# Correlated random effects: `iidkd`

This model is available for dimensions $k = 2$, to 10. We describe in detail the case for $k = 3$ as other ones are similar. This model do the same as models `iid2d`, `iid3d`, `iid4d`, `iid5d`, but uses a different and more efficient parameterisation.

## Parametrization

The $(k = 3)$-dimensional Normal-Wishard model is used if one want to define three vectors of "random effects", $u$ and $v$ and $w$, say, for which $(u_i, v_i, w_i)$ are iid bivariate Normals

$$
\begin{pmatrix} u_i \\ v_i \\ w_i \end{pmatrix} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{W}^{-1}\right)
$$

where the covariance matrix $\mathbf{W}^{-1}$ is parameterised as $\mathbf{W} = \mathbf{L}\mathbf{L}^T$, where

$$
\mathbf{L} = \begin{pmatrix} \exp(\theta_1) & & \\ \theta_4 & \exp(\theta_2) & \\ \theta_5 & \theta_6 & \exp(\theta_3) \end{pmatrix} \tag{1}
$$

and $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6$ can take any value. The number of hyperparameters are $k(k+1)/2$, which is 3, 6, 10, 15, 21, 28, 36, 45, 55, for $k = 2, 3, 4, 5, 6, 7, 8, 9, 10$.

For these models the precision matrix $\mathbf{W}$ is Wishart distributed

$$
\mathbf{W} \sim \text{Wishart}_k(r, \mathbf{R}^{-1}),
$$

with density

$$
\pi(\mathbf{W}) = c^{-1}|\mathbf{W}|^{(r-(k+1))/2} \exp\left\{-\frac{1}{2}\text{Trace}(\mathbf{W}\mathbf{R})\right\}, \quad r > k+1
$$

and

$$
c = 2^{(rk)/2}|\mathbf{R}|^{-r/2}\pi^{(k(k-1))/4} \prod_{j=1}^{k} \Gamma((r+1-j)/2).
$$

Then,

$$
\text{E}(\mathbf{W}) = r\mathbf{R}^{-1}, \quad \text{and} \quad \text{E}(\mathbf{W}^{-1}) = \mathbf{R}/(r-(k+1)).
$$

## Hyperparameters

The hyperparameters are $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6$.

The prior-parameters are

$$
(r, R_1, R_2, R_3, R_4, R_5, R_6)
$$

where

$$
\mathbf{R} = \begin{pmatrix} R_1 & R_4 & R_5 \\ R_4 & R_2 & R_6 \\ R_5 & R_6 & R_3 \end{pmatrix}
$$

The `inla` function reports posterior distribution for the hyperparameters $\{\theta_i\}$, and the conversion into interpretable quantities can be done using simulation as described below.

The prior for $\theta$ is **fixed** to be `wishartkd`, and number of prior parameters required are $1 + k(k+1)/2$. By default the prior-parameters are

$$
(r = 100, \underbrace{1, \ldots, 1}_{k \text{ times}}, 0, \ldots, 0)
$$

## Specification

The model `iidkd` is specified as

```
y ~ f(i, model="iidkd", order=3, n = <length>) + ...
```

where order $= k = 3$, and the `iidkd` model is represented internally as one vector of length $n$,

$$(u_1, u_2 \ldots, u_m, v_1, v_2, \ldots, v_m, w_1, w_2, \ldots, w_m)$$

where $n = 3m$, and $n$ is the (required) argument in `f()`.

For this model the argument `constr=TRUE` is interpreted as 3 sum-to-zero constraints

$$\sum u_i = 0, \quad \sum v_i = 0 \quad \text{and} \quad \sum w_i = 0.$$

## Hyperparameter spesification and default values

(**Note:** The value "2468.8642" is just a code for "replace this by the default value". As the default value depends on `order`, the was the easy way out for the moment.)

**doc** Gaussian random effect in dim=k with Wishart prior

**hyper**

    **theta1**

        **hyperid** 29101

        **name** theta1

        **short.name** theta1

        **initial** 2468.8642

        **fixed** FALSE

        **prior** wishartkd

        **param** 100 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642

        **to.theta** function(x) x

        **from.theta** function(x) x

    **theta2**

        **hyperid** 29102

        **name** theta2

        **short.name** theta2

        **initial** 2468.8642

        **fixed** FALSE

        **prior** none

        **param**

        **to.theta** function(x) x

        **from.theta** function(x) x

    **theta3**

**hyperid** 29103

**name** theta3

**short.name** theta3

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** `function(x) x`

**from.theta** `function(x) x`

## theta4

**hyperid** 29104

**name** theta4

**short.name** theta4

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** `function(x) x`

**from.theta** `function(x) x`

## theta5

**hyperid** 29105

**name** theta5

**short.name** theta5

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** `function(x) x`

**from.theta** `function(x) x`

## theta6

**hyperid** 29106

**name** theta6

**short.name** theta6

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** `function(x) x`

**from.theta** `function(x) x`

## theta7

**hyperid** 29107

**name** theta7

**short.name** theta7

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** function(x) x

**from.theta** function(x) x

**theta8**

**hyperid** 29108

**name** theta8

**short.name** theta8

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** function(x) x

**from.theta** function(x) x

**theta9**

**hyperid** 29109

**name** theta9

**short.name** theta9

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** function(x) x

**from.theta** function(x) x

**theta10**

**hyperid** 29110

**name** theta10

**short.name** theta10

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** function(x) x

**from.theta** function(x) x

**theta11**

**hyperid** 29111

**name** theta11

**short.name** theta11

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** function(x) x

**from.theta** `function(x) x`

**theta12**

    **hyperid** 29112

    **name** theta12

    **short.name** theta12

    **initial** 2468.8642

    **fixed** FALSE

    **prior** none

    **param**

    **to.theta** `function(x) x`

    **from.theta** `function(x) x`

**theta13**

    **hyperid** 29113

    **name** theta13

    **short.name** theta13

    **initial** 2468.8642

    **fixed** FALSE

    **prior** none

    **param**

    **to.theta** `function(x) x`

    **from.theta** `function(x) x`

**theta14**

    **hyperid** 29114

    **name** theta14

    **short.name** theta14

    **initial** 2468.8642

    **fixed** FALSE

    **prior** none

    **param**

    **to.theta** `function(x) x`

    **from.theta** `function(x) x`

**theta15**

    **hyperid** 29115

    **name** theta15

    **short.name** theta15

    **initial** 2468.8642

    **fixed** FALSE

    **prior** none

    **param**

    **to.theta** `function(x) x`

    **from.theta** `function(x) x`

**theta16**

    **hyperid** 29116

    **name** theta16

**short.name** theta16

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** function(x) x

**from.theta** function(x) x

**theta17**

    **hyperid** 29117

    **name** theta17

    **short.name** theta17

    **initial** 2468.8642

    **fixed** FALSE

    **prior** none

    **param**

    **to.theta** function(x) x

    **from.theta** function(x) x

**theta18**

    **hyperid** 29118

    **name** theta18

    **short.name** theta18

    **initial** 2468.8642

    **fixed** FALSE

    **prior** none

    **param**

    **to.theta** function(x) x

    **from.theta** function(x) x

**theta19**

    **hyperid** 29119

    **name** theta19

    **short.name** theta19

    **initial** 2468.8642

    **fixed** FALSE

    **prior** none

    **param**

    **to.theta** function(x) x

    **from.theta** function(x) x

**theta20**

    **hyperid** 29120

    **name** theta20

    **short.name** theta20

    **initial** 2468.8642

    **fixed** FALSE

    **prior** none

**param**

**to.theta** `function(x) x`

**from.theta** `function(x) x`

**theta21**

**hyperid** 29121

**name** theta21

**short.name** theta21

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** `function(x) x`

**from.theta** `function(x) x`

**theta22**

**hyperid** 29122

**name** theta22

**short.name** theta22

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** `function(x) x`

**from.theta** `function(x) x`

**theta23**

**hyperid** 29123

**name** theta23

**short.name** theta23

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** `function(x) x`

**from.theta** `function(x) x`

**theta24**

**hyperid** 29124

**name** theta24

**short.name** theta24

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** `function(x) x`

**from.theta** `function(x) x`

**theta25**

**hyperid** 29125

**name** theta25

**short.name** theta25

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** `function(x) x`

**from.theta** `function(x) x`

## theta26

**hyperid** 29126

**name** theta26

**short.name** theta26

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** `function(x) x`

**from.theta** `function(x) x`

## theta27

**hyperid** 29127

**name** theta27

**short.name** theta27

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** `function(x) x`

**from.theta** `function(x) x`

## theta28

**hyperid** 29128

**name** theta28

**short.name** theta28

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** `function(x) x`

**from.theta** `function(x) x`

## theta29

**hyperid** 29129

**name** theta29

**short.name** theta29

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** function(x) x

**from.theta** function(x) x

**theta30**

**hyperid** 29130

**name** theta30

**short.name** theta30

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** function(x) x

**from.theta** function(x) x

**theta31**

**hyperid** 29131

**name** theta31

**short.name** theta31

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** function(x) x

**from.theta** function(x) x

**theta32**

**hyperid** 29132

**name** theta32

**short.name** theta32

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** function(x) x

**from.theta** function(x) x

**theta33**

**hyperid** 29133

**name** theta33

**short.name** theta33

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** function(x) x

**from.theta** `function(x) x`

**theta34**

    **hyperid** 29134

    **name** theta34

    **short.name** theta34

    **initial** 2468.8642

    **fixed** FALSE

    **prior** none

    **param**

    **to.theta** `function(x) x`

    **from.theta** `function(x) x`

**theta35**

    **hyperid** 29135

    **name** theta35

    **short.name** theta35

    **initial** 2468.8642

    **fixed** FALSE

    **prior** none

    **param**

    **to.theta** `function(x) x`

    **from.theta** `function(x) x`

**theta36**

    **hyperid** 29136

    **name** theta36

    **short.name** theta36

    **initial** 2468.8642

    **fixed** FALSE

    **prior** none

    **param**

    **to.theta** `function(x) x`

    **from.theta** `function(x) x`

**theta37**

    **hyperid** 29137

    **name** theta37

    **short.name** theta37

    **initial** 2468.8642

    **fixed** FALSE

    **prior** none

    **param**

    **to.theta** `function(x) x`

    **from.theta** `function(x) x`

**theta38**

    **hyperid** 29138

    **name** theta38

**short.name** theta38

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** function(x) x

**from.theta** function(x) x

**theta39**

**hyperid** 29139

**name** theta39

**short.name** theta39

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** function(x) x

**from.theta** function(x) x

**theta40**

**hyperid** 29140

**name** theta40

**short.name** theta40

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** function(x) x

**from.theta** function(x) x

**theta41**

**hyperid** 29141

**name** theta41

**short.name** theta41

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** function(x) x

**from.theta** function(x) x

**theta42**

**hyperid** 29142

**name** theta42

**short.name** theta42

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** function(x) x

**from.theta** function(x) x

**theta43**

**hyperid** 29143

**name** theta43

**short.name** theta43

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** function(x) x

**from.theta** function(x) x

**theta44**

**hyperid** 29144

**name** theta44

**short.name** theta44

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** function(x) x

**from.theta** function(x) x

**theta45**

**hyperid** 29145

**name** theta45

**short.name** theta45

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** function(x) x

**from.theta** function(x) x

**theta46**

**hyperid** 29146

**name** theta46

**short.name** theta46

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** function(x) x

**from.theta** function(x) x

**theta47**

**hyperid** 29147

**name** theta47

**short.name** theta47

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** function(x) x

**from.theta** function(x) x

**theta48**

**hyperid** 29148

**name** theta48

**short.name** theta48

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** function(x) x

**from.theta** function(x) x

**theta49**

**hyperid** 29149

**name** theta49

**short.name** theta49

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** function(x) x

**from.theta** function(x) x

**theta50**

**hyperid** 29150

**name** theta50

**short.name** theta50

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** function(x) x

**from.theta** function(x) x

**theta51**

**hyperid** 29151

**name** theta51

**short.name** theta51

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** function(x) x

**from.theta** function(x) x

**theta52**

**hyperid** 29152

**name** theta52

**short.name** theta52

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** function(x) x

**from.theta** function(x) x

**theta53**

**hyperid** 29153

**name** theta53

**short.name** theta53

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** function(x) x

**from.theta** function(x) x

**theta54**

**hyperid** 29154

**name** theta54

**short.name** theta54

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** function(x) x

**from.theta** function(x) x

**theta55**

**hyperid** 29155

**name** theta55

**short.name** theta55

**initial** 2468.8642

**fixed** FALSE

**prior** none

**param**

**to.theta** function(x) x

 

        **from.theta** function(x) x

**constr** FALSE

**nrow.ncol** FALSE

**augmented** TRUE

**aug.factor** 1

**aug.constr** 1 2 3 4 5 6 7 8 9 10

**n.div.by** -1

**n.required** TRUE

**set.default.values** TRUE

**status** experimental

**pdf** iidkd

## Example

Just simulate some data and estimate the parameters back. This is for `order=4`.

```
n <- 300
m <- 4
N <- m*n
rho <- 0.8

Sigma <- matrix(NA, m, m)
diag(Sigma) <- (1/(1:m))^2
for(i in 1:m) {
    if (i+1 <= m) {
        for (j in (i+1):m) {
            Sigma[i, j] <- Sigma[j, i] <- rho * sqrt(Sigma[i, i]*Sigma[j, j])
        }
    }
}

library(mvtnorm)
yy <- rmvnorm(n, sigma = Sigma)
y <- c()
for(i in 1:m) {
    y <- c(y, yy[, i])
}

r <- inla(y ~ f(i, model = "iidkd", order = m, n=N,
                ## set parameters using 'theta1'.
                ## these are the default parameters.
                hyper =  list(theta1 = list(
                                    param = c(100, rep(1, m), rep(0, m*(m-1)/2))))),
        data = data.frame(i = 1:N, y),
        ## fix precision as we have exact observations
        control.family = list(hyper = list(
                                    prec = list(initial = 15, fixed = TRUE))),
        verbose = FALSE)
```

```
## this is how the internal parameters are defined
L <- t(chol(solve(Sigma)))
diag(L) <- log(diag(L))
LL <- t(chol(solve(cov(yy))))
diag(LL) <- log(diag(LL))


## compare the estimated (internal) parameters with MLE and the truth
round(dig = 3, cbind(true = c(diag(L), L[lower.tri(L)]),
                     mle = c(diag(LL), LL[lower.tri(LL)]),
                     inla = r$mode$theta))


## this gives a list of sampled matrices (stdev's and correlations)
xx <- inla.iidkd.sample(10^4, r, "i")
## compute the mean
qq <- matrix(rowMeans(matrix(unlist(xx), nrow = m^2)), m, m)

iSigma <- 1/sqrt(diag(Sigma))
Cor <- diag(iSigma) %*% Sigma %*% diag(iSigma)
round(dig = 3, cbind(inla = c(diag(qq), qq[lower.tri(qq)]),
                     true = c(sqrt(diag(Sigma)), Cor[lower.tri(Cor)])))
```