

# MOTION

15-16 year-olds

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## URM

### Characteristics

A body describes a **uniform rectilinear motion** (URM) when:

- The trajectory is a straight line.
- Its velocity  $v$  is constant. (acceleration  $a = 0$ ).

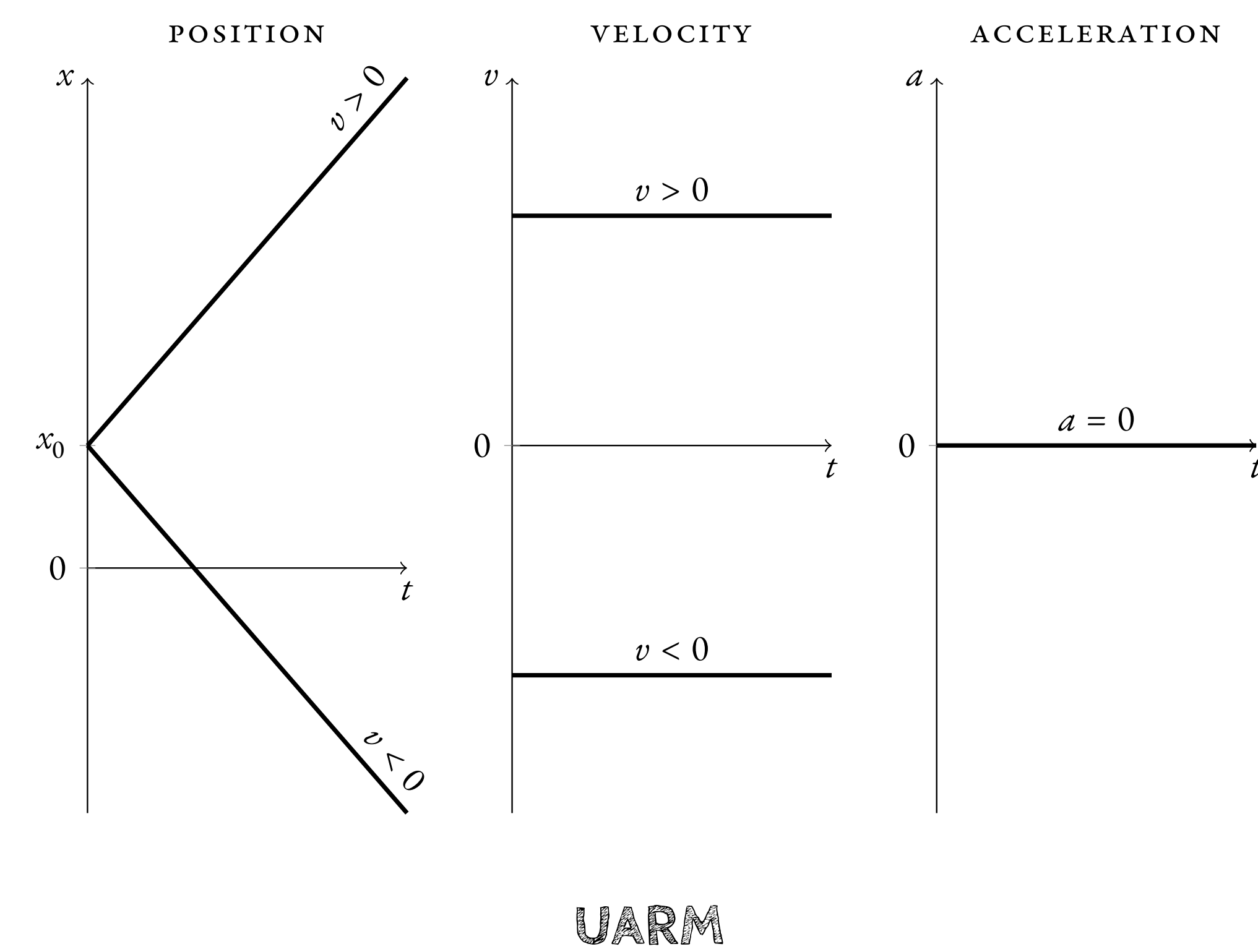
### Equation

The **equation** for the URM s:

$$x(t) = x_0 + v(t - t_0)$$

where  $x$  represents the final position,  $x_0$  the initial position,  $v$  the final velocity,  $t$  the final time and  $t_0$  the initial time.

### Graphs



### Characteristics

A body is describing a **Uniformly Accelerated Rectilinear Motion** (UARM) when:

- The trajectory is a straight line.
- The acceleration  $a$  is constant (velocity  $v$  variable).

### Main equations

The **main equations** for the UARM :

$$\text{Position: } x(t) = x_0 + v_0(t - t_0) + \frac{1}{2}a(t - t_0)^2 \quad (1)$$

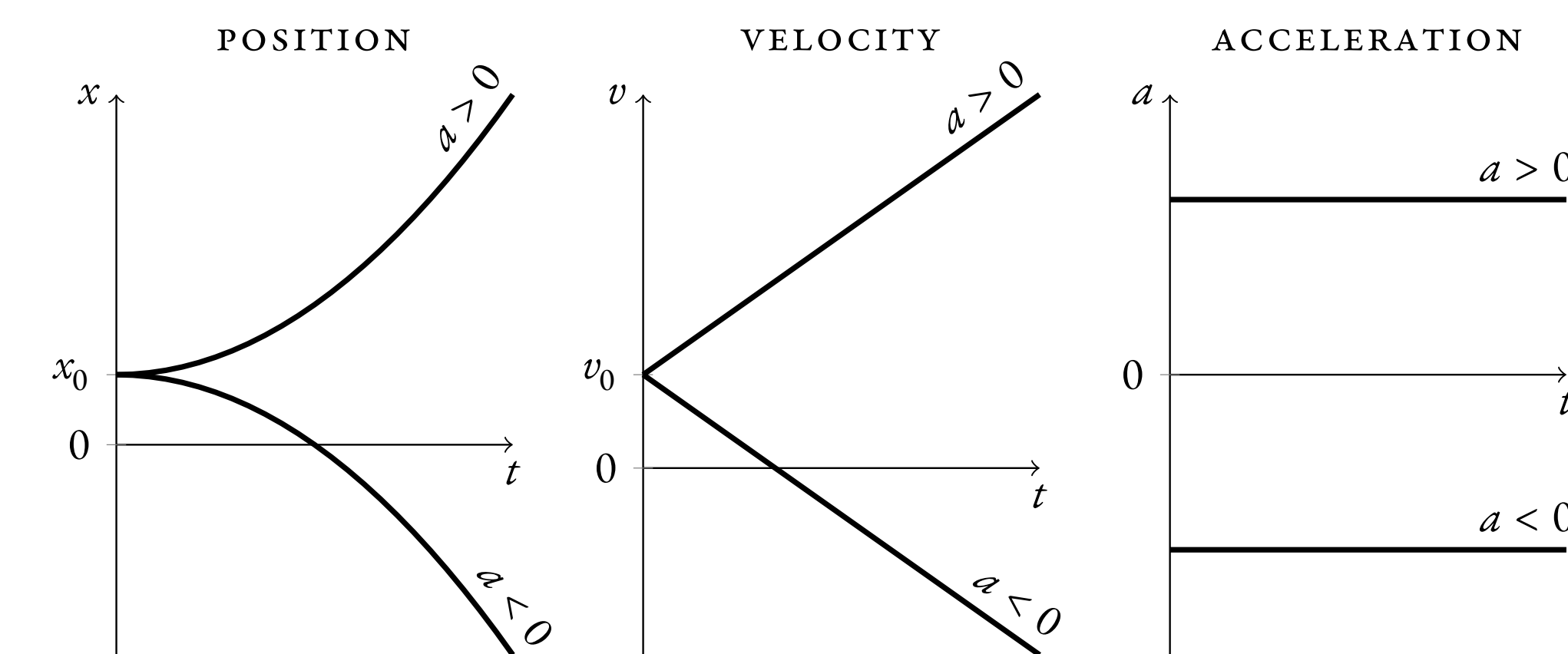
$$\text{Velocity: } v(t) = v_0 + a(t - t_0) \quad (2)$$

$$v^2 - v_0^2 = 2a\Delta x \quad (3)$$

where  $x$  represents the final position,  $x_0$  the initial position,  $v$  the final velocity,  $v_0$  the initial position,  $a$  the acceleration,  $t$  the final time,  $t_0$  the initial time and  $\Delta x = x - x_0$  is the distance covered by the object.

## UARM (cont.)

### Graphs



## Free fall/vertical motion

**Free falling motion** is a type of UARM where the acceleration is the acceleration of **gravity**. On Earth,  $a = -g = -9.8 \text{ m/s}^2$  (the acceleration is negative because it always points downwards).

## Encounters

In these kind of problems two bodies start on different positions and they meet after a while.

We follow **three steps**:

1. We **write** the **position equations** for each body.
2. We **set the meeting condition**, that is, both positions are the same when they meet.
3. We **solve** for the magnitude asked.

## Example

A car 🚗 is moving on a road which is parallel to a train track. The car stops on a red light in the exact moment that a train 🚂 is passing with a constant velocity of 12 m/s. The car stays in the traffic light for 6 s and then it starts moving with an acceleration of 2 m/s<sup>2</sup>. Calculate:

- a) Time needed for the car to catch the train since it stopped at the red light.
- b) Distance covered by the car from the traffic light until it catches the train.
- c) The velocity of the car when it catches the train.

### Solution

- a) The first thing to do is to **write the motion equations for each object**:

$$\text{🚗 (UARM): } x_c = x_{0c} + v_{0c}(t - t_{0c}) + \frac{1}{2}a_c(t - t_{0c})^2$$

$$\text{🚂 (URM): } x_t = x_{0t} + v_t(t - t_{0t})$$

## Example (cont.)

### a) Data:

$$\begin{aligned} x_{0c} &= x_{0t} = 0 \\ v_{0c} &= 0; \quad v_t = 12 \text{ m/s} \\ a_c &= 2 \text{ m/s}^2 \\ t_{0c} &= 6 \text{ s}; \quad t_{0t} = 0 \end{aligned}$$

$$\text{🚗 (UARM): } x_c = 0 + 0 \cdot (t - 6) + \frac{1}{2} \cdot 2 \cdot (t - 6)^2 = (t - 6)^2 = t^2 - 12t + 36$$

$$\text{🚂 (URM): } x_t = 0 + 12 \cdot (t - 0) = 12t$$

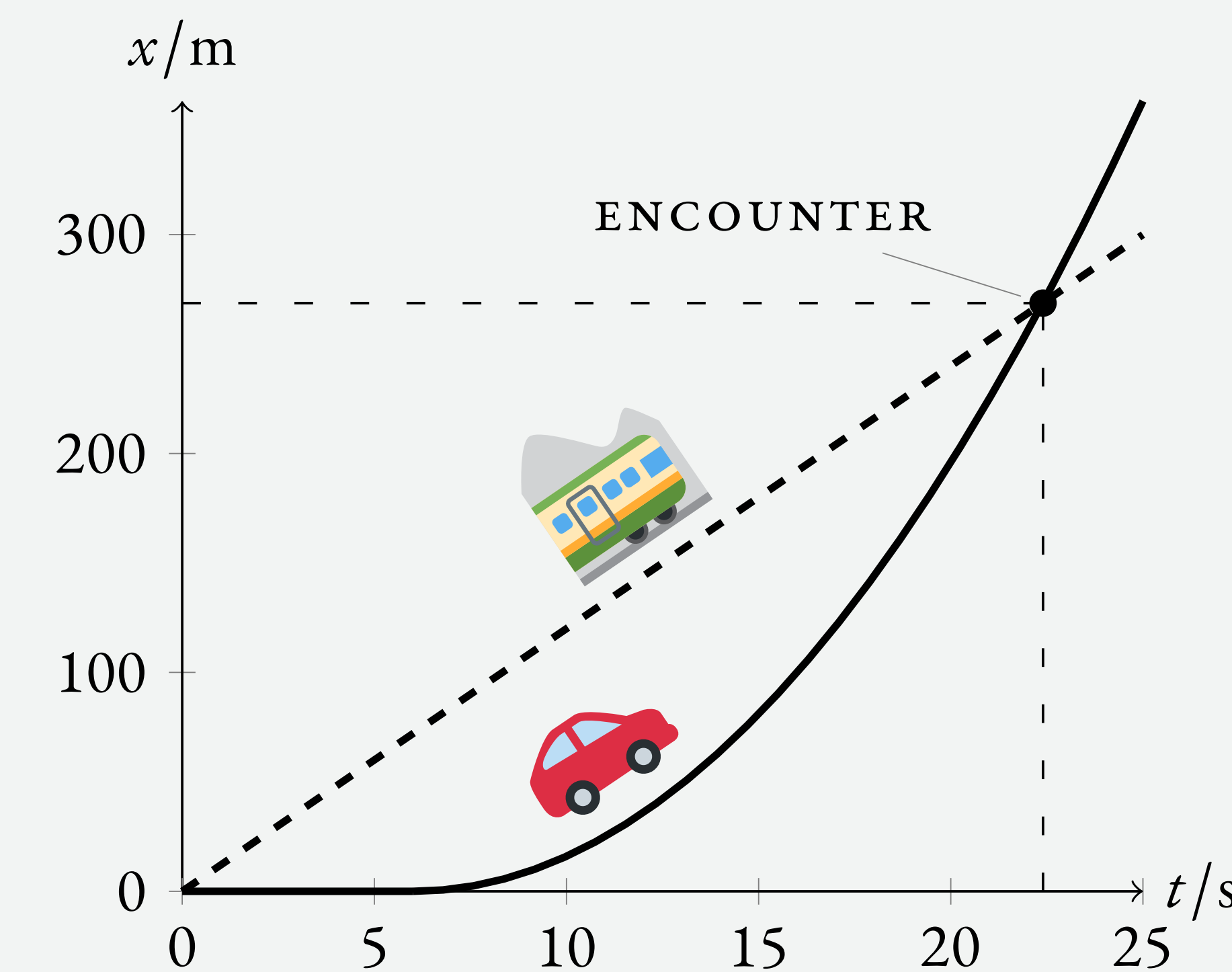
Now we **set the meeting condition**:

$$\begin{aligned} x_c &= x_t \\ t^2 - 12t + 36 &= 12t \\ t^2 - 24t + 36 &= 0 \end{aligned}$$

We solve for the **time**  $t^*$ :

$$t^* = \frac{24 \pm \sqrt{24^2 - 4 \cdot 1 \cdot 36}}{2} = \frac{24 \pm \sqrt{432}}{2} = \begin{cases} 22.4 \text{ s} \\ 1.6 \text{ s} \end{cases}$$

The solution  $t = 1.6 \text{ s}$  is not a valid solution, since it is lower than the 6 s that the car was stopped in the traffic light. We can test our solution drawing the graph distance vs time ( $x - t$ ) for each object:



where we can see that the car is stopped for the first 6 s and then it starts the motion accelerating (parabola) and catching the train after 22.4 s.

- b) To calculate the **distance covered** by the car we substitute the time  $t^* = 22.4 \text{ s}$ , in the position equation having in mind that  $x_{0c} = 0$ :

$$x_c(t^*) = t^{*2} - 12t^* + 36 = 22.4^2 - 12 \cdot 22.4 + 36 = 268.7 \text{ m}$$

- c) The **velocity** of the car when it catches the train can be calculated using the **velocity equation** of the car, substituting  $t = t^*$ :

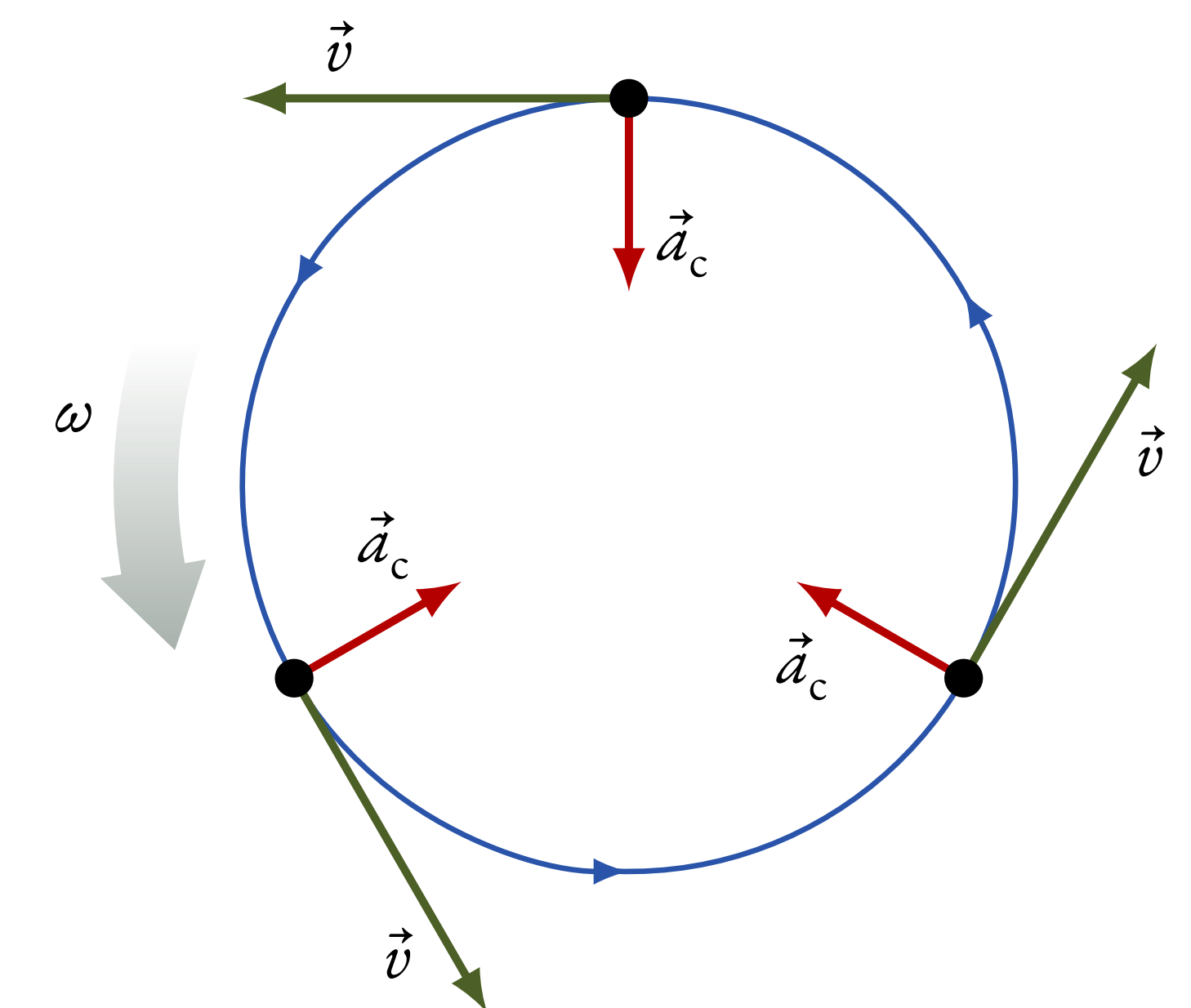
$$v_c(t^*) = v_{0c} + a_c(t^* - t_0) = 0 + 2 \cdot (22.4 - 6) = 32.8 \text{ m/s}$$

## UCM

### Characteristics

The **Characteristics** of the **Uniform Circular Motion** (UCM) are:

- Circular trajectory.
- Velocity's module constant (tangential acceleration  $a_t = 0$ ).



### Equation

The **equation** for the UCM is:

$$\varphi(t) = \varphi_0 + \omega(t - t_0),$$

where  $\varphi$  is the final angular position,  $\varphi_0$  the initial angular position,  $\omega$  the angular velocity,  $t$  the final time and  $t_0$  the initial time.

**Period**  $T$  The time invested by the object in covering a complete revolution is called **period**,  $T$ .

**Frequency**  $f$  is the number of revolutions covered in 1 s.

**Frequency**,  $f$ , it is related to the period by the equation:

$$f = \frac{1}{T} \left[ \frac{1}{s} = s^{-1} = \text{Hz} \right]$$

The angular velocity,  $\omega$ , is related with the period and the frequency by the following expressions:

$$\omega = \frac{\Delta \varphi}{\Delta t} = \frac{2\pi}{T} = 2\pi f$$

Lineal and angular magnitudes are related by the radius  $R$ :

$$\begin{aligned} s &= \varphi R \\ v &= \omega R \end{aligned}$$

### Centripetal Acceleration $\vec{a}_c$

Also called **normal acceleration**, is the acceleration related to the change of direction of the velocity vector. Its module can be calculated as:

$$a_c = \frac{v^2}{R}$$

and it always points to the center of the circumference.