CSE1729 - Introduction to Programming March 30, 2016

Prelim 2

This is a 50 minute exam, commencing at 10:10am and finishing at 11:00am. You may not use any reference material during the exam, including books or notes. You may not use any electronics equipment (cell phones, calculators, computers, etc.) during the exam. *Please read every question carefully before answering* and express your answers as completely as you can.

There are 4 questions on the exam, each worth 10 points; **you may choose *any three* to complete**. Thus the highest score that can be achieved on the exam is 30 points. Please indicate which 3 questions you wish to be graded in the check boxes next to the question numbers below.

Name: ______________________________

Question	Grade?	Score
1		
2		
3		
4		
Total		

TREE CONVENTIONS

Several problems on the exam concern binary trees. As in class, we consider binary trees with the property that each node stores—in addition to its two subtrees—an integer. Recall the convention that we used for such trees: each node is represented by a list

```
(v left right),
```

where `v` is a value, and `left` and `right` are its two subtrees.

Please adopt this convention in your responses to the exam questions. You may make free use of the "convenience" functions:

```
(define (maketree v left-tree right-tree)
   (list v left-tree right-tree))

(define (value T) (car T))
(define (left T)  (cadr T))
(define (right T) (caddr T))
```

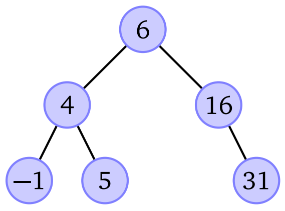
The first of these functions builds a tree from a value and a pair of trees; the following three functions extract, from a tree T, the value of the root and its left and right subtrees, respectively. Finally, we used the SCHEME empty list to represent the empty tree.

We have focused on two important additional conditions on binary trees:

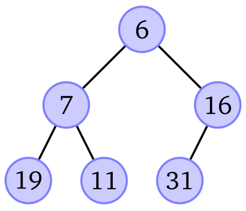
Binary search trees. A binary search tree is a binary tree with the extra condition that for every node n of the tree, all nodes in the left subtree of n have values strictly less than the value of n, while all nodes in the right subtree have values strictly larger than the value of n.

Heaps. A heap is a binary tree with the extra condition that for every node n of the tree, the value of each child of n is strictly larger than the value of n.

For the sake of simplicity, you may assume throughout the exam that all elements appearing in (and inserted in) trees, heaps, and lists are distinct—there are no duplicates.



A binary search tree



A heap

Figure 1: On the left, a tree that has binary search tree structure; on the right, a tree that has heap structure.

1. (**10 points.**) Define SCHEME functions with the following specifications.

   (a) (**4 points.**) The built-in SCHEME function `max` returns the maximum of two numbers. Define a SCHEME function `listmax` which takes a list of numbers as input, and returns their maximum.

   (b) (**2 points.**) Define a SCHEME function `filter` which takes a list L and a number k as inputs and returns a list containing the numbers in L that are less than k. For example `(filter '(1 5 2 6 3 7) 4)` should return `(1 2 3)`, as these are the elements of `(1 5 2 6 3 7)` that are less than 4.

(c) (**4 points.**) Define a SCHEME function `treesize` which, given a binary tree, returns the *number* of nodes in the tree. (Note that both of the trees in Figure 1 have 6 nodes.)

2. (**10 points.**) Recall that a *heap* is a binary tree with the property that for every node of the tree, its value is (strictly) smaller the values of its children. Thus, the smallest value of a heap always appears at the root.

   (a) (**5 points.**) Given a heap `H` and a number `v`, define a SCHEME function that properly inserts `v` into `H`; your function should return the resulting heap. Use the alternating subtree heuristic we discussed in class, so that elements are inserted into subtrees by insertion into the left subtree after *exchanging* the two subtrees.

   (b) (**1 points.**) What's the point of the "alternating subtree heuristic" discussed above? (That is, why not just insert the element into the left subtree?)

(c) (**1 points.**) How does the "alternating subtree heuristic" compare with the policy of inserting the element into the *smaller* of the two subtrees?

(d) (**3 points.**) Define a SCHEME function `heapify` which, given a list of numbers (call it `numbers`), constructs and returns a heap from the numbers in the list.

3. (**10 points.**) Consider the problem of finding the *kth smallest element* of a given list of numbers—that is, the number that would appear in the kth position if the list was placed in sorted order. Of course, one obvious way to solve the problem is to sort the list and then return the kth element.

However, there is a nice recursive algorithm for this process which we will call **QuickSelect**; its operation is very like **QuickSort** and, in general, it finds the kth element of an (unsorted) list more quickly than completely sorting the list.

To recap: Given a number k and a list L, **QuickSelect** should return the kth smallest element of L (that is, the kth element of the list that would result if L were sorted). A detailed description of the **QuickSelect** algorithm:

QuickSelect(k, L):

- Select a *pivot* element p from the list L. (p may be any element of the list.)
- **Partition** the list L around the element p. This results in two lists, *Small* and *Big*: the list *Small* contains all elements smaller than p; the list *Big* contains all elements larger than p.
- Let ℓ be the number of elements in *Small*. (Note then that p is the $\ell + 1$st element of the original list when placed in sorted order because there are exactly ℓ elements less than p.)
- If $\ell + 1 = k$, return p.
- If $k \leq \ell$, return **QuickSelect**(*Small*, k).
- If $\ell + 1 < k$, return **QuickSelect**(*Big*, $k - \ell - 1$).

(a) (**4 points.**) Give a full implementation of `partition`, a SCHEME function which takes a list `numbers` and a pivot value `pivot`. It should return a pair, consisting of two lists `smaller` and `larger` which contain the elements that are smaller than and larger than the pivot, respectively. (N.b., this is exactly the same function used in **QuickSort**!)

(b) (**4 points.**) Give a full implementation of **QuickSelect**. You may use the `partition` function from the previous problem, even if you did not solve it. (Hint: I recommend that you start with a `let*` statement to set the pivot to be the first element of the list, call `partition`, and determine the length of the part of the partition containing "small" elements—you may use the built-in `length` function for this purpose.)

(c) (**2 points.**). Note that both **QuickSort** and **QuickSelect** use a pivot to partition the elements of the list. However, there is an important difference in how they recurse on the pieces. Explain, and indicate why this might make **QuickSelect** faster than **QuickSort**.

4. (**10 points.**) Counting small values in ordered trees.

   (a) (**3 points.**) Define a SCHEME function `countlessthan` which, given a binary search tree T and a number k, returns the *number of elements* in T less than or equal to k. Observe that if $k <$ `value`(T) then none of the elements in `(right T)` are less than k—use this idea to optimize your code.

(b) (**3 points.**) Consider a fancy adaptation of the binary search tree concept in which each node additionally stores the *size* of the whole tree rooted at the node (*size* here means the number of nodes). Thus, each node actually stores two numbers: the data value (which is an arbitrary number) and a second number equal to the size of the tree rooted at the node. We can adapt our node data structure to maintain this new information by adding to the list: in particular, each node is now represented as

```
(value left right totalsize)
```

and we may add the "convenience" function

```
(define (size T) (if (null? T) 0 (cadddr T)))
```

(The other convenience functions can retain their usual definitions, except for `maketree`, which now takes four parameters.) Of course, in order to keep the size data up to date, they have to be updated appropriately during insertion. Show how to rewrite the standard binary search tree `insert` function to additionally update the size registers. Specifically, define a SCHEME function `insert` which, given a binary search tree T with correct size data (as described above) and a value v, returns the tree that results by inserting v into T—of course, the returned tree should have correct size data!

(c) (**4 points.**) Finally, write a SCHEME function `countlessthan` which, given a fancy binary search tree (in which all the size registers indeed correctly hold the tree sizes) and a number k, returns the number of elements less than k. Show how to use the `size` function to give a fast implementation that avoids exploring certain subtrees.

Scratch Space

Scratch Space