

# 15-16 year-olds

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URM

### Characteristics

A body describes a uniform rectilinear motion (URM) when:

- The trajectory is a straight line.
- Its velocity v is constant. (acceleration a = 0).

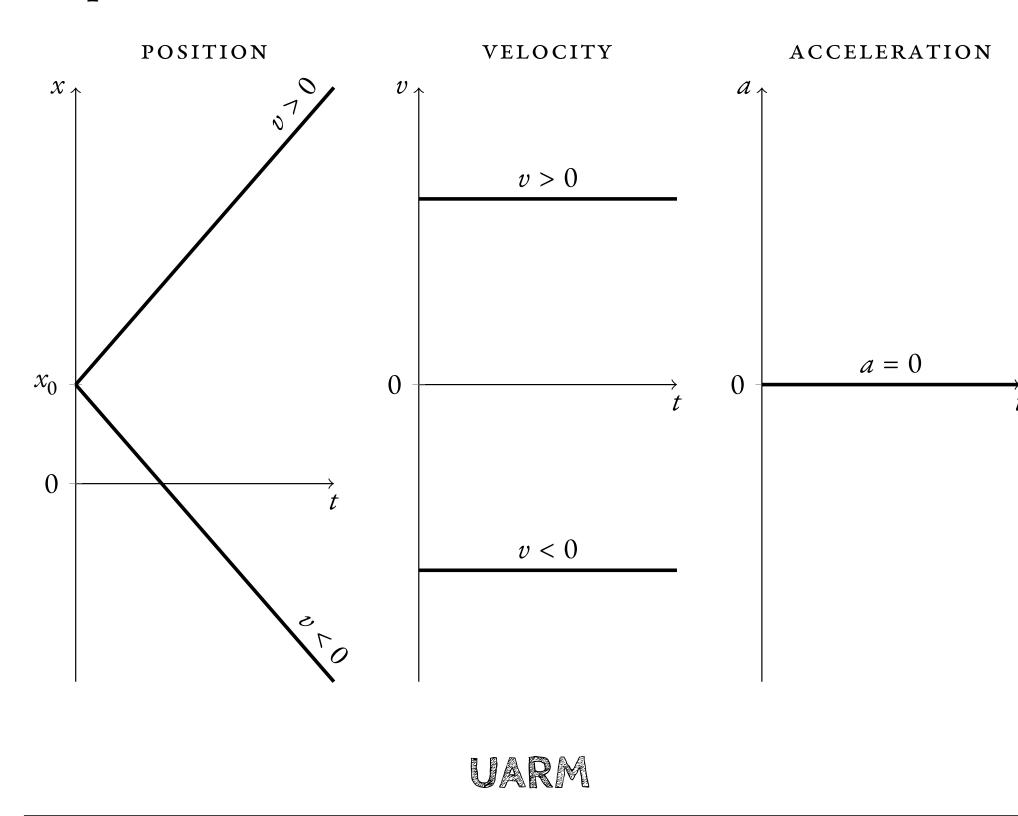
### **Equation**

The **equation** for the URM s:

$$x(t) = x_0 + v(t - t_0)$$

where x represents the final position,  $x_0$  the initial position, v the final velocity, t the final time and  $t_0$  the initial time.

### Graphs



## Characteristics

A body is describing a **Uniformly Accelerated Rectilinear Motion** (UARM) when:

- The trajectory is a straight line.
- The acceleration a is constant (velocity v variable).

#### Main equations

The **main equations** for the UARM:

**Position:** 
$$x(t) = x_0 + v_0(t - t_0) + \frac{1}{2}a(t - t_0)^2$$
 (1)

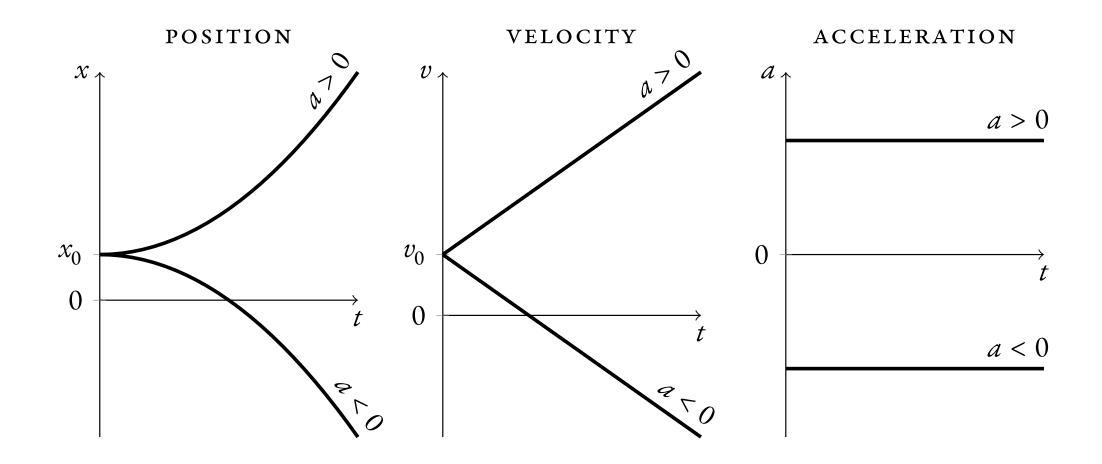
Velocity: 
$$v(t) = v_0 + a(t - t_0)$$
 (2)

$$v^2 - v_0^2 = 2a\Delta x$$

where x represents the final position,  $x_0$  the initial position, v the final velocity,  $v_0$  the initial position, a the acceleration, t the final time,  $t_0$  the initial time and  $\Delta x = x - x_0$  is the distance covered by the object.

# uarm [cont.]

### Graphs



# Free fall/vertical motion

**Free falling motion** is a type of UARM where the acceleration is the acceleration of **gravity**. On Earth,  $a = -g = -9.8 \,\text{m/s}^2$  (the acceleration is negative because it always points downwards).

# Encounters

In these kind of problems two bodies start on different positions and they meet after a while.

We follow three steps:

- 1. We write the position equations for each body.
- 2. We **set** the **meeting** condition, that is, both positions are the same when they meet.
- 3. We **solve** for the magnitude asked.

# Example

A car  $\Leftrightarrow$  is moving on a road which is parallel to a train track. The car stops on a red light in the exact moment that a train  $\rightleftharpoons$  is passing with a constant velocity of 12 m/s. The car stays in the traffic light for 6 s and then it starts moving with an acceleration of  $2 \text{ m/s}^2$ . Calculate:

- a) Time needed for the car to catch the train since it stopped at the red light.
- b) Distance covered by the car from the traffic light until it catches the train.
- c) The velocity of the car when it catches the train.

#### Solution

a) The first thing to do is to write the **motion equations** for each object:

(UARM): 
$$x_c = x_{0_c} + v_{0_c}(t - t_{0_c}) + \frac{1}{2}a_c(t - t_{0_c})^2$$
  
(URM):  $x_t = x_{0_t} + v_t(t - t_{0_t})$ 

# Example [cont.]

a) Data:

$$x_{0_{c}} = x_{0_{t}} = 0$$

$$v_{0_{c}} = 0; \quad v_{t} = 12 \text{ m/s}$$

$$a_{c} = 2 \text{ m/s}^{2}$$

$$t_{0_{c}} = 6 \text{ s}; \quad t_{0_{t}} = 0$$

$$\text{(UARM): } x_{c} = 0 + 0 \cdot (t - 6) + \frac{1}{2} \cdot 2 \cdot (t - 6)^{2}$$

$$= (t - 6)^{2} = t^{2} - 12t + 36$$

$$\text{(URM): } x_{t} = 0 + 12 \cdot (t - 0) = 12t$$

Now we set the **meeting condition**:

$$x_{c} = x_{t}$$

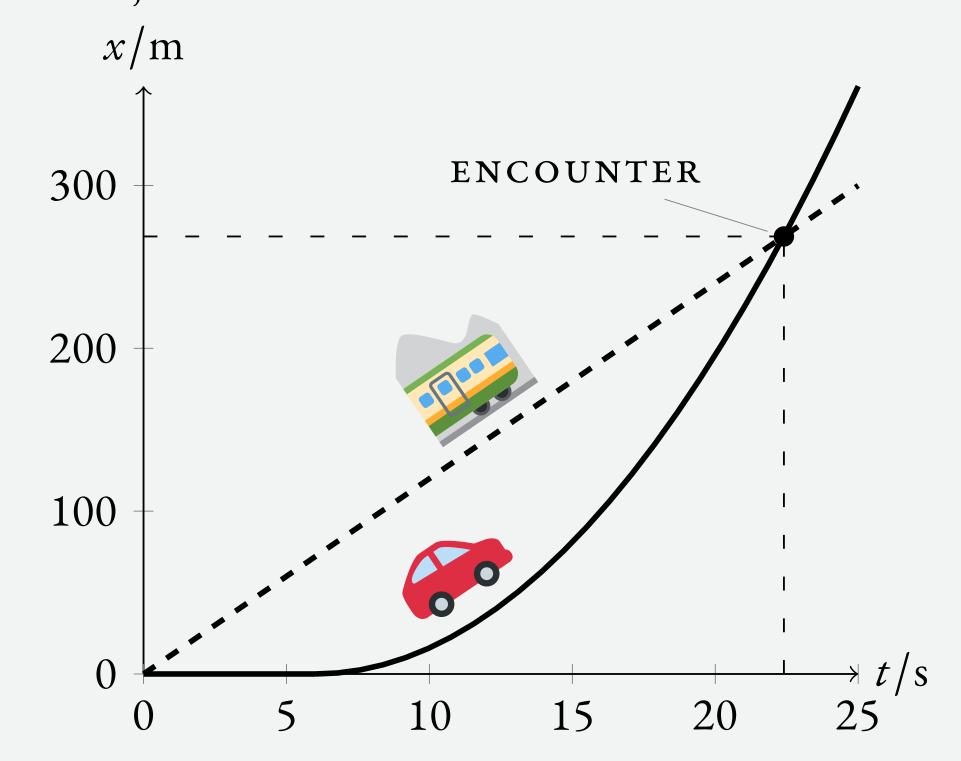
$$t^{2} - 12t + 36 = 12t$$

$$t^{2} - 24t + 36 = 0$$

We solve for the **time**  $t^*$ :

$$t^* = \frac{24 \pm \sqrt{24^2 - 4 \cdot 1 \cdot 36}}{2} = \frac{24 \pm \sqrt{432}}{2} = \begin{cases} 22.4 \\ 1.65 \end{cases}$$

The solution t = 1.6 s is not a valid solution, since it is lower than the 6 s that the car was stopped in the traffic light. We can test our solution drawing the graph distance vs time (x - t) for each object:



where we can see that the car is stopped for the first 6 s and the it starts the motion accelerating (parabola) and catching the train after 22.4 s.

b) To calculate the **distance covered** by the car we substitute the time  $t^* = 22.4$  s, in the position equation having in mind that en  $x_0 = 0$ :

$$x_{c}(t^{*}) = t^{*2} - 12t^{*} + 36 = 22.4^{2} - 12 \cdot 22.4 + 36 = 268.7 \text{ m}$$

c) The **velocity** of the car when it catches the train can be calculated using the **velocity equation** of the car, substituting  $t = t^*$ :

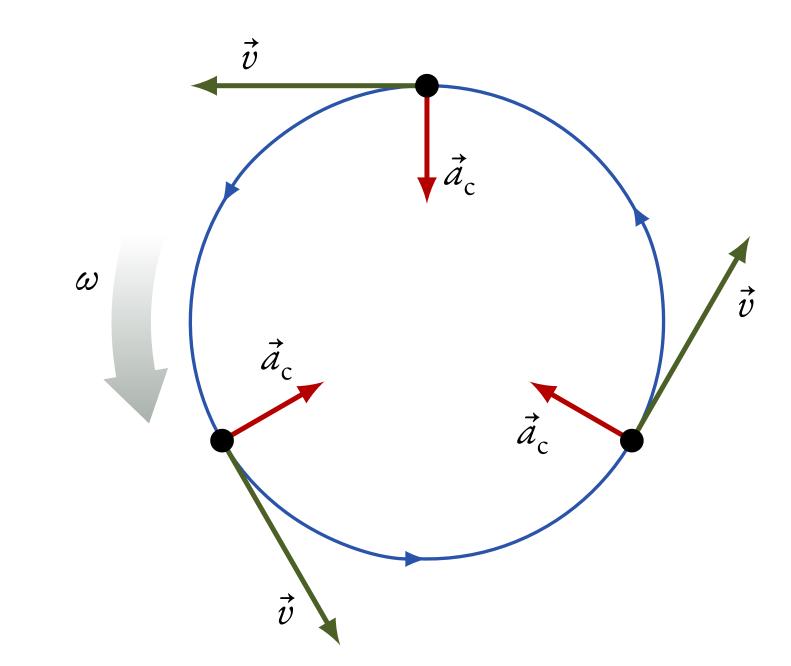
$$v_c(t^*) = v_{0_c} + a_c(t^* - t_0) = 0 + 2 \cdot (22.4 - 6) = 32.8 \text{ m/s}$$

# UCM

#### Characteristics

The **Characteristics** of the **Uniform Circular Motion** (UCM) are:

- Circular trajectory.
- Velocity's module constant (tangential acceleration  $a_t = 0$ ).



#### Equation

The **equation** for the UCM is:

$$\varphi(t) = \varphi_0 + \omega(t - t_0),$$

where  $\varphi$  is the final angular position,  $\varphi_0$  the initial angular position,  $\omega$  the angular velocity, t the final time and  $t_0$  the initial time.

Period T The time invested by the object in covering a complete revolution is called **period**, T.

Frequency f is the number of revolutions covered in 1 s. Frequency, f, it is related to the period by the equation:

$$f = \frac{1}{T} \left[ \frac{1}{s} = s^{-1} = Hz \right]$$

The angular velocity,  $\omega$ , is related with the period and the frequency by the following expressions:

$$\omega = \frac{\varDelta \varphi}{\varDelta t} = \frac{2\pi}{T} = 2\pi f$$

Lineal and angular magnitudes are related by the radius R:

$$s = \varphi R$$
$$v = \omega R$$

## Centripetal Acceleration $\vec{a}_c$

Also called **normal acceleration**, is the acceleration related to the change of direction of the velocity vector. Its module can be calculated as:

$$a_{\rm c} = \frac{v^2}{R}$$

and it always points to the center of the circumference.