

# FORCES

15-16 year-olds

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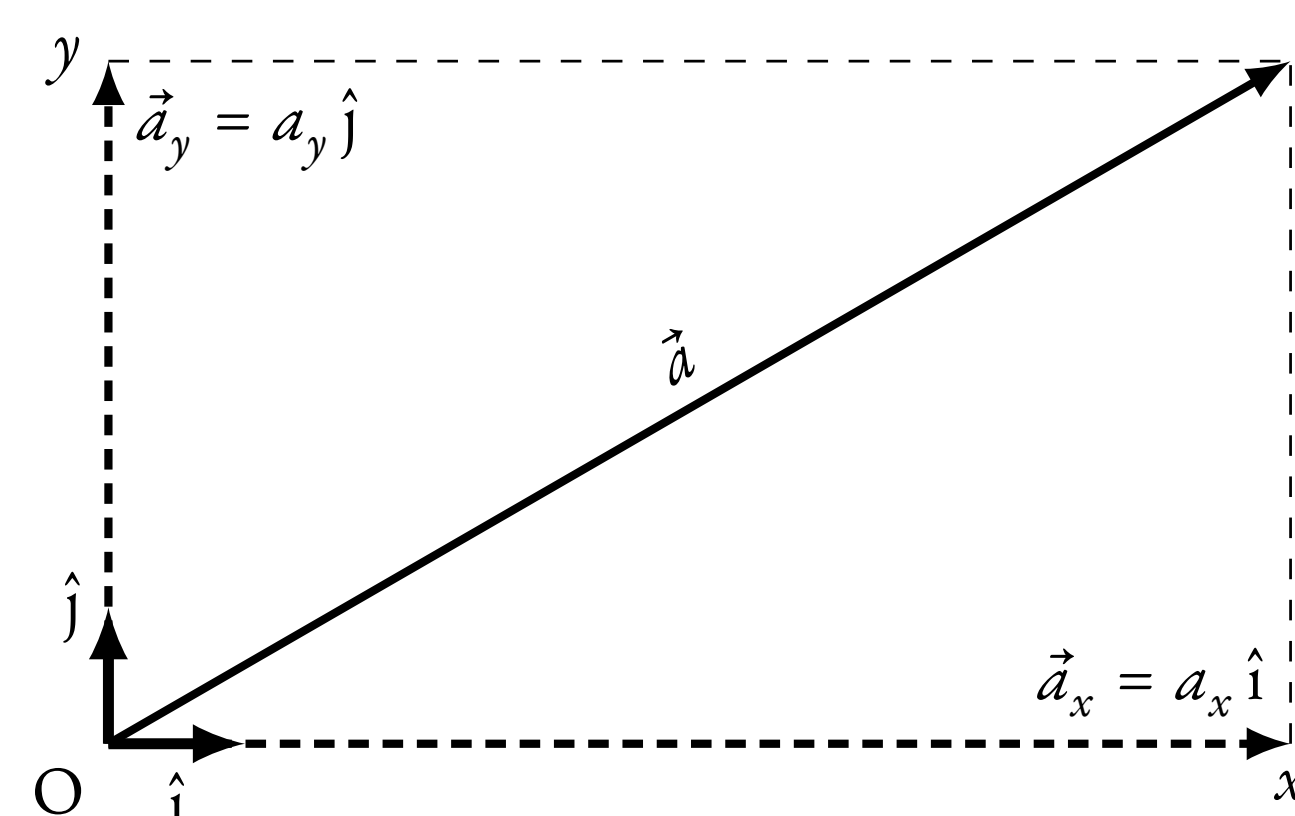


## Forces as Vectors

**Forces** are **vector magnitudes**, which means that they are defined by a **vector**, which, in turn, is defined by:

**Magnitude** Also called modulus, it's the length of the line segment.

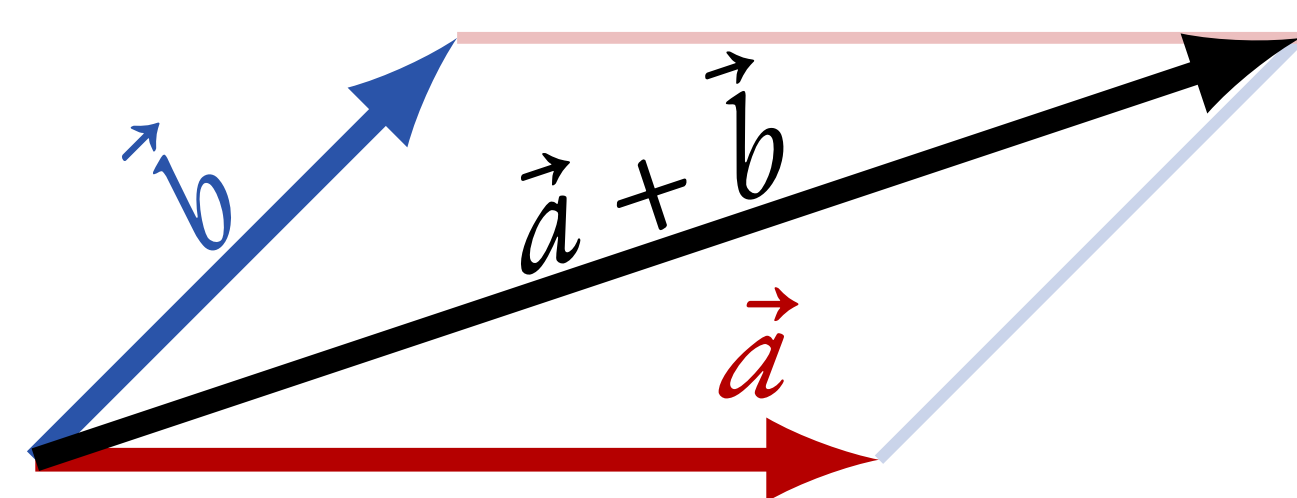
**Direction** The direction (angle) of the arrow from tail to head.



Two-dimensional vectors can be written as  $\vec{a} = a_x \hat{i} + a_y \hat{j}$ , where  $\hat{i}$  and  $\hat{j}$  are unit vectors (modulus = 1) on the  $x$  and  $y$  axis. The magnitude (modulus) of  $\vec{a}$ ,  $|\vec{a}|$ , can be calculated using the Pythagorean Theorem  $|\vec{a}| = \sqrt{a_x^2 + a_y^2}$ .

### Addition and subtraction of vectors

Graphically, we must draw one vector after the other and then join the initial and final points:



Mathematically, we add or subtract each coordinate (component) separately:

$$\vec{a} + \vec{b} = (a_x + b_x) \hat{i} + (a_y + b_y) \hat{j}$$

## Newton's Laws

### 1st Law (Law of Inertia)

*"If no net force is applied to an object, the object will stay at rest or will move at a constant velocity."*

### 2nd Law (Dynamic's Fundamental Law)

*"The change in the state of motion of a body is directly proportional to, and in the same direction, as the net force that is acting upon that body."*

Mathematically, it is written as

$$\sum \vec{F} = m\vec{a} \quad (\text{acceleration is directly proportional to the net force})$$

In the **SI** force is measured in **newton** (N):  $1 \text{ N} = 1 \text{ kg m s}^{-2}$ .

### 3rd Law (Action-Reaction Principle)

*"For every action there is always a reaction, of the same magnitude but in the opposite direction."*

If a body A exerts a force over another body B, B will exert the same force on A but in the opposite direction ( $\vec{F}_{AB} = -\vec{F}_{BA}$ ).

## Forces of Special Interest

### Weight $\vec{F}_w$

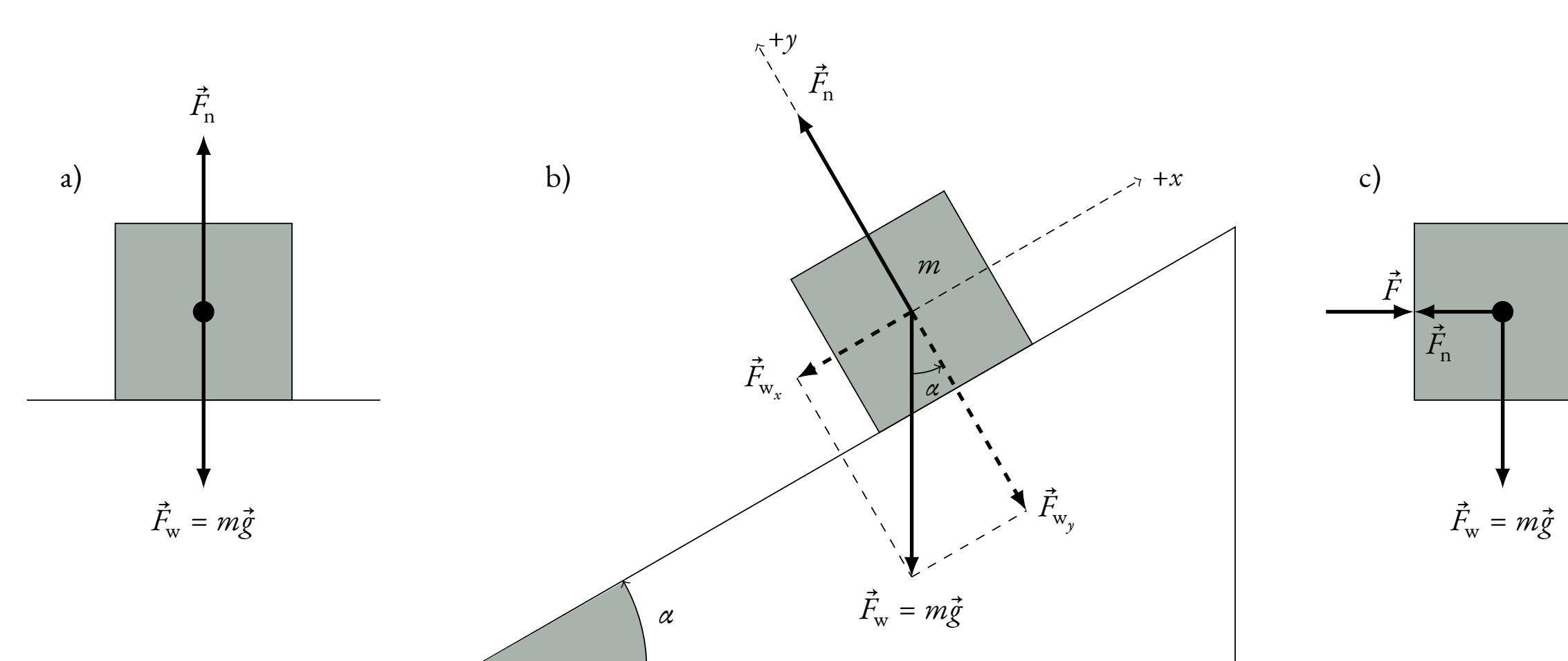
**Weight** is the force exerted by the Earth on every object. It is calculated as:

$$\vec{F}_w = m\vec{g},$$

where  $m$  is the mass of the object and  $\vec{g}$  is the acceleration of gravity. It always points to the center of the Earth (downwards).

### Normal Force $\vec{F}_n$

Also known as **reaction** force, it is defined as the force exerted by a surface on every object standing on it. It has the same magnitude but opposite direction as the force exerted by the object on the surface.



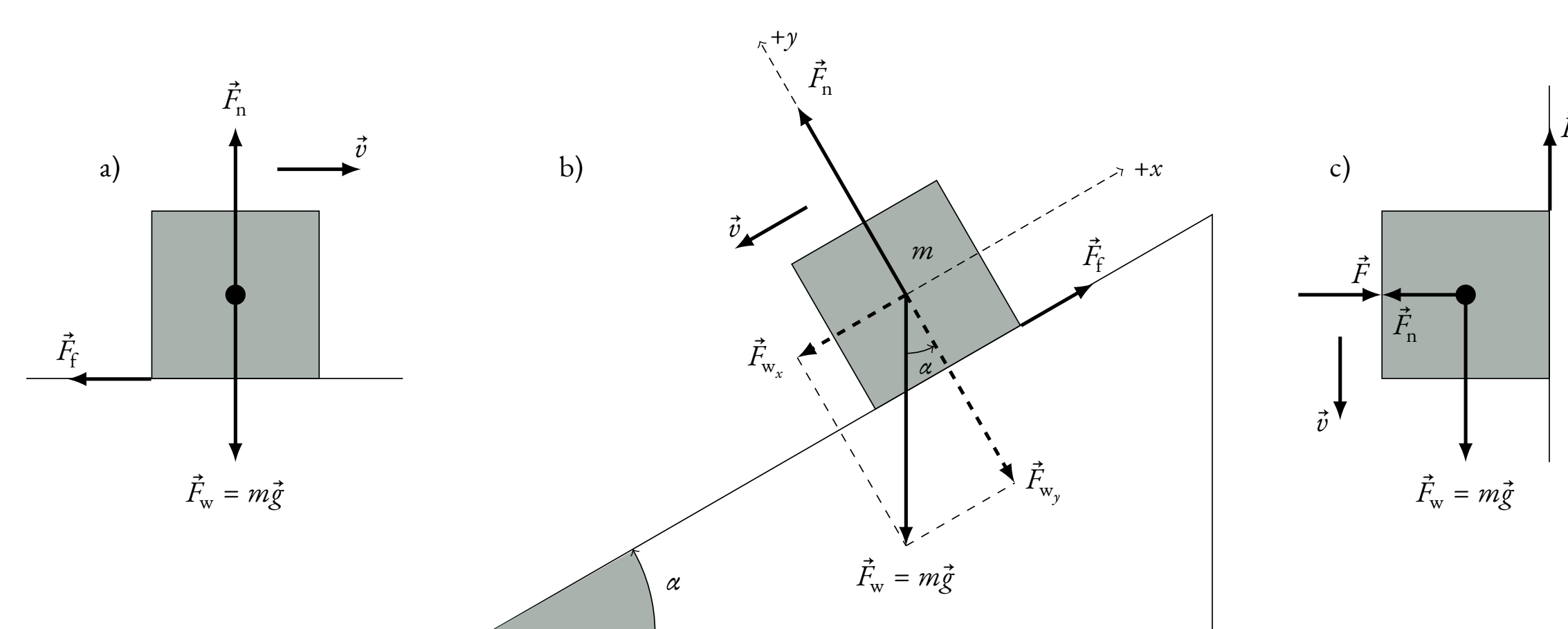
**Figure 1.** Normal force on a) a horizontal surface, b) a slide and c) a vertical surface.

### Friction $\vec{F}_f$

**Friction force** is the force between two surfaces in contact, opposing to the relative motion between both surfaces. The friction force is directly proportional to the normal force  $F_n$ :

$$F_f = \mu F_n,$$

where  $\mu$  is the coefficient of friction.



**Figure 2.** Friction force on a) a horizontal surface, b) a slide and c) a vertical surface.

### Centripetal Force $\vec{F}_c$

**Centripetal force** is the force or component of the force acting on an object moving on a curved path and pointing to the center of the trajectory. Its modulus is calculated using the **centripetal acceleration** and the **2nd Law of Newton**:

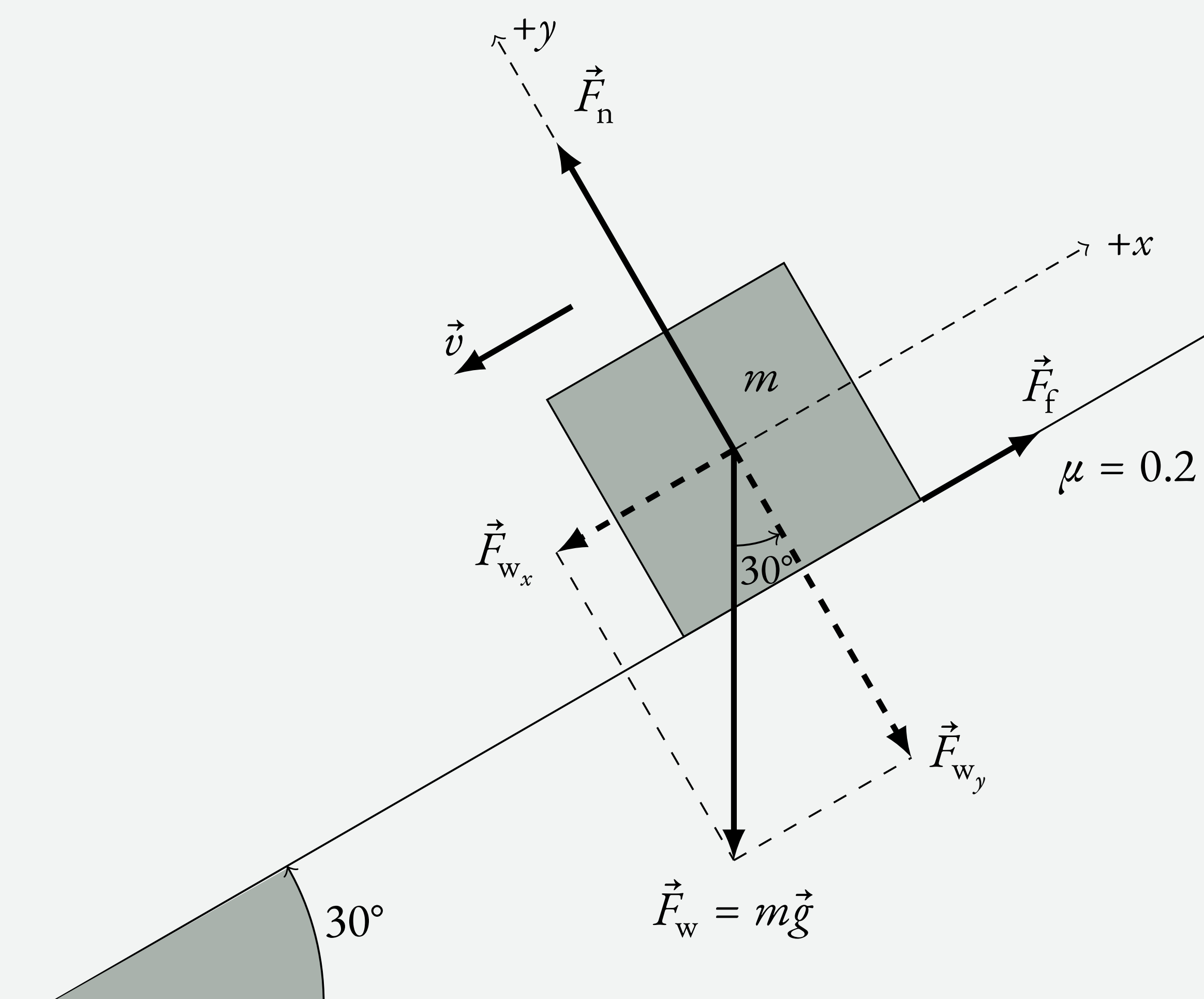
$$F_c = ma_c = m \cdot \frac{v^2}{R} = \frac{mv^2}{R}$$

## Example

A body is falling down a slide inclined  $30^\circ$  with a coefficient of friction  $\mu = 0.2$ . Calculate the velocity and the space covered by the object after 5 s, if it was initially at rest.

### Solution

First we make a drawing of the problem:



The **forces** acting are:

- Weight  $\vec{F}_w = -F_{w_x} \hat{i} - F_{w_y} \hat{j}$ , where:

$$F_{w_x} = mg \sin \alpha = 9.8m \sin 30^\circ = 4.9m \text{ N}$$

$$F_{w_y} = mg \cos \alpha = 9.8m \cos 30^\circ = 4.9\sqrt{3}m \text{ N}$$

- Normal  $\vec{F}_n = F_n \hat{j}$
- Friction force  $\vec{F}_f = \mu F_n \hat{i} = 0.2 F_n \hat{i}$

We write the **2nd Law of Newton** for each **component**:

$$\text{Component } x \rightarrow F_f - F_{w_x} = ma \quad (1)$$

$$\text{Component } y \rightarrow F_n - F_{w_y} = 0 \quad (2)$$

Solving  $F_n = F_{w_y} = 4.9\sqrt{3}m$  from (2) and substituting in (1), using also that  $F_f = 0.2F_n$  and that  $F_{w_x} = 4.9m$ :

$$0.2 \cdot 4.9\sqrt{3}m - 4.9m = ma \rightarrow a = -3.2 \text{ m/s}^2$$

$$\vec{a} = -3.2 \hat{i} \text{ m/s}^2$$

The **velocity** after 5 s can be calculated with the **velocity equation**:

$$v = v_0 + at = 0 - 3.2 \cdot 5 = -16.0 \text{ m/s}$$

$$\vec{v} = -16.0 \hat{i} \text{ m/s}$$

For the **space covered** we can use the **motion equation**:

$$\Delta x = |x - x_0| = \left| v_0 \cdot t + \frac{1}{2}at^2 \right| = \left| 0 - \frac{1}{2} \cdot 3.2 \cdot 5^2 \right| = 40.0 \text{ m}$$