Correlated random effects: `iidkd`

This model is available for dimensions $k = 2$, to 10. We describe in detail the case for $k = 3$ as other ones are similar. This model do the same as models `iid2d`, `iid3d`, `iid4d`, `iid5d`, but uses a more efficient parameterisation.

Parametrization

The ($k = 3$)-dimensional Normal-Wishard model is used if one want to define two vectors of "random effects", u and v, say, for which (u_i, v_i) are iid bivariate Normals

$$\begin{pmatrix} u_i \\ v_i \\ w_i \end{pmatrix} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{W}^{-1}\right)$$

where the covariance matrix $\mathbf{W}^{-1}$ is parameterised as $\mathbf{W} = \mathbf{L}\mathbf{L}^T$, where

$$\mathbf{L} = \begin{pmatrix} \exp(\theta_1) & & \\ \theta_4 & \exp(\theta_2) & \\ \theta_5 & \theta_6 & \exp(\theta_3) \end{pmatrix} \tag{1}$$

and $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6$ can take any value. The number of hyperparameters are $k(k+1)/2$, which is 3, 6, 10, 15, 21, 28, 36, 45, 55, for $k = 2, 3, 4, 5, 6, 7, 8, 9, 10$.

For these models the precision matrix $\mathbf{W}$ is Wishart distributed

$$\mathbf{W} \sim \text{Wishart}_k(r, \mathbf{R}^{-1}),$$

with density

$$\pi(\mathbf{W}) = c^{-1}|\mathbf{W}|^{(r-(k+1))/2} \exp\left\{-\frac{1}{2}\text{Trace}(\mathbf{W}\mathbf{R})\right\}, \quad r > k+1$$

and

$$c = 2^{(rk)/2}|\mathbf{R}|^{-r/2}\pi^{(k(k-1))/4} \prod_{j=1}^{k} \Gamma((r+1-j)/2).$$

Then,

$$\text{E}(\mathbf{W}) = r\mathbf{R}^{-1}, \quad \text{and} \quad \text{E}(\mathbf{W}^{-1}) = \mathbf{R}/(r-(k+1)).$$

Hyperparameters

The hyperparameters are $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6$.

The prior-parameters are

$$(r, R_1, R_2, R_3, R_4, R_5, R_6)$$

where

$$\mathbf{R} = \begin{pmatrix} R_1 & R_4 & R_5 \\ R_4 & R_2 & R_6 \\ R_5 & R_6 & R_3 \end{pmatrix}$$

The `inla` function reports posterior distribution for the hyperparameters $\{\theta_i\}$, and the conversion into interpretable quantities can be done using simulation as described below.

The prior for θ is **fixed** to be `wishartkd`, and number of prior parameters required are $1 + k(k+1)/2$. By default the prior-parameters are

$$(r = 100, \underbrace{1, \ldots, 1}_{k \text{ times}}, 0, \ldots, 0)$$

Specification

The model `iidkd` is specified as

```
y ~ f(i, model="iidkd", order=3, n = <length>) + ...
```

where order $= k = 3$, and the `iidkd` model is represented internally as one vector of length n,

$$(u_1, u_2 \ldots, u_m, v_1, v_2, \ldots, v_m, w_1, w_2, \ldots, w_m)$$

where $n = 3m$, and n is the (required) argument in `f()`.

For this model the argument `constr=TRUE` is interpreted as 3 sum-to-zero constraints

$$\sum u_i = 0, \quad \sum v_i = 0 \quad \text{and} \quad \sum w_i = 0.$$

Hyperparameter spesification and default values

doc Gaussian random effect in dim=k with Wishart prior

hyper

theta1

- **hyperid** 29101
- **name** theta1
- **short.name** theta1
- **initial** 0
- **fixed** TRUE
- **prior** wishartkd
- **param** 100 NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA
- **to.theta** `function(x) x`
- **from.theta** `function(x) x`

theta2

- **hyperid** 29102
- **name** theta2
- **short.name** theta2
- **initial** 0
- **fixed** TRUE
- **prior** none
- **param**
- **to.theta** `function(x) x`
- **from.theta** `function(x) x`

theta3

- **hyperid** 29103
- **name** theta3
- **short.name** theta3
- **initial** 0
- **fixed** TRUE
- **prior** none

param

to.theta `function(x) x`

from.theta `function(x) x`

theta4

hyperid 29104

name theta4

short.name theta4

initial 0

fixed TRUE

prior none

param

to.theta `function(x) x`

from.theta `function(x) x`

theta5

hyperid 29105

name theta5

short.name theta5

initial 0

fixed TRUE

prior none

param

to.theta `function(x) x`

from.theta `function(x) x`

theta6

hyperid 29106

name theta6

short.name theta6

initial 0

fixed TRUE

prior none

param

to.theta `function(x) x`

from.theta `function(x) x`

theta7

hyperid 29107

name theta7

short.name theta7

initial 0

fixed TRUE

prior none

param

to.theta `function(x) x`

from.theta `function(x) x`

theta8

hyperid 29108
name theta8
short.name theta8
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta9
hyperid 29109
name theta9
short.name theta9
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta10
hyperid 29110
name theta10
short.name theta10
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta11
hyperid 29111
name theta11
short.name theta11
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta12
hyperid 29112
name theta12
short.name theta12
initial 0

fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta13

hyperid 29113
name theta13
short.name theta13
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta14

hyperid 29114
name theta14
short.name theta14
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta15

hyperid 29115
name theta15
short.name theta15
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta16

hyperid 29116
name theta16
short.name theta16
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`

from.theta `function(x) x`

theta17

hyperid 29117
name theta17
short.name theta17
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta18

hyperid 29118
name theta18
short.name theta18
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta19

hyperid 29119
name theta19
short.name theta19
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta20

hyperid 29120
name theta20
short.name theta20
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta21

hyperid 29121
name theta21

short.name theta21
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta22

hyperid 29122
name theta22
short.name theta22
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta23

hyperid 29123
name theta23
short.name theta23
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta24

hyperid 29124
name theta24
short.name theta24
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta25

hyperid 29125
name theta25
short.name theta25
initial 0
fixed TRUE
prior none

param

to.theta `function(x) x`

from.theta `function(x) x`

theta26

hyperid 29126

name theta26

short.name theta26

initial 0

fixed TRUE

prior none

param

to.theta `function(x) x`

from.theta `function(x) x`

theta27

hyperid 29127

name theta27

short.name theta27

initial 0

fixed TRUE

prior none

param

to.theta `function(x) x`

from.theta `function(x) x`

theta28

hyperid 29128

name theta28

short.name theta28

initial 0

fixed TRUE

prior none

param

to.theta `function(x) x`

from.theta `function(x) x`

theta29

hyperid 29129

name theta29

short.name theta29

initial 0

fixed TRUE

prior none

param

to.theta `function(x) x`

from.theta `function(x) x`

theta30

hyperid 29130
name theta30
short.name theta30
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta31
hyperid 29131
name theta31
short.name theta31
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta32
hyperid 29132
name theta32
short.name theta32
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta33
hyperid 29133
name theta33
short.name theta33
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta34
hyperid 29134
name theta34
short.name theta34
initial 0

fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta35

hyperid 29135
name theta35
short.name theta35
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta36

hyperid 29136
name theta36
short.name theta36
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta37

hyperid 29137
name theta37
short.name theta37
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta38

hyperid 29138
name theta38
short.name theta38
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`

from.theta `function(x) x`

theta39

hyperid 29139
name theta39
short.name theta39
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta40

hyperid 29140
name theta40
short.name theta40
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta41

hyperid 29141
name theta41
short.name theta41
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta42

hyperid 29142
name theta42
short.name theta42
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta43

hyperid 29143
name theta43

short.name theta43
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta44

hyperid 29144
name theta44
short.name theta44
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta45

hyperid 29145
name theta45
short.name theta45
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta46

hyperid 29146
name theta46
short.name theta46
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta47

hyperid 29147
name theta47
short.name theta47
initial 0
fixed TRUE
prior none

param
to.theta `function(x) x`
from.theta `function(x) x`

theta48
hyperid 29148
name theta48
short.name theta48
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta49
hyperid 29149
name theta49
short.name theta49
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta50
hyperid 29150
name theta50
short.name theta50
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta51
hyperid 29151
name theta51
short.name theta51
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta52

hyperid 29152
name theta52
short.name theta52
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta53
hyperid 29153
name theta53
short.name theta53
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta54
hyperid 29154
name theta54
short.name theta54
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

theta55
hyperid 29155
name theta55
short.name theta55
initial 0
fixed TRUE
prior none
param
to.theta `function(x) x`
from.theta `function(x) x`

constr FALSE

nrow.ncol FALSE

augmented TRUE

aug.factor 1

aug.constr 1 2 3 4 5 6 7 8 9 10

n.div.by -1

n.required TRUE

set.default.values TRUE

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Example