Aggregated Gaussian

Parametrisation

This is shorthand to allow for aggregated Gaussian observations, where we have repeated observations with the same mean and (known scaled) precision. These can be aggregated into an equivalent likelihood reducing the computional effort.

Let $y = (y_1, \ldots, y_n)$ be iid observations of a Gaussian with mean μ and precisions $s_i\tau$, where the s_i's are fixed and known scaling parameters (default $s_i = 1$), then

$$
\begin{aligned}
f(y|\mu,\tau) &= \prod_{i=1}^{n} (2\pi)^{-1/2} (s_i\tau)^{1/2} \exp\left(-\frac{1}{2}(s_i\tau)\,(y_i-\mu)^2\right) & (1)\\
&= (2\pi)^{-n/2}\tau^{n/2}\left(\prod_{i=1}^{n} s_i^{1/2}\right)\exp\left(-\frac{1}{2}\tau\sum_{i=1}^{n} s_i(y_i-\mu)^2\right) & (2)\\
&= (2\pi)^{-n/2}\tau^{n/2}\left(\prod_{i=1}^{n} s_i^{1/2}\right)\exp\left(-\frac{1}{2}m\tau\left[(\bar{y}-\mu)^2+v\right]\right) & (3)
\end{aligned}
$$

where

$$
\begin{aligned}
m &= \sum_{i=1}^{n} s_i\\
\bar{y} &= \frac{1}{m}\sum_{i=1}^{n} s_i y_i\\
v &= \frac{1}{m}\sum_{i=1}^{n} s_i y_i^2 - \bar{y}^2
\end{aligned}
$$

Link-function

The mean μ is linked to the linear predictor η by

$$\mu = \eta$$

Hyperparameters

The hyperparameter is θ, where

$$\theta = \log\tau$$

and the prior is defined on θ.

Specification

This family require the response to be an `inla.mdata`-object, where each row[1] is

$$(v,\ \frac{1}{2}\sum_{i=1}^{n}\log(s_i),\ m,\ n,\ \bar{y})$$

This object is most easily constructed using the `inla.agaussian()` function, which gives the object to use directly.

- family = `agaussian`

[1]It is a list of vectors, so not strictly a "row".

- Required arguments: `inla.mdata(`Y, S`)`. Argument S is optional, and is not given, then s_i for i.

Here, Y is a matrix where each row is a vector of observations. NA's in each row are remove. The matrix S is optional

Hyperparameter spesification and default values

doc The aggregated Gaussian likelihoood

hyper

theta

hyperid 66001
name log precision
short.name prec
initial 4
fixed FALSE
prior loggamma
param 1 5e-05
to.theta `function(x) log(x)`
from.theta `function(x) exp(x)`

survival FALSE

discrete FALSE

link default identity logit loga cauchit log logoffset

pdf agaussian

Example

```
n <- 10
y <- rnorm(n)
Y <- inla.agaussian(y)

r <- inla(Y ~ 1,
          data = list(Y = Y),
          family = "agaussian")
rr <- inla(y ~ 1,
           data = data.frame(y),
           family = "gaussian")
print(r$mlik - rr$mlik)

inla.dev.new()
par(mfrow = c(1, 2))
plot(r$internal.marginals.hyperpar[[1]], pch = 19, main = "prec")
lines(rr$internal.marginals.hyperpar[[1]], lwd = 3)
plot(r$marginals.fixed$'(Intercept)', pch = 19, main = "intercept")
lines(rr$marginals.fixed$'(Intercept)', lwd = 3)
```

```
####################################
####################################

n <- 5
s <- 1:n ## scale the precision
y <- rnorm(n)
yy <- rnorm(n, sd = sqrt(1/s))
Y <- inla.agaussian(rbind(y, yy),
                    rbind(rep(1, n), s))

r <- inla(Y ~ 1,
          data = list(Y = Y),
          family = "agaussian",
          control.compute = list(cpo = TRUE, dic = TRUE))
rr <- inla(yyy ~ 1,
           data = data.frame(yyy = c(y, yy)),
           scale = c(rep(1, n), s),
           control.compute = list(cpo = TRUE, dic = TRUE),
           family = "gaussian")
print(r$mlik - rr$mlik)

inla.dev.new()
par(mfrow = c(1, 2))
plot(r$internal.marginals.hyperpar[[1]], pch = 19, main = "prec")
lines(rr$internal.marginals.hyperpar[[1]], lwd = 3)
plot(r$marginals.fixed$'(Intercept)', pch = 19, main = "intercept")
lines(rr$marginals.fixed$'(Intercept)', lwd = 3)
```

Notes

- Thanks to JW for suggesting this formulation and for providing the details.