

## Likelihood fmri

### Parametrisation

This is special parameterisation of the non-central  $\chi_\nu$  distribution. Let  $\{x_i\}$  are iid Gaussians with mean  $\mu$  and precision  $\tau$ , then

$$z = \sqrt{\sum_{i=1}^{\nu} \tau x_i^2}$$

is non-central  $\chi$ -distribution with (integer and fixed by design)  $\nu > 0$  degrees of freedom, and non-centrality parameter

$$\rho = \sqrt{\nu\tau\mu^2}.$$

The observation  $y$  is  $y = z/\sqrt{\tau}$  and we're interested in the underlying true signal

$$\lambda = \rho/\sqrt{\tau} = \sqrt{\nu\mu^2}$$

### Link-function

The linkfunction is given as

$$\log(\lambda) = \eta$$

where  $\eta$  is the linear predictor.

### Hyperparameters

The hyperparameters are  $\theta = (\theta_1, \theta_2)$ , where

$$\tau = \exp(\theta_1)$$

is the precision, and

$$\nu = \theta_2$$

For technical reasons,  $\nu$  is implemented as a hyper-parameter, but is required to be fixed. Hence, the initial value for  $\theta_2$  defines the (fixed) value for  $\nu$ .

The prior is given on  $\theta_1$ .

### Specification

- family = `fmri` or family = `fmrismrv`
- Required arguments: `y` (and optional `scale` for `fmri` to scale  $\tau$ )

### Hyperparameter specification and default values

**doc** fmri distribution (special nc-chi)

**hyper**

**thetal**

**hyperid** 103101

**name** precision

**short.name** prec

**initial** 0

**fixed** FALSE

```

    prior loggamma
    param 10 10
    to.theta function(x) log(x)
    from.theta function(x) exp(x)
theta2
    hyperid 103202
    name dof
    short.name df
    initial 4
    fixed TRUE
    prior normal
    param 0 1
    to.theta function(x) x
    from.theta function(x) x

status experimental

survival FALSE

discrete FALSE

link default log

pdf fmri

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hyper

```

```

theta1
    hyperid 104101
    name precision
    short.name prec
    initial 0
    fixed FALSE
    prior loggamma
    param 10 10
    to.theta function(x) log(x)
    from.theta function(x) exp(x)
theta2
    hyperid 104201
    name dof
    short.name df
    initial 4
    fixed TRUE
    prior normal
    param 0 1
    to.theta function(x) x

```

```

from.theta function(x) x
status experimental
survival TRUE
discrete FALSE
link default log
pdf fmri

```

## Example

In the following example we estimate the parameters in a simulated example.

```

n <- 300
x <- rnorm(n, sd = 0.3)
df <- 1
prec <- 3
eta <- 1 + x
lambda <- exp(eta)
y <- sqrt(rchisq(n, df = df, ncp = prec * lambda^2) /prec)

r <- inla(y ~ 1 + x,
          data = data.frame(y, x),
          family = "fmri",
          control.family = list(hyper = list(df = list(initial = df))),
          control.inla = list(cmin = 0,
                              int.strategy = "eb",
                              strategy = "adaptive"),
          verbose = TRUE)
summary(r)

## 'cmin=0' seems to be required only for initial values that can give
## 'crazy' values. We can rerun without this re-starting at the prev fit,
## to validate
r$.args$control.inla$cmin <- -Inf
r$.args$control.inla$int.strategy <- "auto"
rr <- inla.rerun(r)
summary(rr)

```

## Notes

Thanks to LS for providing all the details and a robust implementation of this likelihood.