CSE 1729 - Introduction to Principles of Programming Due: September 11, 2017

Problem Set 1

Making it easy to read your results

We will be using a specific labeling convention with Racket definition files to make grading/verification clearer and less error-prone.

This will involve the use of strings. A string in Scheme is a sequence of characters within quotation marks. We will use strings later for various things, but for now it is sufficient to know that a string evaluates to a string that looks like itself. So if you put a string in a Racket definition pane, then hit Run, that string will show up in the lower window as the result of its evaluation.

We will use this as follows: before the code from a problem, we will identify which problem it is, and for each test case we will put in a string identifying the test case. Here is a simple example: suppose question 3 asks you to write a square function in Scheme. What you put in the Racket definitions will be something like the following:

```
"problem 3a."
(define (square x)
  (* x x))
"tests"
"(square 4)"
(square 4)
"(square -5)"
(square -5)
"(square (square 3))"
(square (square 3))
```

What you (and we) will see in the lower window when you hit the Run button is:

```
"problem 3a."
"tests"
"(square 4)"
16
"(square -5)"
25
"(square (square 3))"
81
>
```

Now it is easy to see what your test results are. Your definitions should be in the same order as in the assignment, with tests within each question.

1. Re-write the following arithmetic expressions as Scheme expressions and show the result of the Scheme interpreter when invoked on your expressions.

   (a) $(22 + 42) \times (54 \times 99)$.

   (b) $((22 + 42) \times 54) \times 99$.

   (c) $64 \times 102 + 16 \times (44/22)$.

   (d) Is the expression in the following limerick, written by recreational mathematician James Mercer, correct?

      A dozen, a gross, and a score
      Plus three times the square root of four
      Divided by seven
      Plus five times eleven
      Is nine squared and not a bit more

      That is, if you evaluate the following equation as a Scheme expression with the logical operator $=$ for the equality symbol, does it evaluate to true (`#t`)?

$$\frac{12 + 144 + 20 + 3\sqrt{4}}{7} + (5 \times 11) = 9^2 + 0$$

      Note: You can use the built-in Scheme function `sqrt` to compute the square root of 4 as follows: `(sqrt 4)`

2. Reflect on the expressions above.

   (a) Of course, the first two expressions evaluate to the same number. In what sense are they different? How is this reflected in the Scheme expression?

   (b) In an unparenthesized infix arithmetic expression, like $3 + 4 * 5$, we rely on a *convention* to determine which operation we apply first (rules of precedence). Are rules of precedence necessary for arithmetic operations Scheme?

3. Write Scheme definitions for the functions below. Use the interpreter to try them out on a couple of test cases to check that they work, and include this output with your solutions.

   NOTE: Scheme provides built-in support for exponentiation (via the `expt` function, defined so that `(expt x y)` yields x^y). For the exercises below, however, please construct the functions $x \mapsto x^k$ using only $*$ and function application.

   (a) `cube`, the function $\texttt{cube}(x) = x^3$.

   (b) `p`, the polynomial function $\texttt{p}(x) = (x^5 + 11x^4 + 24x^3 - x + 21)^3$.
   (Hint: Of course it is possible to expand $(x^5 + 11x^4 + 24x^3 - x + 21)^3$ as a polynomial of degree 15 and write a Scheme function to compute this by brute force. You can avoid much of this computation by defining `p` in stages—first define the polynomial $q(x) = x^5 + 11x^4 + 24x^3 - x + 21$ as a Scheme function; now use this function to define `p`.)

   (c) Using your function `cube`, write the function $\texttt{tenth}(x) = x^{10}$.

   (d) Using the function `tenth`, write the function $\texttt{hundredth}(x) = x^{100}$. Recall that $100 = 10 \times 10$.

   (e) How might you check to see whether your `hundredth` function gives you the right answer?

(f) Reflect on your definition of `hundredth` above. What would have been the difficulty of defining this merely in terms of $*$?

4. Write and test the following functions that deal with points and lines in the Cartesian plane.

   (a) `(y-value x b m)`, a function of three parameters (an x value, a y-intercept b, and a slope m) that returns the y value of the line at that x, that is $mx + b$.

   (b) `(points-slope x1 y1 x2 y2)`, a function of four parameters (the x and y values of two points) that calculates the slope of a line through those points (x_1, y_1) and (x_2, y_2). You may assume that the two points are distinct. (Note: this function is not required to work if the slope of the line is undefined.)

   (c) `(points-intercept x1 y1 x2 y2)`, a function of four parameters (the x and y values of two points) that calculates the y-intercept of a line through points (x_1, y_1) and (x_2, y_2). You may assume that the two points are distinct. (Note: this function is not required to work if the line does not have a y-intercept.)

   (d) `(on-parallels? x1 y1 x2 y2 x3 y3 x4 y4)`, a function of eight parameters (the x and y values of four points), returns true if the line through points (x_1, y_1) and (x_2, y_2) is parallel to the line through points (x_3, y_3) and (x_4, y_4). You may assume that points (x_1, y_1) and (x_2, y_2) are distinct, as are (x_3, y_3) and (x_4, y_4). (Note: this function can use any of the functions defined above, but should give a correct answer even in the cases where `points-slope` or `points-intercept` would fail.

5. Remember the *Quadratic formula*, which can be used to find the roots of a quadratic eguation? For a quadratic equation $ax^2 + bx + c = 0, a \neq 0$, the formula is

   $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

   Notice that this gives us two different roots (because of the $\pm$) whenever $b^2 - 4ac \neq 0$.

   Write the following Scheme functions.

   (a) `(root1 a b c)` that gives us the root corresponding to the plus in the $\pm$ in the quadratic formula (that is, calculate $\frac{-b+\sqrt{b^2-4ac}}{2a}$).

   (b) `(root2 a b c)` that gives us the root corresponding to the minus in the $\pm$ in the quadratic formula (that is, calculate $\frac{-b-\sqrt{b^2-4ac}}{2a}$).

   (c) `(number-of-roots a b c)` which calculates the number of distinct roots to the equation $ax^2 + bx + c = 0, a \neq 0$ (which will either be 1 or 2).

   (d) `(real-roots? a b c)` is a boolean function that evaluates to `#t` when the roots of $ax^2 + bx + c = 0, a \neq 0$ are real numbers. Note that you do not have to calculate the roots to determine whether they are real or complex numbers.

   Note: there are some common calculations done in the above functions; it may make sense to write some auxiliary functions to make your code simpler.