Maximal Entanglement in Quantum Computation

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Outline

Motivation

Simulation of quantum physics

I therefore believe it's true that with a suitable class of quantum machines you could imitate any quantum system, including the physical reality.

–Richard P. Feynman,

"Simulating physics with computers", 1982.

Quantum simulation

- Quantum computation
- Quantum annealing
- Adiabatic QC
- ...

Classical simulation

- Quantum Montecarlo
- Artificial Neural Networks
- Tensor Networks
- ...

Quantum advantage frontier depends on the performance of classical algorithms.

Classical computation methods

Pros: we have supercomputers (MareNostrum,...)
Cons:

- Quantum Montecarlo: sign problem
- Artificial Neural Networks: training
- Tensor Networks: only efficient for low entanglement[1]

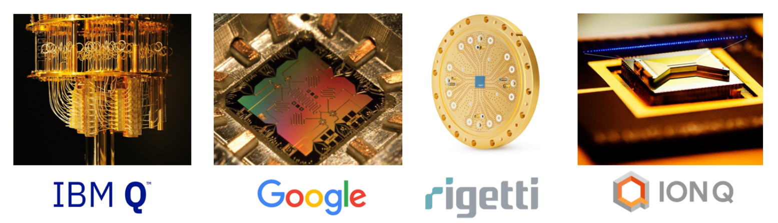
and

- Exponential growth of resources: to simulate n qubits (quantum systems of dimension 2) we need $2 \cdot 2^n - 2$ parameters!!!

Quantum computation must solve these problems to have some advantage over classical computation.

[1] G. Vidal, Phys. Rev. Lett. **91**, 147902 (2003)

Proliferation of quantum computers



...

Characterized by: qubit decoherence and relaxation times, gate fidelities.

- Is that enough? Are current quantum computers performing as expected from their characterization parameters?
- How can we compare their performance?
- Are these devices good enough to show quantum advantage at some point?

Entanglement

Entanglement

Two or more systems are entangled if their quantum state can not be described independently from the other.

$$|\psi\rangle_{AB} \neq |\psi\rangle_A \otimes |\psi\rangle_B$$

Example:

Product state

$$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|00\rangle_{AB} + |01\rangle_{AB}\right) = |0\rangle_A \otimes \frac{1}{\sqrt{2}}\left(|0\rangle_B + |1\rangle_B\right)$$

Entangled state

$$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|00\rangle_{AB} + |11\rangle_{AB}\right) \neq |\psi\rangle_A \otimes |\psi\rangle_B$$

Entanglement in quantum algorithms

- Entanglement has been found at the core of exponential speed up of quantum algorithms such Shor's factorization algorithm[2]
- There is a majorization arrow in terms of entanglement in quantum algorithms such Grover or Phase Estimation[3]: at each step of the computation, entanglement increases.

In addition, we should keep in mind:

- Maximal violation of Bell Inequalities in $d = 2$ systems are realized with maximally entangled states. That is, **classical physics can not describe quantum mechanics because of entanglement.**

[2] R. Jozsa and N. Linden, Proc. R. Soc. London A **459**, 2011 (2003).
[3] J. I. Latorre and M. A. Martín-Delgado, Phys. Rev. A **66**, 022305 (2002)

Conclusion:

No high entanglement, no party

(where "party = quantum advantage")

Spin chain in a Quantum Computer

Disentangling Hamiltonian

$$\begin{array}{ccc} \text{Entangling hamiltonian } H & \longrightarrow & \text{Non-interacting hamiltonian } \tilde{H} \\ \text{(hard to simulate)} & & \text{(easy to simulate)} \end{array}$$

$$\boxed{H = U_{dis} \tilde{H} U_{dis}^{\dagger}}$$

- H Eigenstates correspond to the computational basis:
  $\rightarrow$ easy to prepare
- By applying the unitary operation U_{dis} (quantum gate) we obtain H eigenstates:
  $\rightarrow$ we have access to the whole spectrum
  1. Simulate time evolution $e^{-itH} = U_{dis} e^{-it\tilde{H}} U_{dis}^{\dagger}$
  2. Simulate thermal state $e^{-\beta H} = U_{dis} e^{-\beta \tilde{H}} U_{dis}^{\dagger}$

The XY model

First-neighbor interaction (with Periodic Boundary Conditions) with a transverse field

$$H = J\sum_{i=1}^{n}\left(\frac{1+\gamma}{2}\sigma_i^x\sigma_{i+1}^x + \frac{1-\gamma}{2}\sigma_i^y\sigma_{i+1}^y\right) + \lambda\sum_{i=1}^{n}\sigma_i^z$$

Two **quantum phase transitions**:

- $J = \lambda$: (anti)ferromagnetic – paramagnetic phase transition.
- $\gamma = 0$ anisotropic phase transition.

Analytical solution

1. Jordan-Wigner transformation

Maps the spin operators to fermionic modes. In terms of the wave function

$$|\Psi\rangle = \sum_{\{i_n\}=0}^{1} \psi_{\mathbf{i_1},\mathbf{i_2},\cdots,\mathbf{i_n}} |i_1, i_2, ..., i_n\rangle \xrightarrow{J.W.} |\Psi\rangle_c = \sum_{\{i_n\}=0}^{1} \psi_{\mathbf{i_1},\mathbf{i_2},\cdots,\mathbf{i_n}} (c_1^\dagger)^{i_1}(c_2^\dagger)^{i_2}...(c_n^\dagger)^{i_n}|\Omega_c\rangle$$

$\rightarrow$ Wave function coefficients do not change!

2. Fourier transform

Exploits translational invariance and takes H into a momentum space Hamiltonian.

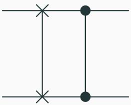
3. Bogoliubov transformation

Decouples the modes with opposite momentum.

Quantum circuit to diagonalize XY Hamiltonian

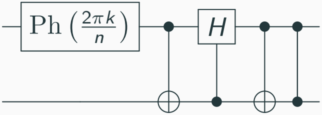
i) **Jordan-Wigner:**

$$fSWAP = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \rightarrow$$



ii) **Fourier transform:** Fast Fourier Transform[4]

$$F_k^n = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{e^{-\frac{2\pi ik}{n}}}{\sqrt{2}} & -\frac{e^{-\frac{2\pi ik}{n}}}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & -e^{-\frac{2\pi ik}{n}} \end{pmatrix} \rightarrow$$

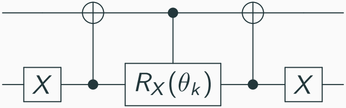


[4] A. J. Ferris, Phys. Rev. Lett. **113**, 010401 (2014)

Quantum circuit to diagonalize XY Hamiltonian

iii) **Bogoliubov transformation:**

$$B_k^n = \begin{pmatrix} \cos\left(\frac{\theta_k}{2}\right) & 0 & 0 & i\sin\left(\frac{\theta_k}{2}\right) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ i\sin\left(\frac{\theta_k}{2}\right) & 0 & 0 & \cos\left(\frac{\theta_k}{2}\right) \end{pmatrix} \rightarrow$$

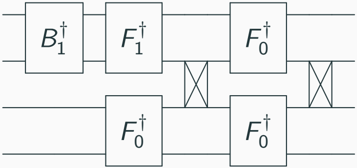


U_{dis} circuit

$$\tilde{H} \equiv H_a \xrightarrow{U_{Bog}} H_b \xrightarrow{U_{FT}} H_c \longrightarrow H \Longrightarrow \boxed{U_{dis} = U_{FT} U_{Bog}}$$ [5]

For $n = 4$:

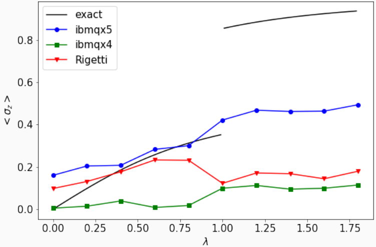
Product state $\longrightarrow$

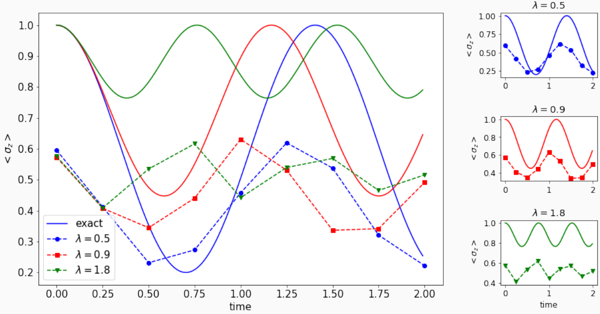


Entangled state

[5] F. Verstraete, J. I. Cirac, J. I. Latorre, Phys. Rev. A **79**, 032316 (2009).

Results: Ising model





Quantum circuits for Absolutely Maximally Entangled states

Absolutely Maximally Entangled states

Absolutely Maximally Entangled states (AMEs)

Pure states that are maximally entangled in all their bipartitions.

$\nexists$ AME(n, d) $\forall$ d and n[6]:

- $d = 2$ (qubits): $\exists$ AME for $n = 2, 3, 5$ and 6.
- $d = 3$ (qutrits): $\exists$ AME for $n = 2, \cdots 7, 9$ and 10.
- $d = 4$ (ququarts): $\exists$ AME for $n = 2, 3, 4, 5, 6, 9$ and 10.
- **For a given d and n, AME existence is an open problem.**

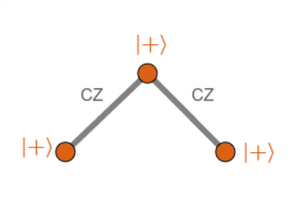
[6] F. Huber, Table of AME states, `http://www.tp.nt.uni-siegen.de/+fhuber/ame.html`

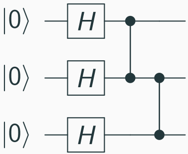
Graph states (qubits)

Graph states can be constructed from a graph, where each vertex performs the operation

$$|+\rangle = H|0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right),$$

and each edge a CZ gate.

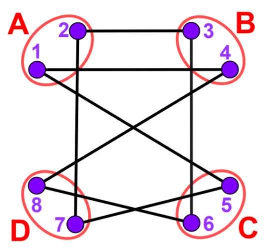


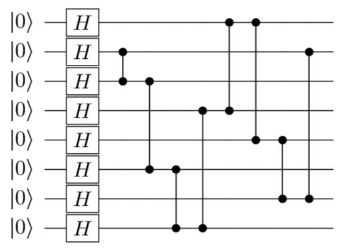


$$\frac{1}{2\sqrt{2}}(|000\rangle + |001\rangle + |010\rangle - |011\rangle$$
$$|100\rangle + |101\rangle - |110\rangle + |111\rangle)$$

AME graph state circuit

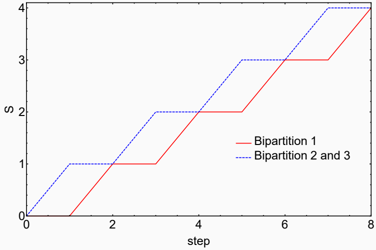
Construct an AME state with a simple quantum circuit.
Example: AME(4,4).



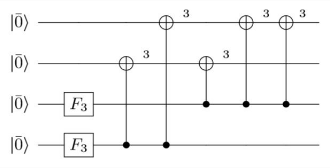


Entanglement majorization

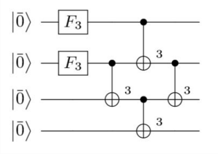
Compute average entropy at each step. Example: AME(4,4)



Use this property to find optimal circuits in general. Example: AME(4,3).



$\rightarrow$



Conclusions

Summary

- We can simulate **exactly** integrable spin models in quantum computers.
- Quantum computers must support highly entangled states to show quantum advantage.
  $\rightarrow$ **Hard test:** generation of Absolutely Maximally Entangled states.
- We show that majorization is behind the efficiency of a quantum circuit to entangle all its parts.
- The run of the proposed circuits on a quantum computer could be used as a benchmark method to test the performance of a quantum device.

Thanks!

More info:

"Exact Ising Model Simulation on a Quantum computer"
ACL, Quantum 2, 114 (2018).

"Quantum Circuits for maximally entangled states"
ACL,J. I. Latorre and D. Goyeneche, arXiv:1904.07955 [quant-ph]

"Maximal Entanglement. Applications in Quantum Information and Particle Physics"
ACL, arXiv:1906.XXXXX [quant-ph] (hopefully!).