

# Priority Inheritance with Backtracking for Iterative Multi-agent Path Finding

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# Multi-agent Path Finding (MAPF)

given

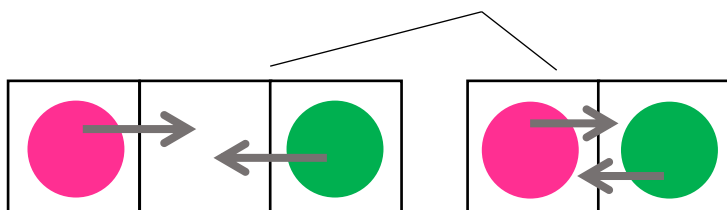
□ graph

● agents (starts)

■ goals

obtain

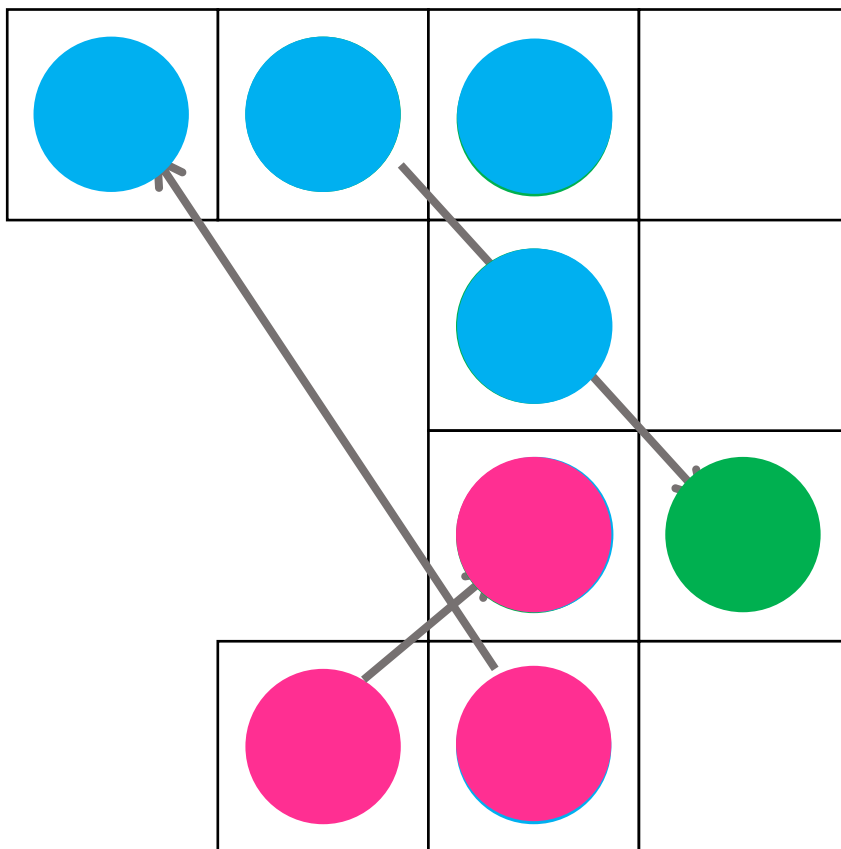
1. paths without collisions



s.t.

**all** agents are on their goals **simultaneously**

computationally **DIFFICULT** to obtain optimal solutions



# Multi-agent Path Finding (MAPF)

Systems with moving agents will be more and more common.



YouTube/Mind Blowing Videos



Twitter/People's Daily, China

In practical scenarios, MAPF must be solved  
**iteratively**

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OUR FOCUS

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one-shot MAPF

iterative MAPF

# Iterative Multi-agent Path Finding

given

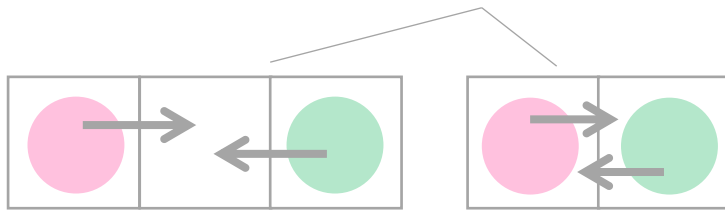
□ graph

● agents (starts)

ⓧ tasks (set of goals)

obtain

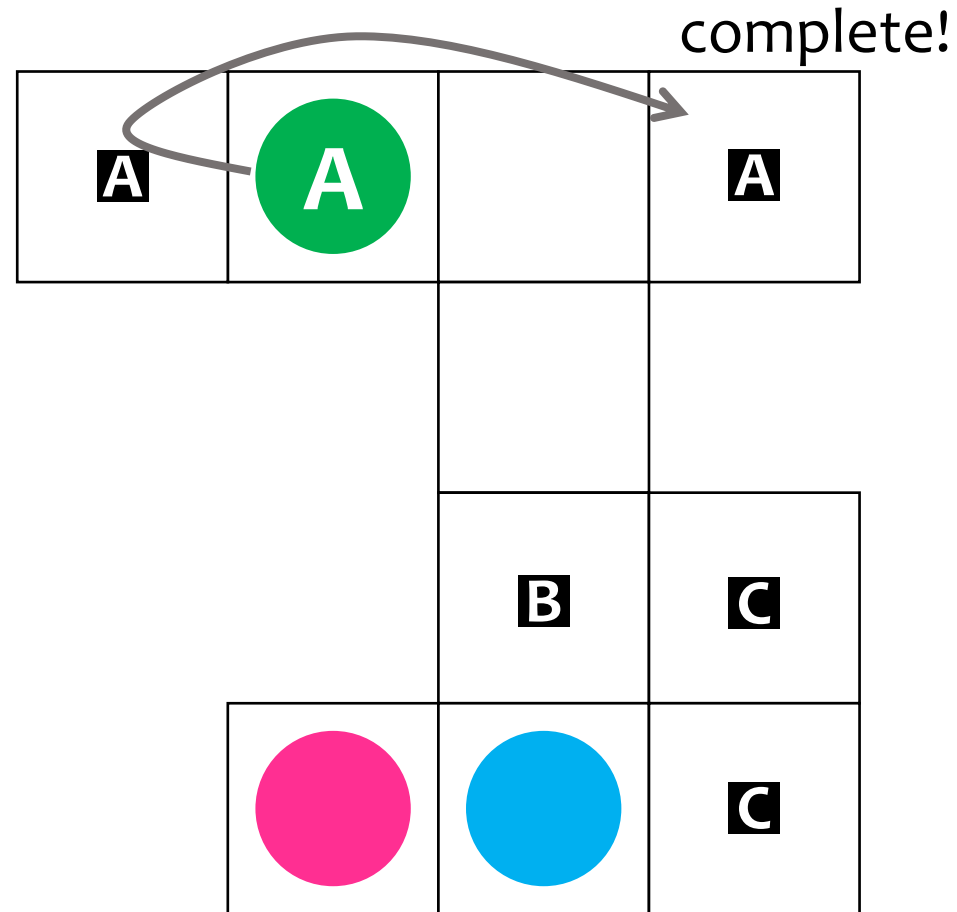
1. paths without collisions



2. task allocation

s.t.

all tasks are **completed** in finite time



# Overview

Priority Inheritance with Backtracking

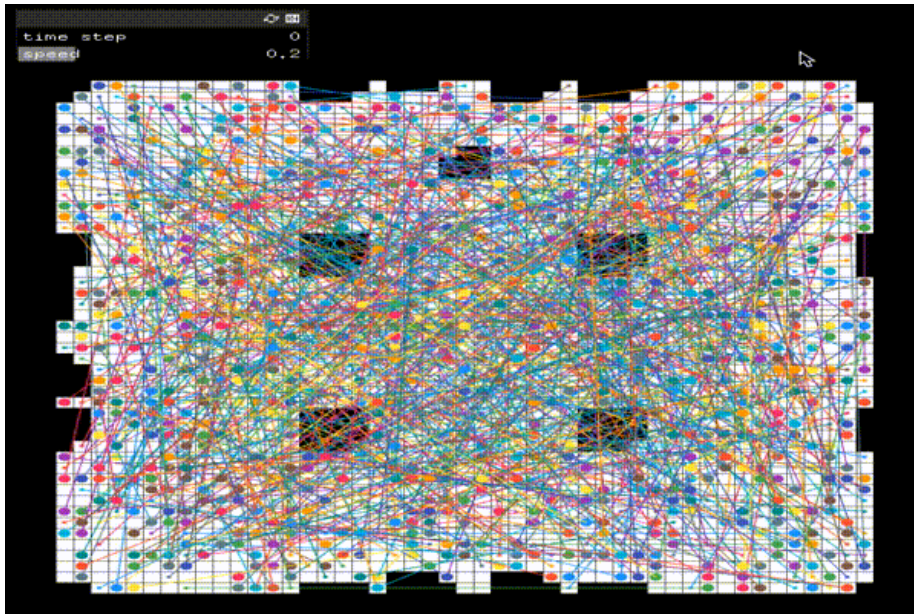
- ✓ for solving iterative MAPF, propose an algorithm **PIBT** that ensures **reachability**

(in biconnected-like graph)

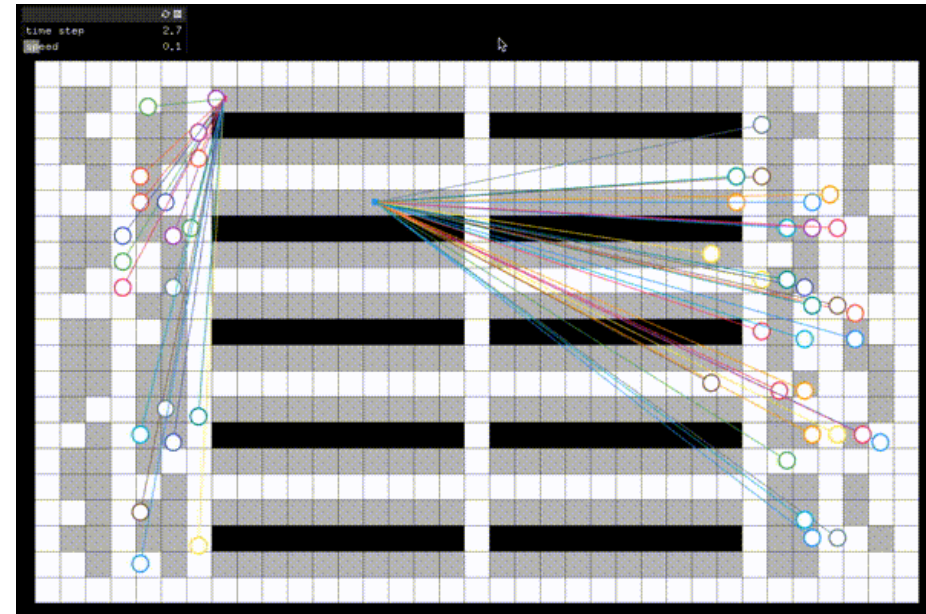
all agents reach their goals in finite time after being given

fast, scalable

adaptive for iterative use



500 agents, within 0.5 sec



Multi-agent Pickup & Delivery

# Design Choice of Algorithm

tradeoff  
optimality vs speed

style  
centralized vs decentralized

communication  
globally vs locally

+

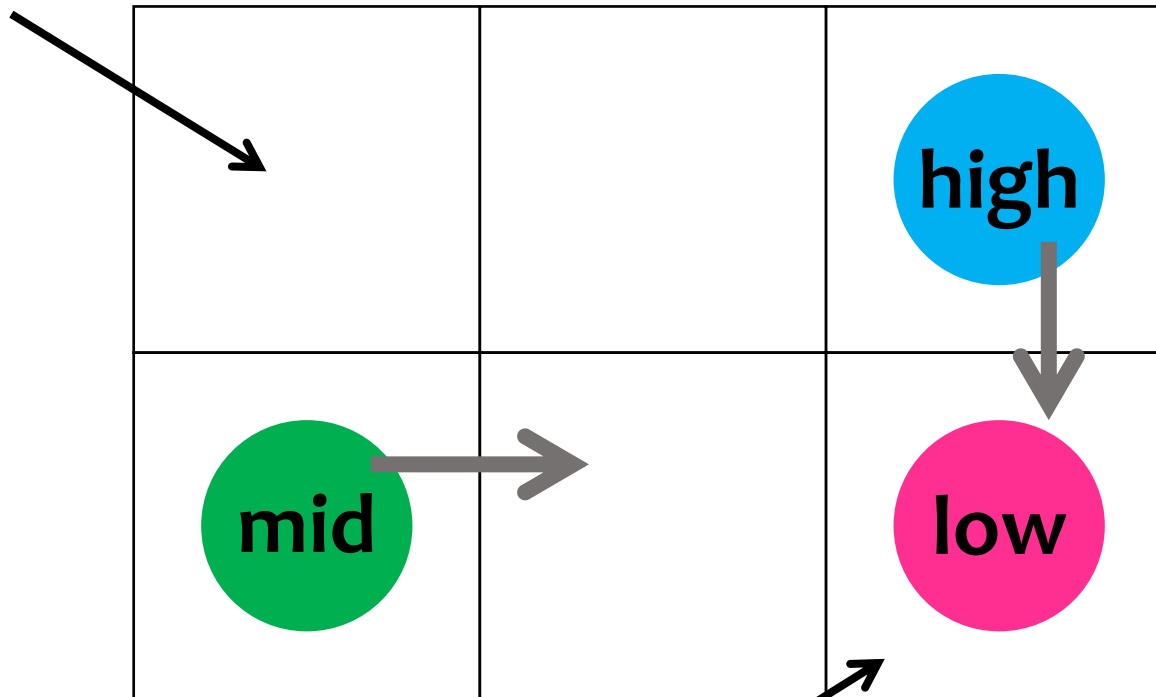
Adaptivity for Iterative Use

# Prioritized Planning

incomplete

single time-window  
(PIBT relies on)

node = resource



**STUCK**

**priority inversion** in resource scheduling

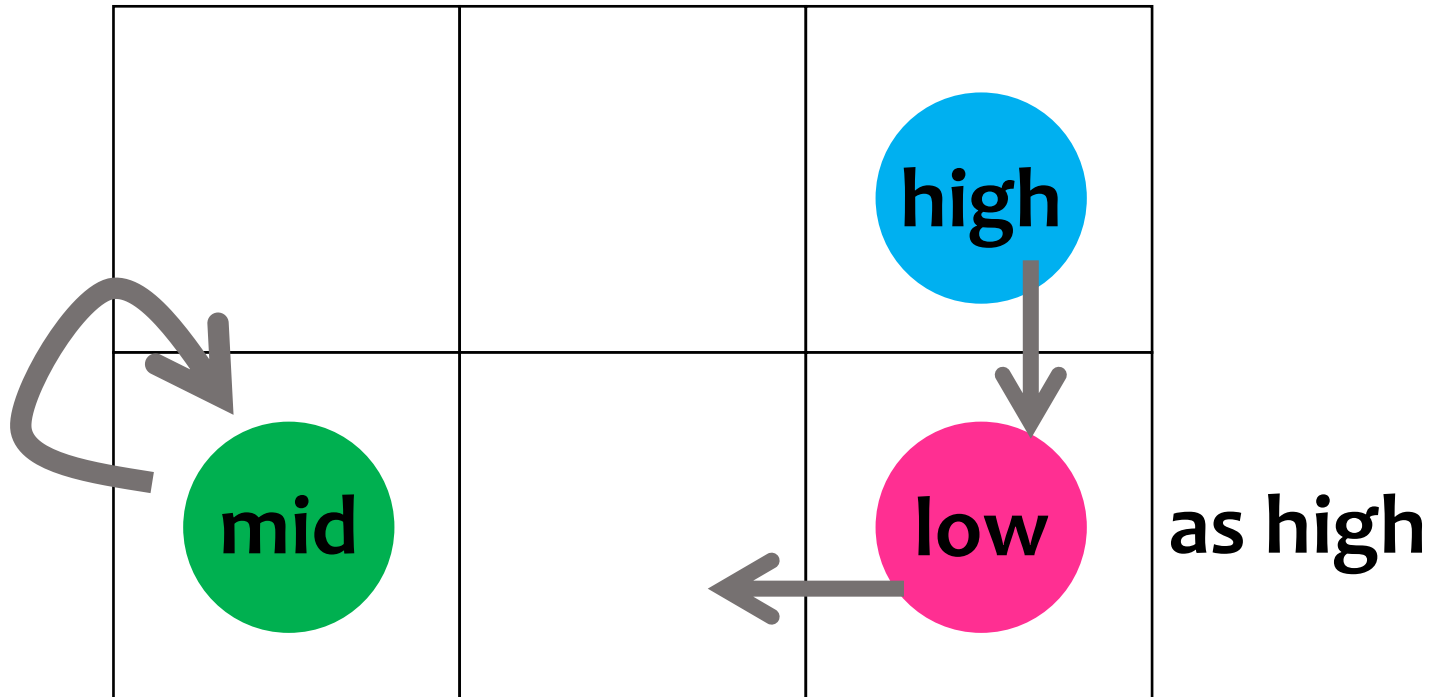
# Priority Inheritance

countermeasure to priority inversion



**priority inheritance**

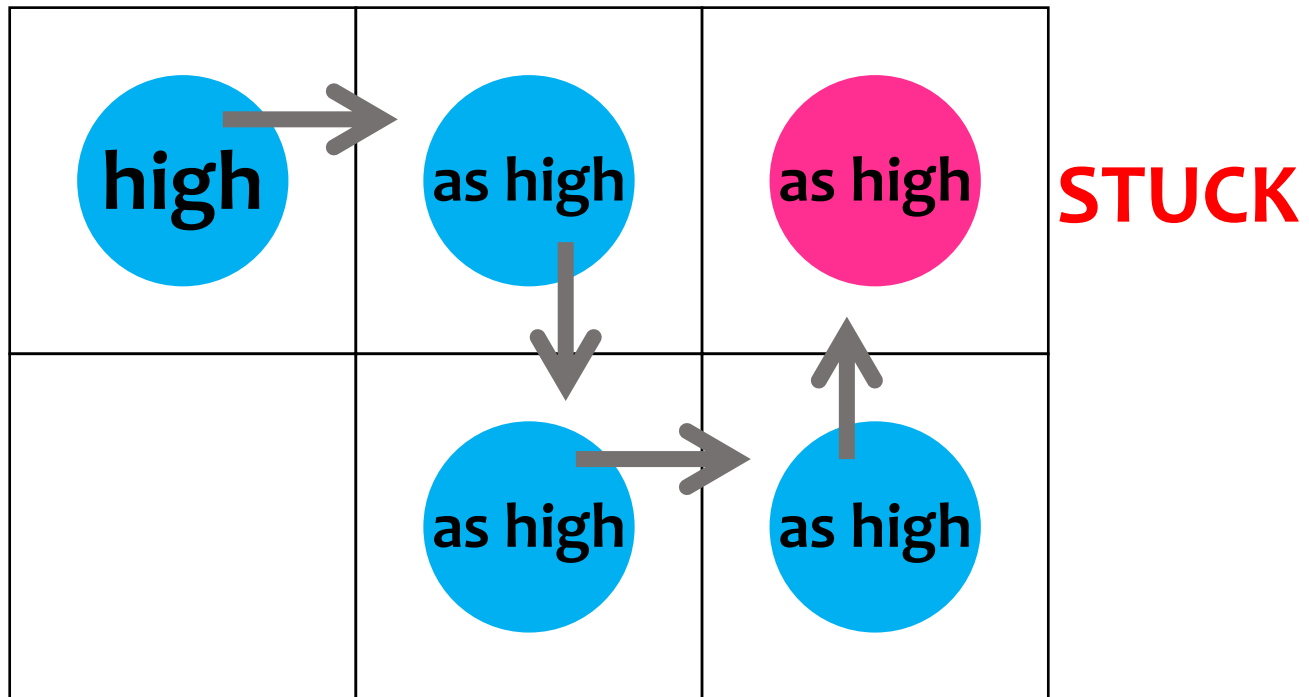
[Sha et al., 1990]





# Stuck Again

Priority inheritance is **insufficient** to prevent all stuck situations.



# with Backtracking

Agents with priority inheritance have to wait for **backtracking**.



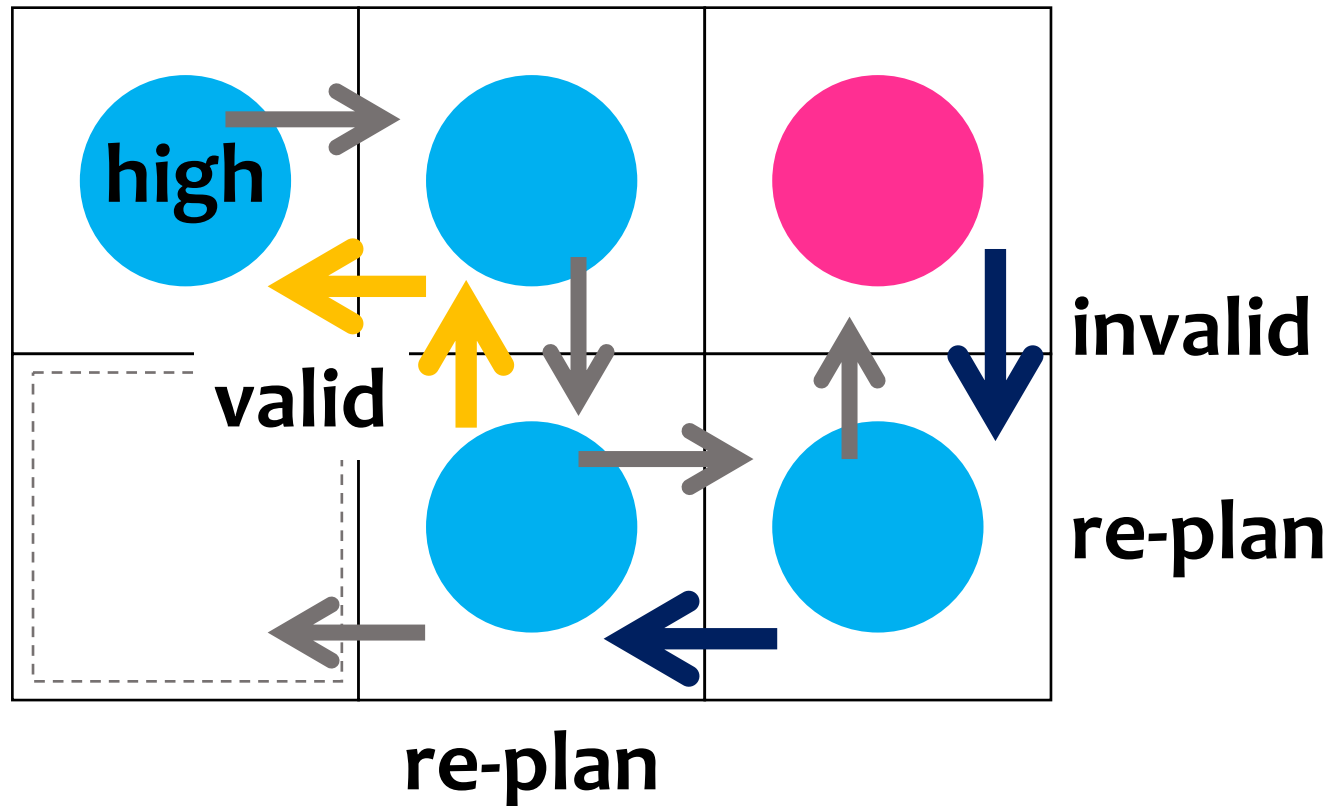
valid

You can move



invalid

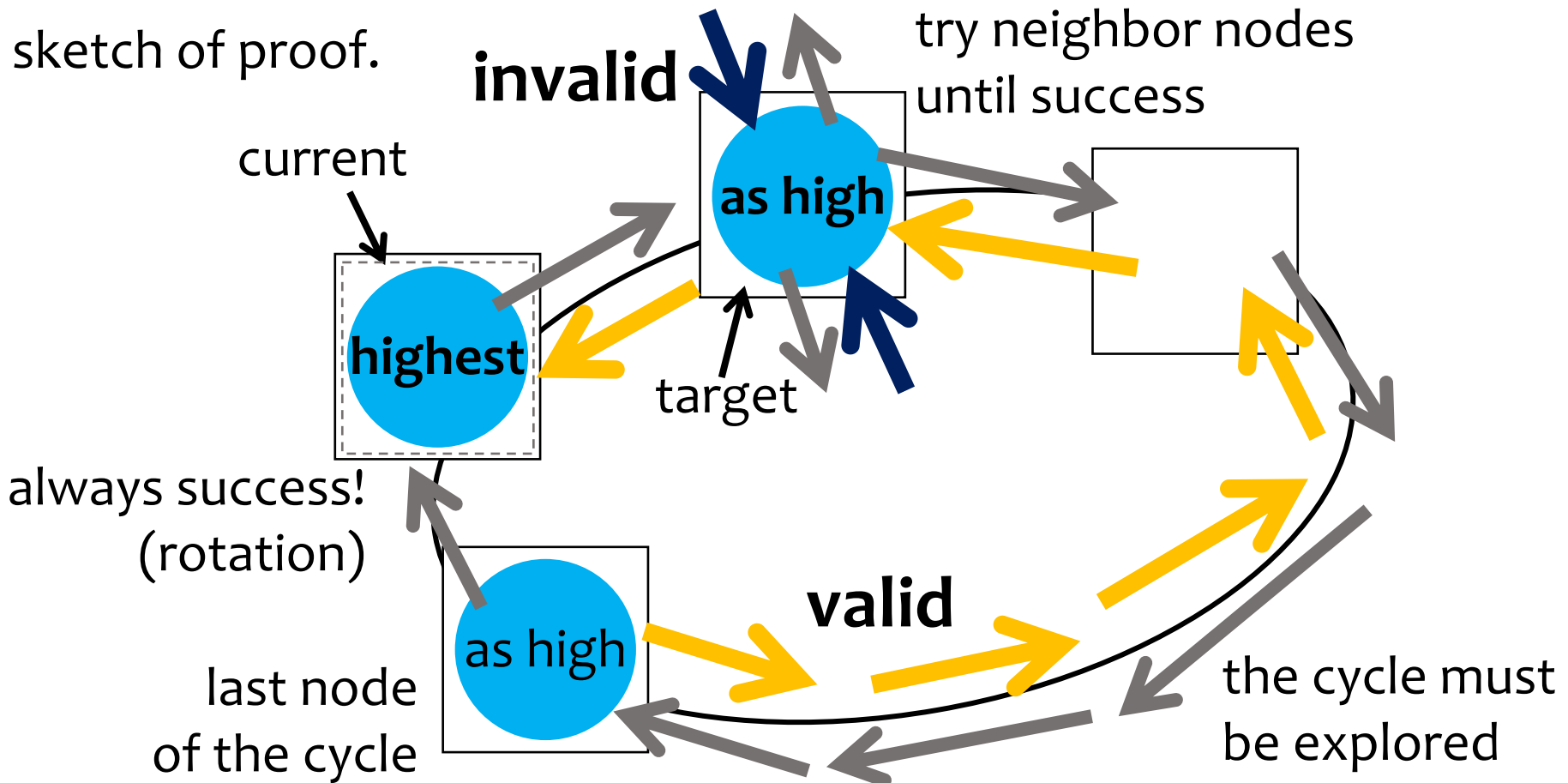
You must re-plan, I will stay



# Guarantee of Local Movement

Lemma 1

The agent with **highest priority** successfully moves to a target node if the two nodes (current, target) belong to a simple cycle.



# Local to Global Movement

Lemma 1



(depends on graph)

the agent with highest priority can move to an **arbitrary neighbor** node in next timestep.



such an agent always reach its **goal** with the **shortest path**



how to select?



by **dynamic priorities**,

(it is enabled to ensure that)

**all agents** reach their own goals in finite time

# Dynamic Priorities

Once an agent reaches its goal, it **drops** priority.



Agents that have not reached goal have higher priority.

unique between agents  
for every timestep

$$p_i(t) \leftarrow \eta_i(t) + \epsilon_i$$

for an agent  $a_i$  at timestep  $t$

**elapsed timestep**

since  $a_i$  last updated its goal,  
or, be zero if no goal

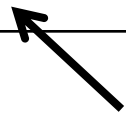
tie-breaker, in  $[0, 1)$

**unique** between agents

# Reachability

Theorem 2

If a graph  $G$  has a **simple cycle for all pairs of adjacent nodes**, then, with PIBT, **all agents** reach their own goals within  $\text{diam}(G) \cdot |A|$  timesteps **after being given**.



#agents

example

- undirected: biconnected
- directed: ring

**NOT** ensure that all agents be on their goals **simultaneously**!

# Exp.1 one-shot MAPF



300 agents

ost003d, 194x194 [Sturtevant, 2012]



PIBT



Parallel Push & Swap

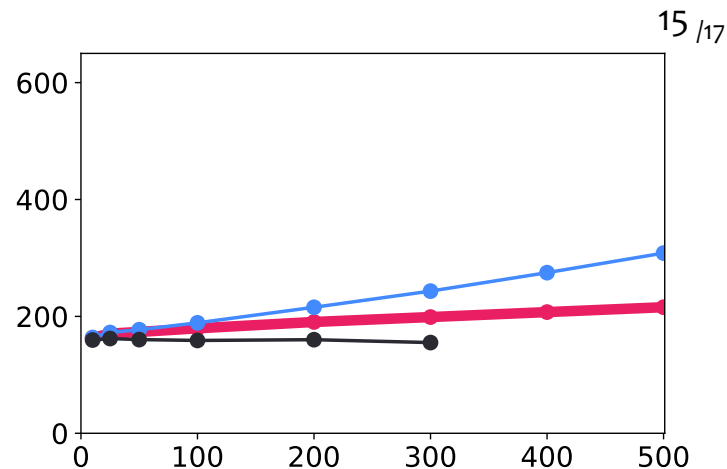
[Sajid et al., SoCS12]



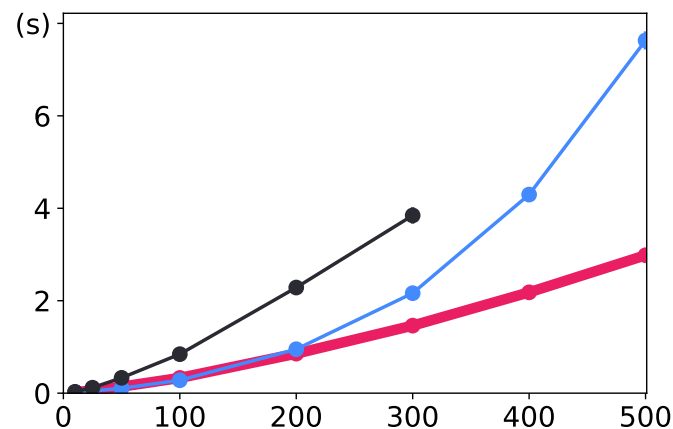
WHCA\*-10

[Silver, AIIDE05]

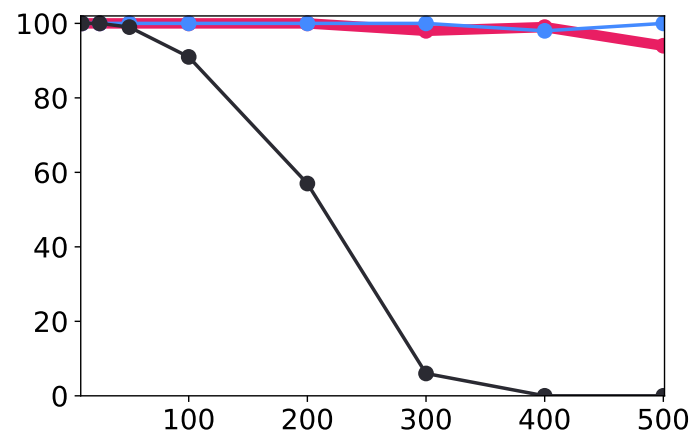
path cost  
(steps)



computation  
time  
(sec.)



success  
(%)



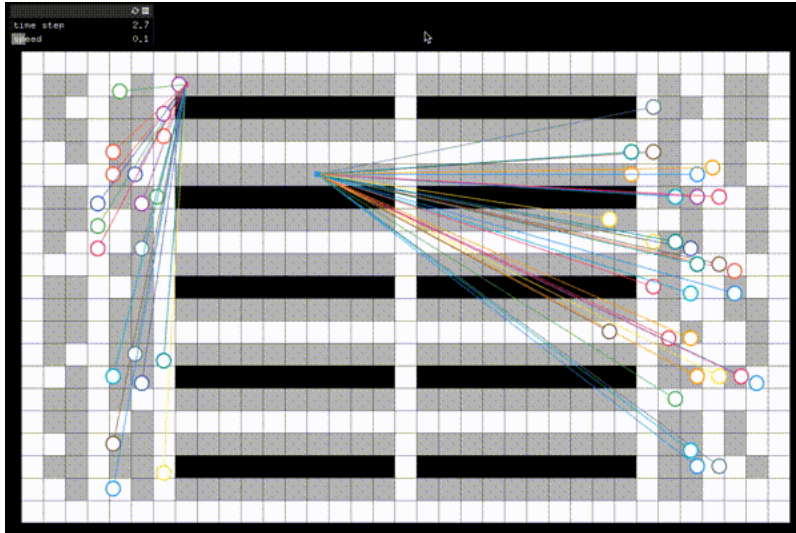
#agents

\*average over only success cases within the solver (in 100 times)

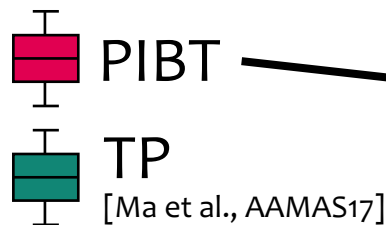
# Exp.2 Multi-agent Pickup & Delivery

[Ma et al., AAMAS17]

task = (pickup loc., delivery loc.)

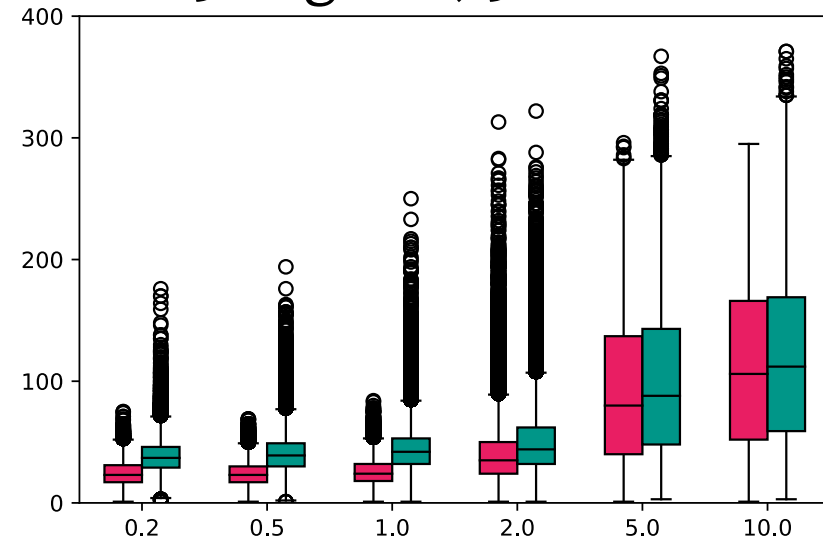


PIBT with task allocation  
let free agents move to  
the **nearest pickup** loc.

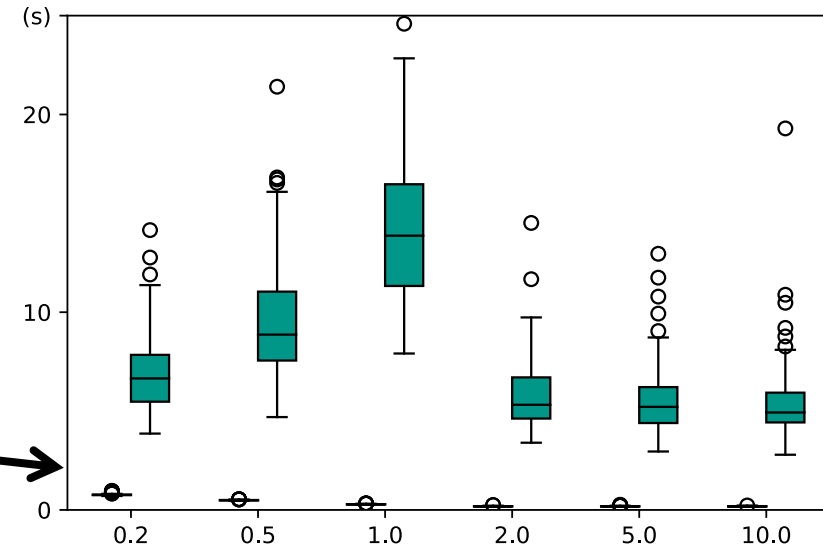


50 agents, 500 tasks

service  
time  
(steps)



comp.  
time  
(sec)



(task) frequency

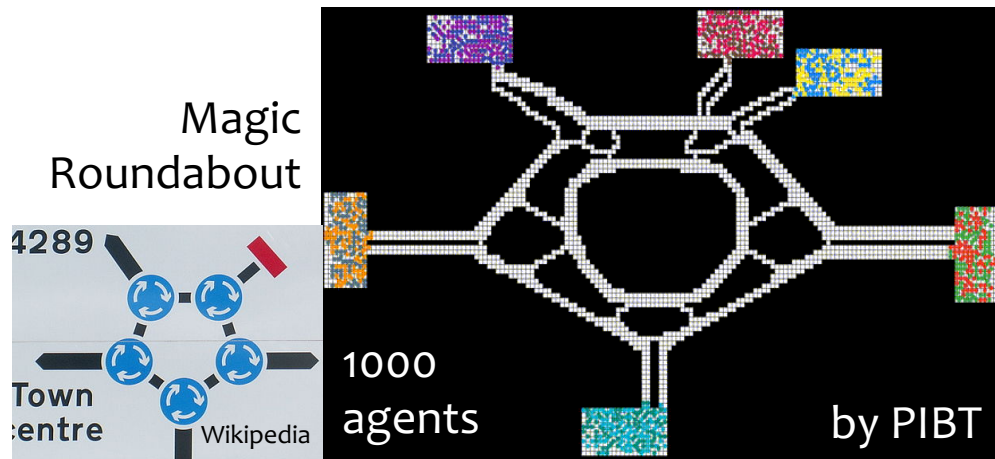


# Conclusion

- ✓ **PIBT** is an algorithm for solving **iterative MAPF**
- ✓ PIBT ensures **reachability** in **biconnected**-like graph

future work

- expand time window  
technical paper is available,  
called **winPIBT** [arXiv:1905.10149]
- relax graph conditions
- adapt to full asynchrony



## PIBT

priority inheritance  
backtracking

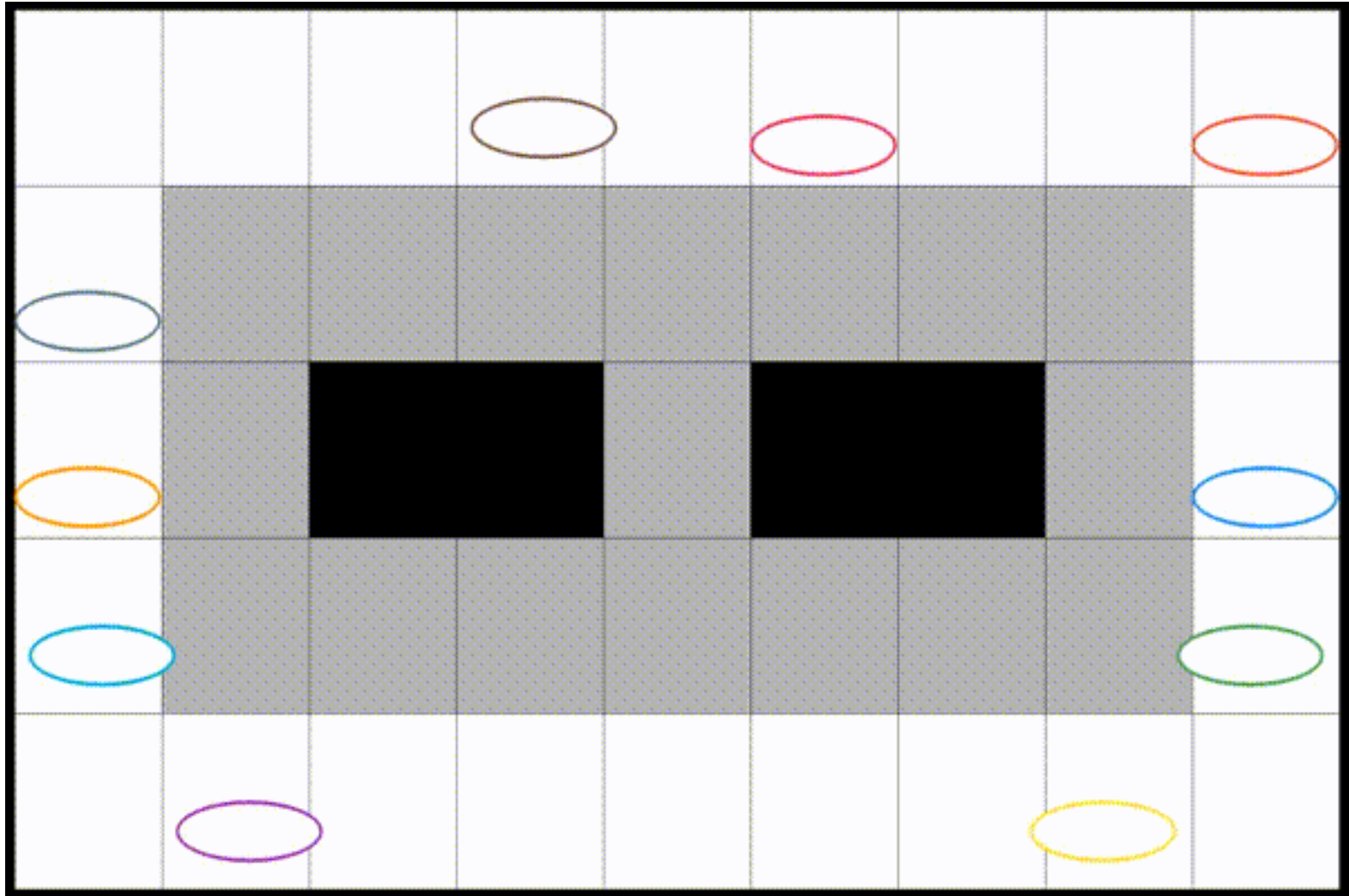
prioritized planning

dynamic priorities

# Appendix

# Multi-agent Sushi Pickup & Delivery

inspired by conveyor belt sushi



# Simple Sketch of PIBT

An agent starts its planning with

- receiving priority inheritance
- becoming the highest in unplanned agents

1. Initialize/update candidate nodes for next timestep

Sends invalid as backtracking if no candidates

2. Pickup one node from candidates

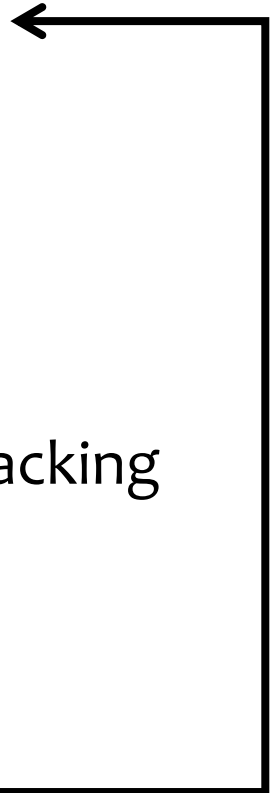
3. If no agent there, move there (fin.)

Otherwise, do priority inheritance and wait for backtracking

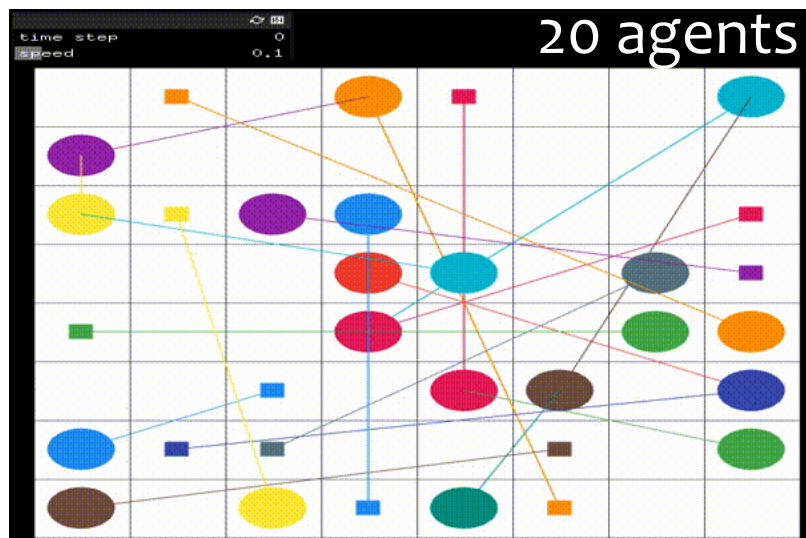
4. Receive backtracking

valid: move there (fin.)

invalid: back to 1



# Exp.1 one-shot MAPF



PIBT

[Okumura et al., IJCAI19]



Parallel Push & Swap

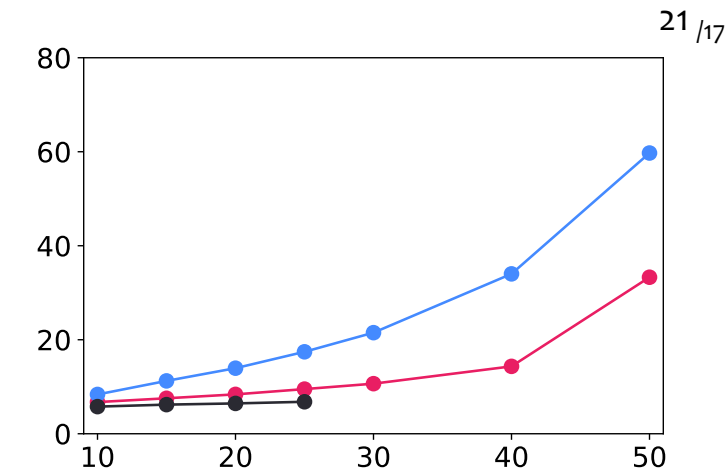
[Sajid et al., SoCS12]



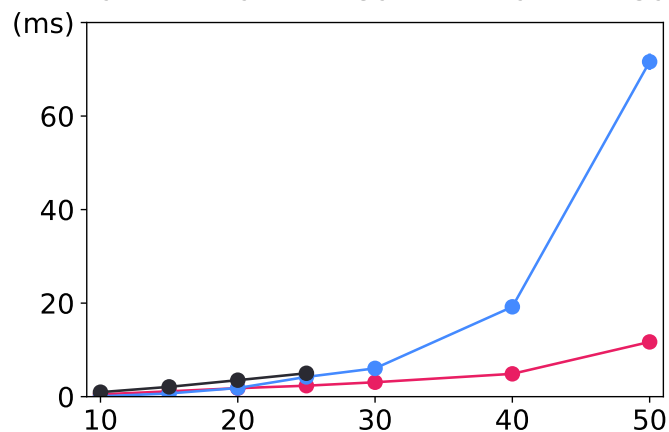
WHCA-10

[Silver, AIIDE05]

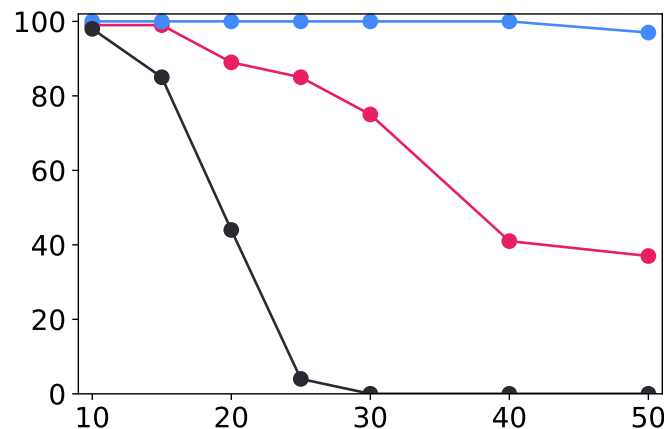
path cost  
(steps)



runtime  
(ms)



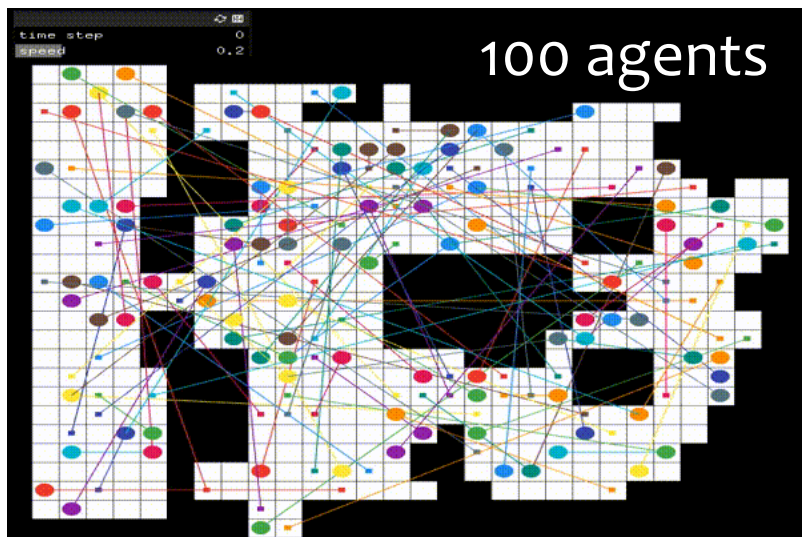
success  
(%)



\*average over only success cases within the solver (in 100 times)

#agents

# Exp.1 one-shot MAPF



lak105d, 25x31 [Sturtevant, 2012]



PIBT

[Okumura et al., IJCAI19]



Parallel Push & Swap

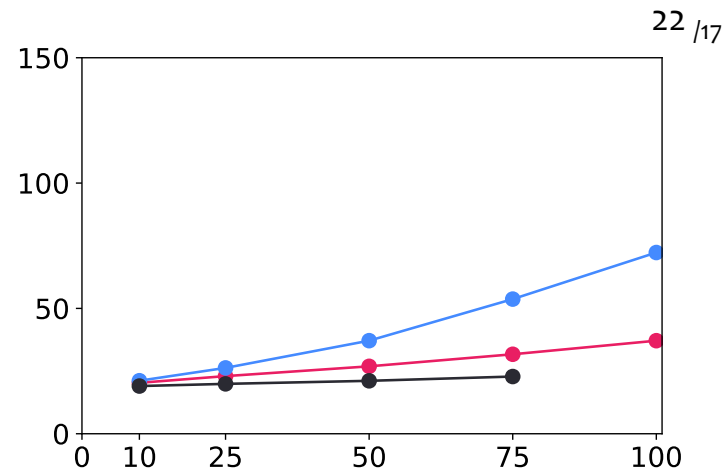
[Sajid et al., SoCS12]



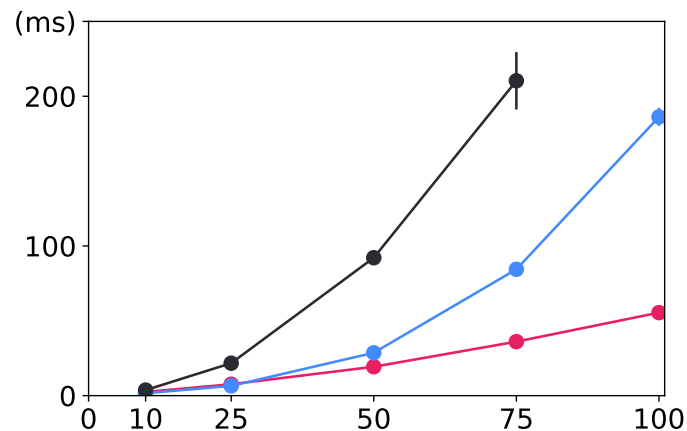
WHCA-10

[Silver, AIIDE05]

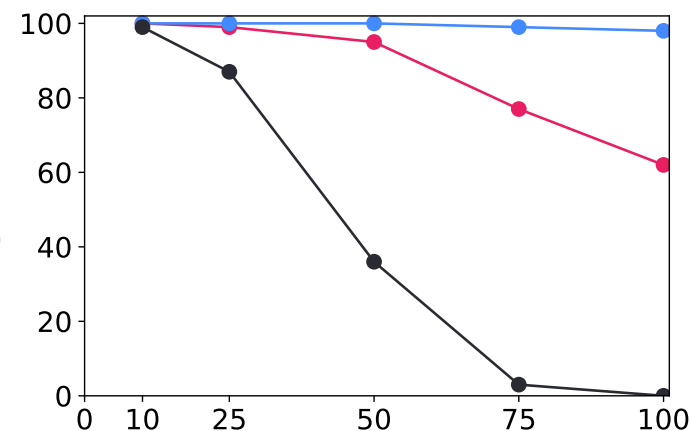
path cost  
(steps)



runtime  
(ms)



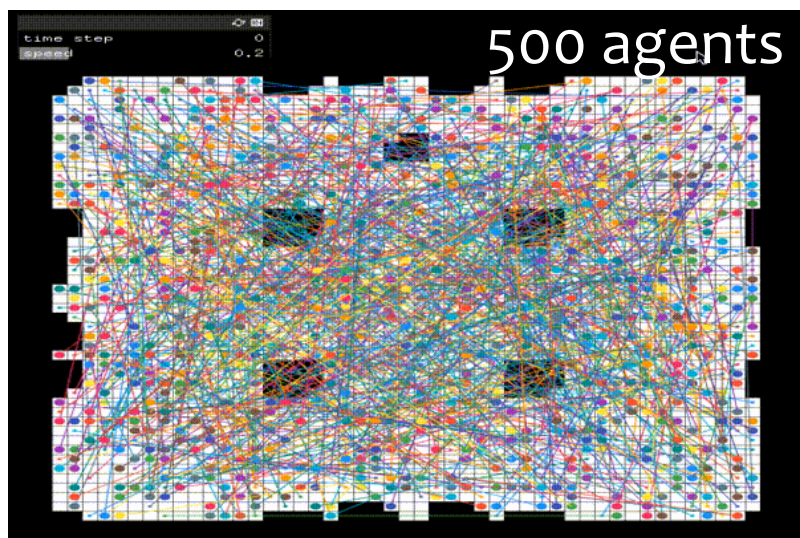
success  
(%)



\*average over only success cases within the solver (in 100 times)

#agents

# Exp.1 one-shot MAPF



arena, 49x49 [Sturtevant, 2012]



PIBT

[Okumura et al., IJCAI19]



Parallel Push & Swap

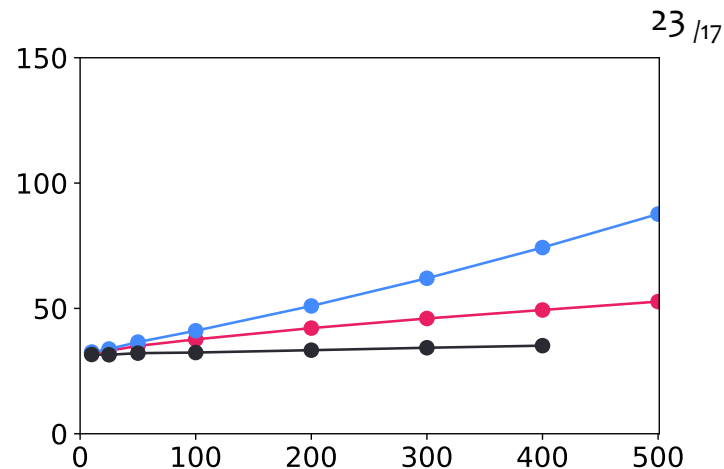
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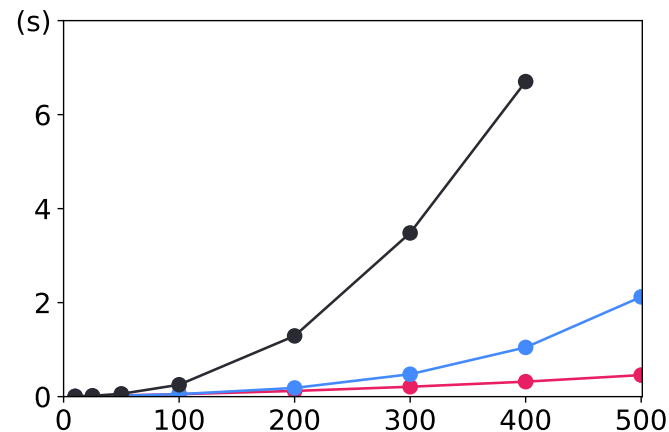
WHCA-10

[Silver, AIIDE05]

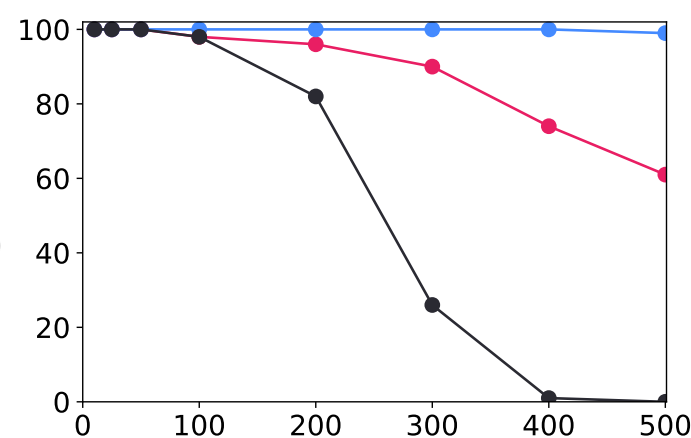
path cost  
(steps)



runtime  
(s)



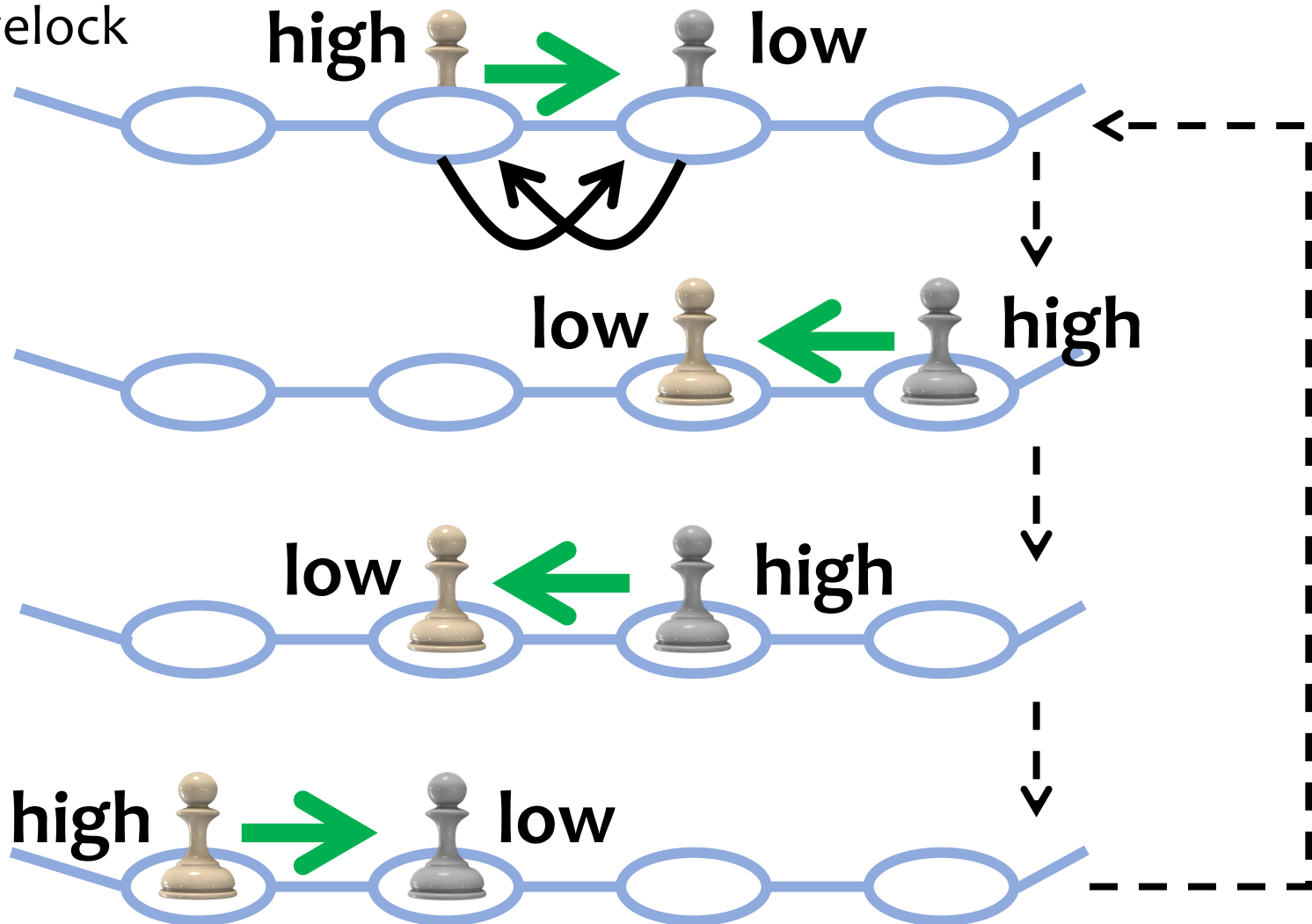
success  
(%)



\*average over only success cases within the solver (in 100 times)

# Cons of PIBT

cons-1. livelock



cons-2. tree (without cycles) → potentially cause deadlock



# Computational Complexity

Proposition

Assume that PIBT performs in a centralized way.

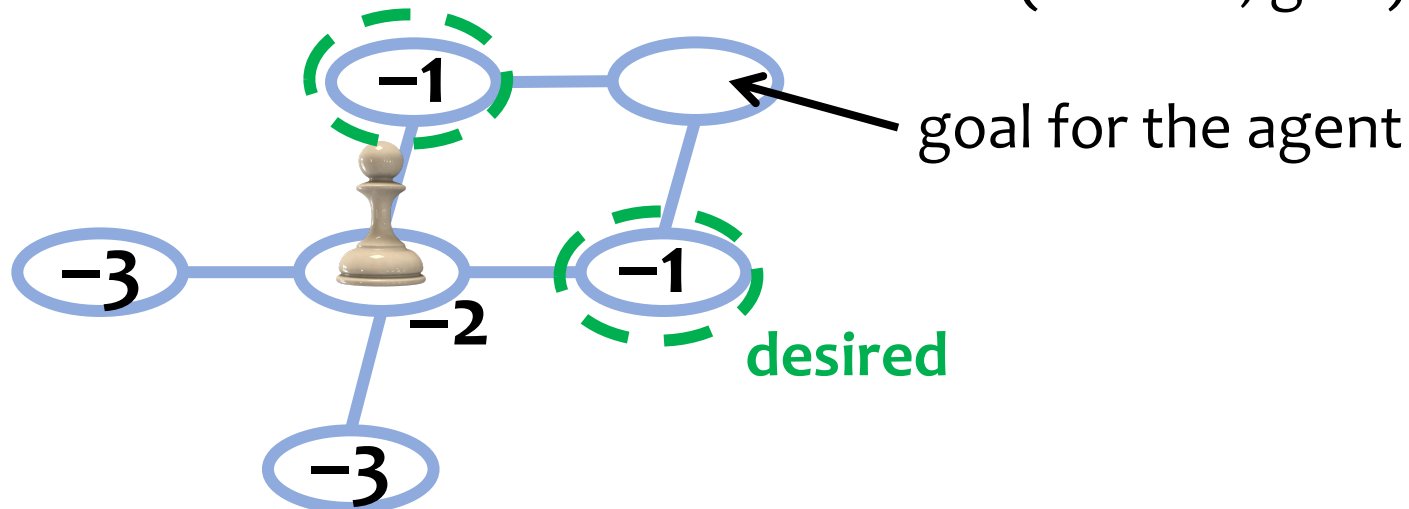
The computational complexity of PIBT **in one step** is

$$O(|A| \cdot \Delta(G) \cdot F)$$

maximum degree of  $G$

Maximum time required to  
choose the next target node

e.g., let node evaluation function be  $-\text{cost}(\text{current}, \text{goal})$



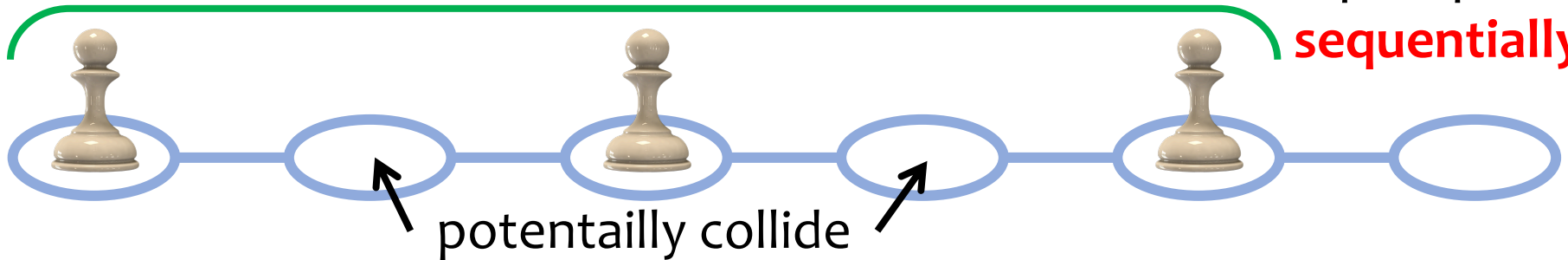
# Communication

PIBT can be performed **locally** to some extent. (for decentralized)

Case 1.

interacting agents

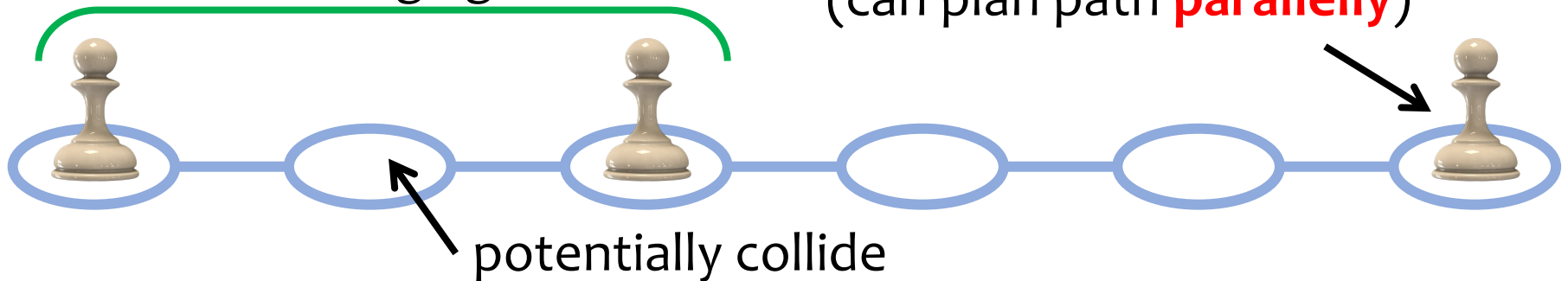
must plan paths  
**sequentially**



Case 2.

interacting agents


**unaffected** by the left group  
(can plan path **parallelly**)



Communication assumption is sufficient that two agents in close proximity can talk directly and utilize multi-hop communication.

# Adapt Decentralized Fashion

PIBT relatively smoothly adapts to decentralized fashion.  
This transition requires three steps.

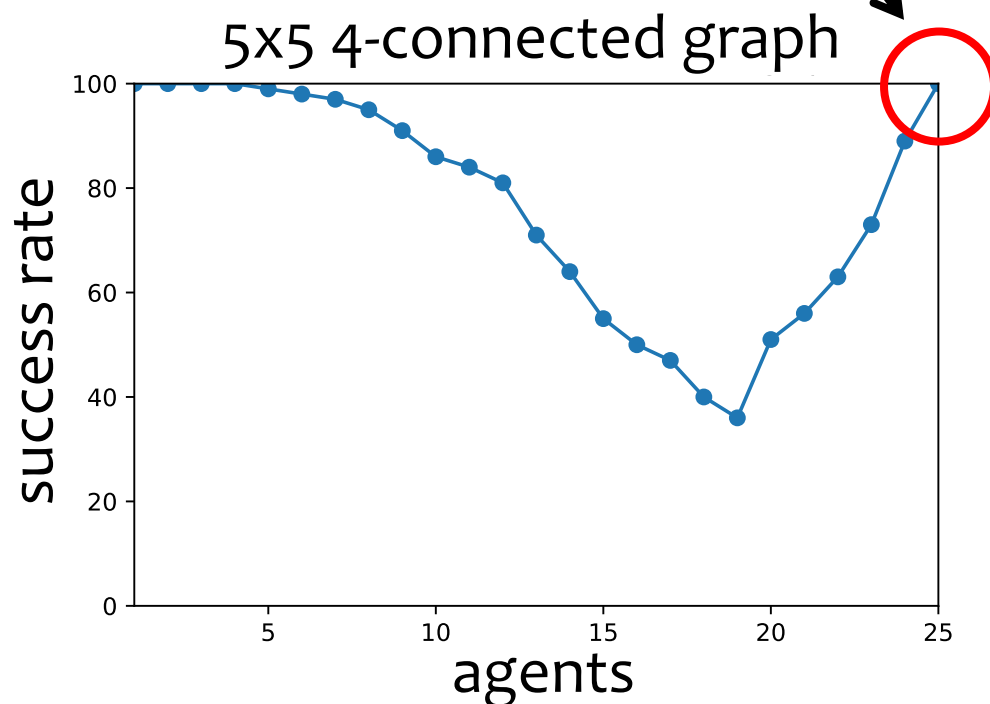
- ✓ detect interacting groups  might be difficult
- ✓ collect priorities  
Updating rule of priorities relaxes this effort,  
e.g., take care only when  $\eta_i(t)$  (primally term) becomes zero.
- ✓ plan paths sequentially within groups

Priority inheritance and backtracking is performed  
by **token passing**.

message counts:  $O(|A|)$

# PIBT on MAPF in Simple Grids

I don't know why but PIBT seems to solve MAPF in **100%** without any vacant in **simple grids**.



In 8x8 4-connected graph, we obtain similar results.