The Chronicles of Entangland: the foundation of the SC town

Alba Cervera-Lierta

Qiskit Europe Hackathon

May 26th 2021

the matter lab

VELUT ÆVO ARBOR

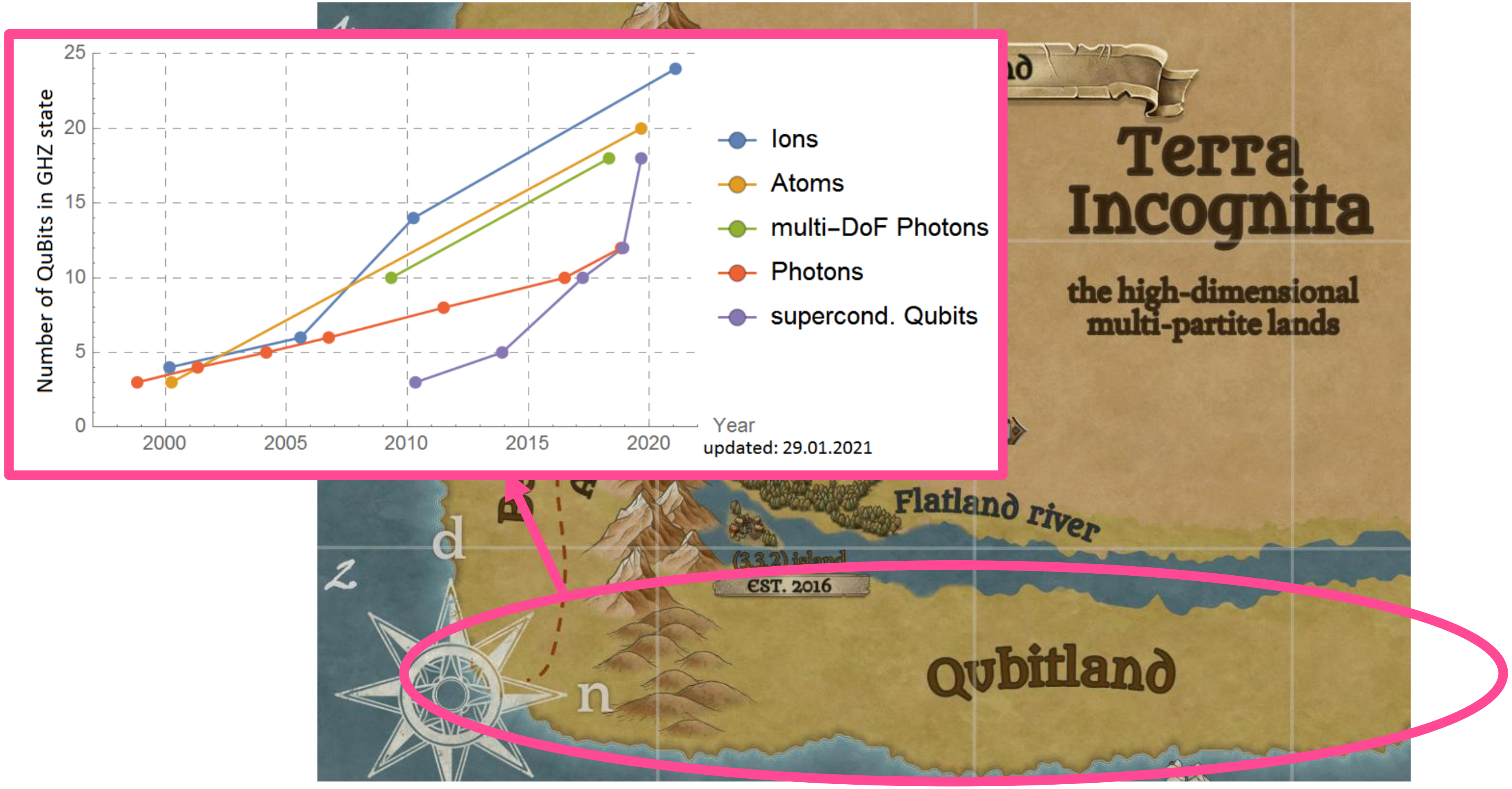
Index

Entangland

Local dimension d

Number of parties n

Entangland: qubitland



References:
https://mariokrenn.wordpress.com/
2021/01/29/reference-list-for-records-in-large-entanglement-generation-number-of-qubits-in-ghz-states/

Entangland: high-dimensional regions

Generation and confirmation of a (100 × 100)-dimensional entangled quantum system
Mario Krenn et. al., PNAS **111** (17) 6243-6247 (2014)

Experimental Greenberger–Horne–Zeilinger entanglement beyond qubits
Manuel Erhard et. al., Nature Photonics **12**, pages759–764 (2018)

Multi-photon entanglement in high dimensions,
Mehul Malik *et al. Nature Photon* **10,** 248–252 (2016)

Experimental creation of multi-photon high-dimensional layered quantum states
Xiao-Min Hu et. al., npj Quantum Information **6**, 88 (2020)

A. Cervera-Lierta, M. Krenn, A. Aspuru-Guzik, A. Galda, arXiv:2104.05627 [quant-ph]

GHZ plain

$d = 3$

$n = 3$

The Greenberger-Horne-Zeilinger state:

$$|GHZ\rangle = \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} |kkk\rangle$$

For qubits ($d = 2$)

$$|GHZ\rangle = \frac{1}{\sqrt{2}} (|000\rangle + |111\rangle)$$

For qutrits ($d = 3$)

$$|GHZ\rangle = \frac{1}{\sqrt{3}} (|000\rangle + |111\rangle + |222\rangle)$$

A. Cervera-Lierta, M. Krenn, A. Aspuru-Guzik, A. Galda, arXiv:2104.05627 [quant-ph]

GHZ plain

What are the physics in the GHZ plain?

Quantum foundations: non-locality tests (GHZ contradictions)

	Theory	Experiment
Qubits	*Bell's theorem without inequalities,* D. M. Greenberger, A. Horne, A. Zeilinger American Journal of Physics 58, 1131 **(1990)**	*Experimental test of quantum nonlocality in three-photon Greenberger–Horne–Zeilinger entanglement* J-W Pan, D. Bouwmeester, M. Daniell, H. Weinfurter, A. Zeilinger Nature 403, 515–519 **(2000)**
Qudits	*Rotational covariance and Greenberger-Horne-Zeilinger theorems for three or more particles of any dimension* J. Lawrence Phys. Rev. A 89, **(2014)**	

High-dimensional lands

What are the physics in the High-dimensional lands?

Applications: Quantum Error Correction

Enhanced Fault-Tolerant Quantum Computing in d-Level Systems
E. T. Campbell, Phys. Rev. Lett. **113**, 230501 (2014).

Magic-State Distillation in All Prime Dimensions Using Quantum Reed-Muller Codes
E. T. Campbell, H. Anwar, D. E. Browne, Phys. Rev. X **2**, 041021 (2012).

Optimal quantum error correcting codes from absolutely maximally entangled states
Z. Raissi, C. Gogolin, A, Riera, A. Acín, J. Phys. A: Math. Theor. **51** 075301 (2018)

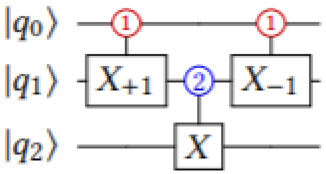
High-dimensional lands

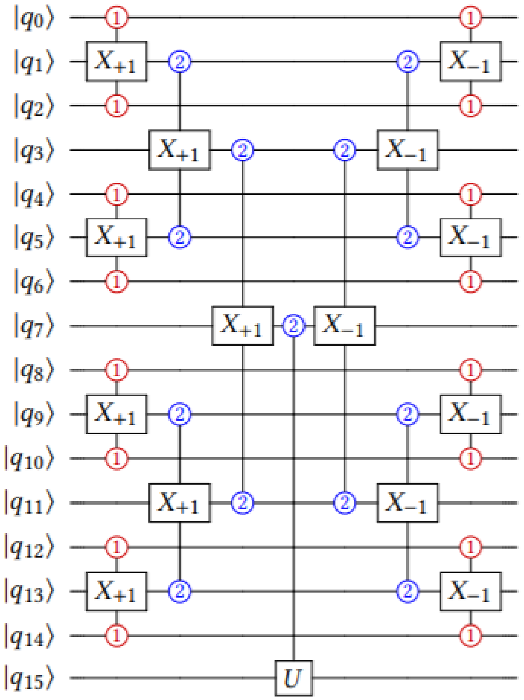
What are the physics in the High-dimensional lands?

Applications: Quantum Computation

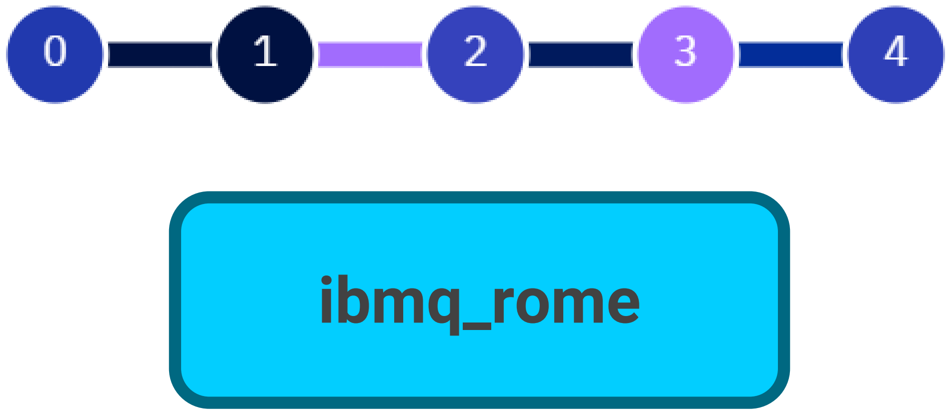
Reduce the circuit depth of a qubit Toffoli gate by manipulating states in the 3rd level.

Asymptotic improvements to quantum circuits via qutrits
P. Gokhale, J. M Baker, C. Duckering, N. C Brown, K. R Brown, F. T Chong,
Proceedings of the 46th International Symposium on Computer Architecture , 554–566 (2019).





The vessel

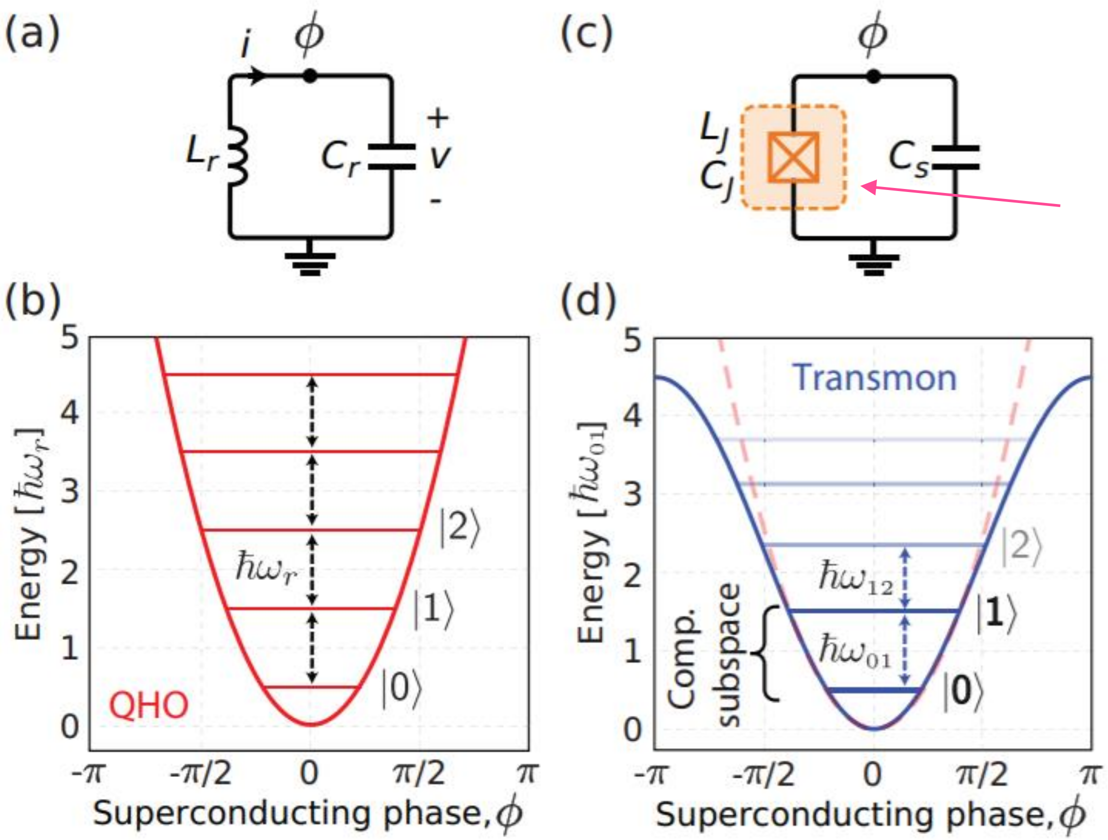


	Q_0	Q_1	Q_2	Q_3	Q_4
Qutrit $\|0\rangle \leftrightarrow \|1\rangle$ frequency, $\omega_{01}/2\pi$ (GHz)	4.969	4.770	5.015	5.259	4.998
Qutrit $\|1\rangle \leftrightarrow \|2\rangle$ frequency, $\omega_{12}/2\pi$ (GHz)	4.631	4.443	4.677	4.926	4.658
Lifetime $T_1^{\|1\rangle\to\|0\rangle}$ (μs)	90	102	44	70	82
Echo time $T_{2\mathrm{Echo}}$, $\|1\rangle/\|0\rangle$ (μs)	71	83	86	159	138
Readout error	2.8e-2	3.0e-2	3.3e-2	3.2e-2	3.8e-2
Prob. Prep. $\|0\rangle$ Meas. $\|1\rangle$	2.3e-2	2.3e-2	1.5e-2	1.3e-2	3.2e-2
Prob. Prep. $\|1\rangle$ Meas. $\|0\rangle$	3.4e-2	3.8e-2	5.1e-2	5.1e-2	4.4e-2
X gate error	2.5e-4	2.2e-4	4.3e-4	3.5e-4	3.5e-4
$u2$ gate duration (ns)	36	36	36	36	36

	[0, 1]	[1, 0]	[1, 2]	[2, 1]	[2, 3]	[3, 2]	[3, 4]	[4, 3]
CNOT gate error	7.4e-3	7.4e-3	1.8e-2	1.8e-2	9.7e-3	9.7e-3	9.5e-3	9.5e-3
CNOT gate duration (ns)	320	356	1109	1145	377	341	476	512

TABLE III. Calibration data for `ibmq_rome`.

Superconducting transmon ~~qubits~~ qudits



Non-linear inductance (Josephson Junction)

Quantum Harmonic Oscillator

Quantum anharmonic Oscillator

Transmons contain more than two energy levels.

By finding the proper frequency transitions we can address and manipulate high-dimensional states.

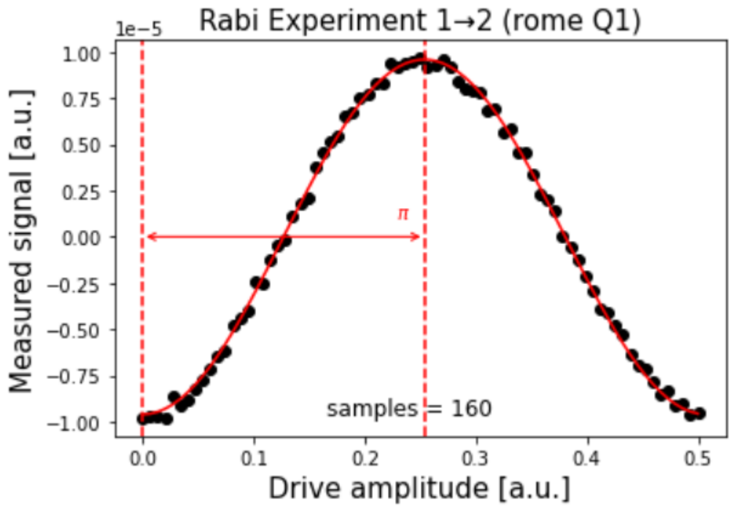
Philip Krantz et. al., Applied Physics Reviews **6**, 021318 (2019)

Pulse-level manipulation

1. Find the $\pi_{1\rightarrow 2}$ pulses by running a Rabi experiment.

```
def rabi_12_schedules(qubit, drive_samples, drive_sigma, drive_amp):
    rabi_12_sched = []
    for drive_amps in drive_amp:
        base_12_pulse = Gaussian(duration=drive_samples,
                                 sigma=drive_sigma,
                                 amp=drive_amps,
                                 name='Rabi_12')
        rabi_12_pulse = apply_sideband(base_12_pulse, f01[qubit], f12[qubit], drive_samples)

        sched = Schedule(name='Rabi_Q{}'.format(qubit))
        sched |= inst_map.get('x', qubits=[qubit]) # 0 -> 1
        sched |= Play(rabi_12_pulse, DriveChannel(qubit)) << sched.duration # 1 -> 2 Rabi
        sched |= inst_map.get('measure', qubit) << sched.duration
        rabi_12_sched.append(sched)
    return(rabi_12_sched)
```




```
Pi amplitude 1 → 2 Q1: 0.25374164783066344
```

```
rabi_12_sched = rabi_12_schedules(QQ[i], drive_samples, drive_sigma, drive_amp)
rabi_12_program = assemble(rabi_12_sched, backend, shots=shots, meas_level=1, meas_return='avg')
rabi_12_job = backend.run(rabi_12_program)
```

Pulse-level manipulation

1. Find the $\pi_{1\to2}$ pulses by running a Rabi experiment.

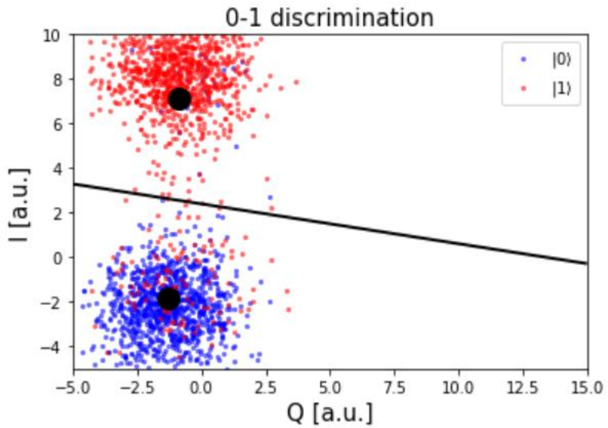
2. Define the gates in the (12) subspace using the $\pi_{1\to2}$ pulse

```
def def_gate_12(name, inst_map, qubits, drive_samples, amp, drive_sigma, beta, freq01, freq12):
    pulse_base = Drag(duration=drive_samples,
                      amp=amp,
                      sigma=drive_sigma,
                      beta=beta,
                      name=name)
    gate_pulse = apply_sideband(pulse_base, freq01*GHz, freq12*GHz, drive_samples)

    # add them to inst_map
    inst_map.has(name, qubits=(qubits, ))
    # custom schedule
    sched = Schedule(name = name)
    sched |= Play(gate_pulse, DriveChannel(qubits))
    # remove if already exists (to correctly rewrite)
    if inst_map.has(name, qubits=(qubits, )):
        inst_map.remove(name, qubits=(qubits, ))
    inst_map.add(name, qubits=(qubits, ), schedule=sched)
```

```
def_gate_12('Xp_12', inst_map, QQ_total[i], drive_samples, pi_amp_12[i], drive_sigma, beta_12[i], f01[i], f12[i])
def_gate_12('X90p_12', inst_map, QQ_total[i], drive_samples, pi_amp_12[i]/2.0, drive_sigma, beta_12[i], f01[i], f12[i])
```

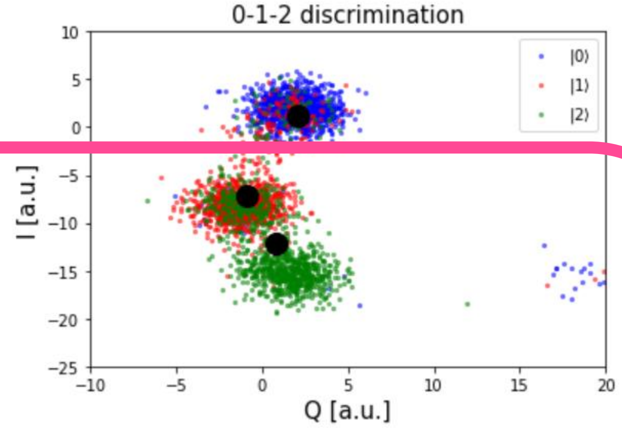
Measurement protocol



Option 1: Build a 0-1-2 discriminator

🙂 The same experiment (Schedule) can measure all basis elements.

🙁 The centroids move with time (need to recalibrate often) and overlap between them (high assigment error)



Option 2: "Lower" the basis elements and measure the |000> state

$$X_+ = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \qquad X_- = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

e.g. to measure the state $|012\rangle$, we apply at the end of the circuit the gate X_+ on qutrit 3 and X_- on qutrit 2.

🙂 We can use the default qubit discriminator (well calibrated by IBMQ)

🙁 We need a single experiment to measure each basis element

Plots: Qiskit textbook

The map

Find the optimal quantum circuit with the available resources.

Gate set

Gate	Matrix
$R_y^{(01)}(\theta)$	$\begin{pmatrix} \cos(\theta/2) & -\sin(\theta/2) & 0 \\ \sin(\theta/2) & \cos(\theta/2) & 0 \\ 0 & 0 & 1 \end{pmatrix}$
$R_y^{(12)}(\theta)$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta/2) & -\sin(\theta/2) \\ 0 & \sin(\theta/2) & \cos(\theta/2) \end{pmatrix}$
$R_x^{(12)}(\theta)$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta/2) & -i\sin(\theta/2) \\ 0 & -i\sin(\theta/2) & \cos(\theta/2) \end{pmatrix}$

(a) Single-qutrit gates.

Control	Target	Output
$\|0\rangle$	$\|0\rangle$	$\|00\rangle$
$\|0\rangle$	$\|1\rangle$	$\|01\rangle$
$\|0\rangle$	$\|2\rangle$	$\|02\rangle$
$\|1\rangle$	$\|0\rangle$	$\|10\rangle$
$\|1\rangle$	$\|1\rangle$	$\|11\rangle$
$\|1\rangle$	$\|2\rangle$	$i\|12\rangle$
$\|2\rangle$	$\|0\rangle$	$a\|20\rangle + b\|21\rangle$
$\|2\rangle$	$\|1\rangle$	$b^*\|20\rangle + c\|21\rangle$
$\|2\rangle$	$\|2\rangle$	$e^{i\varphi}\|22\rangle$

(b) Truth table IBM default CNOT gate.

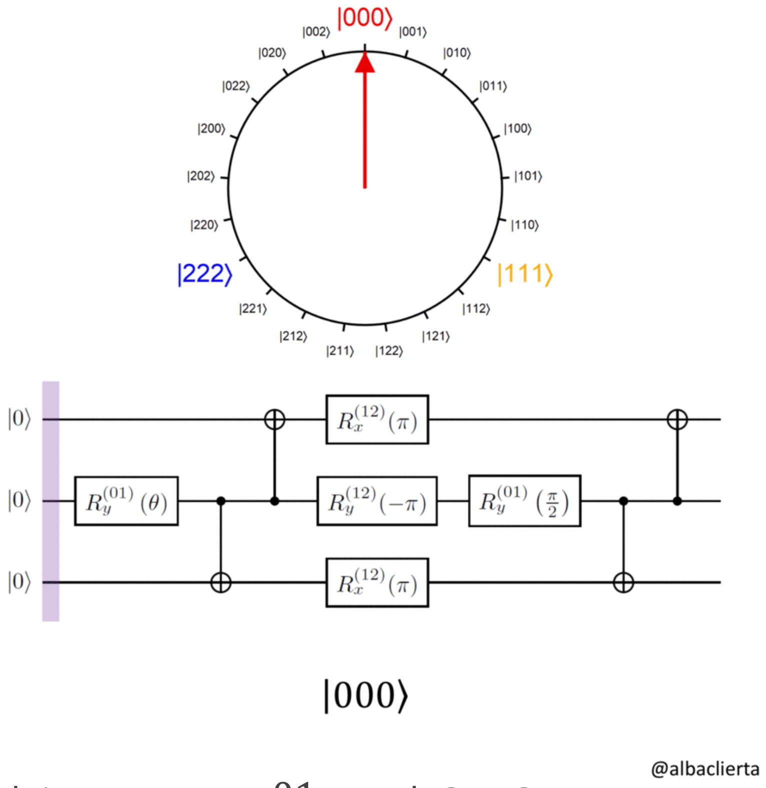
It doesn't act as a perfect qubit CNOT when the control is in the |2> state

$R_y^{(01)}(\theta) \equiv U3(\theta, 0, 0)$

$R_y^{(12)}(\theta)$ and $R_x^{(12)}(\theta)$ are defined by us using the $\pi_{1\to 2}$ pulse

CNOT is the default qubit CNOT provided by IBM, but…

The qutrit GHZ circuit



Qubit gates: R_y^{01} and CNOT*.

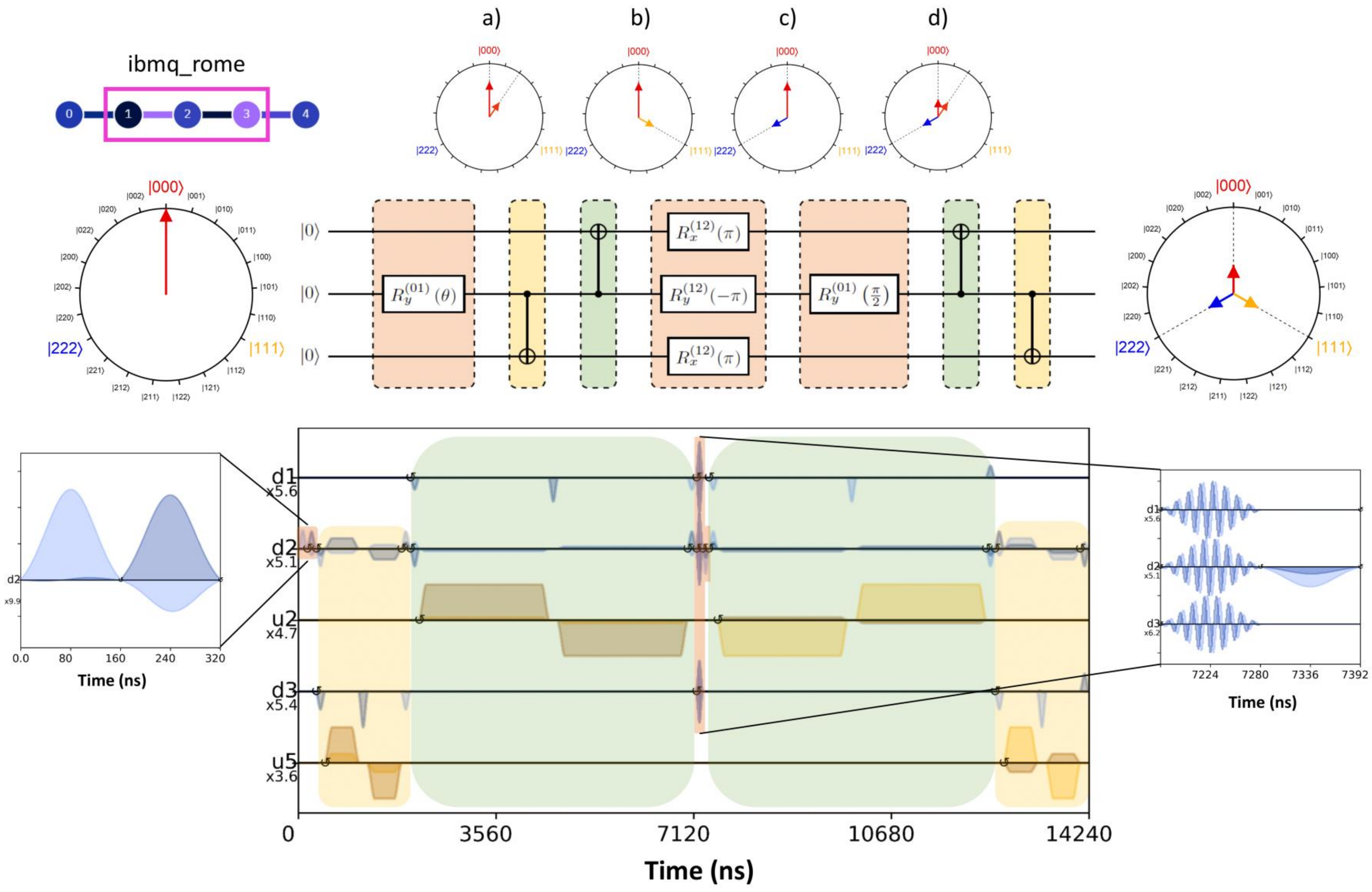
Qutrit gates: $R_\alpha^{(12)}(\theta) = e^{-i\frac{\theta}{2}\sigma_\alpha^{(12)}}$

We search for the optimal circuit (in terms of gates and circuit depth) to generate the GHZ state using only:

$$R_{x,y}^{01}(\pm\pi),\ R_{x,y}^{01}\left(\pm\frac{\pi}{2}\right),\ R_{x,y}^{12}(\pm\pi),\ R_{x,y}^{12}\left(\pm\frac{\pi}{2}\right),\ CNOT^*$$

And allowing only one gate out of this set: $R_y^{01}\left(2\tan^{-1} 1/\sqrt{2}\right)$ to create the $1/\sqrt{3}$ coefficent.

Pulse Schedule and circuit



The entanglement astrolabe

Certify the generation of a high-dimensional multipartite entangled quantum state.

Tomography

To compute the fidelity w.r.t. the GHZ state, we only need to measure the diagonal terms + 3 off-diagonal terms of the density matrix:

$$\mathrm{Tr}\left(\rho|GHZ\rangle\langle GHZ|\right)$$

$$F_{exp} = \frac{1}{3}\left(\sum_{i=0}^{2}\langle iii|\rho|iii\rangle + 2\sum_{\substack{i,j=0\\ i<j}}^{2}\mathrm{Re}\langle iii|\rho|jjj\rangle\right)$$

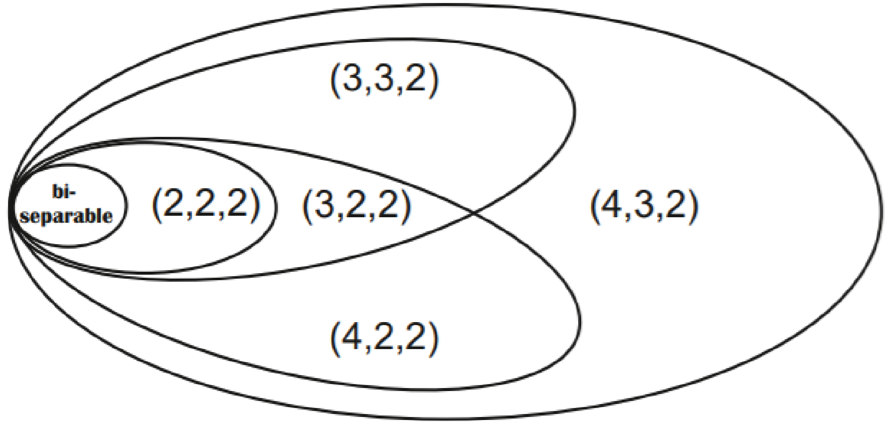
$$\langle 000|\rho|111\rangle,\ \langle 000|\rho|222\rangle \text{ and } \langle 111|\rho|222\rangle$$

We have to project into the $\sigma_{x,y}^{(ij)}$ basis to compute the expectation values.

$$\begin{aligned}\mathrm{Re}\left(\langle ijk|\rho|lmn\rangle\right) = \frac{1}{8}\Big(&\langle\sigma_x^{(il)}\sigma_x^{(jm)}\sigma_x^{(kn)}\rangle\\ &-\langle\sigma_y^{(il)}\sigma_y^{(jm)}\sigma_x^{(kn)}\rangle\\ &-\langle\sigma_y^{(il)}\sigma_x^{(jm)}\sigma_y^{(kn)}\rangle\\ &-\langle\sigma_x^{(il)}\sigma_y^{(jm)}\sigma_y^{(kn)}\rangle\Big)\end{aligned}$$

$$\begin{aligned}\mathrm{Im}\left(\langle ijk|\rho|lmn\rangle\right) = \frac{1}{8}\Big(&\langle\sigma_y^{(il)}\sigma_y^{(jm)}\sigma_y^{(kn)}\rangle\\ &-\langle\sigma_x^{(il)}\sigma_x^{(jm)}\sigma_y^{(kn)}\rangle\\ &-\langle\sigma_x^{(il)}\sigma_y^{(jm)}\sigma_x^{(kn)}\rangle\\ &-\langle\sigma_y^{(il)}\sigma_x^{(jm)}\sigma_x^{(kn)}\rangle\Big)\end{aligned}$$

Entanglement witness



$$F_{max} = \max_{\sigma \in (3,3,2)} \mathrm{Tr}\left(\sigma |GHZ\rangle\langle GHZ|\right)$$

$$F_{max} = \max_{\mathrm{rank}(\sigma_{\bar{A}})} \mathrm{Tr}\left(\sigma |\psi\rangle\langle\psi|\right) = \sum_{i=1}^{\xi} \lambda_i^2.$$

What is the maximal fidelity achievable by a state with Schmidt Rank (3,3,2)?

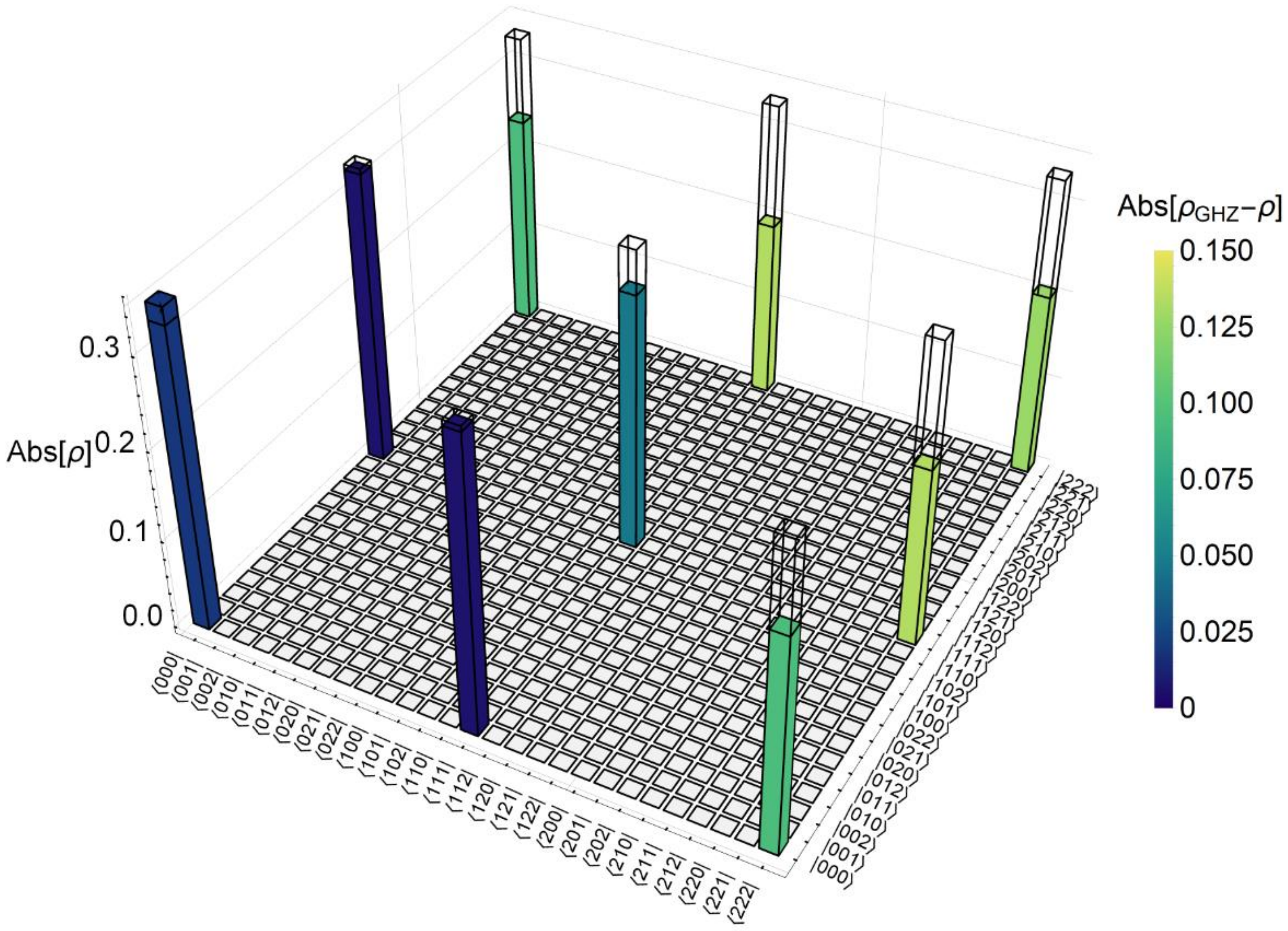
$$F_{max} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

If we measure a fidelity > 66%, we have a genuine 3-dimensional 3-partite entangled state.

M. Huber, J. I. de Vicente, Phys. Rev. Lett. **110**, 030501 (2013)

The foundation of Superconducting circuits town

Results



$$F_{raw} = 0.69 \pm 0.02$$

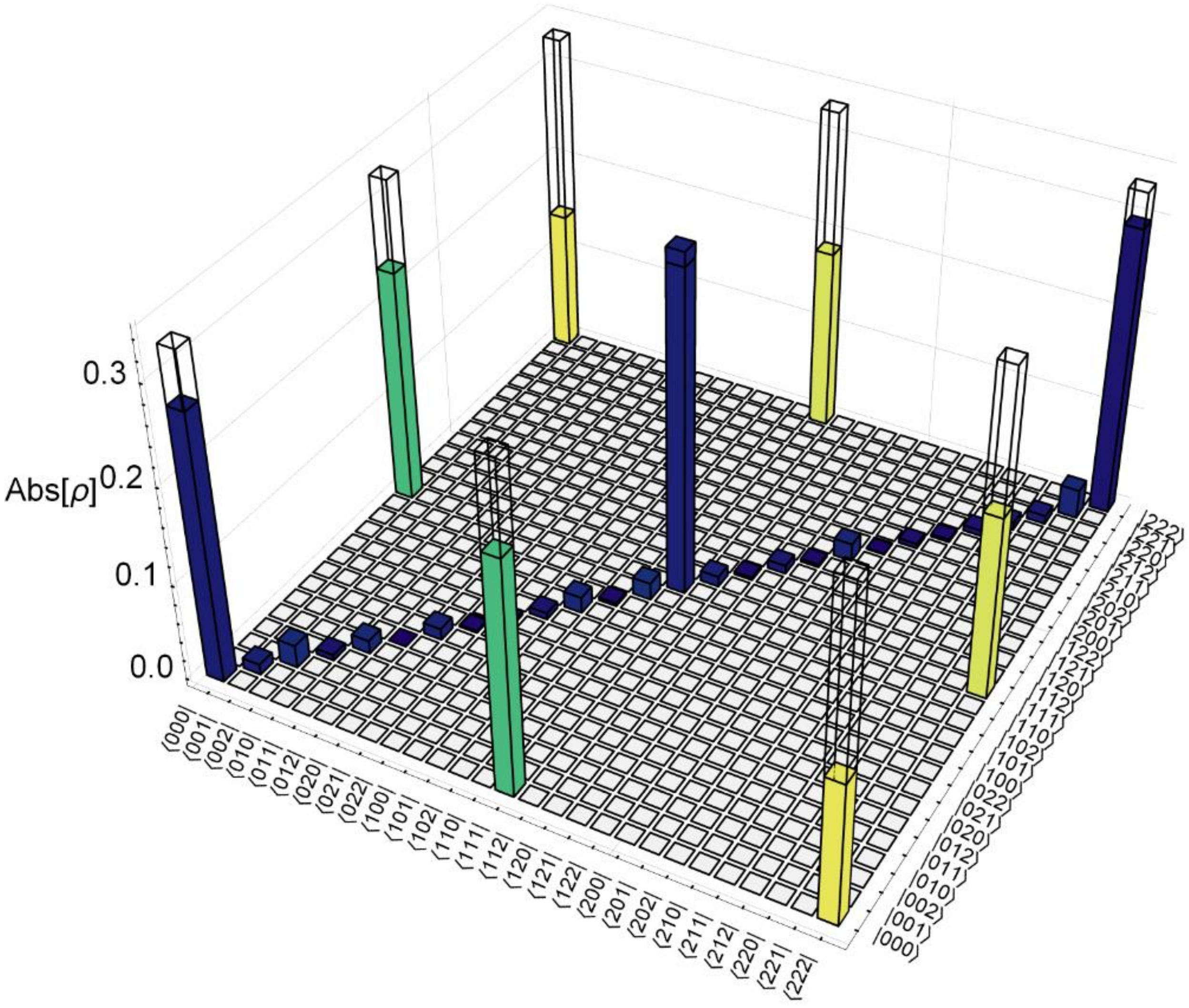
$$F_{exp} = 0.78 \pm 0.01$$

(with measurement error mitigation)

Rome [3,2,1] transmons

A fidelity $> \frac{2}{3}$ corresponds to a genuine three-partite high-dimensional state.

Fresh results! (May 17th)



$$F_{raw} = 0.69 \pm 0.01$$

$$F_{mit} = 0.76 \pm 0.01$$

Rome [4,3,2] transmons

(We also resport up to 72% raw fidelity in other devices like Casablanca)

arXiv v2 comming soon

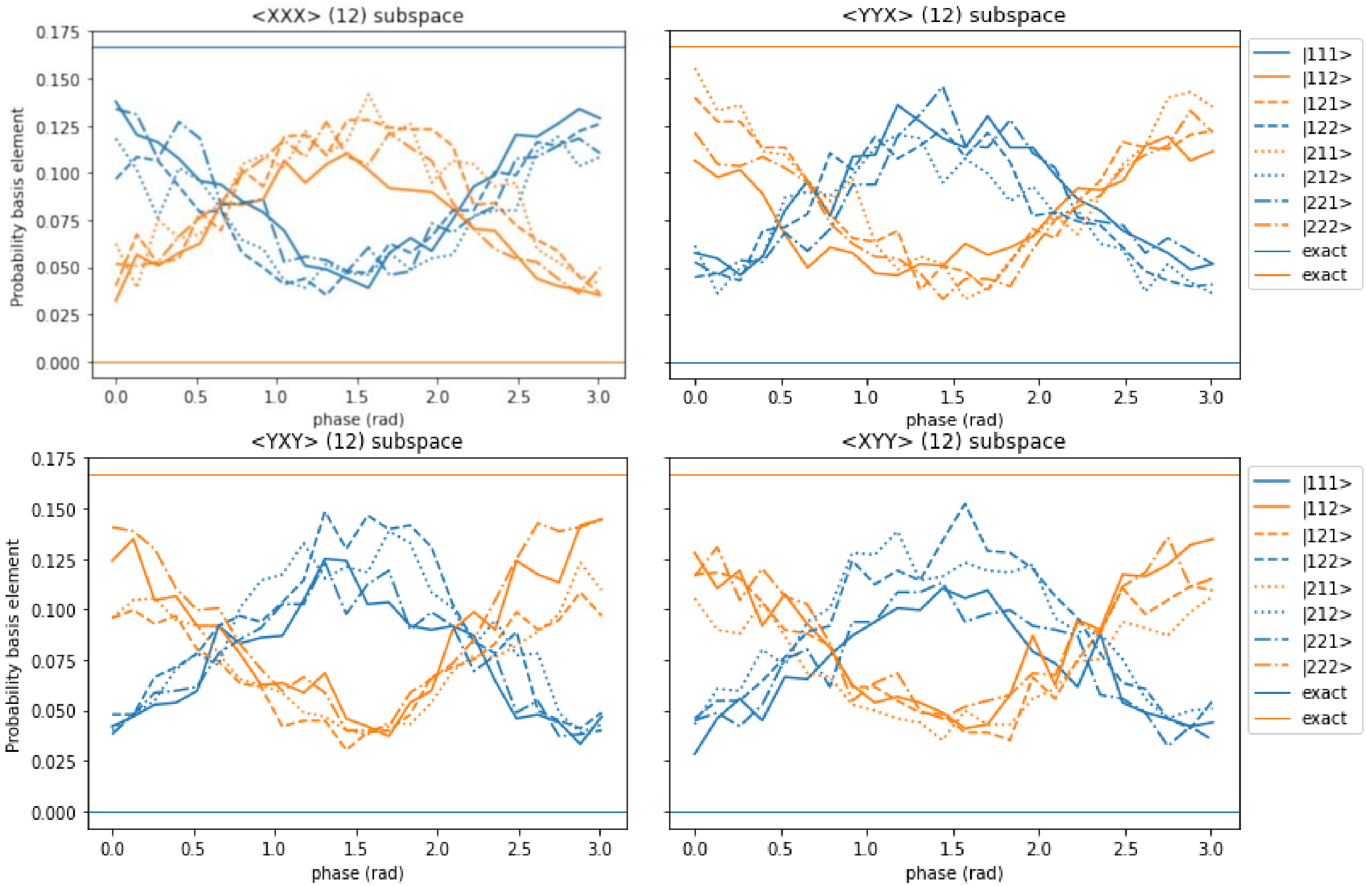
Tracking the phase

Due to the relative phase introduced by the CNOT* and rotational reference frame imposed in the (01) subspace:

Instead of applying $R^{12}_{x,y}$ gates, we apply R^{12}_{n} where n is a unit vector in the (x, y) plane.

To compensate this phase acumulation, we apply a phase gate on one of the qutrits and scan for different phases.

$$\begin{aligned}\mathrm{Re}\left(\langle ijk|\rho|lmn\rangle\right) = \frac{1}{8}\Big(&\langle\sigma_x^{(il)}\sigma_x^{(jm)}\sigma_x^{(kn)}\rangle \\ &-\langle\sigma_y^{(il)}\sigma_y^{(jm)}\sigma_x^{(kn)}\rangle \\ &-\langle\sigma_y^{(il)}\sigma_x^{(jm)}\sigma_y^{(kn)}\rangle \\ &-\langle\sigma_x^{(il)}\sigma_y^{(jm)}\sigma_y^{(kn)}\rangle\Big)\end{aligned}$$



arXiv v2 comming soon

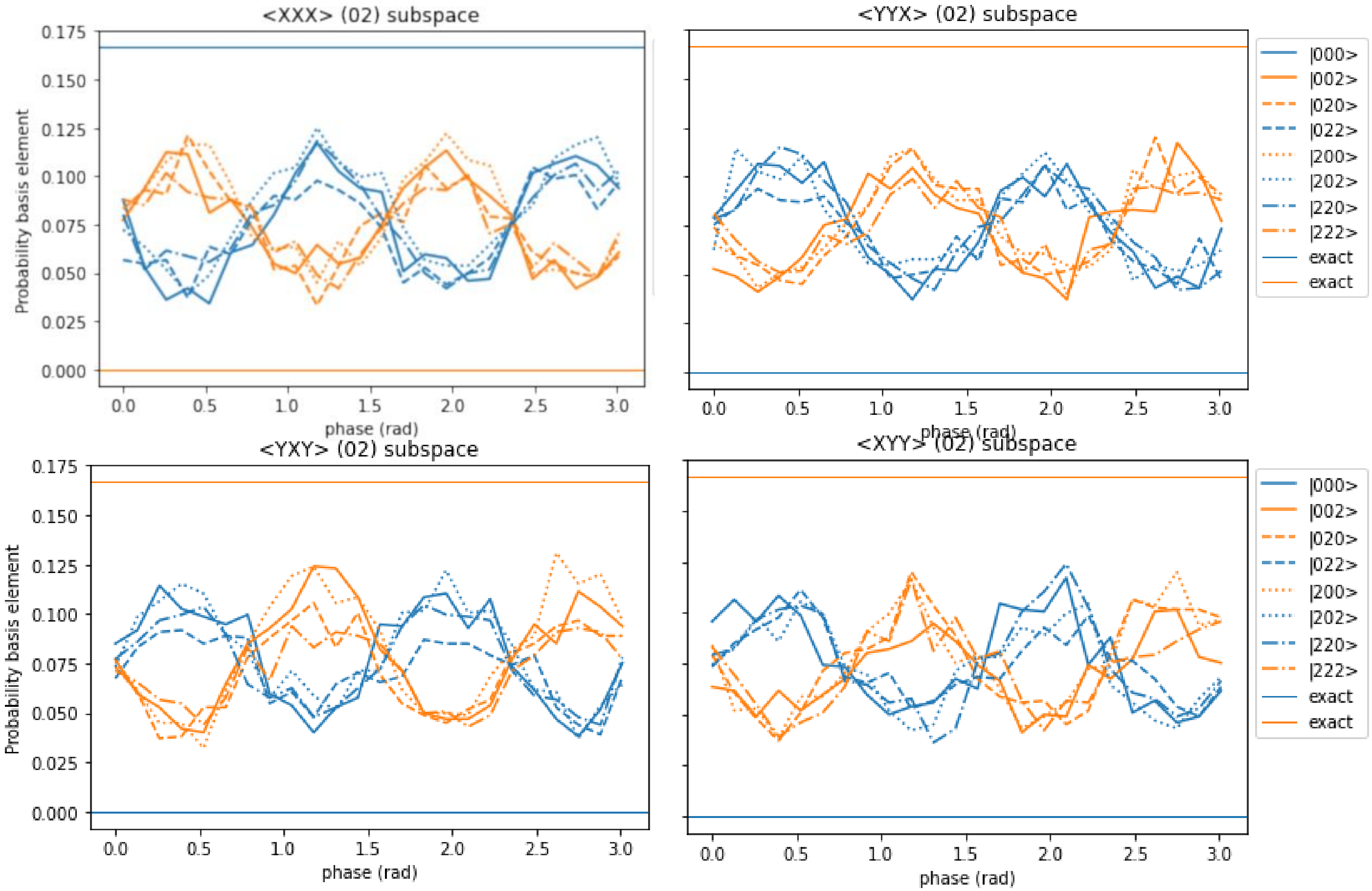
Tracking the phase

The relative phase introduced by the CNOT*

Other noisy phenomena (cross-talk, AC Stark shift, …): instead of applying $R^{12}_{x,y}$ gates, we apply R^{12}_{n} where n is a unit vector in the (x, y) plane.

To compensate this phase acumulation, we apply a phase gate on one of the qutrits and scan for different phases.

$$\mathrm{Re}\left(\langle ijk|\rho|lmn\rangle\right) = \frac{1}{8}\Big(\langle\sigma_x^{(il)}\sigma_x^{(jm)}\sigma_x^{(kn)}\rangle - \langle\sigma_y^{(il)}\sigma_y^{(jm)}\sigma_x^{(kn)}\rangle - \langle\sigma_y^{(il)}\sigma_x^{(jm)}\sigma_y^{(kn)}\rangle - \langle\sigma_x^{(il)}\sigma_y^{(jm)}\sigma_y^{(kn)}\rangle\Big)$$



arXiv v2 comming soon

Epilog

Summary

This is the **first** experiment that generates the a high-dimensional multi-partite state with superconducting circuits.

This is the **first** experiment that generates a high-dimensional multi-partite state with a non-photonic platform.

The photonic experiment took **weeks**, ours took **seconds**.

This opens the path to explore high-dimensional physics out of reach for other physical platforms.

This experiment was carried out in the cloud!

A. Cervera-Lierta, M. Krenn, A. Aspuru-Guzik, A. Galda, arXiv:2104.05627 [quant-ph]

Aknowledgments

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Qiskit IBM Q™

the matter lab

A. Cervera-Lierta, M. Krenn, A. Aspuru-Guzik, A. Galda, arXiv:2104.05627 [quant-ph]