Maximal Entanglement

Applications to Quantum Computation and Particle Physics

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Outline

Maximal Entanglement in High Energy physics

QI paradigm

Quantum Information is bringing new insights:

$$H|\psi\rangle = E|\psi\rangle$$

- Traditional emphasis on operators $\rightarrow H$
- QI emphasis on states $\rightarrow |\psi\rangle$

Motivation

Example: Entanglement is maximal at Quantum Phase Transition

- *Von Neumann Entropy* $\rho_A = \mathrm{Tr}_B |\psi\rangle_{AB}\langle\psi|,\ S(\rho_A) = -\mathrm{Tr}_A \rho_A \log \rho_A$
- *Fixed points*: scaling with block size L $\quad S(\rho_A) = \frac{c}{3}\log L \ll L$ [1]
- Some entropy scaling examples:

$S \sim n$	Random states, QMA problems,
$S \sim .8858n$	Prime state
$S \sim n^{\frac{d-1}{d}}$	Area law in d-dimensions
$S \sim \frac{c}{3}\log n$	Critical scaling in $d=1$
$S \sim \log(\xi) = ct$	Finitely correlated states away from criticality

[1] Callan-Wilczek 94, Vidal-Latorre-Rico-Kitaev 02

Maximal Entanglement in QED

Focus

Two-particle scattering processes at tree level

Entanglement of helicity degrees of freedom

$$|\psi\rangle_{final} = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$$

assuming $|0\rangle$, $|1\rangle$ helicity or polarization states.

$$|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$$

Figure of merit to quantify entanglement: **concurrence**

$$\Delta = 2|\alpha\delta - \beta\gamma|,$$

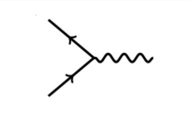
by construction, $0 \leq \Delta \leq 1$.

Question

Can a product state become entangled?

Maximal Entanglement in QED

Generation of entanglement: s channel



$$j^{\mu}_{ss'} = e\bar{v}^{s'}(p')\gamma^{\mu}u^{s}(p)$$

Process: $e^+e^- \to \mu^+\mu^-$ at high energy

Incoming:

$$j^{\mu}_{RL} = 2ep_0\,(0, 1, i, 0)$$

Outgoing:

$$j^{\mu}_{RL} = 2ep_0\,(0, \cos\theta, i, -\sin\theta)$$
$$j^{\mu}_{LR} = 2ep_0\,(0, \cos\theta, -i, \sin\theta)$$

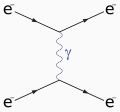
$$|RL\rangle \to (1+\cos\theta)|RL\rangle + (-1+\cos\theta)|LR\rangle$$

$$\theta = \pi/2 \to \Delta = 1$$

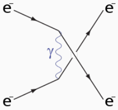
Maximal Entanglement in QED

Generation of entanglement: indistinguishability

Process: $e^- e^- \to e^- e^-$ at high energy



t channel



u channel

$$\mathcal{M}\left(|RL\rangle \to |RL\rangle\right) = -2e^2 \frac{u}{t}$$
$$\mathcal{M}\left(|RL\rangle \to |LR\rangle\right) = 0$$

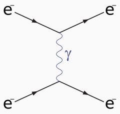
$$\mathcal{M}\left(|RL\rangle \to |RL\rangle\right) = 0$$
$$\mathcal{M}\left(|RL\rangle \to |LR\rangle\right) = -2e^2 \frac{t}{u}$$

$$|RL\rangle \to \frac{u}{t}|RL\rangle - \frac{t}{u}|LR\rangle$$

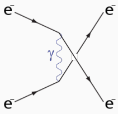
$t = u \ (\theta = \pi/2) \to \Delta = 1$

Generation of entanglement: indistinguishability

Process: $e^- e^- \to e^- e^-$ at any energy



t channel



u channel

$$\Delta_{|RL\rangle} = \frac{2tu\left(tu + m^2\frac{(t-u)^2}{t+u}\right)}{2m^2(t-u)^2\left(2m^2 - 2(t+u) + \frac{tu}{t+u}\right) + (t^4+u^4)} \xrightarrow{t=u} 1$$

$$\Delta_{|RR\rangle} \xrightarrow{E \ll m,\ t=u} 1 + \mathcal{O}(p^2/m^2)$$

QED interaction can generate maximal entanglement in almost all processes and at different energy regimes.

Is this a property of nature interactions?

It from bit

Could a symmetry emerge from a
Maximum Entanglement Principle
?

It from bit philosophy by J. A. Wheeler:

"All things physical are information-theoretic in origin"

MaxEnt principle

"Nature is such that maximally entangled states exist"

Max Entanglement = Max Entropy = Max Surprise = NO Local Realism

MaxEnt Principle = Nature cannot be described by classical physics

Bell Inequalities will be violated

Test: QED coupling

QED Lagrangian at tree level (high-energy limit, $m = 0$):

$$\mathcal{L} = \underbrace{i\bar{\psi}\gamma^\mu\partial_\mu\psi}_{\text{Dirac eq.}}^{\text{free fermions}} + \underbrace{\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{Maxwell eq.}}^{\text{free photons}} + \overbrace{-eA_\mu\bar{\psi}G^\mu\psi}^{\text{interaction term}}$$

G^μ: 4×4 arbitrary matrices

Gauge invariance: $G^\mu = \gamma^\mu$

Which are the couplings, G^μ, that generate MaxEnt?

Unconstrained QED

1. In general, G^μ may not be Lorentz invariant. Expand in a basis of 16 matrices:

$$G^\mu = a^\mu \mathbb{I} + a^{\mu\nu}\gamma_\nu + i a^{\mu 5}\gamma^5 + a^{\mu\nu 5}\gamma^5\gamma_\nu + a^{\mu\nu\rho}[\gamma_\nu, \gamma_\rho]$$

2. Assuming conservation of $\mathcal{P}$, $\mathcal{T}$ and $\mathcal{C}$ symmetries:

$$G^\mu = a^{\mu\nu}\gamma^\nu \qquad a_{\mu\nu} \in \mathbb{R} \qquad a_{0i} = a_{i0} = 0$$

3. Computation of amplitudes of all tree-level processes:

$$\mathcal{M}_{|initial\rangle \to |final\rangle} = f(\theta, a_{\mu\nu})$$

Unconstrained QED

Constrain G^{μ} imposing MaxEnt in **ALL** tree level processes

$$\max_{a^{\mu\nu}} \{\Delta_{Bhabha}, \Delta_{Compton}, \Delta_{pair\ \ annhilation}, \Delta_{Moller}, \ldots\}$$

Each process will deliver different kind of MaxEnt at different angles

$\rightarrow$ Choose optimal settings

(Logic: Bell Ineq. seek to discard classical physics using optimal settings)

Unconstrained QED

$$e^- e^+ \to \mu^- \mu^+$$

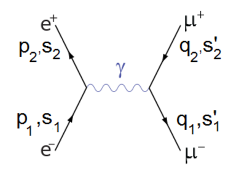
Amplitudes quadratic in a's:

$$\begin{aligned}
\mathcal{M}_{|RL\rangle \to |RL\rangle} &= \left(-a_{i2}^2 - a_{i1}^2 \cos\theta + a_{i1}a_{i3}\sin\theta\right) + i\left(a_{i1}a_{i2}(1-\cos\theta) + a_{i2}a_{i3}\sin\theta\right) \\
\mathcal{M}_{|RL\rangle \to |LR\rangle} &= \left(-a_{i2}^2 + a_{i1}^2 \cos\theta - a_{i1}a_{i3}\sin\theta\right) + i\left(a_{i1}a_{i2}(1+\cos\theta) - a_{i2}a_{i3}\sin\theta\right) \\
\mathcal{M}_{|RL\rangle \to |RR\rangle} &= \mathcal{M}_{|RL\rangle \to |LL\rangle} = 0
\end{aligned}$$

Arbitrary angle dependent solutions are discarded by other processes

MaxEnt $\quad \begin{matrix} \theta = \pi/2 \\ \Delta = 1 \end{matrix} \quad \Longrightarrow \quad \begin{matrix} A = aa^T \geq 0 \\ A_{22}A_{13} - A_{12}A_{23} = 0 \end{matrix}$

QED $\quad a_{ij} = \begin{cases} 0 & \forall i \neq j \\ 1 & \forall i = j \end{cases} \quad \Longrightarrow \quad A_{ij} = \begin{cases} 0 & \forall i \neq j \\ 1 & \forall i = j \end{cases}$



MaxEnt consistency

MaxEnt is well defined for a given process since it is a quadratic maximization, but

Is this consistent?

Yes

Does MaxEnt pull in different directions depending on the process?

No

Is there a unique maximum?

$G^\mu = \left(\pm\gamma^0, \pm\gamma^1, \pm\gamma^2, \pm\gamma^3\right)$

Observations

- **NO** incompatible pulls!! MaxEnt can be achieved consistently in different channels.
- Entanglement generated either in s channel or in superposition of t and u channels.
- A process may display MaxEnt at some angle with a contrived solution for a's. This solution will fail in other processes.
- Using COM or LAB reference frames do not change the analysis.
- Need of three-body processes to discard wrong signs.

Furthermore,

QED is an isolated maximum

All deformations around QED produce lower entanglement

Apparently, MaxEnt can fix the structure of an interaction like QED

Could we use it to obtain an estimation of free parameters in other interactions?

MaxEnt in weak interactions

Weak neutral current

$$J_\mu^{NC} = \bar{u}_f \gamma_\mu \left(g_V^f - \gamma^5 g_A^f \right) u_f$$

$$g_A^f = T_3^f/2 \qquad g_V^f = T_3^f/2 - Q_f \sin^2\theta_w$$

For electrons: $T_3^\ell = -1/2$, $Q_\ell = -1$.
Experimentally, $\sin^2\theta_w \simeq 0.23$

Guessing

MaxEnt might be achievable on a line in the plane $\theta - \theta_w$
Non-trivial tests: $e^+e^- \to \mu^+\mu^-$ (Z/γ interference)
Special case, no kinematics: Z decay

Z decay to leptons

$m \ll M_Z$, $g_R = (g_V - g_A)/2$ and $g_L = (g_V + g_A)/2$

Longitudinal polarization:

$$\left.\begin{aligned} \mathcal{M}_{|0\rangle \to |RL\rangle} &= g_R M_Z \sin\theta \\ \mathcal{M}_{|0\rangle \to |LR\rangle} &= g_L M_Z \sin\theta \end{aligned}\right\} \; \Delta_0 = \frac{2|g_L g_R|}{g_L^2 + g_R^2}$$

$\Delta_0 = 1$ if $|g_L| = |g_R| \Rightarrow g_A = 0$ or $g_V = 0$.

$$g_A = T_3/2 \neq 0 \Rightarrow g_V = 0 \Rightarrow \sin^2\theta_w = \frac{T_3}{2Q} \xrightarrow{\substack{\text{for charged} \\ \text{leptons}}} \sin^2\theta_w = 1/4.$$

MaxEnt in weak interactions

Z decay to leptons

If Z is right or left handed:

$$\Delta_{\substack{R\\L}} = \frac{2|g_L g_R| \sin^2\theta}{|2(g_L^2 - g_R^2)\cos\theta \pm (g_L^2 + g_R^2)(1+\cos^2\theta)|}$$

$$\Delta_{\substack{R\\L}} = 1 \text{ if } \begin{cases} \dfrac{g_R}{g_L} = \pm\cot^2(\theta/2) \\ \\ \dfrac{g_R}{g_L} = \pm\tan^2(\theta/2) \end{cases}$$

Assuming g_R and g_L are independent of the initial polarization:

$$\frac{g_R}{g_L} = \pm 1 \Rightarrow |g_L| = |g_R| \Rightarrow g_V = 0 \Rightarrow \sin^2\theta_w = 1/4$$

MaxEnt in weak interactions

$e^+e^- \to \mu^+\mu-$ **mediated by** Z

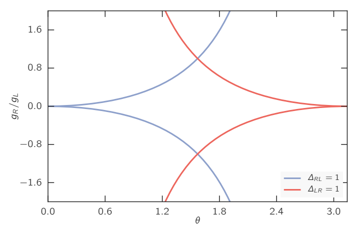
$$\Delta_{RL} \sim \frac{\sin^2\theta|g_L g_R|}{2\left(s^4 g_L^2 + c^4 g_R^2\right)} \quad \Delta_{LR} \sim \frac{\sin^2\theta|g_L g_R|}{2\left(c^4 g_L^2 + s^4 g_R^2\right)}$$

$c = \cos(\theta/2)$, $s = \sin(\theta/2)$

$$s^2 g_L \pm c^2 g_R = 0 \to \Delta_{RL} = 1$$
$$c^2 g_L \pm s^2 g_R = 0 \to \Delta_{LR} = 1$$

Imposing MaxEnt at the same COM angle:

$$\theta = \frac{\pi}{2},\ \sin^2\theta_w = \frac{1}{4}$$



MaxEnt in weak interactions

$e^+e^- \to \mu^+\mu-$ **with** Z/γ **interference**

Photon contribution add terms to both RL and LR, which are independent of $\sin^2\theta_w$

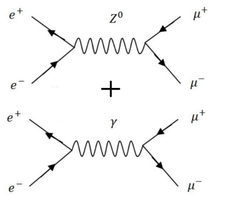
$$\mathcal{M} \sim \left(\mathcal{M}_Z^{RL}(\theta,\theta_w) + \mathcal{M}_\gamma^{RL}(\theta)\right)|RL\rangle + \left(\mathcal{M}_Z^{LR}(\theta,\theta_w) + \mathcal{M}_\gamma^{LR}(\theta)\right)|LR\rangle$$

$$\Delta_{RL} = \frac{4\sin^2\theta}{6\cos\theta + 5(1+\cos^2\theta)} \qquad \Delta_{RL} = 1 \to \quad \theta = \arccos\left(-\frac{1}{3}\right)$$

$$\Delta_{LR} = \frac{\sin^2\theta\sin^2\theta_w}{c^4 + 4s^4\sin^4\theta_w} \qquad \Delta_{LR} = 1 \to \quad \theta_w = \arcsin\left(\frac{1}{\sqrt{2}}\cot(\theta/2)\right)$$

Imposing MaxEnt at the same COM angle:

$$\theta = \arccos\left(-\frac{1}{3}\right), \ \sin^2\theta_w = \frac{1}{4}$$



Summary

Maximal entanglement:

- Discards classical physics by principle redictive
- Consistent with QED, which is an isolated solution
- MaxEnt is found in every channel where it was possible
- Open Questions:
  - Relax C, P and T to CPT symmetry?
  - Other interaction theories: chiral, gravity, effective,...
  - RG? IR divergences?
  - Formulate on probabilities and Bell inequalities?

Weak mixing angle

- MaxEnt in weak interactions predict $\sin^2\theta_w = 0.25$.
- Discarding MaxEnt for Z decay with longitudinal polarization, it is always possible to achieve MaxEnt $\forall\theta_w$.
- How to get closer to experimental value $\sin^2\theta_w^{exp} \simeq 0.23$?
  - Going to next order
  - Compute more processes: maximization of entanglement over θ_w

Quantum Phase Transition in a Quantum Computer

Motivation

Entangling hamiltonian H $\longrightarrow$ Non-interacting hamiltonian $\tilde{H}$
(hard to simulate) (easy to simulate)

$$\boxed{H = U_{dis}\tilde{H}U_{dis}^{\dagger}}$$

$$\begin{array}{c} H|\varphi_i\rangle = \epsilon_i|\varphi_i\rangle \\ \tilde{H}|\psi_i\rangle = \epsilon_i|\psi_i\rangle \end{array} \longrightarrow \begin{array}{c} HU_{dis}|\psi_i\rangle = \epsilon_i U_{dis}|\psi_i\rangle \\ H|\varphi_i\rangle = \epsilon_i|\varphi_i\rangle \end{array}$$

$\tilde{H} = \sum_i \epsilon_i \sigma_i^z$ (diagonal hamiltonian):

- Eigenstates $|\psi_i\rangle$ correspond to the computational basis:
  $\rightarrow$ easy to prepare
- Applying unitary operation U_{dis} we obtain H eigenstates $|\varphi_i\rangle$:
  $\rightarrow$ we have access to the whole spectrum
- Simulate time evolution $e^{-itH} = U_{dis}e^{-it\tilde{H}}U_{dis}^{\dagger}$
- Simulate thermal state $e^{-\beta H} = U_{dis}e^{-\beta\tilde{H}}U_{dis}^{\dagger}$

The XY model

First-neighbor interaction (with Periodic Boundary Conditions) with a transverse field

$$H = J \sum_{i=1}^{n} \left(\frac{1+\gamma}{2} \sigma_i^x \sigma_{i+1}^x + \frac{1-\gamma}{2} \sigma_i^y \sigma_{i+1}^y \right) + \lambda \sum_{i=1}^{n} \sigma_i^z$$

Two quantum phase transitions:

- $J = \lambda$: (anti)ferromagnetic – paramagnetic phase transition. Same universality class as the Ising model phase transition.
- $\gamma = 0$ anisotropic phase transition between σ_x and σ_y ordered phases.

It is an exact solvable model. The XY Hamiltonian can be diagonalized applying the Jordan-Wigner transformation, the Fourier transform and the Bogoliubov transformation.

Jordan-Wigner transformation

Maps the spin operators into fermionic modes:

$$c_j = \left(\prod_{m<j} \sigma_m^z\right) \frac{\sigma_j^x - i\sigma_j^y}{2}, \qquad c_j^\dagger = \left(\prod_{m<j} \sigma_m^z\right) \frac{\sigma_j^x + i\sigma_j^y}{2}$$

$$\{c_i, c_j\} = 0, \qquad \{c_i, c_j^\dagger\} = \delta_{ij}, \qquad c_i|\Omega_c\rangle = 0$$

In the thermodynamic limit ($n \to \infty$):

$$\boxed{H_c = \frac{J}{2} \sum_{i=1}^{n} \left[c_{i+1}^\dagger c_i + c_i^\dagger c_{i+1} + \gamma \left(c_i^\dagger c_{i+1}^\dagger + c_i c_{i+1} \right) \right] + \lambda \sum_{i=1}^{n} \left(c_i^\dagger c_i - \frac{1}{2} \right)}$$

In terms of the wave function

$$|\Psi\rangle = \sum_{\{i_n\}=0}^{1} \psi_{\mathbf{i_1},\mathbf{i_2},\cdots,\mathbf{i_n}} |i_1, i_2, ..., i_n\rangle \xrightarrow{J.W.} |\Psi\rangle_c = \sum_{\{i_n\}=0}^{1} \psi_{\mathbf{i_1},\mathbf{i_2},\cdots,\mathbf{i_n}} (c_1^\dagger)^{i_1} (c_2^\dagger)^{i_2} ... (c_n^\dagger)^{i_n} |\Omega_c\rangle$$

$\rightarrow$ Wave function coefficients do not change!

Fourier transform

Exploits translational invariance and takes H_c into a momentum space hamiltonian H_b:

$$b_k = \frac{1}{\sqrt{n}} \sum_{j=1}^{n} e^{i\frac{2\pi}{n}jk} c_j, \quad b_k^\dagger = \frac{1}{\sqrt{n}} \sum_{j=1}^{n} e^{-i\frac{2\pi}{n}jk} c_j^\dagger, \quad k = -\frac{n}{2}+1, \cdots, \frac{n}{2}$$

$$\boxed{H_b = \sum_{i=0}^{n-1} \left[\left(J\cos\left(\frac{2\pi k}{n} + \lambda\right)\right) b_k^\dagger b_k + J\gamma \frac{e^{i\frac{2\pi k}{n}}}{2}) \left(b_k^\dagger b_{-k}^\dagger + b_k b_{-k} \right) \right] - \lambda \frac{n}{2}}$$

For $\gamma = 0$ (XX model), the Hamiltonian is already diagonal.

Bogoliubov transformation

Decouples the modes with opposite momentum

$$a_k = \cos\left(\frac{\theta_k}{2}\right) b_k + i \sin\left(\frac{\theta_k}{2}\right) b^\dagger_{-k}$$

$$a^\dagger_{-k} = -i \sin\left(\frac{\theta_k}{2}\right) b^\dagger_k + \cos\left(\frac{\theta_k}{2}\right) b^\dagger_{-k} +$$

where $\theta_k = 2 \arctan\left(\frac{J\gamma \sin\left(\frac{2\pi k}{n}\right)}{J\cos\left(\frac{2\pi k}{n}\right) + \lambda}\right)$.

$$\boxed{H_a = \sum_k \omega_k \left(a^\dagger_k a_k - \frac{1}{2}\right)}$$

with

$$\omega_k = \sqrt{\left(J\cos\left(\frac{2\pi k}{n}\right) + \lambda\right)^2 + J^2\gamma^2 \sin^2\left(\frac{2\pi k}{n}\right)}$$

Ground state energy: $\epsilon_0 = -\sum_k \omega_k$

Quantum circuit to diagonalize XY Hamiltonian

The diagonalization method introduced before solves the model in the thermodynamic limit. Our quantum computers are finite, so we have to simulate this limit.

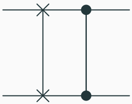
$\rightarrow$ Introduce extra terms in XY Hamiltonian to cancel periodic boundary conditions

$$H = J \sum_{i=1}^{n} \left(\frac{1+\gamma}{2} \sigma_i^x \sigma_{i+1}^x + \frac{1-\gamma}{2} \sigma_i^y \sigma_{i+1}^y \right) + \lambda \sum_{i=1}^{n} \sigma_i^z$$
$$+ J \left(\frac{1+\gamma}{2} \sigma_1^y \sigma_2^z \cdots \sigma_{n-1}^z \sigma_n^y + \frac{1-\gamma}{2} \sigma_1^x \sigma_2^z \cdots \sigma_{n-1}^z \sigma_n^x \right)$$

Quantum circuit to diagonalize XY Hamiltonian

i) **Jordan-Wigner:** the wave function is not affected so we do not need to implement any operation on the quantum register.
But we have to keep in mind that we are dealing with femionic modes that obey anticommutation relations. Any swap between occupied modes carries a minus sign:

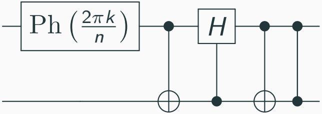
$$fSWAP = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \rightarrow$$



Quantum circuit to diagonalize XY Hamiltonian

ii) **Fourier transform:** If $n = 2^m$, we can implement the Fast Fourier Transform[2]

$$F_k^n = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{e^{-\frac{2\pi i k}{n}}}{\sqrt{2}} & -\frac{e^{-\frac{2\pi i k}{n}}}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & -e^{-\frac{2\pi i k}{n}} \end{pmatrix} \rightarrow$$

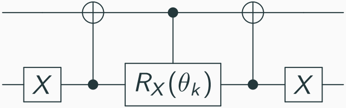


[2] A. J. Ferris, Phys. Rev. Lett. **113**, 010401 (2014)

Quantum circuit to diagonalize XY Hamiltonian

iii) **Bogoliubov transformation:**

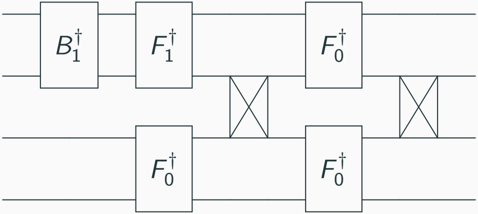
$$B_k^n = \begin{pmatrix} \cos\left(\frac{\theta_k}{2}\right) & 0 & 0 & i\sin\left(\frac{\theta_k}{2}\right) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ i\sin\left(\frac{\theta_k}{2}\right) & 0 & 0 & \cos\left(\frac{\theta_k}{2}\right) \end{pmatrix} \rightarrow$$



U_{dis} circuit

$$\tilde{H} \equiv H_a \xrightarrow{U_{Bog}} H_b \xrightarrow{U_{FT}} H_c \longrightarrow H \Longrightarrow \boxed{U_{dis} = U_{FT}\, U_{Bog}}$$ [3]

For $n = 4$:



[3] F. Verstraete, J. I. Cirac, J. I. Latorre, Phys. Rev. A **79**, 032316 (2009).

Time evolution

$$\langle\Psi(t)|\mathcal{O}|\Psi(t)\rangle = \sum_n \sum_m e^{-it(E_n - E_m)} \langle\Psi_0|E_m\rangle\langle E_m|\mathcal{O}|E_n\rangle\langle E_n|\Psi_0\rangle$$

If $[H, \mathcal{O}] \neq 0$ and $|\Psi_0\rangle$ is not an stationary state, we can observe an oscillation with t with phase $(E_n - E_m)$.

1. Choose $|\Psi_0\rangle$ written in terms of the $\tilde{H}$ basis: $|\Psi_0\rangle = \sum_i c_i |\psi_i\rangle$
2. Apply time-evolution operator:
   $|\Psi(t)\rangle = e^{-it\tilde{H}}|\Psi_0\rangle = \sum_i c_i e^{-it\epsilon_i}|\psi_i\rangle$
3. Implement the U_{dis} gate to transform the state into the XY basis:

$$\boxed{|\Phi(t)\rangle = U_{dis}|\Psi(t)\rangle = U_{dis} \sum_i c_i e^{-it\epsilon_i}|\psi_i\rangle}$$

Example: time evolution $|\uparrow\uparrow\uparrow\uparrow\rangle$ state

Ising model ($\gamma = 1$)

1. $|\varphi_0\rangle = |\uparrow\uparrow\uparrow\uparrow\rangle \equiv |0000\rangle_{Is}$ in the Ising basis. In the computational basis:

$$|\psi_0\rangle = U_{dis}^{\dagger}|0000\rangle_{Is} = \cos\phi|0000\rangle + i\sin\phi|1100\rangle,$$

with $\phi = \frac{1}{2}\arccos\left(\frac{\lambda}{\sqrt{1+\lambda^2}}\right)$.

2. **Time evolution:** add the corresponding phases $e^{-it\epsilon_i}$ where ϵ_i are the energies of the states $|0000\rangle$ and $|1100\rangle$:

$$|\psi(t)\rangle \propto (\cos\phi|00\rangle + i e^{4it\sqrt{1+\lambda^2}}\sin\phi|11\rangle) \otimes |00\rangle$$

3. Apply U_{dis} to recover the Ising state:

$$|\varphi(t)\rangle = U_{dis}|\psi(t)\rangle$$

Thermal simulation

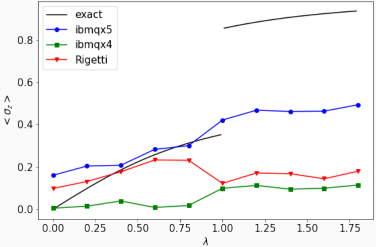
- Quantum system exposed to a heat bath with finite $\beta = 1/T$:

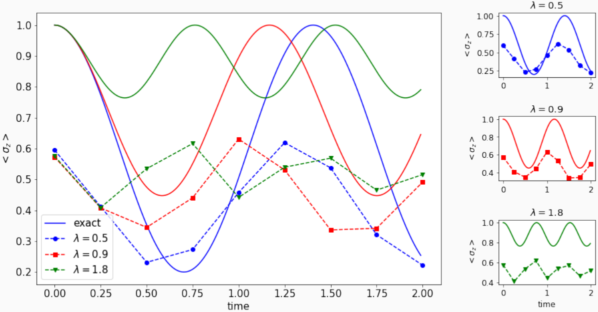
$$\langle \mathcal{O}(\beta) \rangle = \mathrm{Tr}[\mathcal{O}\rho(\beta)] = \frac{1}{\mathcal{Z}} \sum_i e^{-\beta \epsilon_i} \langle E_i | \mathcal{O} | E_i \rangle,$$

  with $\mathcal{Z} = \sum_i e^{-\beta \epsilon_i}$.

- $|E_i\rangle$ are the states of the computational basis (easy to prepare).
- Two strategies to simulate thermal states:
  i) Exact simulation: take as initial state all computational basis states and average them with their corresponding energies (as in time evolution)
  ii) Sampling: a random generator returns one of the computational basis states following the Boltzman distribution.

Results: Ising model





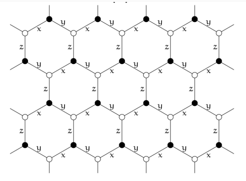
(ibmqx5)

Other models

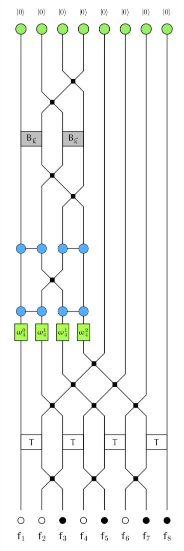
Other exactly-solvable models:

- Kitaev-Honeycomb model[4]

$$H = -J_x \sum_{\mathrm{x-links}} \sigma_i^x \sigma_j^x - J_y \sum_{\mathrm{y-links}} \sigma_i^y \sigma_j^y - J_z \sum_{\mathrm{z-links}} \sigma_i^z \sigma_j^z$$



- Heisenberg Model: *Bethe Ansatz*?



[4] P. Schmoll and R. Orús, Phys. Rev. B **95**, 045112 (2017)

Summary

- It is possible to design an efficient quantum circuit that solves *exactly* integrable hamiltonians
- Available quantum computers such as those from IBM and Rigetti Computing can implement these circuits and simulate few qubit systems
- Example, $n = 4$ Ising spin chain circuit: magnetization for $\lambda < 1$ is well simulated and for $\lambda > 1$ catches the correct trend as well as time evolution for different λ
- Still high error rates and low T_1 and T_2 to simulate larger systems

$\rightarrow$ **Useful to test quantum computers**
$\rightarrow$ **Can we apply what we have learned to simulate non integrable models?**

Quantum circuits for Absolutely Maximally Entangled states

Simulation of quantum physics

Quantum simulation

- Quantum computation
- Quantum annealing
- Adiabatic QC
- ...

Classical simulation

- Quantum Montecarlo
- Artificial Neural Networks
- Tensor Networks
- ...

Quantum advantage frontier depends on the performance of classical algorithms.

Classical computation methods

Pros: we have supercomputers (MareNostrum,...)

Cons:

- Quantum Montecarlo: sign problem
- Artificial Neural Networks: training
- Tensor Networks: only efficient for low entanglement[5]

Quantum computation must solve these problems to have some advantage over classical computation.

[5] G. Vidal, Phys. Rev. Lett. **91**, 147902 (2003)

Entanglement in quantum algorithms

Entanglement has been found at the core of exponential speed up of quantum algorithms such Shor's factorization algorithm[6]

There is a majorization arrow in terms of entanglement in quantum algorithms such Grover or Phase Estimation[7]

[6] R. Jozsa and N. Linden, Proc. R. Soc. London A **459**, 2011 (2003).
[7] J. I. Latorre and M. A. Martín-Delgado, Phys. Rev. A **66**, 022305 (2002)

Bell Inequalities

We should keep in mind:

Violation of Bell Inequalities (BI) prove that there is no hidden variable theory that can describe quantum physics.

Maximal violation of BI in $d = 2$ systems are realized with GHZ states[8]. In $d > 2$, by highly entangled states[9] (deformation of GHZ-type states)

[8] N. D. Mermin, Phys. Rev. Lett. **65**, 1838 (1990)
[9] D. Alsina et. al., Phys. Rev. A **94**, 032102 (2016)

Conclusion:

No high entanglement, no party

(where “party = quantum advantage”)

Absolutely Maximally Entangled states

Absolutely Maximally Entangled states (AMEs)

Pure states that are maximally entangled in all their bipartitions. In other words, all reduced densiy matrices are proportional to the identity.

$\nexists$ AME(n, d) $\forall$ d and n[10]:

n \ d	2	3	4	5	...
2	BS	BS	BS	BS	...
3	GHZ	GHZ	GHZ	GHZ	...
4	$\nexists$	AME(4,3)	AME(4,4)	AME(4,5)	...
5	AME(5,2)	AME(5,3)	AME(5,4)	AME(5,5)	...
6	AME(6,2)	AME(6,3)	AME(6,4)	AME(6,5)	...

[10] F. Huber, Table of AME states, `http://www.tp.nt.uni-siegen.de/+fhuber/ame.html`

Absolutely Maximally Entangled states

- $\nexists$ AMEs of $d = 2$ for $n > 6$.
- For a given n one can always increase d to find an AMEs.
- AMEs of minimal support are those that can be composed with $d^{\lfloor n/2 \rfloor}$ fully separable orthogonal states with equal positive coefficients of $d^{-\lfloor n/2 \rfloor}/2$. Futhermore, they have a one-to-one correspondence with maximum distance separable codes.
- There are different ways to obtain AMEs. In particular, we use *graph states*.

Graph states

Graph states can be constructed from a graph, where each vertex is a $F_d|0\rangle = (|0\rangle + |1\rangle + \cdots |d-1\rangle)/\sqrt{d}$ state and each edge a CZ gate.

Depending on the graph, we will obtain states with different entanglement. AMEs have a corresponding graph, although we will obtain an AMEs of maximal support.

Some AME graphs work for any d. AME graphs of non-prime dimension $d = d_1 d_2 \cdots$ can be obtained from the corresponding graphs of prime dimension d_1, d_2, ...[11]

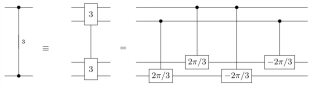
[11] W. Helwig, arXiv:1306.2879 [quant-ph]

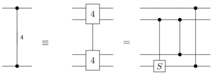
For $d = 2$, each vertex is generated with a Hadamard gate (F_2) and each edge with a CZ gate.

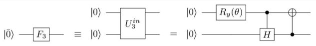
Quantum computers of $d > 2$ are hard to construct, but we can simulate qudit states with qubits:

$$\begin{array}{ccc} |0\rangle & \rightarrow & |00\rangle \\ |1\rangle & \rightarrow & |01\rangle \\ |2\rangle & \rightarrow & |10\rangle \\ & \dots & \end{array}$$

$\rightarrow$ adapt qudit CZ and F_d gates

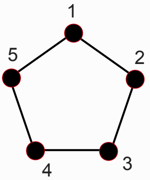


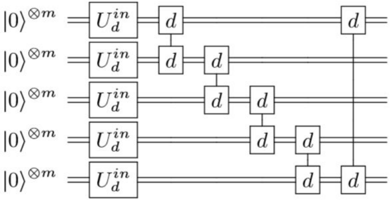


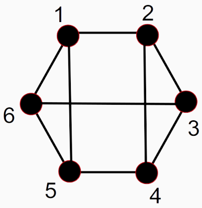


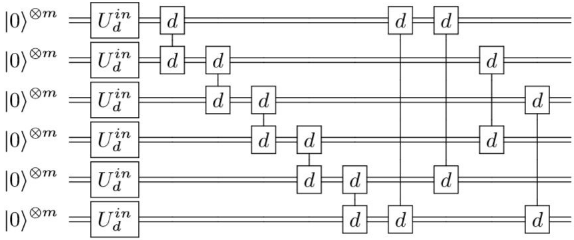
...

AME graph states with qubits

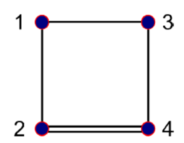


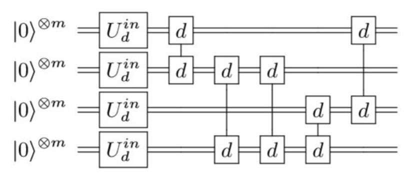


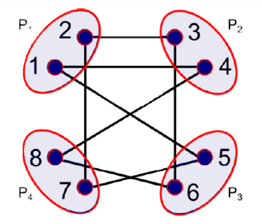


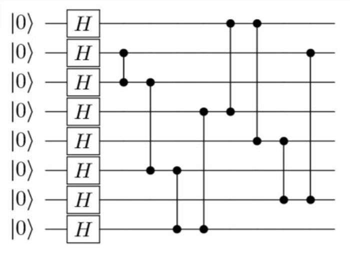


AME graph states with qubits



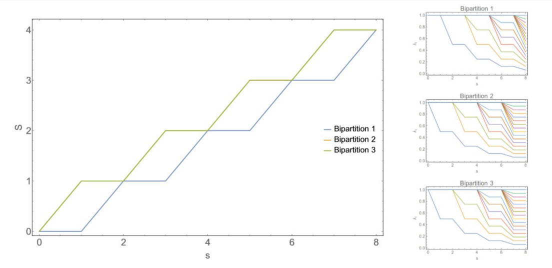




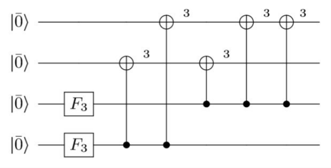


Entanglement majorization

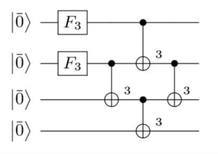
Compute average entropy at each step. Example: AME(4,4)



Find optimal circuits



$\rightarrow$



Summary

- Quantum computers must support highly entangled states to show quantum advantage.
- A hard test for a quantum device can be the generation of Absolutely Maximally Entangled states.
- We propose to construct AMEs of any dimension $d = 2$ and to simulate AMEs of $d > 2$ with qubits.
- Moreover, we show that majorization is behind the efficiency of a quantum circuit to entangle all its parts.
- The run of the proposed circuits on a quantum computer could be used as a benchmark method to test the performance of a quantum computer.

Observation

Furthermore,

- Many quantum algorithms use pre-stablished ansantzs to entangle its parts (e.g. VQE, quantum autoencoders, VQC,...): are these ansatz generating the required amount of entanglement? Can AMEs or graph states structures be used to improve the performance of these algorithms?

Thanks!

- **“Maximal Entanglement in High Energy Physics”**
  ACL, J. I. Latorre, J. Rojo and L. Rottoli, SciPost Phys. 3, 036 (2017).
- **“Exact Ising Model Simulation on a Quantum computer”**
  ACL, Quantum 2, 114 (2018).
- **“Quantum circuits for Absolutely Maximally Entangled states”**
  ACL, D. Goyeneche and J. I. Latorre, in preparation.