MOTION

15-16 year-olds

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URM

Characteristics

A body describes a **uniform rectilinear motion** (URM) when:

- The trajectory is a straight line.
- Its velocity v is constant. (acceleration $a = 0$).

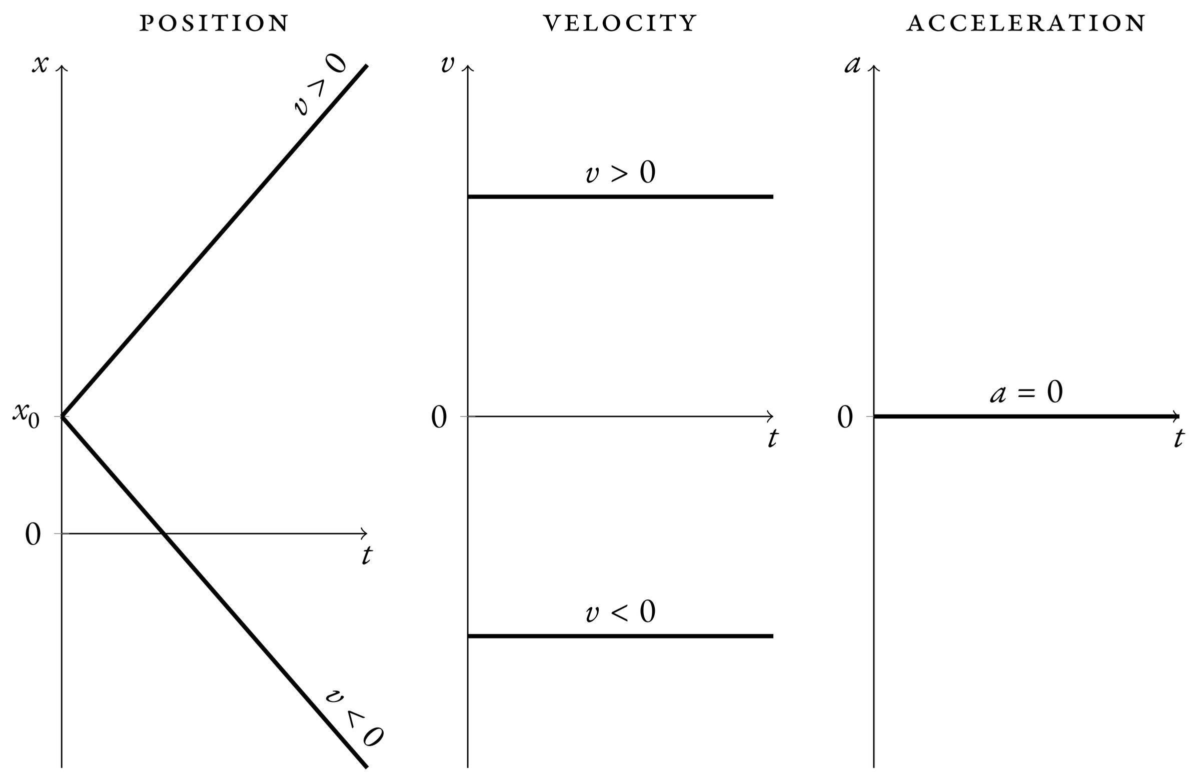
Equation

The **equation** for the URM s:

$$x(t) = x_0 + v(t - t_0)$$

where x represents the final position, x_0 the initial position, v the final velocity, t the final time and t_0 the initial time.

Graphs



UARM

Characteristics

A body is describing a **Uniformly Accelerated Rectilinear Motion** (UARM) when:

- The trajectory is a straight line.
- The acceleration a is constant (velocity v variable).

Main equations

The **main equations** for the UARM :

$$\textbf{Position: } x(t) = x_0 + v_0(t - t_0) + \frac{1}{2}a(t - t_0)^2 \quad (1)$$

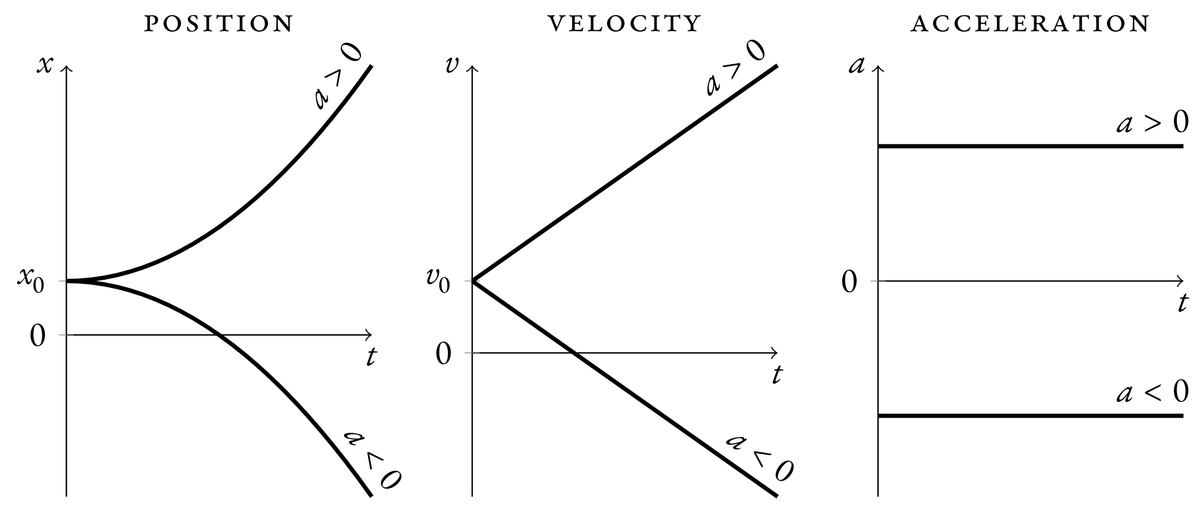
$$\textbf{Velocity: } v(t) = v_0 + a(t - t_0) \quad (2)$$

$$v^2 - v_0^2 = 2a\Delta x \quad (3)$$

where x represents the final position, x_0 the initial position, v_0 the initial velocity, v the final velocity, a the acceleration, t the final time, t_0 the initial time and $\Delta x = x - x_0$ is the distance covered by the object.

UARM (cont.)

Graphs



Free fall/vertical motion

Free falling motion is a type of UARM where the acceleration is the acceleration of **gravity**. On Earth, $a = -g = -9.8\ \text{m/s}^2$ (the acceleration is negative because it always points downwards).

Encounters

In these kind of problems two bodies start on different positions and they meet after a while.

We follow **three steps**:

1. We **write** the **position equations** for each body.
2. We **set** the **meeting** condition, that is, both positions are the same when they meet.
3. We **solve** for the magnitude asked.

Example

A car 🚗 is moving on a road which is parallel to a train track. The car stops on a red light in the exact moment that a train 🚋 is passing with a constant velocity of 12 m/s. The car stays in the traffic light for 6 s and then it starts moving with an acceleration of 2 m/s². Calculate:

a) Time needed for the car to catch the train since it stopped at the red light.
b) Distance covered by the car from the traffic light until it catches the train.
c) The velocity of the car when it catches the train.

Solution

a) The first thing to do is to write the **motion equations** for each object:

$$🚗\ \textbf{(UARM)}: x_c = x_{0_c} + v_{0_c}(t - t_{0_c}) + \frac{1}{2}a_c(t - t_{0_c})^2$$

$$🚋\ \textbf{(URM)}: x_t = x_{0_t} + v_t(t - t_{0_t})$$

Example (cont.)

a) **Data**:

$$x_{0_c} = x_{0_t} = 0$$
$$v_{0_c} = 0; \quad v_t = 12\ \text{m/s}$$
$$a_c = 2\ \text{m/s}^2$$
$$t_{0_c} = 6\ \text{s}; \quad t_{0_t} = 0$$

$$🚗\ \textbf{(UARM)}: x_c = 0 + 0 \cdot (t - 6) + \frac{1}{2} \cdot 2 \cdot (t - 6)^2 = (t - 6)^2 = t^2 - 12t + 36$$

$$🚋\ \textbf{(URM)}: x_t = 0 + 12 \cdot (t - 0) = 12t$$

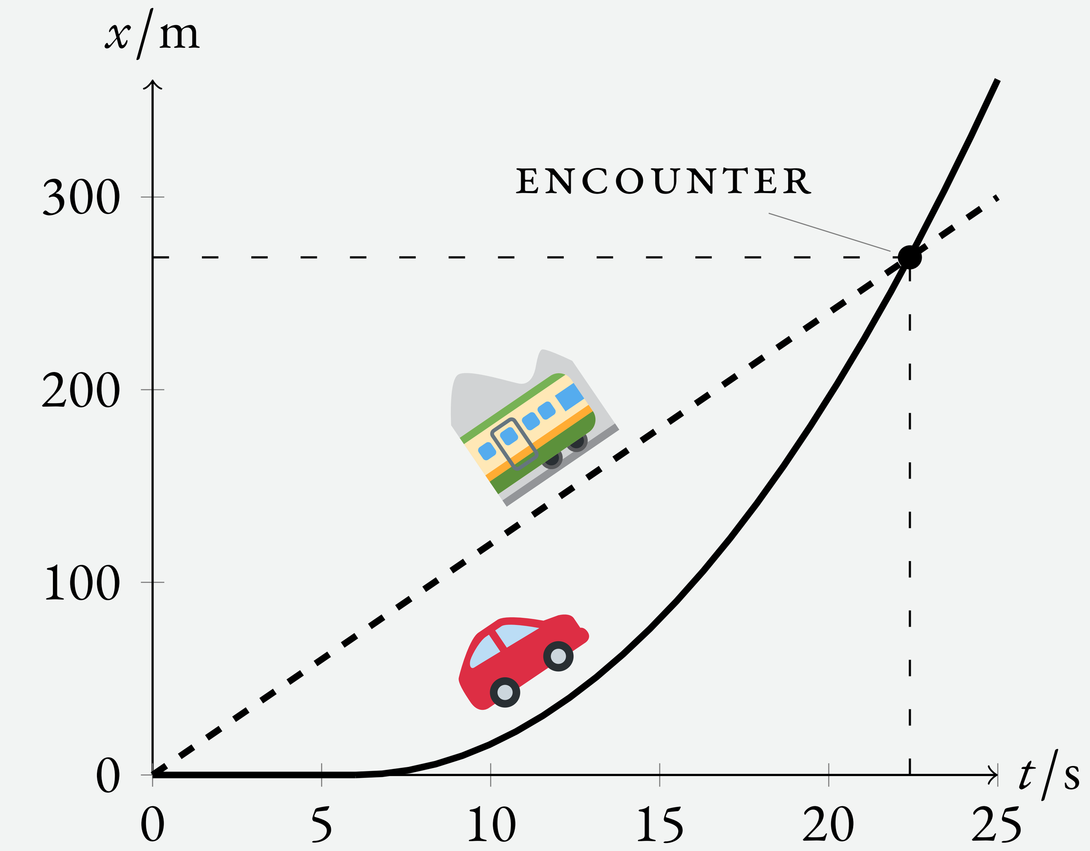
Now we set the **meeting condition**:

$$x_c = x_t$$
$$t^2 - 12t + 36 = 12t$$
$$t^2 - 24t + 36 = 0$$

We solve for the **time** t^*:

$$t^* = \frac{24 \pm \sqrt{24^2 - 4 \cdot 1 \cdot 36}}{2} = \frac{24 \pm \sqrt{432}}{2} = \begin{cases} 22.4\ \text{s} \\ \cancel{1.6\ \text{s}} \end{cases}$$

The solution $t = 1.6$ s is not a valid solution, since it is lower than the 6 s that the car was stopped in the traffic light. We can test our solution drawing the graph distance vs time $(x - t)$ for each object:



where we can see that the car is stopped for the first 6 s and the it starts the motion accelerating (parabola) and catching the train after 22.4 s.

b) To calculate the **distance covered** by the car we substitute the time $t^* = 22.4$ s, in the position equation having in mind that en $x_0 = 0$:

$$x_c(t^*) = t^{*2} - 12t^* + 36 = 22.4^2 - 12 \cdot 22.4 + 36 = 268.7\ \text{m}$$

c) The **velocity** of the car when it catches the train can be calculated using the **velocity equation** of the car, substituting $t = t^*$:

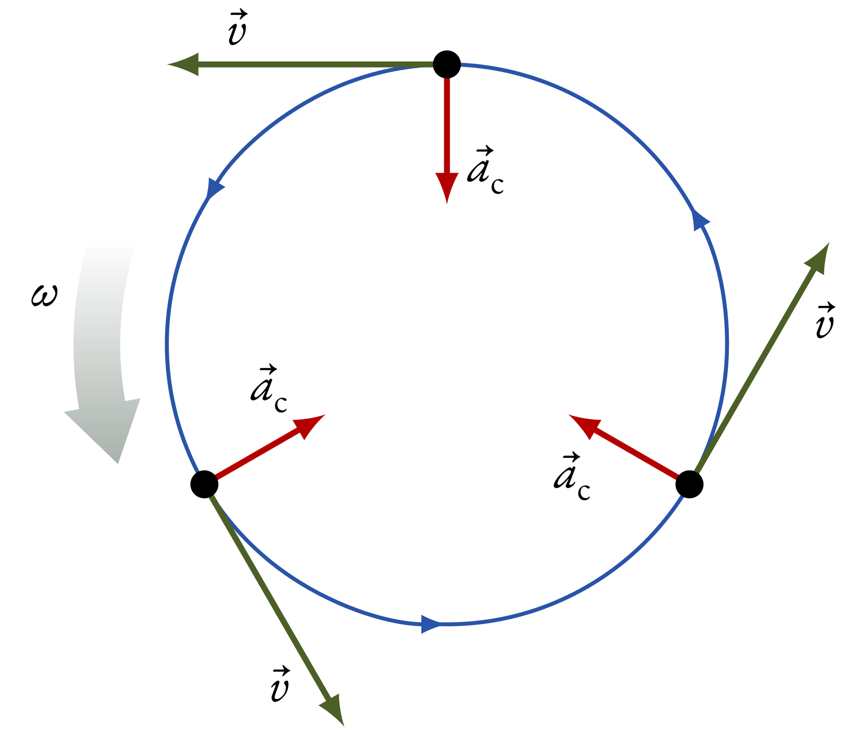
$$v_c(t^*) = v_{0_c} + a_c(t^* - t_0) = 0 + 2 \cdot (22.4 - 6) = 32.8\ \text{m/s}$$

UCM

Characteristics

The **Characteristics** of the **Uniform Circular Motion** (UCM) are:

- Circular trajectory.
- Velocity's module constant (tangential acceleration $a_t = 0$).



Equation

The **equation** for the UCM is:

$$\varphi(t) = \varphi_0 + \omega(t - t_0),$$

where φ is the final angular position, φ_0 the initial angular position, ω the angular velocity, t the final time and t_0 the initial time.

Period T The time invested by the object in covering a complete revolution is called **period**, T.

Frequency f is the number of revolutions covered in 1 s. **Frequency**, f, it is related to the period by the equation:

$$f = \frac{1}{T} \left[\frac{1}{\text{s}} = \text{s}^{-1} = \text{Hz} \right]$$

The angular velocity, ω, is related with the period and the frequency by the following expressions:

$$\omega = \frac{\Delta\varphi}{\Delta t} = \frac{2\pi}{T} = 2\pi f$$

Lineal and angular magnitudes are related by the radius R:

$$s = \varphi R$$
$$v = \omega R$$

Centripetal Acceleration $\vec{a}_c$

Also called **normal acceleration**, is the acceleration related to the change of direction of the velocity vector. Its module can be calculated as:

$$a_c = \frac{v^2}{R}$$

and it always points to the center of the circumference.