

## Correlated random effects: iidkd

This model is available for dimensions  $k = 2$ , to 10. We describe in detail the case for  $k = 3$  as other ones are similar. This model do the same as models `iid2d`, `iid3d`, `iid4d`, `iid5d`, but uses a more efficient parameterisation.

### Parametrization

The ( $k = 3$ )-dimensional Normal-Wishard model is used if one want to define two vectors of “random effects”,  $u$  and  $v$ , say, for which  $(u_i, v_i)$  are iid bivariate Normals

$$\begin{pmatrix} u_i \\ v_i \\ w_i \end{pmatrix} \sim \mathcal{N}(\mathbf{0}, \mathbf{W}^{-1})$$

where the covariance matrix  $\mathbf{W}^{-1}$  is parameterised as  $\mathbf{W} = \mathbf{L}\mathbf{L}^T$ , where

$$\mathbf{L} = \begin{pmatrix} \exp(\theta_1) & & \\ \theta_4 & \exp(\theta_2) & \\ \theta_5 & \theta_6 & \exp(\theta_3) \end{pmatrix} \quad (1)$$

and  $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6$  can take any value. The number of hyperparameters are  $k(k+1)/2$ , which is 3, 6, 10, 15, 21, 28, 36, 45, 55, for  $k = 2, 3, 4, 5, 6, 7, 8, 9, 10$ .

For these models the precision matrix  $\mathbf{W}$  is Wishart distributed

$$\mathbf{W} \sim \text{Wishart}_k(r, \mathbf{R}^{-1}),$$

with density

$$\pi(\mathbf{W}) = c^{-1} |\mathbf{W}|^{(r-(k+1))/2} \exp \left\{ -\frac{1}{2} \text{Trace}(\mathbf{W}\mathbf{R}) \right\}, \quad r > k+1$$

and

$$c = 2^{(rk)/2} |\mathbf{R}|^{-r/2} \pi^{(k(k-1))/4} \prod_{j=1}^k \Gamma((r+1-j)/2).$$

Then,

$$\text{E}(\mathbf{W}) = r\mathbf{R}^{-1}, \quad \text{and} \quad \text{E}(\mathbf{W}^{-1}) = \mathbf{R}/(r - (k+1)).$$

### Hyperparameters

The hyperparameters are  $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6$ .

The prior-parameters are

$$(r, R_1, R_2, R_3, R_4, R_5, R_6)$$

where

$$\mathbf{R} = \begin{pmatrix} R_1 & R_4 & R_5 \\ R_4 & R_2 & R_6 \\ R_5 & R_6 & R_3 \end{pmatrix}$$

The `inla` function reports posterior distribution for the hyperparameters  $\{\theta_i\}$ , and the conversion into interpretable quantities can be done using simulation as described below.

The prior for  $\theta$  is **fixed** to be `wishartkd`, and number of prior parameters required are  $1 + k(k+1)/2$ . By default the prior-parameters are

$$(r = 100, \underbrace{1, \dots, 1}_{k \text{ times}}, 0, \dots, 0)$$

The model `iidkd` is specified as

where  $\text{order} = k = 3$ , and the `iidkd` model is represented internally as one vector of length  $n$ ,

where  $n = 3m$ , and  $n$  is the (required) argument in  $\mathbf{f}()$ .

$$\sum u_i = 0, \quad \sum v_i = 0 \quad \text{and} \quad \sum w_i = 0.$$

**doc** Gaussian random effect in dim=k with Wishart prior

param

```

    to.theta function(x) x
    from.theta function(x) x
theta4
  hyperid 29104
  name theta4
  short.name theta4
  initial 0
  fixed TRUE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta5
  hyperid 29105
  name theta5
  short.name theta5
  initial 0
  fixed TRUE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta6
  hyperid 29106
  name theta6
  short.name theta6
  initial 0
  fixed TRUE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta7
  hyperid 29107
  name theta7
  short.name theta7
  initial 0
  fixed TRUE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta8
  hyperid 29108

```

```

    name theta8
    short.name theta8
    initial 0
    fixed TRUE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta9
    hyperid 29109
    name theta9
    short.name theta9
    initial 0
    fixed TRUE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta10
    hyperid 29110
    name theta10
    short.name theta10
    initial 0
    fixed TRUE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta11
    hyperid 29111
    name theta11
    short.name theta11
    initial 0
    fixed TRUE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta12
    hyperid 29112
    name theta12
    short.name theta12
    initial 0
    fixed TRUE

```

```

    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta13
    hyperid 29113
    name theta13
    short.name theta13
    initial 0
    fixed TRUE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta14
    hyperid 29114
    name theta14
    short.name theta14
    initial 0
    fixed TRUE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta15
    hyperid 29115
    name theta15
    short.name theta15
    initial 0
    fixed TRUE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta16
    hyperid 29116
    name theta16
    short.name theta16
    initial 0
    fixed TRUE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x

```

```

theta17
  hyperid 29117
  name theta17
  short.name theta17
  initial 0
  fixed TRUE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta18
  hyperid 29118
  name theta18
  short.name theta18
  initial 0
  fixed TRUE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta19
  hyperid 29119
  name theta19
  short.name theta19
  initial 0
  fixed TRUE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta20
  hyperid 29120
  name theta20
  short.name theta20
  initial 0
  fixed TRUE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta21
  hyperid 29121
  name theta21
  short.name theta21

```

```

    initial 0
    fixed TRUE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta22
    hyperid 29122
    name theta22
    short.name theta22
    initial 0
    fixed TRUE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta23
    hyperid 29123
    name theta23
    short.name theta23
    initial 0
    fixed TRUE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta24
    hyperid 29124
    name theta24
    short.name theta24
    initial 0
    fixed TRUE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta25
    hyperid 29125
    name theta25
    short.name theta25
    initial 0
    fixed TRUE
    prior none
    param

```

```

    to.theta function(x) x
    from.theta function(x) x
theta26
  hyperid 29126
  name theta26
  short.name theta26
  initial 0
  fixed TRUE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta27
  hyperid 29127
  name theta27
  short.name theta27
  initial 0
  fixed TRUE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta28
  hyperid 29128
  name theta28
  short.name theta28
  initial 0
  fixed TRUE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta29
  hyperid 29129
  name theta29
  short.name theta29
  initial 0
  fixed TRUE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta30
  hyperid 29130

```



```

    name theta30
    short.name theta30
    initial 0
    fixed TRUE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta31
    hyperid 29131
    name theta31
    short.name theta31
    initial 0
    fixed TRUE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta32
    hyperid 29132
    name theta32
    short.name theta32
    initial 0
    fixed TRUE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta33
    hyperid 29133
    name theta33
    short.name theta33
    initial 0
    fixed TRUE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta34
    hyperid 29134
    name theta34
    short.name theta34
    initial 0
    fixed TRUE

```

```

    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta35
    hyperid 29135
    name theta35
    short.name theta35
    initial 0
    fixed TRUE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta36
    hyperid 29136
    name theta36
    short.name theta36
    initial 0
    fixed TRUE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta37
    hyperid 29137
    name theta37
    short.name theta37
    initial 0
    fixed TRUE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta38
    hyperid 29138
    name theta38
    short.name theta38
    initial 0
    fixed TRUE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x

```

#### **theta39**

**hyperid** 29139  
**name** theta39  
**short.name** theta39  
**initial** 0  
**fixed** TRUE  
**prior** none  
**param**  
**to.theta** function(x) x  
**from.theta** function(x) x

#### **theta40**

**hyperid** 29140  
**name** theta40  
**short.name** theta40  
**initial** 0  
**fixed** TRUE  
**prior** none  
**param**  
**to.theta** function(x) x  
**from.theta** function(x) x

#### **theta41**

**hyperid** 29141  
**name** theta41  
**short.name** theta41  
**initial** 0  
**fixed** TRUE  
**prior** none  
**param**  
**to.theta** function(x) x  
**from.theta** function(x) x

#### **theta42**

**hyperid** 29142  
**name** theta42  
**short.name** theta42  
**initial** 0  
**fixed** TRUE  
**prior** none  
**param**  
**to.theta** function(x) x  
**from.theta** function(x) x

#### **theta43**

**hyperid** 29143  
**name** theta43  
**short.name** theta43

```

    initial 0
    fixed TRUE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta44
    hyperid 29144
    name theta44
    short.name theta44
    initial 0
    fixed TRUE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta45
    hyperid 29145
    name theta45
    short.name theta45
    initial 0
    fixed TRUE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta46
    hyperid 29146
    name theta46
    short.name theta46
    initial 0
    fixed TRUE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta47
    hyperid 29147
    name theta47
    short.name theta47
    initial 0
    fixed TRUE
    prior none
    param

```

```

    to.theta function(x) x
    from.theta function(x) x
theta48
  hyperid 29148
  name theta48
  short.name theta48
  initial 0
  fixed TRUE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta49
  hyperid 29149
  name theta49
  short.name theta49
  initial 0
  fixed TRUE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta50
  hyperid 29150
  name theta50
  short.name theta50
  initial 0
  fixed TRUE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta51
  hyperid 29151
  name theta51
  short.name theta51
  initial 0
  fixed TRUE
  prior none
  param
  to.theta function(x) x
  from.theta function(x) x
theta52
  hyperid 29152

```

```

    name theta52
    short.name theta52
    initial 0
    fixed TRUE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta53
    hyperid 29153
    name theta53
    short.name theta53
    initial 0
    fixed TRUE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta54
    hyperid 29154
    name theta54
    short.name theta54
    initial 0
    fixed TRUE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x
theta55
    hyperid 29155
    name theta55
    short.name theta55
    initial 0
    fixed TRUE
    prior none
    param
    to.theta function(x) x
    from.theta function(x) x

constr FALSE

nrow.ncol FALSE

augmented TRUE

aug.factor 1

```

**aug.constr** 1 2 3 4 5 6 7 8 9 10

**n.div.by** -1

**n.required** TRUE

**set.default.values** TRUE

**pdf** iidkd

**Example**