



MOTION

15-16 year-olds

Rodrigo Alcaraz de la Osa. Translation: Rodrigo Alcaraz de la Osa and Alicia Sampedro (@AliciaInfoFyQ)



URM

Characteristics

A body describes a **uniform rectilinear motion** (URM) when:

- The trajectory is a straight line.
- Its velocity v is constant. (acceleration $a = 0$).

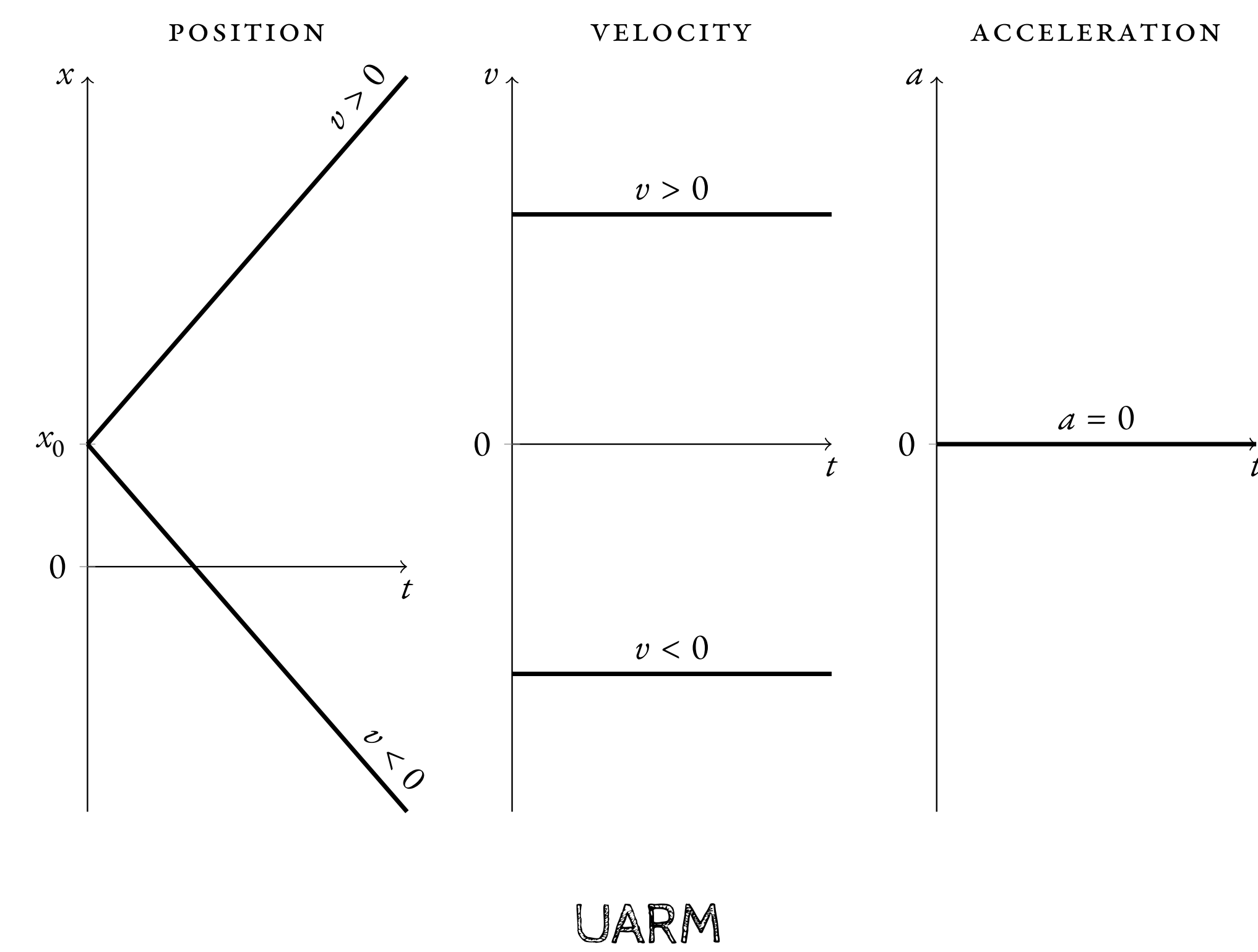
Equation

The **equation** for the URM s:

$$x(t) = x_0 + v(t - t_0)$$

where x represents the final position, x_0 the initial position, v the final velocity, t the final time and t_0 the initial time.

Graphs



Characteristics

A body is describing a **Uniformly Accelerated Rectilinear Motion** (UARM) when:

- The trajectory is a straight line.
- The acceleration a is constant (velocity v variable).

Main equations

The **main equations** for the UARM :

$$\text{Position: } x(t) = x_0 + v_0(t - t_0) + \frac{1}{2}a(t - t_0)^2 \quad (1)$$

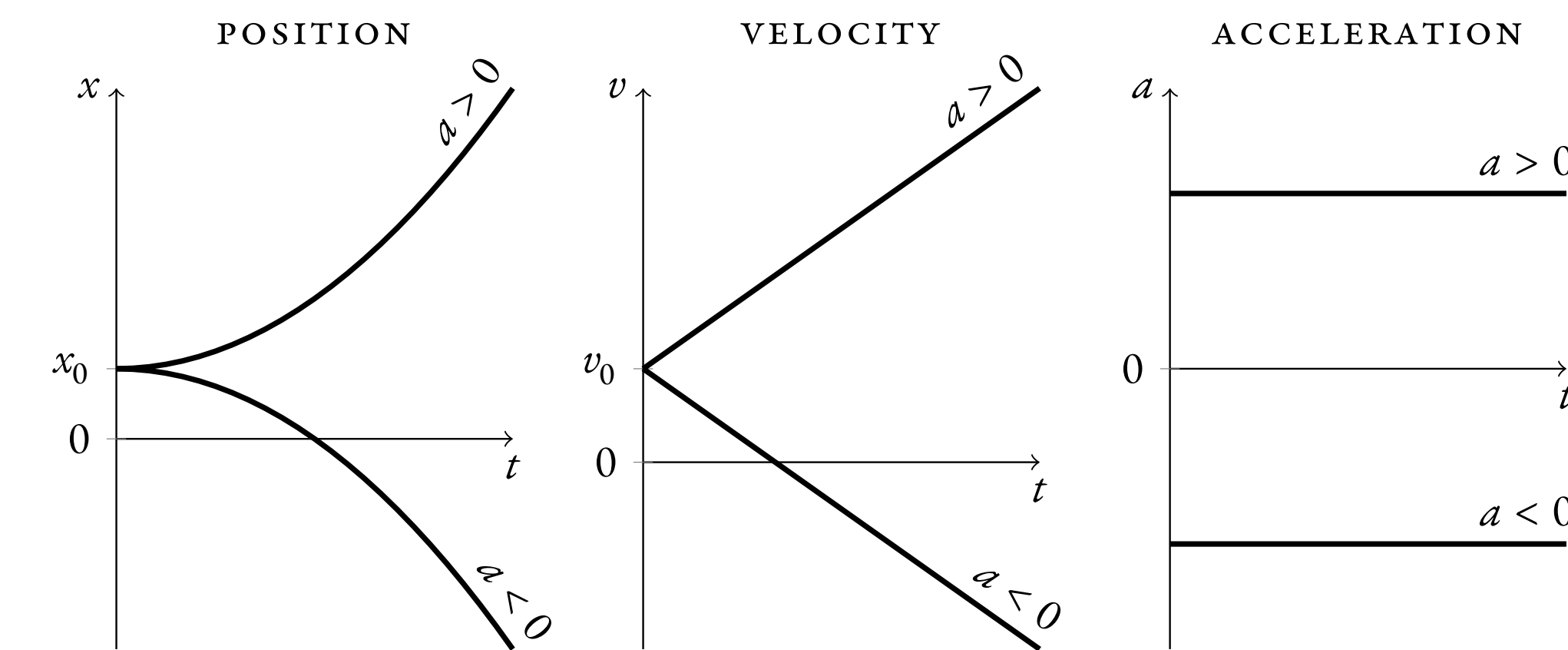
$$\text{Velocity: } v(t) = v_0 + a(t - t_0) \quad (2)$$

$$v^2 - v_0^2 = 2a\Delta x \quad (3)$$

where x represents the final position, x_0 the initial position, v the final velocity, v_0 the initial position, a the acceleration, t the final time, t_0 the initial time and $\Delta x = x - x_0$ is the distance covered by the object.

UARM (cont.)

Graphs



Free falling motion is a type of UARM where the acceleration is the acceleration of **gravity**. On Earth, $a = -g = -9.8 \text{ m/s}^2$ (the acceleration is negative because it always points downwards).

Encounters

In these kind of problems two bodies start on different positions and they meet after a while.

We follow **three steps**:

1. We **write** the **position equations** for each body.
2. We **set the meeting condition**, that is, both positions are the same when they meet.
3. We **solve** for the magnitude asked.

Example

A car is moving on a road which is parallel to a train track. The car stops on a red light in the exact moment that a train is passing with a constant velocity of 12 m/s. The car stays in the traffic light for 6 s and then it starts moving with an acceleration of 2 m/s². Calculate:

- a) Time needed for the car to catch the train since it stopped at the red light.
- b) Distance covered by the car from the traffic light until it catches the train.
- c) The velocity of the car when it catches the train.

Solution

- a) The first thing to do is to **write the motion equations for each object**:

$$\text{Car (UARM): } x_c = x_{0c} + v_{0c}(t - t_{0c}) + \frac{1}{2}a_c(t - t_{0c})^2$$

$$\text{Train (URM): } x_t = x_{0t} + v_t(t - t_{0t})$$

Example (cont.)

a) Data:

$$\begin{aligned} x_{0c} &= x_{0t} = 0 \\ v_{0c} &= 0; \quad v_t = 12 \text{ m/s} \\ a_c &= 2 \text{ m/s}^2 \\ t_{0c} &= 6 \text{ s}; \quad t_{0t} = 0 \end{aligned}$$

$$\begin{aligned} \text{Car (UARM): } x_c &= 0 + 0 \cdot (t - 6) + \frac{1}{2} \cdot 2 \cdot (t - 6)^2 \\ &= (t - 6)^2 = t^2 - 12t + 36 \\ \text{Train (URM): } x_t &= 0 + 12 \cdot (t - 0) = 12t \end{aligned}$$

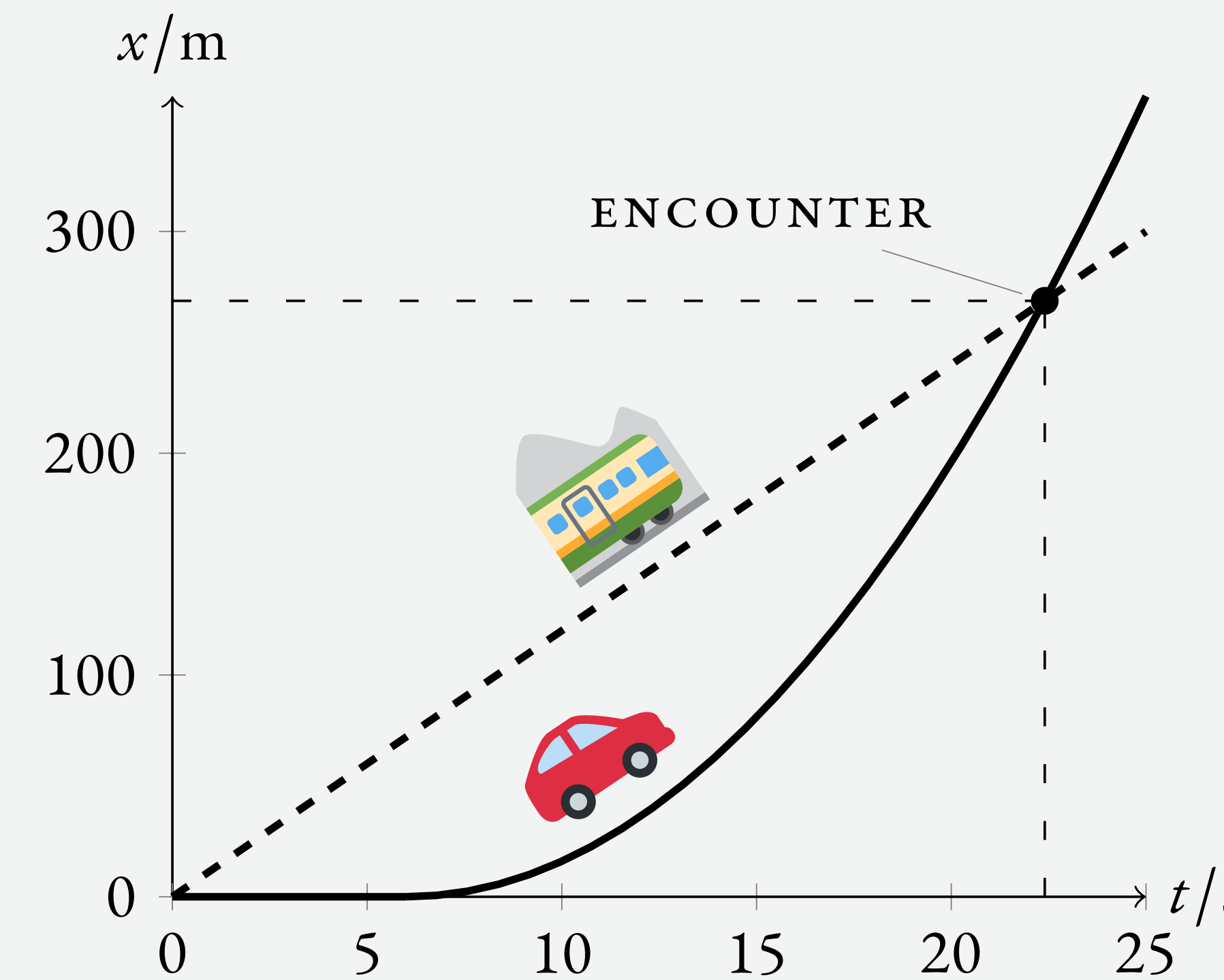
Now we **set the meeting condition**:

$$\begin{aligned} x_c &= x_t \\ t^2 - 12t + 36 &= 12t \\ t^2 - 24t + 36 &= 0 \end{aligned}$$

We solve for the **time** t^* :

$$t^* = \frac{24 \pm \sqrt{24^2 - 4 \cdot 1 \cdot 36}}{2} = \frac{24 \pm \sqrt{432}}{2} = \begin{cases} 22.4 \text{ s} \\ 1.6 \text{ s} \end{cases}$$

The solution $t = 1.6 \text{ s}$ is not a valid solution, since it is lower than the 6 s that the car was stopped in the traffic light. We can test our solution drawing the graph distance vs time ($x - t$) for each object:



where we can see that the car is stopped for the first 6 s and then it starts the motion accelerating (parabola) and catching the train after 22.4 s.

- b) To calculate the **distance covered** by the car we substitute the time $t^* = 22.4 \text{ s}$, in the position equation having in mind that $x_{0c} = 0$:

$$x_c(t^*) = t^{*2} - 12t^* + 36 = 22.4^2 - 12 \cdot 22.4 + 36 = 268.7 \text{ m}$$

- c) The **velocity** of the car when it catches the train can be calculated using the **velocity equation** of the car, substituting $t = t^*$:

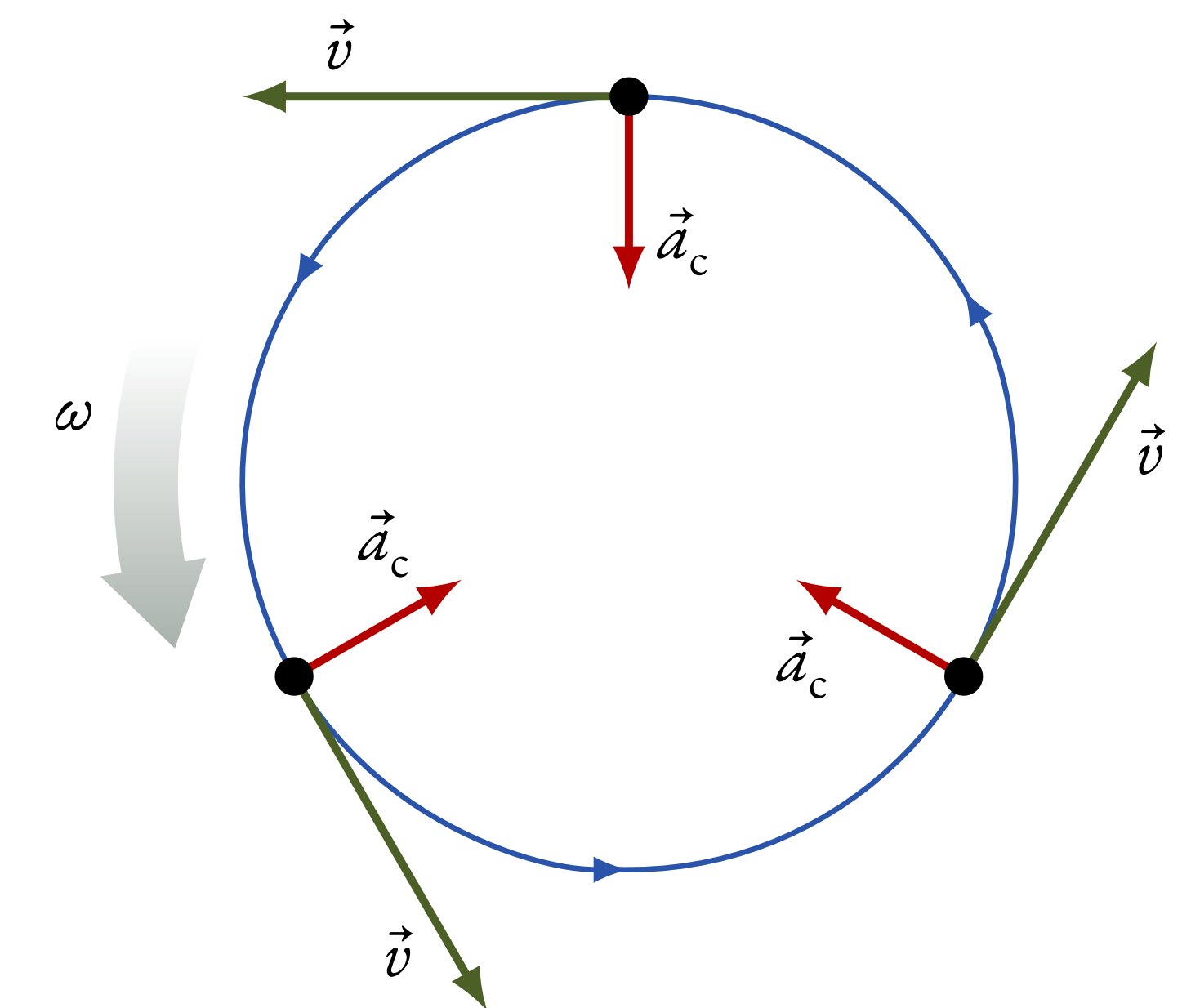
$$v_c(t^*) = v_{0c} + a_c(t^* - t_0) = 0 + 2 \cdot (22.4 - 6) = 32.8 \text{ m/s}$$

UCM

Characteristics

The **Characteristics** of the **Uniform Circular Motion** (UCM) are:

- Circular trajectory.
- Velocity's module constant (tangential acceleration $a_t = 0$).



Equation

The **equation** for the UCM is:

$$\varphi(t) = \varphi_0 + \omega(t - t_0),$$

where φ is the final angular position, φ_0 the initial angular position, ω the angular velocity, t the final time and t_0 the initial time.

Period T The time invested by the object in covering a complete revolution is called **period**, T .

Frequency f is the number of revolutions covered in 1 s.

Frequency, f , it is related to the period by the equation:

$$f = \frac{1}{T} \left[\frac{1}{s} = s^{-1} = \text{Hz} \right]$$

The angular velocity, ω , is related with the period and the frequency by the following expressions:

$$\omega = \frac{\Delta\varphi}{\Delta t} = \frac{2\pi}{T} = 2\pi f$$

Lineal and angular magnitudes are related by the radius R :

$$\begin{aligned} s &= \varphi R \\ v &= \omega R \end{aligned}$$

Centripetal Acceleration \vec{a}_c

Also called **normal acceleration**, is the acceleration related to the change of direction of the velocity vector. Its module can be calculated as:

$$a_c = \frac{v^2}{R}$$

and it always points to the center of the circumference.