Correlated random effects: `iidkd`

This model is available for dimensions $k = 2$, to 10. We describe in detail the case for $k = 3$ as other ones are similar. This model do the same as models `iid2d`, `iid3d`, `iid4d`, `iid5d`, but uses a different and more efficient parameterisation.

Parametrization

The $(k = 3)$-dimensional Normal-Wishard model is used if one want to define three vectors of "random effects", u and v and w, say, for which (u_i, v_i, w_i) are iid bivariate Normals

$$\begin{pmatrix} u_i \\ v_i \\ w_i \end{pmatrix} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{W}^{-1}\right)$$

where the covariance matrix $\mathbf{W}^{-1}$ is parameterised as $\mathbf{W} = \mathbf{L}\mathbf{L}^T$, where

$$\mathbf{L} = \begin{pmatrix} \exp(\theta_1) & & \\ \theta_4 & \exp(\theta_2) & \\ \theta_5 & \theta_6 & \exp(\theta_3) \end{pmatrix} \tag{1}$$

and $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6$ can take any value. The number of hyperparameters are $k(k+1)/2$, which is 3, 6, 10, 15, 21, 28, 36, 45, 55, for $k = 2, 3, 4, 5, 6, 7, 8, 9, 10$.

For these models the precision matrix $\mathbf{W}$ is Wishart distributed

$$\mathbf{W} \ \sim \ \text{Wishart}_k(r, \mathbf{R}^{-1}),$$

with density

$$\pi(\mathbf{W}) = c^{-1}|\mathbf{W}|^{(r-(k+1))/2} \exp\left\{-\frac{1}{2}\text{Trace}(\mathbf{W}\mathbf{R})\right\}, \quad r > k+1$$

and

$$c = 2^{(rk)/2}|\mathbf{R}|^{-r/2}\pi^{(k(k-1))/4} \prod_{j=1}^{k} \Gamma((r+1-j)/2).$$

Then,

$$\text{E}(\mathbf{W}) = r\mathbf{R}^{-1}, \quad \text{and} \quad \text{E}(\mathbf{W}^{-1}) = \mathbf{R}/(r-(k+1)).$$

Hyperparameters

The hyperparameters are $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6$.

The prior-parameters are

$$(r, R_1, R_2, R_3, R_4, R_5, R_6)$$

where

$$\mathbf{R} = \begin{pmatrix} R_1 & R_4 & R_5 \\ R_4 & R_2 & R_6 \\ R_5 & R_6 & R_3 \end{pmatrix}$$

The `inla` function reports posterior distribution for the hyperparameters $\{\theta_i\}$, and the conversion into interpretable quantities can be done using simulation as described below.

The prior for θ is **fixed** to be `wishartkd`, and number of prior parameters required are $1 + k(k+1)/2$. By default the prior-parameters are

$$(r = 100, \underbrace{1, \ldots, 1}_{k \text{ times}}, 0, \ldots, 0)$$

Specification

The model `iidkd` is specified as

```
y ~ f(i, model="iidkd", order=3, n = <length>) + ...
```

where order $= k = 3$, and the `iidkd` model is represented internally as one vector of length n,

$$(u_1, u_2 \ldots, u_m, v_1, v_2, \ldots, v_m, w_1, w_2, \ldots, w_m)$$

where $n = 3m$, and n is the (required) argument in `f()`.

For this model the argument `constr=TRUE` is interpreted as 3 sum-to-zero constraints

$$\sum u_i = 0, \quad \sum v_i = 0 \quad \text{and} \quad \sum w_i = 0.$$

Hyperparameter spesification and default values

(**Note:** The value "2468.8642" is just a code for "replace this by the default value". As the default value depends on `order`, the was the easy way out for the moment.)

doc Gaussian random effect in dim=k with Wishart prior

hyper

theta1

hyperid 29101
name theta1
short.name theta1
initial 2468.8642
fixed FALSE
prior wishartkd
param 100 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642 2468.8642
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to.theta `function(x) x`
from.theta `function(x) x`

theta2

hyperid 29102
name theta2
short.name theta2
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta3

hyperid 29103
name theta3
short.name theta3
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta4

hyperid 29104
name theta4
short.name theta4
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta5

hyperid 29105
name theta5
short.name theta5
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta6

hyperid 29106
name theta6
short.name theta6
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta7

hyperid 29107
name theta7
short.name theta7
initial 2468.8642

fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta8

hyperid 29108
name theta8
short.name theta8
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta9

hyperid 29109
name theta9
short.name theta9
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta10

hyperid 29110
name theta10
short.name theta10
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta11

hyperid 29111
name theta11
short.name theta11
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`

from.theta `function(x) x`

theta12

hyperid 29112
name theta12
short.name theta12
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta13

hyperid 29113
name theta13
short.name theta13
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta14

hyperid 29114
name theta14
short.name theta14
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta15

hyperid 29115
name theta15
short.name theta15
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta16

hyperid 29116
name theta16

short.name theta16
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta17

hyperid 29117
name theta17
short.name theta17
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta18

hyperid 29118
name theta18
short.name theta18
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta19

hyperid 29119
name theta19
short.name theta19
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta20

hyperid 29120
name theta20
short.name theta20
initial 2468.8642
fixed FALSE
prior none

param
to.theta `function(x) x`
from.theta `function(x) x`

theta21
hyperid 29121
name theta21
short.name theta21
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta22
hyperid 29122
name theta22
short.name theta22
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta23
hyperid 29123
name theta23
short.name theta23
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta24
hyperid 29124
name theta24
short.name theta24
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta25

hyperid 29125
name theta25
short.name theta25
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta26

hyperid 29126
name theta26
short.name theta26
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta27

hyperid 29127
name theta27
short.name theta27
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta28

hyperid 29128
name theta28
short.name theta28
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta29

hyperid 29129
name theta29
short.name theta29
initial 2468.8642

fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta30

hyperid 29130
name theta30
short.name theta30
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta31

hyperid 29131
name theta31
short.name theta31
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta32

hyperid 29132
name theta32
short.name theta32
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta33

hyperid 29133
name theta33
short.name theta33
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`

from.theta `function(x) x`

theta34

hyperid 29134
name theta34
short.name theta34
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta35

hyperid 29135
name theta35
short.name theta35
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta36

hyperid 29136
name theta36
short.name theta36
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta37

hyperid 29137
name theta37
short.name theta37
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta38

hyperid 29138
name theta38

short.name theta38
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta39

hyperid 29139
name theta39
short.name theta39
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta40

hyperid 29140
name theta40
short.name theta40
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta41

hyperid 29141
name theta41
short.name theta41
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta42

hyperid 29142
name theta42
short.name theta42
initial 2468.8642
fixed FALSE
prior none

param
to.theta `function(x) x`
from.theta `function(x) x`

theta43
hyperid 29143
name theta43
short.name theta43
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta44
hyperid 29144
name theta44
short.name theta44
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta45
hyperid 29145
name theta45
short.name theta45
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta46
hyperid 29146
name theta46
short.name theta46
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta47

hyperid 29147
name theta47
short.name theta47
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta48

hyperid 29148
name theta48
short.name theta48
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta49

hyperid 29149
name theta49
short.name theta49
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta50

hyperid 29150
name theta50
short.name theta50
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta51

hyperid 29151
name theta51
short.name theta51
initial 2468.8642

fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta52

hyperid 29152
name theta52
short.name theta52
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta53

hyperid 29153
name theta53
short.name theta53
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta54

hyperid 29154
name theta54
short.name theta54
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta55

hyperid 29155
name theta55
short.name theta55
initial 2468.8642
fixed FALSE
prior none
param
to.theta `function(x) x`

from.theta `function(x) x`

constr FALSE

nrow.ncol FALSE

augmented TRUE

aug.factor 1

aug.constr 1 2 3 4 5 6 7 8 9 10

n.div.by -1

n.required TRUE

set.default.values TRUE

status experimental

pdf iidkd

Example

Just simulate some data and estimate the parameters back. This is for `order=4`.

```
library(mvtnorm)

n <- 300
m <- 4
N <- m*n
rho <- 0.8

Sigma <- matrix(NA, m, m)
diag(Sigma) <- (1/(1:m))^2
for(i in 1:m) {
    for (j in 1:m) {
        if (i != j) {
            Sigma[i, j] <- rho^abs(i-j) * sqrt(Sigma[i, i] * Sigma[j, j])
        }
    }
}

y <- c()
yy <- rmvnorm(n, sigma = Sigma)
for(i in 1:m) {
    y <- c(y, yy[, i])
}

r <- inla(y ~ f(i, model = "iidkd", order = m, n=N,
                ## set parameters using 'theta1'.
                ## these are the default parameters.
                hyper =  list(theta1 = list(
                                    param = c(100, rep(1, m), rep(0, m*(m-1)/2))))),
          data = data.frame(i = 1:N, y),
          ## fix precision as we have exact observations
          control.family = list(hyper = list(
                                 prec = list(initial = 15, fixed = TRUE))),
          verbose = FALSE)
```

```
## this is how the internal parameters are defined
L <- t(chol(solve(Sigma)))
diag(L) <- log(diag(L))
LL <- t(chol(solve(cov(yy))))
diag(LL) <- log(diag(LL))

## compare the estimated (internal) parameters with MLE and the truth
round(dig = 3, cbind(true = c(diag(L), L[lower.tri(L)]),
                     mle = c(diag(LL), LL[lower.tri(LL)]),
                     inla = r$mode$theta))

## this gives a list of sampled matrices (stdev's and correlations)
xx <- inla.iidkd.sample(10^4, r, "i")
## compute the mean
qq <- matrix(rowMeans(matrix(unlist(xx), nrow = m^2)), m, m)

iSigma <- 1/sqrt(diag(Sigma))
Cor <- diag(iSigma) %*% Sigma %*% diag(iSigma)
round(dig = 3, cbind(inla = c(diag(qq), qq[lower.tri(qq)]),
                     true = c(sqrt(diag(Sigma)), Cor[lower.tri(Cor)])))
```