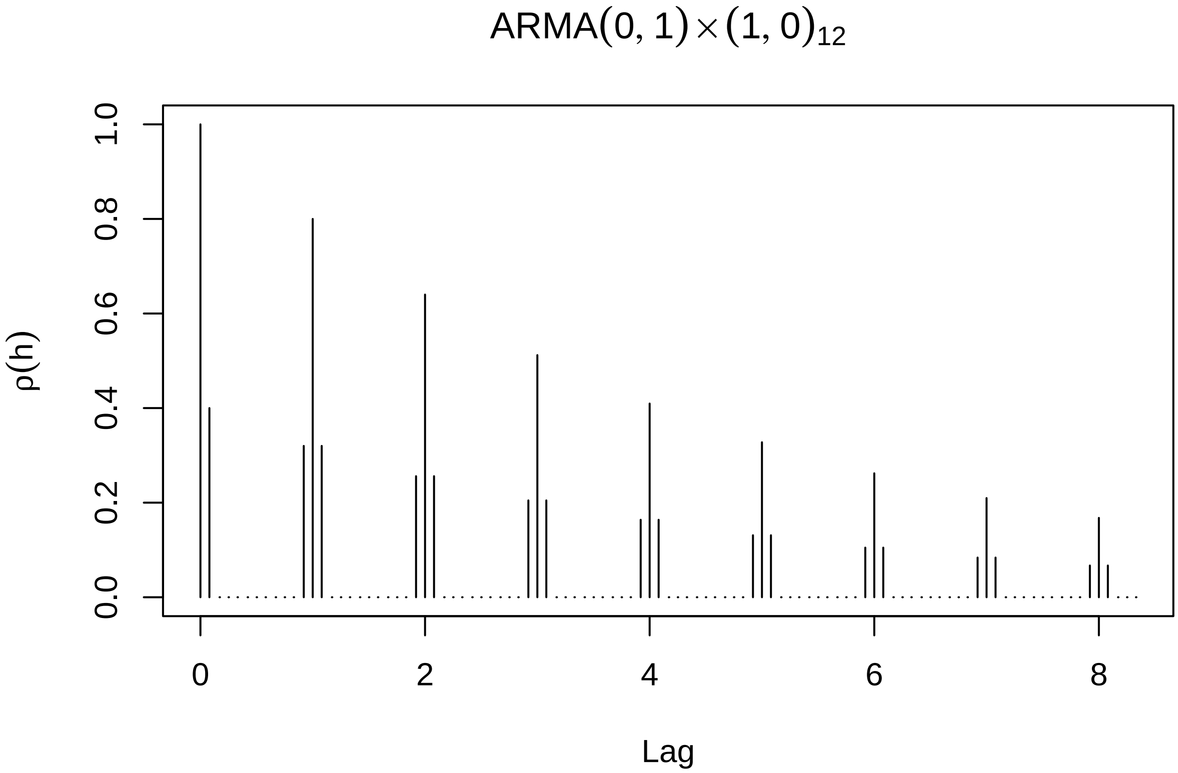
Due: 10/20/2022 11:59pm

Question 1.

Plot the theoretical ACF of the seasonal ARMA $(0,1) \times (1,0)_{12}$ model with $\Phi = 0.8$ and $\theta = 0.5$.

$$\rho(12k) = \Phi^k \implies \rho(12k) = 0.8^k$$
$$\rho(12k-1) = \rho(12k+1) = \frac{\theta}{1+\theta^2}\Phi^k = \frac{0.5}{1+0.5^2}0.8^k = 0.4(0.8^k)$$

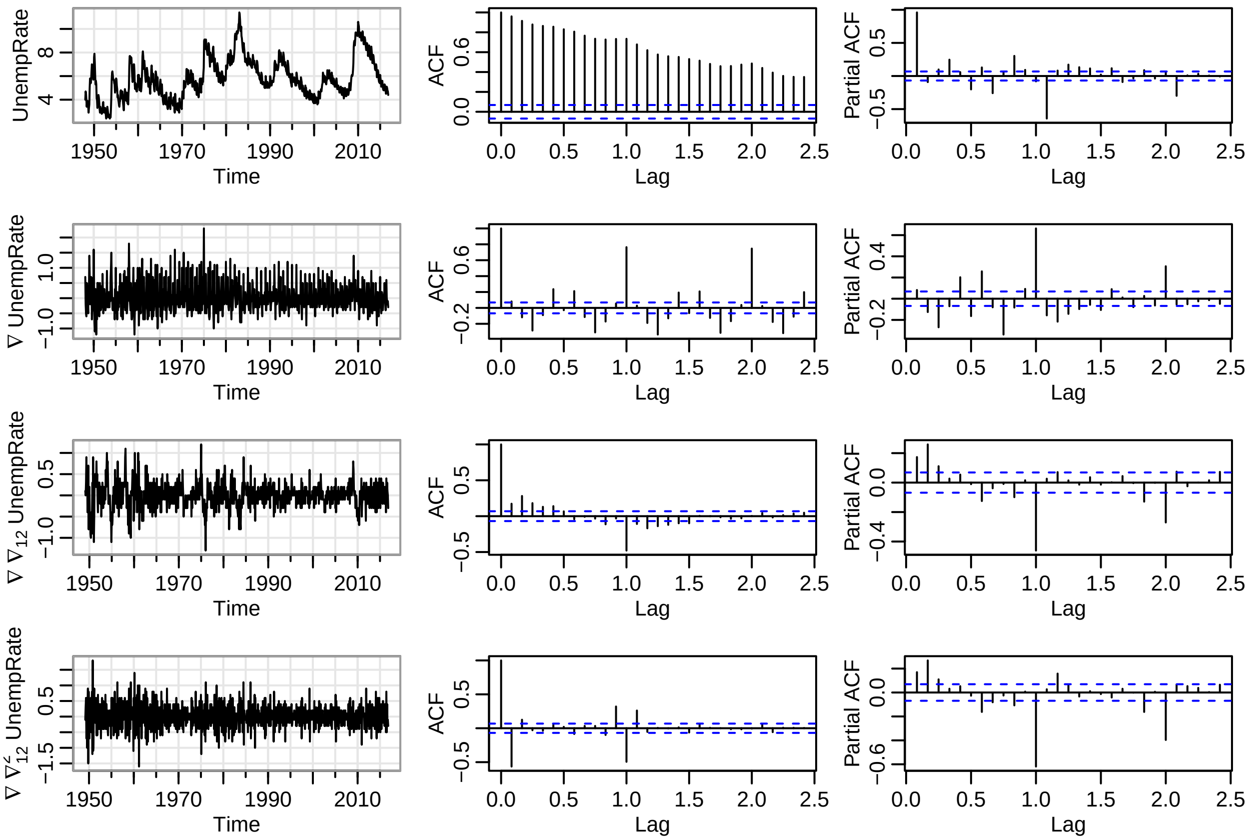
```r
#Creating data frame to save lags and ACF values
data1 = data.frame(matrix(data = rep(0,202), nrow = 101, ncol = 2))
data1 = data1 %>% rename(Lag = "X1") %>% rename(ACF = "X2")
data1$Lag = 0:100
#Computing the ACF values
for(i in 1:101){
  if(data1[i,1] %% 12 == 0){
    #p(12k)
    data1[i,2] = round(0.8^(data1[i,1]/12), 4)
  } else if(data1[i,1] %% 12 == 1){
    #p(12k+1)
    data1[i,2] = round(0.4*0.8^((data1[i,1]-1)/12), 4)
  } else if(data1[i,1] %% 12 == 11){
    #p(12k-1)
    data1[i,2] = round(0.4*0.8^((data1[i,1]+1)/12), 4)
  }
}
data1$Lag = round(data1$Lag / 12, 2)
#Plotting Theoretical ACF
plot(data1$Lag, data1$ACF, xlab = "Lag", ylab = expression(rho(h)), type = "h",
     main = TeX(r"($ARMA(0,1)\times(1,0)_{12}$)"))
```



Question 2.

Suggest a seasonal ARIMA model of your choice to unemployment data, `UnempRate` of `astsa` library.

```
par(mfrow = c(4,3))
#UnempRate
UnempRate = UnempRate
tsplot(UnempRate)
acf(UnempRate)
pacf(UnempRate)
#differenced UnempRate
tsplot(diff(UnempRate), ylab = TeX(r"(\nabla UnempRate)"))
acf(diff(UnempRate))
pacf(diff(UnempRate))
#differenced and seasonal differenced UnempRate
tsplot(diff(diff(UnempRate),12), ylab = TeX(r"(\nabla $\nabla_{12}$ UnempRate)"))
acf(diff(diff(UnempRate), 12))
pacf(diff(diff(UnempRate), 12))
#2nd difference and seasonal difference UnempRate
tsplot(diff(diff(diff(UnempRate),12)), ylab = TeX(r"(\nabla $\nabla_{12}^2$ UnempRate)"))
acf(diff(diff(diff(UnempRate),12)))
pacf(diff(diff(diff(UnempRate),12),12))
```



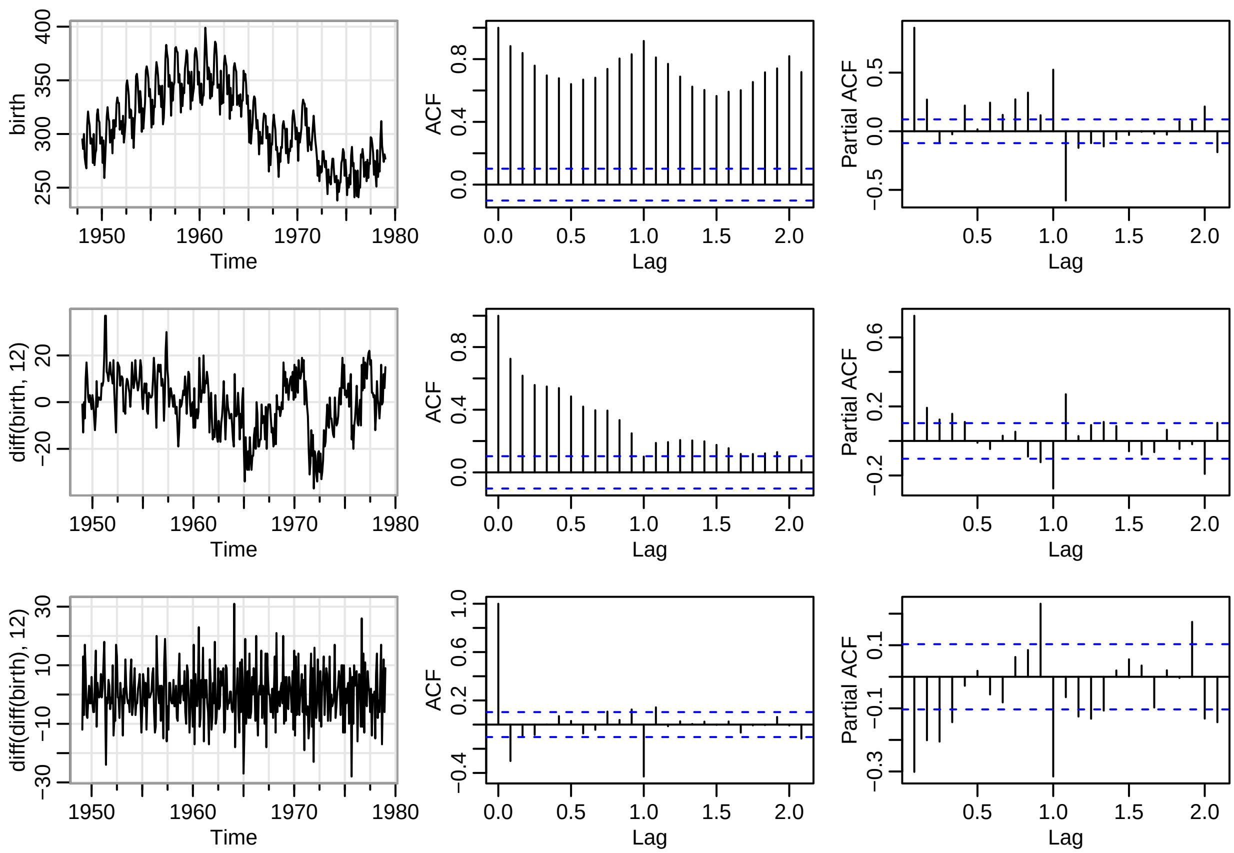
I would suggest a SARIMA$(0,1,1) \times (0,2,1)_{12}$ model for the `UnempRate` data. We can see from the original data that there is both a seasonal and non-seasonal trend. Therefore we will do both a seasonal and non-seasonal difference for the data. However, we can still see that there is a still a slight seasonal trend, so we will take one more seasonal difference. After doing this, we can see that the ACF cuts off after lag 1 and the PACF tails off for the non-seasonal part and ACF cuts off after lag 1 and the PACF tails off for the seasonal part too. That is why I would use the model I suggested above.

Question 3.

Suggest a seasonal ARIMA model of your choice to the U.S. Live Birth Series, `birth` of `astsa` library.

```
par(mfrow = c(3,3))
#birth
birth = birth
tsplot(birth)
acf(birth)
pacf(birth)
#seasonal difference of birth
tsplot(diff(birth, 12))
acf(diff(birth, 12))
pacf(diff(birth, 12))
#seasonal difference and non-seasonal difference of birth
tsplot(diff(diff(birth),12))
```

```r
acf(diff(diff(birth),12))
pacf(diff(diff(birth),12))
```



I would suggest a SARIMA$(0,1,1) \times (0,1,1)_{12}$ model for the `birth` data. We can see from the original data that there is both a seasonal and non-seasonal trend. Therefore we will do both a seasonal and non-seasonal difference for the data. After doing this, we can see that the ACF cuts off after lag 1 and the PACF tails off for the non-seasonal part and ACF cuts off after lag 1 and the PACF tails off for the seasonal part too. That is why I would use the model I suggested above.

Question 4.

An ARMA model has AR characteristic polynomial

$$(1 - 1.6z + 0.7z^2)(1 - 0.8z^{12})$$

(a) Is the model stationary? State your reasons.

```r
#Inputting the polynomial coefficients
m4 = c(1,-1.6,0.7,0,0,0,0,0,0,0,0,0,-0.8,1.28,-0.56)
#Computing the roots
polyroot(m4)
```

```
##  [1]  0.5093846+0.8822801i -0.8822801+0.5093846i -0.5093846-0.8822801i
##  [4]  0.8822801-0.5093846i  0.0000000+1.0187693i -1.0187693+0.0000000i
##  [7]  0.0000000-1.0187693i  1.0187693-0.0000000i -0.5093846+0.8822801i
## [10] -0.8822801-0.5093846i  0.5093846-0.8822801i  0.8822801+0.5093846i
## [13]  1.1428571+0.3499271i  1.1428571-0.3499271i
```

```r
#Finding the modulus of the roots
Mod(polyroot(m4))
```

```
##  [1] 1.018769 1.018769 1.018769 1.018769 1.018769 1.018769 1.018769 1.018769
##  [9] 1.018769 1.018769 1.018769 1.018769 1.195229 1.195229
```

```r
#Checking that all roots are outside of the unit circle.
min(Mod(polyroot(m4)))
```

```
## [1] 1.018769
```

The process is stationary as the modulus of all roots of the AR characteristic polynomial are outside of the unit circle.

(b) Identify the model as a certain seasonal ARIMA model.

- $\boxed{\text{SARIMA}(2,0,0)\times(1,0,0)_{12}}$ OR
- $\boxed{\text{SARMA}(2,0)\times(1,0)_{12}}$

Question 5.

Identify the following as certain multiplicative seasonal ARIMA models:

(a)
$$X_t = 0.5X_{t-1} + X_{t-4} - 0.5X_{t-5} + W_t - 0.3W_{t-1}$$

$$\begin{aligned}
X_t - 0.5X_{t-1} - X_{t-4} + 0.5X_{t-5} &= W_t - 0.3X_{t-1} \\
(1 - 0.5B - B^4 + 0.5B^5)X_t &= (1 - 0.3B)W_t \\
(1 - 0.5B)(1 - B^4)X_t &= (1 - 0.3B)W_t
\end{aligned}$$

$\boxed{\text{SARIMA}(1,0,1)\times(0,1,0)_4}$

(b)
$$X_t = X_{t-1} + X_{t-12} - X_{t-13} + W_t - 0.5W_{t-1} - 0.5W_{t-12} + 0.25W_{t-13}$$

$$\begin{aligned}
X_t - X_{t-1} - X_{t-12} + X_{t-13} &= W_t - 0.5W_{t-1} - 0.5W_{t-12} + 0.25W_{t-13} \\
(1 - B - B^{12} + B^{13})X_t &= (1 - 0.5B - 0.5B^{12} + 0.25B^{13})W_t \\
(1 - B)(1 - B^{12})X_t &= (1 - 0.5B)(1 - 0.5B^{12})W_t
\end{aligned}$$

$\boxed{\text{SARIMA}(0,1,1)\times(0,1,1)_{12}}$