Maximal entanglement in high-energy physics

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the matter lab

Outlook

Quantum information paradigm

Maximal Entanglement in QED

Unconstrained QED

Maximal Entanglement in weak interactions

Conclusions

The quantum information paradigm

$$H|\psi\rangle = E|\psi\rangle$$

Traditional emphasis on operators

- Criticality
- RG flows on coupling constants
- Conformal Symmetry
- ...

Quantum information emphasis on states

- Scaling of entropy
- RG flows on states
- Distribution of entanglement: MERA
- ...

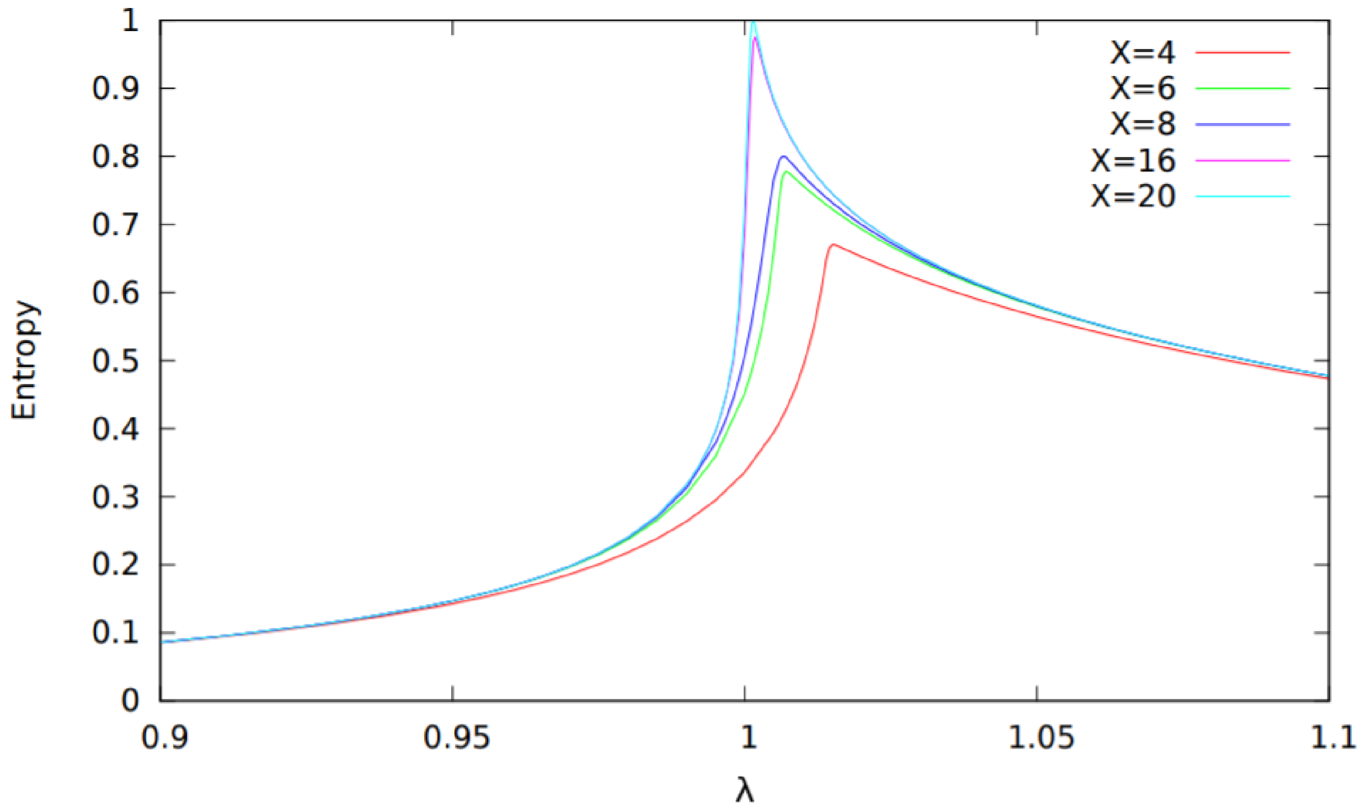
Example: quantum phase transitions

$$H_{QI} = \sum_i \sigma_i^x \sigma_{i+1}^x + \lambda \sigma_i^z$$

Entropy is maximal at a QPT

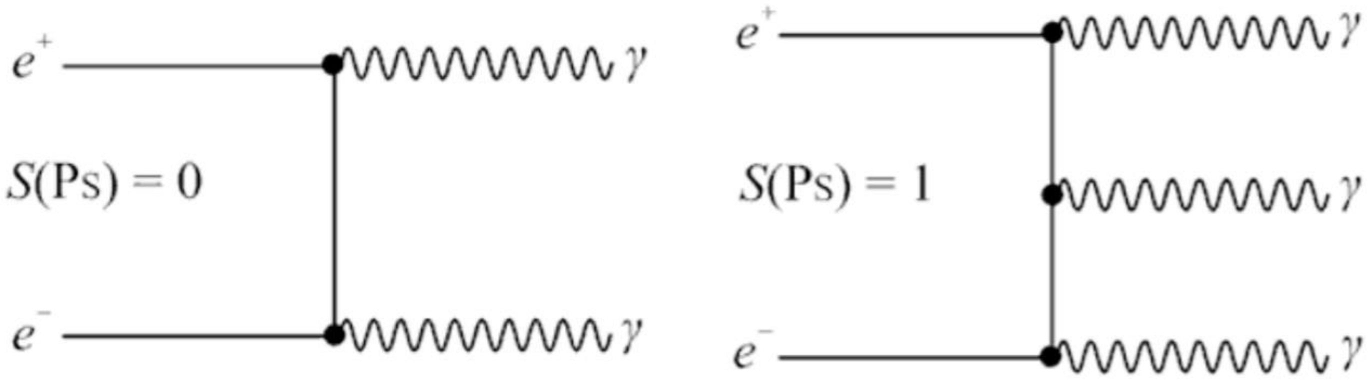
$$\lambda \to 1$$

$\Rightarrow$ Maximal entropy
$\Rightarrow$ Maximal entanglement
$\Rightarrow$ Conformal symmetry



A. Osterloh, Luigi Amico, G. Falci & Rosario Fazio
Nature **416**, 608 (2002)
L. Tagliacozzo, Thiago. R. de Oliveira, S. Iblisdir, J. I. Latorre
Phys. Rev. B **78**, 024410 (2008)

Ortopositronium decay



We can study the multipartite entanglement properties of these systems (e.g. tests of local realism)

$$|\psi_p\rangle = \frac{1}{\sqrt{2}}(|++\rangle - |--\rangle)$$

$$\begin{aligned}|\psi_0(\hat{k}_1,\hat{k}_2,\hat{k}_3)\rangle = &(1-\hat{k}_1\cdot\hat{k}_2)(|++-\rangle + |--+\rangle)\\ &+(1-\hat{k}_1\cdot\hat{k}_3)(|+-+\rangle + |-+-\rangle)\\ &+(1-\hat{k}_2\cdot\hat{k}_3)(|-++\rangle + |+--\rangle)\end{aligned}$$

$$\begin{aligned}|\psi_1(\hat{k}_1,\hat{k}_2,\hat{k}_3)\rangle = &(1-\hat{k}_1\cdot\hat{k}_2)(|++-\rangle - |--+\rangle)\\ &+(1-\hat{k}_1\cdot\hat{k}_3)(|+-+\rangle - |-+-\rangle)\\ &+(1-\hat{k}_2\cdot\hat{k}_3)(|-++\rangle - |+--\rangle)\end{aligned}$$

$$S_z = \pm 1$$

A. Acín, J. I. Latorre, P. Pascual, Phys. Rev. A **63**, 042107 (2001)

Quantifying entanglement

Focus

Two-particle scattering processes at tree level
Entanglement of helicity degrees of freedom

$$|\psi\rangle_{final} = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$$

assuming $|0\rangle$, $|1\rangle$ helicity or polarization states.

$$|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$$

Figure of merit to quantify entanglement: **concurrence**

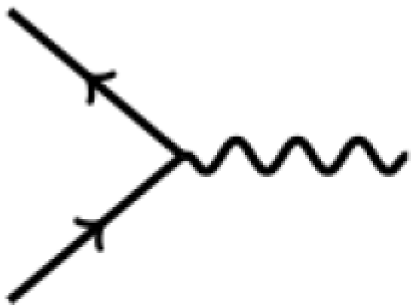
$$\Delta = |\alpha\delta - \beta\gamma|,$$

by construction, $0 \leq \Delta \leq 1$.

Question

Can a product state become entangled?

Entanglement generation: the s channel



$$j^{\mu}_{ss'} = e\bar{v}^{s'}(p')\gamma^{\mu}u^{s}(p)$$

Process: $e^+e^- \to \mu^+\mu^-$ at high energy

Incoming:

$$j^{\mu}_{RL} = 2ep_0\,(0, 1, i, 0)$$
$$j^{\mu}_{LR} = 2ep_0\,(0, 1, -i, 0)$$

Outgoing:

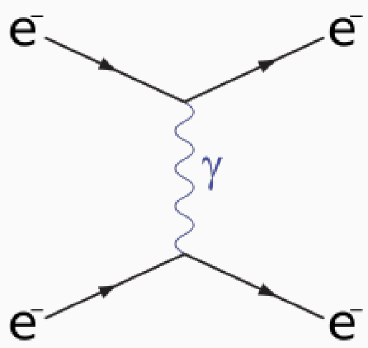
$$j^{\mu}_{RL} = 2ep_0\,(0, \cos\theta, i, -\sin\theta)$$
$$j^{\mu}_{LR} = 2ep_0\,(0, \cos\theta, -i, \sin\theta)$$

$$|RL\rangle \to (1+\cos\theta)|RL\rangle + (-1+\cos\theta)|LR\rangle$$

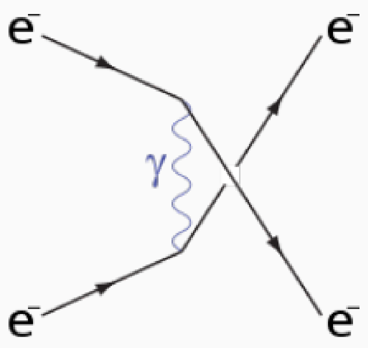
$$\theta = \pi/2 \to \Delta = 1$$

Entanglement generation: indistinguishability

Process: $e^- e^- \to e^- e^-$ at high energy



t channel



u channel

$$\mathcal{M}(|RL\rangle \to |RL\rangle) = -2e^2 \frac{u}{t}$$
$$\mathcal{M}(|RL\rangle \to |LR\rangle) = 0$$

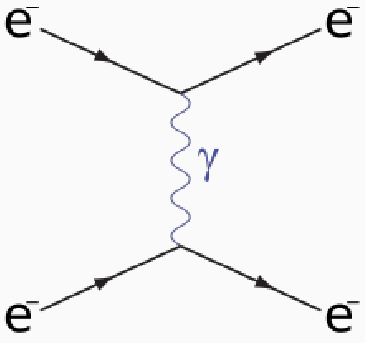
$$\mathcal{M}(|RL\rangle \to |RL\rangle) = 0$$
$$\mathcal{M}(|RL\rangle \to |LR\rangle) = -2e^2 \frac{t}{u}$$

$$|RL\rangle \to \frac{u}{t}|RL\rangle - \frac{t}{u}|LR\rangle$$

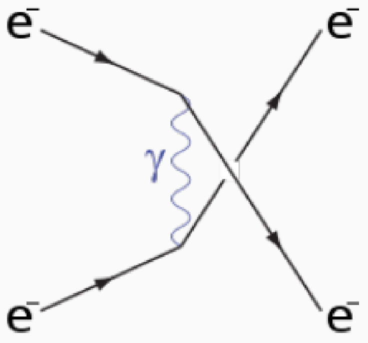
$$t = u \; (\theta = \pi/2) \to \Delta = 1$$

Entanglement generation: indistinguishability

Process: $e^- e^- \to e^- e^-$ at any energy



t channel



u channel

$$\Delta_{|RL\rangle} = \frac{2tu\left(tu + m^2 \frac{(t-u)^2}{t+u}\right)}{2m^2(t-u)^2\left(2m^2 - 2(t+u) + \frac{tu}{t+u}\right) + (t^4 + u^4)} \xrightarrow{t=u} 1$$

$$\Delta_{|RR\rangle} \xrightarrow{E \ll m,\ t=u} 1 + \mathcal{O}(p^2/m^2)$$

ACL, J. I. Latorre, J. Rojo and L. Rottoli, SciPost Phys. 3, 036 (2017).

QED interaction can generate maximal entanglement in almost all processes and at different energy regimes.

Is this a property of nature interactions?

It from bit

Could a symmetry emerge from a
Maximum Entanglement Principle
?

It from bit philosophy by J. A. Wheeler

“All things physical are information-theoretic in origin”

J. A. Wheeler, Proceedings III International Symposium on Foundations of Quantum Mechanics, Tokyo, 345-368 (1989)

Maximal Entanglement conjecture

"Nature is such that maximally entangled states exist"

Max Entanglement → Max Entropy → Max Surprise → NO Local Realism

MaxEnt Principle → Nature cannot be described by classical physics

Bell Inequalities will be violated

Test: QED coupling

QED lagrangian at three-level (high-energy limit, $m = 0$)

	free fermions		free photons		interaction term
$\mathcal{L} =$	$i\bar{\psi}\gamma^\mu\partial_\mu\psi$	$+$	$\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$	$+$	$-eA_\mu\bar{\psi}G^\mu\psi$
	Dirac eq.		Maxwell eq.		

G^μ: 4×4 arbitrary matrices

Gauge invariance imposes $G^\mu = \gamma^\mu$

What are the couplings G^μ that generate maximal entanglement?

Unconstrained QED

In general G^μ may not be Lorentz invariant. Expand in a basis of 16 matrices:

$$G^\mu = a^\mu \mathbb{I} + a^{\mu\nu}\gamma_\nu + i a^{\mu 5}\gamma^5 + a^{\mu\nu 5}\gamma^5\gamma_\nu + a^{\mu\nu\rho}[\gamma_\nu, \gamma_\rho]$$

Assuming conservation of P, T and C symmetries:

$$G^\mu = a^{\mu\nu}\gamma^\nu \qquad a_{\mu\nu} \in \mathbb{R} \qquad a_{0i} = a_{i0} = 0$$

Computation of amplitudes of all tree-level processes:

$$\mathcal{M}_{|initial\rangle \to |final\rangle} = f(\theta, a_{\mu\nu})$$

Unconstrained QED

Constrain G^{μ} imposing MaxEnt in **ALL** tree level processes

$$\max_{a^{\mu\nu}} \{\Delta_{Bhabha}, \Delta_{Compton}, \Delta_{pair\ \ annhilation}, \Delta_{Moller}, \dots\}$$

Each process will deliver different kind of MaxEnt at different angles

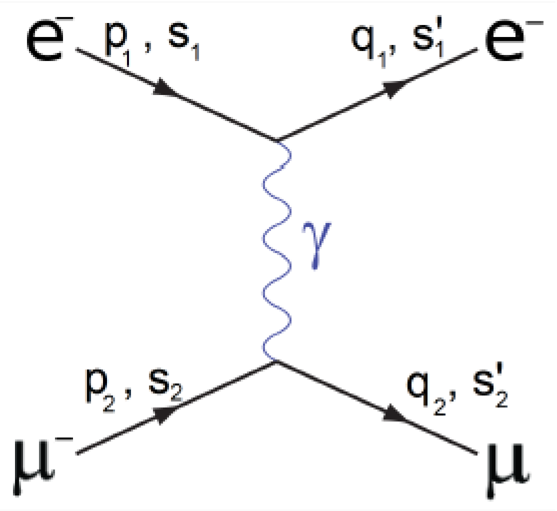
→ Choose optimal settings

(Logic: Bell Ineq. seek to discard classical physics using optimal settings)

Unconstrained Mott scattering

$$e^- \mu^- \to e^- \mu^-$$

$$\begin{aligned}
\mathcal{M}_{|RL\rangle \to |RR\rangle} &= 0 \\
\mathcal{M}_{|RL\rangle \to |RL\rangle} &= f(a) \\
\mathcal{M}_{|RL\rangle \to |LR\rangle} &= 0 \\
\mathcal{M}_{|RL\rangle \to |LL\rangle} &= 0
\end{aligned}$$



No entanglement can be generated!
No constraints emerge from this process

Unconstrained e^-e^+ annihilation to muons

$$e^-e^+ \to \mu^-\mu^+$$

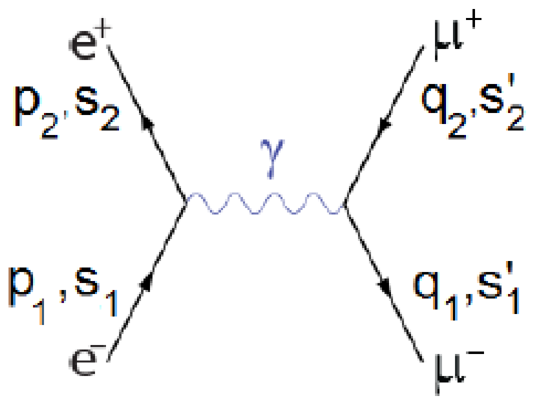
Amplitudes quadratic in a's:

$$\begin{aligned}
\mathcal{M}_{|RL\rangle\to|RL\rangle} &= \left(-a_{i2}^2 - a_{i1}^2\cos\theta + a_{i1}a_{i3}\sin\theta\right) + i\left(a_{i1}a_{i2}(1-\cos\theta) + a_{i2}a_{i3}\sin\theta\right)\\
\mathcal{M}_{|RL\rangle\to|LR\rangle} &= \left(-a_{i2}^2 + a_{i1}^2\cos\theta - a_{i1}a_{i3}\sin\theta\right) + i\left(a_{i1}a_{i2}(1+\cos\theta) - a_{i2}a_{i3}\sin\theta\right)\\
\mathcal{M}_{|RL\rangle\to|RR\rangle} &= \mathcal{M}_{|RL\rangle\to|LL\rangle} = 0
\end{aligned}$$

Arbitrary angle dependent solutions are discarded by other processes

MaxEnt $\quad \begin{matrix}\theta = \pi/2\\ \Delta = 1\end{matrix} \implies \begin{matrix} A = aa^T \geq 0\\ A_{22}A_{13} - A_{12}A_{23} = 0\end{matrix}$

QED $\quad a_{ij} = \begin{cases}0 & \forall i \neq j\\ 1 & \forall i = j\end{cases} \implies A_{ij} = \begin{cases}0 & \forall i \neq j\\ 1 & \forall i = j\end{cases}$



MaxEnt consistency

MaxEnt is well defined for a given process since it is a quadratic maximization, but

Is this consistent?

Yes

Does MaxEnt pull in different directionis depending on the process?

No

Is there a unique maximum?

$$G^{\mu} = (\pm\gamma^0, \pm\gamma^1, \pm\gamma^2, \pm\gamma^3)$$

ACL, J. I. Latorre, J. Rojo and L. Rottoli, SciPost Phys. 3, 036 (2017).

Final solution: QED

Considering all tree level 2-particles processes (Bhabha, Moller, Compton, pair annihilation, ...)

$$
(G^0, G^1, G^2, G^3) = \begin{cases} \left.\begin{array}{l} (\pm\gamma^0, \gamma^1, \gamma^2, \gamma^3) \\ (\pm\gamma^0, -\gamma^1, -\gamma^2, -\gamma^3) \end{array}\right\} \text{QED} \\ \\ \left.\begin{array}{l} (\pm\gamma^0, -\gamma^1, \gamma^2, \gamma^3) \\ (\pm\gamma^0, \gamma^1, -\gamma^2, -\gamma^3) \end{array}\right\} ? \end{cases}
$$

All two-level processes are blind to the signs

$$
\gamma^i \to -\gamma^i \\
e \to -e
$$

$-\gamma^1$ solution:

→ No rotational invariance!
→ Fermion scattering processes are identical to QED
→ Leads to a non-conservation of current
→ Could be discarded at higher orders or appealing to rotational symmetry?

Final solution: QED

→ NO incompatible pulls!! MaxEnt can be achieved consistently in different channels.

→ Entanglement generated either in s channel or in superposition of t and u channels.

→ A process may display MaxEnt at some angle with a contrived solution for a's. This solution will fail in other processes.

→ Using COM or LAB reference frames do not change the analysis.

→ Need of three-body processes or higher orders to discard wrong signs.

Furthermore,

→ QED is an isolated maximum

→ All deformations around QED produce lower entanglement

Apparently, MaxEnt can fix the structure of an interaction like QED

Could we use it to obtain an estimation of free parameters in other interactions?

Weak interactions

Weak neutral current

$$J_\mu^{NC} = \bar{u}_f \gamma_\mu \left(g_V^f - \gamma^5 g_A^f\right) u_f$$

$$g_A^f = T_3^f/2 \qquad g_V^f = T_3^f/2 - Q_f \sin^2\theta_w$$

For electrons: $T_3^\ell = -1/2,\ Q_\ell = -1.$

Experimentally, $\sin^2\theta_w \simeq 0.23$

Guessing

- MaxEnt might be achievable on a line in the plane $\theta - \theta_w$
- Non-trivial tests: Bhabha (Z/γ interference)
- Special case, no kinematics: Z decay

Z decay to leptons

$$m \ll M_Z,\ g_R = (g_V - g_A)/2 \text{ and } g_L = (g_V + g_A)/2$$

Longitudinal polarization:

$$\left.\begin{aligned} \mathcal{M}_{|0\rangle\to|RL\rangle} &= g_R M_Z \sin\theta \\ \mathcal{M}_{|0\rangle\to|LR\rangle} &= g_L M_Z \sin\theta \end{aligned}\right\} \Delta_0 = \frac{2|g_L g_R|}{g_L^2 + g_R^2}$$

$$\Delta_0 = 1 \text{ if } |g_L| = |g_R| \Rightarrow g_A = 0 \text{ or } g_V = 0.$$

$$g_A = T_3/2 \neq 0 \Rightarrow g_V = 0 \Rightarrow \sin^2\theta_w = \frac{T_3}{2Q} \xrightarrow[]{\text{for charged leptons}} \sin^2\theta_w = 1/4.$$

Z decay to leptons

$m \ll M_Z$, $g_R = (g_V - g_A)/2$ and $g_L = (g_V + g_A)/2$

Circular polatization

$$\mathcal{M}_{|R\rangle\to|RL\rangle} = g_R M_Z \sqrt{2}\sin^2(\theta/2) \qquad \mathcal{M}_{|L\rangle\to|RL\rangle} = g_R M_Z \sqrt{2}\cos^2(\theta/2)$$
$$\mathcal{M}_{|R\rangle\to|LR\rangle} = -g_L M_Z \sqrt{2}\cos^2(\theta/2) \qquad \mathcal{M}_{|L\rangle\to|LR\rangle} = -g_L M_Z \sqrt{2}\sin^2(\theta/2)$$

$$\Delta_{\substack{R\\L}} = \frac{2|g_L g_R|\sin^2\theta}{|2(g_L^2 - g_R^2)\cos\theta \pm (g_L^2 + g_R^2)(1+\cos^2\theta)|}$$

$$\Delta_{\substack{R\\L}} = 1 \text{ if } \begin{cases} \dfrac{g_R}{g_L} = \pm\cot^2(\theta/2) \\ \dfrac{g_R}{g_L} = \pm\tan^2(\theta/2) \end{cases}$$

Assuming g_R and g_L are independent of the initial polatization:

$$\frac{g_R}{g_L} = \pm 1 \Rightarrow |g_L| = |g_R|$$

$$\Rightarrow g_V = 0 \Rightarrow \sin^2\theta_w = 1/4$$

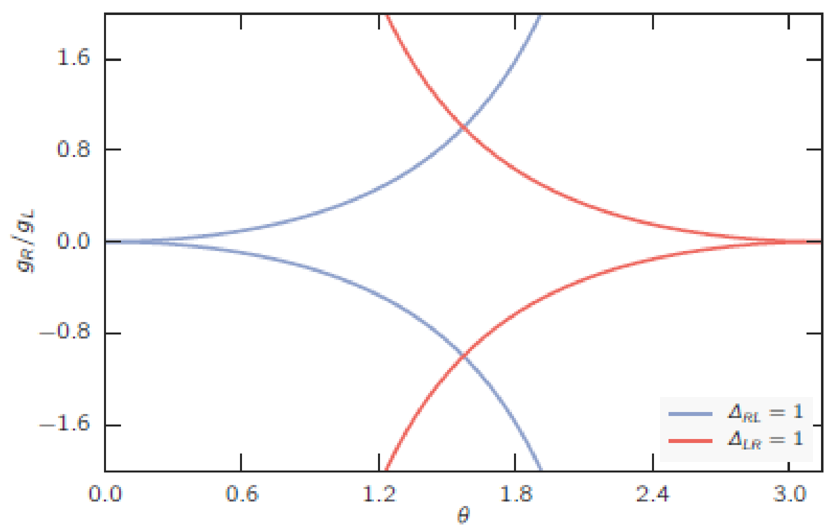
$e^-e^+ \to \mu^-\mu^+$ Z mediated

$m \ll M_Z,$

$$\begin{aligned}
\mathcal{M}_{RL} &\sim \quad (1+\cos\theta)g_R^2 \quad |RL\rangle + \quad (-1+\cos\theta)g_R g_L \quad |LR\rangle \\
\mathcal{M}_{LR} &\sim (-1+\cos\theta)g_R g_L \quad |RL\rangle + \quad (1+\cos\theta)g_L^2 \quad |LR\rangle
\end{aligned}$$

$$\Delta_{RL} \sim \frac{\sin^2\theta|g_L g_R|}{2\,(s^4 g_L^2 + c^4 g_R^2)} \qquad \Delta_{LR} \sim \frac{\sin^2\theta|g_L g_R|}{2\,(c^4 g_L^2 + s^4 g_R^2)}$$

$c = \mathrm{co}$



Imposing maximal entanglement at the same COM angle:

$$\begin{aligned}
s^2 g_L \pm c^2 g_R = 0 &\to \Delta_{RL} = 1 \\
c^2 g_L \pm s^2 g_R = 0 &\to \Delta_{LR} = 1
\end{aligned}$$

$$\theta = \frac{\pi}{2}, \quad \sin^2\theta_w = \frac{1}{4}$$

ACL, J. I. Latorre, J. Rojo and L. Rottoli, SciPost Phys. 3, 036 (2017).

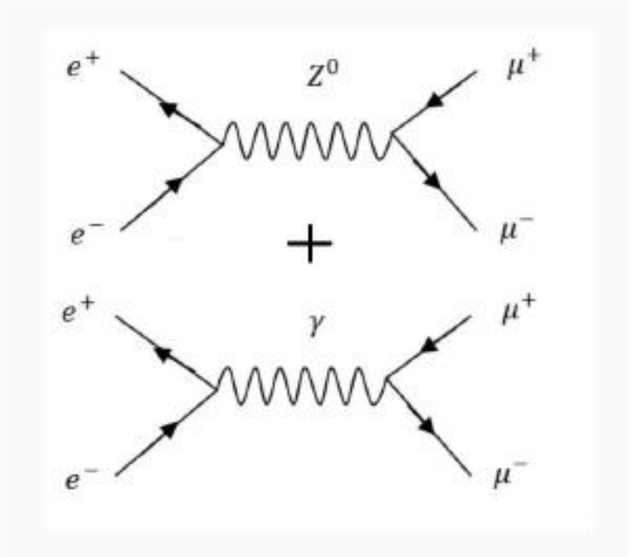
$e^-e^+ \to \mu^-\mu^+$ Z/γ interference

Photon contribution add terms to both RL and LR, which are independent of $sin^2\theta_w$

$$\mathcal{M} \sim \left(\mathcal{M}_Z^{RL}(\theta,\theta_w) + \mathcal{M}_\gamma^{RL}(\theta)\right)|RL\rangle + \left(\mathcal{M}_Z^{Lr}(\theta,\theta_w) + \mathcal{M}_\gamma^{LR}(\theta)\right)|LR\rangle$$

$$\Delta_{RL} = \frac{4\sin^2\theta}{6\cos\theta + 5(1+\cos^2\theta)} \qquad \Delta_{RL} = 1 \to \quad \theta = \arccos\left(-\frac{1}{3}\right)$$

$$\Delta_{LR} = \frac{\sin^2\theta\sin^2\theta_w}{c^4 + 4s^4\sin^4\theta_w} \qquad \Delta_{LR} = 1 \to \quad \theta_w = \arcsin\left(\frac{1}{\sqrt{2}}\cot(\theta/2)\right)$$



Imposing MaxEnt at the same COM angle

$$\theta = \arccos\left(-\frac{1}{3}\right), \quad \sin^2\theta_w = \frac{1}{4}$$

ACL, J. I. Latorre, J. Rojo and L. Rottoli, SciPost Phys. 3, 036 (2017).

Summary

Maximal entanglement:

- Discards classical physics by principle predictive
- Consistent with QED, which is an isolated solution
- MaxEnt is found in every channel where it was possible

Consequences?

- Can we use it as a tool to estimate the value of SM free parameters?
- Weak interactions: MaxEnt in tree-level weak interactions predict $sin^2\theta_w = 0.25$.

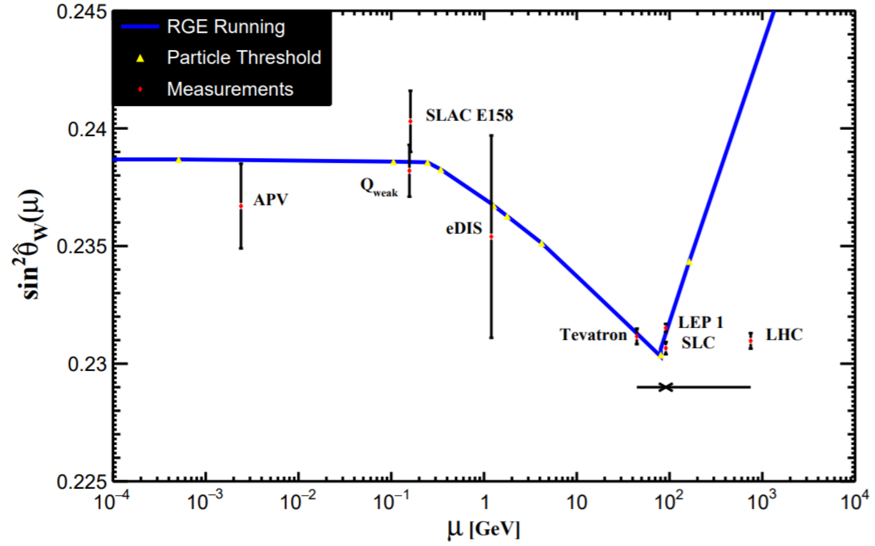
Next steps

→ Multipartite entanglement? (e.g. ortopositronium decay)

→ Higher orders in perturbation theory

  → Renormalization scheme?
  → IR divergences ?

→ Compute more processes: entanglement maximization over θ_w



Scheme	Notation	Value	Uncertainty
On-shell	s_W^2	0.22337	±0.00010
$\overline{\text{MS}}$	$\hat{s}_Z^2$	0.23121	±0.00004
$\overline{\text{MS}}$ND	$\hat{s}_{\text{ND}}^2$	0.23141	±0.00004
$\overline{\text{MS}}$	$\hat{s}_0^2$	0.23857	±0.00005
Effective angle	$\bar{s}_\ell^2$	0.23153	±0.00004

$$s_W^2 \equiv 1 - \frac{M_W^2}{M_Z^2} \qquad \hat{s}_Z^2 \equiv \sin^2\hat{\theta}_W(M_Z)$$

$$\sin^2\hat{\theta}_W(\mu) \equiv \frac{\hat{g}'^2(\mu)}{\hat{g}^2(\mu)+\hat{g}'^2(\mu)} \qquad \hat{s}_0^2 \equiv \sin^2\hat{\theta}_W(0)$$

$$\bar{s}_f^2 \equiv \sin^2\bar{\theta}_{Wf} \equiv \hat{\kappa}_f \hat{s}_Z^2 = \kappa_f s_W^2$$

Particle Data Group, Prog. Theor. Exp. Phys. 2020, 083C01 (2020).

ACL, J. I. Latorre, J. Rojo and L. Rottoli, SciPost Phys. 3, 036 (2017).

Open questions

- Relax C, P and T to CPT symmetry?

- Other interaction theories: QCD, chiral, gravity, …

    - QCD: no asymptotic states (confinement), what does it mean to have an entangled state?
    - CKM relation to mass rations?
    - Neutrino oscillations?
    - Gravity: Feynman rules for graviton interactions?

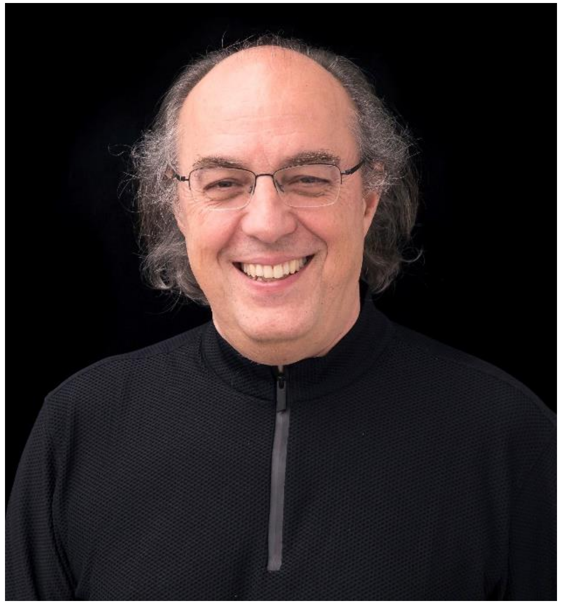
- Formulation in terms of probabilities and Bell inequalities?

- Other degrees of freedom instead of helicities and polarizations
    - Position/momenta space
    - Flavour
    - Color
    - …

Color in gluon-gluon scattering:
no extra information from maximal entanglement
(see arXiv:1906.12099 [quant-ph])

Aknowledgements

José Ignacio Latorre



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Luca Rottoli