Correlated random effects: `iidkd`

This model is available for dimensions $k = 2$, to 10. We describe in detail the case for $k = 3$ as other ones are similar. This model do the same as models `iid2d`, `iid3d`, `iid4d`, `iid5d`, but uses a more efficient parameterisation.

Parametrization

The $(k = 3)$-dimensional Normal-Wishard model is used if one want to define three vectors of "random effects", u and v and w, say, for which (u_i, v_i, w_i) are iid bivariate Normals

$$\begin{pmatrix} u_i \\ v_i \\ w_i \end{pmatrix} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{W}^{-1}\right)$$

where the covariance matrix $\mathbf{W}^{-1}$ is parameterised as $\mathbf{W} = \mathbf{L}\mathbf{L}^T$, where

$$\mathbf{L} = \begin{pmatrix} \exp(\theta_1) & & \\ \theta_4 & \exp(\theta_2) & \\ \theta_5 & \theta_6 & \exp(\theta_3) \end{pmatrix} \tag{1}$$

and $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6$ can take any value. The number of hyperparameters are $k(k + 1)/2$, which is 3, 6, 10, 15, 21, 28, 36, 45, 55, for $k = 2, 3, 4, 5, 6, 7, 8, 9, 10$.

For these models the precision matrix $\mathbf{W}$ is Wishart distributed

$$\mathbf{W} \sim \text{Wishart}_k(r, \mathbf{R}^{-1}),$$

with density

$$\pi(\mathbf{W}) = c^{-1}|\mathbf{W}|^{(r-(k+1))/2} \exp\left\{-\frac{1}{2}\text{Trace}(\mathbf{W}\mathbf{R})\right\}, \quad r > k + 1$$

and

$$c = 2^{(rk)/2}|\mathbf{R}|^{-r/2}\pi^{(k(k-1))/4} \prod_{j=1}^{k} \Gamma((r + 1 - j)/2).$$

Then,

$$\text{E}(\mathbf{W}) = r\mathbf{R}^{-1}, \quad \text{and} \quad \text{E}(\mathbf{W}^{-1}) = \mathbf{R}/(r - (k + 1)).$$

Hyperparameters

The hyperparameters are $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6$.

The prior-parameters are

$$(r, R_1, R_2, R_3, R_4, R_5, R_6)$$

where

$$\mathbf{R} = \begin{pmatrix} R_1 & R_4 & R_5 \\ R_4 & R_2 & R_6 \\ R_5 & R_6 & R_3 \end{pmatrix}$$

The `inla` function reports posterior distribution for the hyperparameters $\{\theta_i\}$, and the conversion into interpretable quantities can be done using simulation as described below.

The prior for θ is **fixed** to be `wishartkd`, and number of prior parameters required are $1 + k(k + 1)/2$. By default the prior-parameters are

$$(r = 100, \underbrace{1, \ldots, 1}_{k \text{ times}}, 0, \ldots, 0)$$

1

Specification

The model `iidkd` is specified as

```
y ~ f(i, model="iidkd", order=3, n = <length>) + ...
```

where order $= k = 3$, and the `iidkd` model is represented internally as one vector of length n,

$$(u_1, u_2 \ldots, u_m, v_1, v_2, \ldots, v_m, w_1, w_2, \ldots, w_m)$$

where $n = 3m$, and n is the (required) argument in `f()`.

For this model the argument `constr=TRUE` is interpreted as 3 sum-to-zero constraints

$$\sum u_i = 0, \quad \sum v_i = 0 \quad \text{and} \quad \sum w_i = 0.$$

Hyperparameter spesification and default values

(**Note:** The value "2468.8642" is just a code for "replace this by the default value". As the default value depends on `order`, the was the easy way out for the moment.)

doc Gaussian random effect in dim=k with Wishart prior

hyper

    theta1

        hyperid 29101

        name theta1

        short.name theta1

        initial 2468.8642

        fixed FALSE

        prior wishartkd

        param 100 2468.8642

        to.theta function(x) x

        from.theta function(x) x

    theta2

        hyperid 29102

        name theta2

        short.name theta2

        initial 2468.8642

        fixed FALSE

        prior none

        param

        to.theta function(x) x

        from.theta function(x) x

    theta3

hyperid 29103

name theta3

short.name theta3

initial 2468.8642

fixed FALSE

prior none

param

to.theta `function(x) x`

from.theta `function(x) x`

theta4

hyperid 29104

name theta4

short.name theta4

initial 2468.8642

fixed FALSE

prior none

param

to.theta `function(x) x`

from.theta `function(x) x`

theta5

hyperid 29105

name theta5

short.name theta5

initial 2468.8642

fixed FALSE

prior none

param

to.theta `function(x) x`

from.theta `function(x) x`

theta6

hyperid 29106

name theta6

short.name theta6

initial 2468.8642

fixed FALSE

prior none

param

to.theta `function(x) x`

from.theta `function(x) x`

theta7

hyperid 29107

name theta7

short.name theta7

initial 2468.8642

fixed FALSE

prior none

param

to.theta function(x) x

from.theta function(x) x

theta8

hyperid 29108

name theta8

short.name theta8

initial 2468.8642

fixed FALSE

prior none

param

to.theta function(x) x

from.theta function(x) x

theta9

hyperid 29109

name theta9

short.name theta9

initial 2468.8642

fixed FALSE

prior none

param

to.theta function(x) x

from.theta function(x) x

theta10

hyperid 29110

name theta10

short.name theta10

initial 2468.8642

fixed FALSE

prior none

param

to.theta function(x) x

from.theta function(x) x

theta11

hyperid 29111

name theta11

short.name theta11

initial 2468.8642

fixed FALSE

prior none

param

to.theta function(x) x

from.theta `function(x) x`

theta12

    hyperid 29112

    name theta12

    short.name theta12

    initial 2468.8642

    fixed FALSE

    prior none

    param

    to.theta `function(x) x`

    from.theta `function(x) x`

theta13

    hyperid 29113

    name theta13

    short.name theta13

    initial 2468.8642

    fixed FALSE

    prior none

    param

    to.theta `function(x) x`

    from.theta `function(x) x`

theta14

    hyperid 29114

    name theta14

    short.name theta14

    initial 2468.8642

    fixed FALSE

    prior none

    param

    to.theta `function(x) x`

    from.theta `function(x) x`

theta15

    hyperid 29115

    name theta15

    short.name theta15

    initial 2468.8642

    fixed FALSE

    prior none

    param

    to.theta `function(x) x`

    from.theta `function(x) x`

theta16

    hyperid 29116

    name theta16

short.name theta16

initial 2468.8642

fixed FALSE

prior none

param

to.theta function(x) x

from.theta function(x) x

theta17

hyperid 29117

name theta17

short.name theta17

initial 2468.8642

fixed FALSE

prior none

param

to.theta function(x) x

from.theta function(x) x

theta18

hyperid 29118

name theta18

short.name theta18

initial 2468.8642

fixed FALSE

prior none

param

to.theta function(x) x

from.theta function(x) x

theta19

hyperid 29119

name theta19

short.name theta19

initial 2468.8642

fixed FALSE

prior none

param

to.theta function(x) x

from.theta function(x) x

theta20

hyperid 29120

name theta20

short.name theta20

initial 2468.8642

fixed FALSE

prior none

param

to.theta function(x) x

from.theta function(x) x

theta21

hyperid 29121

name theta21

short.name theta21

initial 2468.8642

fixed FALSE

prior none

param

to.theta function(x) x

from.theta function(x) x

theta22

hyperid 29122

name theta22

short.name theta22

initial 2468.8642

fixed FALSE

prior none

param

to.theta function(x) x

from.theta function(x) x

theta23

hyperid 29123

name theta23

short.name theta23

initial 2468.8642

fixed FALSE

prior none

param

to.theta function(x) x

from.theta function(x) x

theta24

hyperid 29124

name theta24

short.name theta24

initial 2468.8642

fixed FALSE

prior none

param

to.theta function(x) x

from.theta function(x) x

theta25

hyperid 29125

name theta25

short.name theta25

initial 2468.8642

fixed FALSE

prior none

param

to.theta `function(x) x`

from.theta `function(x) x`

theta26

hyperid 29126

name theta26

short.name theta26

initial 2468.8642

fixed FALSE

prior none

param

to.theta `function(x) x`

from.theta `function(x) x`

theta27

hyperid 29127

name theta27

short.name theta27

initial 2468.8642

fixed FALSE

prior none

param

to.theta `function(x) x`

from.theta `function(x) x`

theta28

hyperid 29128

name theta28

short.name theta28

initial 2468.8642

fixed FALSE

prior none

param

to.theta `function(x) x`

from.theta `function(x) x`

theta29

hyperid 29129

name theta29

short.name theta29

initial 2468.8642

fixed FALSE

prior none

param

to.theta function(x) x

from.theta function(x) x

theta30

hyperid 29130

name theta30

short.name theta30

initial 2468.8642

fixed FALSE

prior none

param

to.theta function(x) x

from.theta function(x) x

theta31

hyperid 29131

name theta31

short.name theta31

initial 2468.8642

fixed FALSE

prior none

param

to.theta function(x) x

from.theta function(x) x

theta32

hyperid 29132

name theta32

short.name theta32

initial 2468.8642

fixed FALSE

prior none

param

to.theta function(x) x

from.theta function(x) x

theta33

hyperid 29133

name theta33

short.name theta33

initial 2468.8642

fixed FALSE

prior none

param

to.theta function(x) x

from.theta `function(x) x`

theta34

    hyperid 29134

    name theta34

    short.name theta34

    initial 2468.8642

    fixed FALSE

    prior none

    param

    to.theta `function(x) x`

    from.theta `function(x) x`

theta35

    hyperid 29135

    name theta35

    short.name theta35

    initial 2468.8642

    fixed FALSE

    prior none

    param

    to.theta `function(x) x`

    from.theta `function(x) x`

theta36

    hyperid 29136

    name theta36

    short.name theta36

    initial 2468.8642

    fixed FALSE

    prior none

    param

    to.theta `function(x) x`

    from.theta `function(x) x`

theta37

    hyperid 29137

    name theta37

    short.name theta37

    initial 2468.8642

    fixed FALSE

    prior none

    param

    to.theta `function(x) x`

    from.theta `function(x) x`

theta38

    hyperid 29138

    name theta38

short.name theta38

initial 2468.8642

fixed FALSE

prior none

param

to.theta `function(x) x`

from.theta `function(x) x`

theta39

hyperid 29139

name theta39

short.name theta39

initial 2468.8642

fixed FALSE

prior none

param

to.theta `function(x) x`

from.theta `function(x) x`

theta40

hyperid 29140

name theta40

short.name theta40

initial 2468.8642

fixed FALSE

prior none

param

to.theta `function(x) x`

from.theta `function(x) x`

theta41

hyperid 29141

name theta41

short.name theta41

initial 2468.8642

fixed FALSE

prior none

param

to.theta `function(x) x`

from.theta `function(x) x`

theta42

hyperid 29142

name theta42

short.name theta42

initial 2468.8642

fixed FALSE

prior none

param

to.theta `function(x) x`

from.theta `function(x) x`

theta43

hyperid 29143

name theta43

short.name theta43

initial 2468.8642

fixed FALSE

prior none

param

to.theta `function(x) x`

from.theta `function(x) x`

theta44

hyperid 29144

name theta44

short.name theta44

initial 2468.8642

fixed FALSE

prior none

param

to.theta `function(x) x`

from.theta `function(x) x`

theta45

hyperid 29145

name theta45

short.name theta45

initial 2468.8642

fixed FALSE

prior none

param

to.theta `function(x) x`

from.theta `function(x) x`

theta46

hyperid 29146

name theta46

short.name theta46

initial 2468.8642

fixed FALSE

prior none

param

to.theta `function(x) x`

from.theta `function(x) x`

theta47

hyperid 29147

name theta47

short.name theta47

initial 2468.8642

fixed FALSE

prior none

param

to.theta function(x) x

from.theta function(x) x

theta48

hyperid 29148

name theta48

short.name theta48

initial 2468.8642

fixed FALSE

prior none

param

to.theta function(x) x

from.theta function(x) x

theta49

hyperid 29149

name theta49

short.name theta49

initial 2468.8642

fixed FALSE

prior none

param

to.theta function(x) x

from.theta function(x) x

theta50

hyperid 29150

name theta50

short.name theta50

initial 2468.8642

fixed FALSE

prior none

param

to.theta function(x) x

from.theta function(x) x

theta51

hyperid 29151

name theta51

short.name theta51

initial 2468.8642

fixed FALSE

prior none

param

to.theta function(x) x

from.theta function(x) x

theta52

hyperid 29152

name theta52

short.name theta52

initial 2468.8642

fixed FALSE

prior none

param

to.theta function(x) x

from.theta function(x) x

theta53

hyperid 29153

name theta53

short.name theta53

initial 2468.8642

fixed FALSE

prior none

param

to.theta function(x) x

from.theta function(x) x

theta54

hyperid 29154

name theta54

short.name theta54

initial 2468.8642

fixed FALSE

prior none

param

to.theta function(x) x

from.theta function(x) x

theta55

hyperid 29155

name theta55

short.name theta55

initial 2468.8642

fixed FALSE

prior none

param

to.theta function(x) x

from.theta function(x) x

constr FALSE

nrow.ncol FALSE

augmented TRUE

aug.factor 1

aug.constr 1 2 3 4 5 6 7 8 9 10

n.div.by -1

n.required TRUE

set.default.values TRUE

status experimental

pdf iidkd

Example

Just simulate some data and estimate the parameters back. This is for `order=4`.

```
n <- 300
m <- 4
N <- m*n
rho <- 0.8

Sigma <- matrix(NA, m, m)
diag(Sigma) <- (1/(1:m))^2
for(i in 1:m) {
    if (i+1 <= m) {
        for (j in (i+1):m) {
            Sigma[i, j] <- Sigma[j, i] <- rho * sqrt(Sigma[i, i]*Sigma[j, j])
        }
    }
}

library(mvtnorm)
yy <- rmvnorm(n, sigma = Sigma)
y <- c()
for(i in 1:m) {
    y <- c(y, yy[, i])
}

r <- inla(y ~ f(i, model = "iidkd", order = m, n=N,
                ## set parameters using 'theta1'.
                ## these are the default parameters.
                hyper =  list(theta1 = list(
                                    param = c(100, rep(1, m), rep(0, m*(m-1)/2))))),
        data = data.frame(i = 1:N, y),
        ## fix precision as we have exact observations
        control.family = list(hyper = list(
                                    prec = list(initial = 15, fixed = TRUE))),
        verbose = TRUE)
```

```
## this is how the internal parameters are defined
L <- t(chol(solve(Sigma)))
diag(L) <- log(diag(L))
LL <- t(chol(solve(cov(yy))))
diag(LL) <- log(diag(LL))


## compare the estimated (internal) parameters with MLE and the truth
round(dig = 3, cbind(true = c(diag(L), L[lower.tri(L)]),
                     mle = c(diag(LL), LL[lower.tri(LL)]),
                     inla = r$mode$theta))


## this is how to compute stdev and correlations from the internal parametes.
## these ones are more interpretable. one have to know where the parameters
## are in the list though, but here it is easy...

convert.internal <- function(theta, dim, offset = 0)
{
    ntheta.off.diag <- dim*(dim-1)/2L
    L <- matrix(0, dim, dim)
    diag(L) <- exp(theta[offset + 1:dim])
    L[lower.tri(L)] <- theta[offset + dim + 1:ntheta.off.diag]
    S <- solve(L %*% t(L))
    iSigma <- 1/sqrt(diag(S))
    Cor <- diag(iSigma) %*% S %*% diag(iSigma)
    return(c(sqrt(diag(S)), Cor[lower.tri(Cor)]))
}


xx <- inla.hyperpar.sample(10000, r)
qq <- rowMeans(apply(xx, 1, convert.internal, dim = m))
iSigma <- 1/sqrt(diag(Sigma))
Cor <- diag(iSigma) %*% Sigma %*% diag(iSigma)
round(dig = 3, cbind(inla = c(qq),
                     true = c(sqrt(diag(Sigma)), Cor[lower.tri(Cor)])))
```