FU FOUNDATION SCHOOL OF ENGINEERING AND APPLIED SCIENCE
DEPARTMENT OF ELECTRICAL ENGINEERING
Master of Science in Electrical Engineering

Homework 2

Computational Methods in Finance
IEOR 4732

Author:

Alban Dietrich (ad4017)

12 March 2023

Contents

1 Case Study 2

In this problem we will study the pricing of an up-and-out call (UOC).

We define $w(x,\tau)$ as the value of a derivative security that satisfies the PIDE given in the question. We can also find this PIDE in lecture 5.

$$\frac{dw}{d\tau}(x,\tau) - (r-q)\frac{dw}{dx}(x,\tau) + rw(x,\tau) - \int_{-\infty}^{\infty}\left[w(x+y,\tau) - w(x,\tau) - \frac{dw}{dx}(x,\tau)\left(e^y - 1\right)\right]k(y)dy = 0 \tag{1}$$

We defined $k(y)$ as:

$$k(y) = \frac{e^{-\lambda_p y}}{\nu y^{Y+1}}\mathbf{1}_{y>0} + \frac{e^{-\lambda_n |y|}}{\nu |y|^{Y+1}}\mathbf{1}_{y<0} \tag{2}$$

We can notice above two intervals $y > 0$ and $y < 0$. As the Levy process $k(y)$ is singular at $y = 0$, we will divide the evaluation of the integral in 2 domains: $|y| > \epsilon$ and $|y| \leq \epsilon$.

Let us define other parameters:

$$\lambda_p = \left(\frac{\theta^2}{\sigma^4} + \frac{2}{\sigma^2\nu}\right)^{1/2} - \frac{\theta}{\sigma^2} \tag{3}$$

$$\lambda_n = \left(\frac{\theta^2}{\sigma^4} + \frac{2}{\sigma^2\nu}\right)^{1/2} + \frac{\theta}{\sigma^2} \tag{4}$$

We define $x = \ln(S)$ and $\tau = T - t$.

We consider the following initial condition:

$$w(x,0) = (e^x - K)^+ \tag{5}$$

Finally we have the following boundary conditions $\forall\tau$:

$$w(x_0,\tau) = 0 \tag{6}$$

$$w(x_N,\tau) = w(B,\tau) = 0 \tag{7}$$

This problem has the following parameters: $S_0 = 1900$, $K = 2000$, $B = 2200$ (the upper barrier), $r = 0.25\%$, $q = 1.5\%$, $T = 0.5\,\text{year}$, $\sigma = 25\%$, $\nu = 0.31$, $\theta = -0.25$ and $Y = 0.4$.

To solve this PIDE, we will use the explicit-implicit finite difference scheme covered during lecture 5.

The detailed code is provided in the appendix.

1.1 Explicit-implicit finite difference scheme

Let us discuss the steps to create a code to solve this PIDE.

The first step is to defined the parameters given above.

After we can define the discretization parameters as follows:

$$\Delta x = \frac{x_{\max} - x_{\min}}{N} \tag{8}$$

$$\Delta \tau = \frac{T - 0}{M} \tag{9}$$

We decide to choose $S_{\min} = 1$ (but we can also choose other values for $S_{\min}$) and $S_{\max} = B$, thus $x_{\min} = \ln(S_{\min}) = 0$ and $x_{\max} = \ln(S_{\max}) = \ln(B)$.

The discretization can be changed by modifying the parameters N and M. The bigger those parameters, the thinner the discretization.

In lecture 5 we find a difference equation for the PIDE by following these steps:

- Evaluate the integral term: $\int_{-\infty}^{\infty} \left[w(x+y,\tau) - w(x,\tau) - \frac{dw}{dx}(x,\tau)\,(e^y - 1)\right] k(y)dy$.
  - We divide the integral into 2 regions as Levy measure $k(y)dy$ is singular at $y = 0$: $|y| > \epsilon$ and $|y| \leq \epsilon$.
  - Obvious choice for $\epsilon = \Delta x$.
  - After certain manipulation and Taylor approximations (for $w(x+y,\tau)$ and e^y), we obtain a simplified expression for the integral:
    * for $y <= \epsilon$:

$$\frac{\sigma^2(\epsilon)}{2}\frac{d^2 w(x,\tau)}{dx^2} - \frac{\sigma^2(\epsilon)}{2}\frac{dw(x,\tau)}{dx} \tag{10}$$

$$\text{where} \quad \sigma^2(\epsilon) = \int_{|y|\leq\epsilon} y^2 k(y) \tag{11}$$

    * for $y > \epsilon$:

$$\int_{|y|>\epsilon} \left(w(x+y,\tau) - w(x,\tau)\right) k(y)dy + \frac{dw(x,\tau)}{dx}\omega(\epsilon) \tag{12}$$

$$\text{where} \quad \omega(\epsilon) = \int_{|y|>\epsilon} y^2 k(y)dy \tag{13}$$

- We apply the implicit discretization for the derivative terms:

$$\frac{dw(x_i,\tau_j)}{d\tau} \simeq \frac{w_{i,j+1} - w_{i,j}}{\Delta\tau} \tag{14}$$

$$\frac{dw(x_i,\tau_j)}{dx} \simeq \frac{w_{i+1,j+1} - w_{i,j+1}}{\Delta x} \tag{15}$$

$$\frac{d^2 w(x_i,\tau_j)}{dx^2} \simeq \frac{w_{i+1,j+1} - 2w_{i,j+1} + w_{i-1,j+1}}{2(\Delta x)^2} \tag{16}$$

- We can discretize $\sigma^2(\epsilon)$ and $\omega(\epsilon)$ by defining:

$$g_1(\xi) = \int_{\xi}^{\infty} \frac{e^{-z}}{z^{\alpha}} dz \tag{17}$$

$$g_2(\xi) = \int_{\xi}^{\infty} \frac{e^{-z}}{z^{\alpha+1}} dz \tag{18}$$

- We divide the integral $\int_{|y|>\Delta x} (w(x_i+y,\tau_j) - w(x_i,\tau_j))\, k(y)dy$ on the region $|y| > \Delta x$ into 4 parts. We apply an explicit treatment on them.
  - $[-\infty, x_o - x_i]$: see lecture 5, slide 26
  - $[x_o - x_i, -\Delta x]$: see lecture 5, slides 20-22
  - $[\Delta x, x_N - x_i]$: lecture 5, slides 24-25
  - $[x_N - x_i, +\infty]$: lecture 5, slide 27
- After discretization and substitution, we obtain the difference equation.

Here below is the difference equation which corresponds to the PIDE that we have to solve to price an up-and-out call. We assume that we know $w_{i,j}$ at a time τ_j. We thus solve a linear system to find $w_{i,j+1}$ for all i. At point (x_i, τ_{j+1}):

$$l_{i,j+1}w_{i-1,j+1} + d_{i,j+1}w_{i,j+1} + u_{i,j+1}w_{i+1,j+1} = w_{i,j} + \frac{\Delta\tau}{\nu}R_{i,j} \tag{19}$$

Where:

$$l_{i,j+1} = -B_l \tag{20}$$

$$d_{i,j+1} = 1 + r\Delta\tau + B_l + B_u + \frac{\Delta\tau}{\nu}\left(\lambda_n^Y g_2(i\Delta x\lambda_n) + \lambda_n^Y g_2((N-i)\Delta x\lambda_p)\right) \tag{21}$$

$$u_{i,j+1} = -B_u \tag{22}$$

$$B_l = \frac{\sigma^2(\Delta x)\Delta\tau}{2\Delta x^2} - \left(r - q + \omega(\Delta x) - \frac{\sigma^2(\Delta x)}{2}\right)\frac{\Delta\tau}{2\Delta x} \tag{23}$$

$$B_u = \frac{\sigma^2(\Delta x)\Delta\tau}{2\Delta x^2} + \left(r - q + \omega(\Delta x) - \frac{\sigma^2(\Delta x)}{2}\right)\frac{\Delta\tau}{2\Delta x} \tag{24}$$

The values of the $R_{i,j}$ elements are given in lecture 5, slide 29.

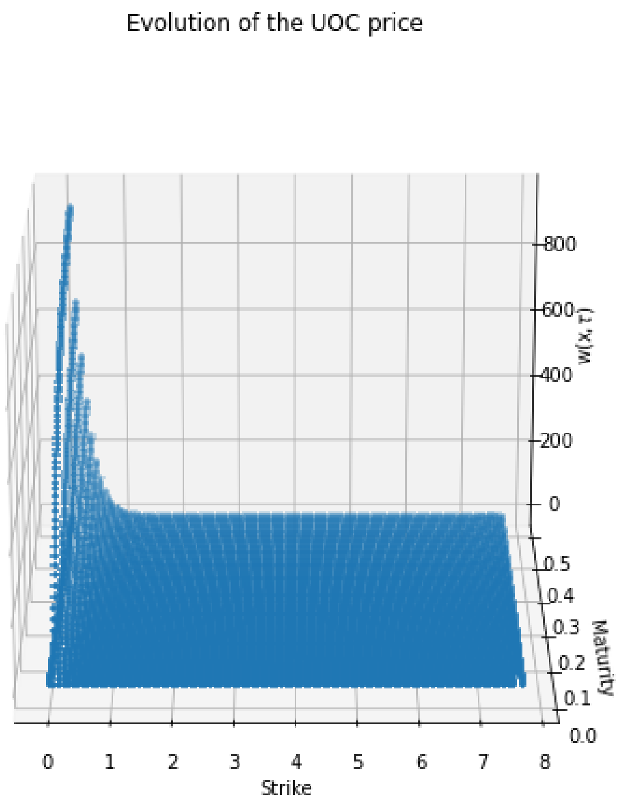
Before solving this equation, we pre-calculate and store some vectors to speed up the calculations. This corresponds to some values of the g_1 and g_2 functions.

A remark concerning these functions. They depend on a parameter $0 \le \alpha < 1$. In this code we chose $\alpha = Y$.

1.2 Results

In this section, we will discussed the results obtained. Again for more details see the appendix with the provided code and results.

By choosing the parameters mentioned before and $x_{\min} = 0$, $x_{\max} = ln(B)$, $N = 100$, $M = 100$, we obtain the following graph for the UOC price:



We can notice that we obtain the same behaviour as the plot showed in lecture 5.

Now we want to calculate the UOC option premium. We calculate w at S_0 and maturity $T = 0.5$, i.e. $w\left(\ln(S_0), 0.5\right)$. To do so we use an interpolation function (see code in appendix for more details):

```
UOC = np.interp(np.log(S0), np.array(array_x)[:,M], w[:,M])
print(UOC)
1.7449607702117078
```

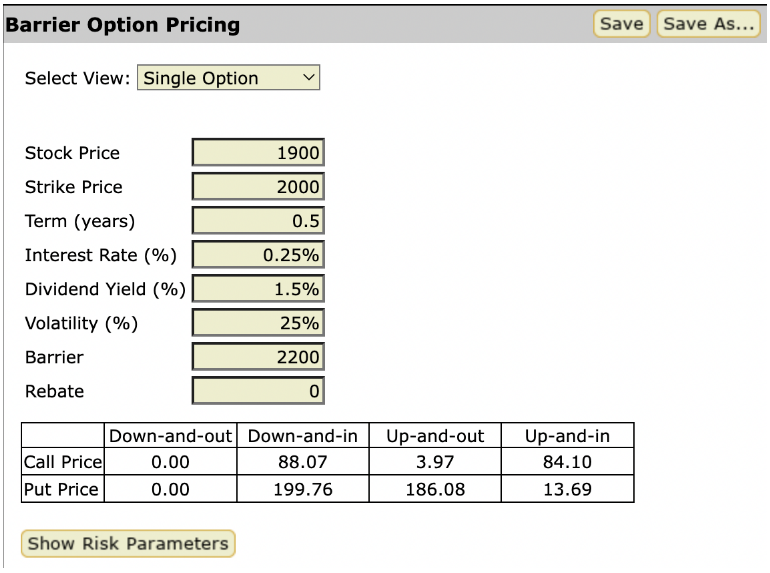
We obtain an UOC premium of $\simeq 1.745$.

We can see in the notebook that the execution time is about 6 seconds which is quite fast.

Even with the change of variable $x = ln(S)$ and $\tau = T - t$, we know that $w\left(x, \tau\right) = V(S, t)$ (see slide 5, lecture 5) which is the option price.

By using an online calculator for the Black Scholes (BS) model, we find a value of 3.97 (see figure below[1]). As we are using a different model, it is normal that we don't get the exact same value but we will see if we can get closer to this value.

[1]The online calculator can be found here: `http://www.coggit.com/freetools`.



To improve our result we could increase the discretization parameters N and M or reduce the interval $[x_{\min}, x_{\max}]$ by taking a bigger value for $x_{\min}$. By doing so, for the same discretization N, we have a better resolution.

Let us take $S_{\min} = K/2$ with $N = 100$ and $M = 100$. By doing so we find:

```
UOC = np.interp(np.log(S0), np.array(array_x)[:,M], w[:,M])
print(UOC)
```

```
4.139740590658877
```

So we have an UOC premium of $\simeq 4.140$ which is closer to the result found with the BS model.

To check our results, another method has been used to solve the problem. This method use the tridiagonal solver. The code is provided in the appendix.

For $S_{\min} = K/2$ with $N = 200$ and $M = 20$, we find:

```
UOC = np.interp(np.log(S0), x, W)
print("The UOC premium for strike %s the option premium is %6.4f" % (2000,UOC))
```

```
The UOC premium for strike 2000 the option premium is 3.2507
```

So we find an UOC premium of $\simeq 3.251$ which is also close to the value found by the BS model.

The execution time here is only about 1.5 seconds (and for the same parameters as before $N = 100$ and $M = 100$, we have about 3 seconds). So even if the result is not as good as the one found before, this model is much faster.

1.3 Conclusion

In conclusion, we have reviewed and implemented the explicit-implicit finite difference scheme (see code in appendix to see the implementation in details). For the specific situation and parameters given in the guidelines, we have found a good value (close to the one found by the BS model). We could do so by choosing good parameters with a good resolution. The choice of these parameters are thus very important.

The execution time (for $N = 100$ and $M = 100$) of the algorithm is about 6 seconds which is a quite good performance.

Finally, we also studied another algorithm by using a tridiagonal solver. This algorithm gave a good output and a shorter execution time.

2 Appendix

Below is the code used to solve the problem asked.

Case Study 2

Alban Dietrich, UNI: ad4017, 12 March 2023.

We first import some libraries.

In [1]:
```
import numpy as np
import scipy.integrate as integrate
import matplotlib.pyplot as plt
import time
```

Let us define the parameters that we will use.

In [2]:
```
# Set parameters
S0 = 1900
K = 2000
B = 2200
r = 0.0025
q = 0.015
T = 0.5
sig = 0.25
v = 0.31
theta = -0.25
Y = 0.4

# Parameter for g1 & g2 functions
alpha = Y
```

We define the minimum and maximum values of x to determine the interval we want to explore.

In [3]:
```
S_max = B
S_min = 1
x_max = np.log(S_max)
x_min = np.log(S_min)
```

We do the following changes of variables: $x = ln(S)$ and $\tau = T - t$.

Before starting the discretization of the PIDE, we look into the evalutaion of the integral term (a detailed explanation of the explicit-implicit finite difference scheme is given in the report).

As the Levy process $k(y)$ is singular at $y = 0$, we will divide the evaluation of the integral in 2 domains: $|y| > \epsilon$ and $|y| <= \epsilon$. An obvious choice is $\epsilon = \Delta x$.

Let us define the discretization parameters:

In [4]:
```
N = 100
M = 100

Delta_x = (x_max-x_min)/N
Delta_tau = (T-0)/M
```

Let us initialize the value of a derivative security and other functions we will use after.

In [5]:
```
w = np.zeros((N+1, M+1))
l = np.zeros((N+1, M+2))
d = np.zeros((N+1, M+2))
u = np.zeros((N+1, M+2))
R = np.zeros((N+1, M+1))
```

Let us define the value of the x_i and τ variables we will use.

In [6]:
```
array_x = np.zeros((N+1, M+1))
array_t = np.zeros((N+1, M+1))

for i in range(N+1):
    for j in range(M+1):
        array_x[i][j] = x_min + i*Delta_x
        array_t[i][j] = j*Delta_tau
```

Let us apply the initial condition.

In [7]:
```
for i in range(N+1):
    x = x_min + i*Delta_x
    w[i][0] = max((np.exp(x)-K),0)
```

And the boudary conditions.

In [8]:
```
for i in range(M+1):
    w[0][i] = 0
    w[N][i] = 0
```

Let us define some constants:

In [9]:
```
lambda_p = (theta**2/sig**4+2/(sig**2*v))**(1/2)-theta/sig**2
lambda_n = (theta**2/sig**4+2/(sig**2*v))**(1/2)+theta/sig**2
```


Let us define the g_1 and g_2 functions.

In [10]:
```
def g1(xi):
    return integrate.quad(lambda z: np.exp(-z)/z**alpha,xi,np.inf)[0]
```

In [11]:
```
def g2(xi):
    return integrate.quad(lambda z: np.exp(-z)/z**(alpha+1),xi,np.inf)[0]
```

To speed up the calculation, we precalculate the following values:

In [12]:
```
g1_lambda_n_Delta_x = [0]
g1_lambda_p_Delta_x = [0]
g2_lambda_n_Delta_x = [0]
g2_lambda_p_Delta_x = [0]
g2_lambda_n_plus_Delta_x = [0]
g2_lambda_p_minus_Delta_x = [0]

for k in range(1, N+1):
    g1_lambda_n_Delta_x.append(g1(k*lambda_n*Delta_x))
    g1_lambda_p_Delta_x.append(g1(k*lambda_p*Delta_x))
    g2_lambda_n_Delta_x.append(g2(k*lambda_n*Delta_x))
    g2_lambda_p_Delta_x.append(g2(k*lambda_p*Delta_x))
    g2_lambda_n_plus_Delta_x.append(g2(k*(lambda_n+1)*Delta_x))
    g2_lambda_p_minus_Delta_x.append(g2(k*(lambda_p-1)*Delta_x))
```

Let us define more functions $\sigma^2(\epsilon)$ and $\omega(\epsilon)$:

In [13]:
```
def sigma2(eps):
    return 1/v*lambda_p**(Y-2)*(-(lambda_p*eps)**(1-Y)*np.exp(-lambda_p*eps)+(1-Y)*(g1(0)-g1(la
mbda_p*eps)))\
+1/v*lambda_n**(Y-2)*(-(lambda_n*eps)**(1-Y)*np.exp(-lambda_n*eps)+(1-Y)*(g1(0)-g1(lambda_n*eps
)))
```

In [14]:
```
def omega(eps):
    return lambda_p**Y/v*g2(lambda_p*eps)-(lambda_p-1)**Y/v*g2((lambda_p-1)*eps)\
+ lambda_n**Y/v*g2(lambda_n*eps)-(lambda_n+1)**Y/v*g2((lambda_n+1)*eps)
```

Let us define more constants (B_l and B_u):

In [15]:
```
Bl = sigma2(Delta_x)*Delta_tau/(2*Delta_x**2)-(r-q+omega(Delta_x)-1/2*sigma2(Delta_x))*Delta_ta
u/(2*Delta_x)
Bu = sigma2(Delta_x)*Delta_tau/(2*Delta_x**2)+(r-q+omega(Delta_x)-1/2*sigma2(Delta_x))*Delta_ta
u/(2*Delta_x)
```

We can assign these values to the elements $l_{i,j+1}$ and $u_{i,j+1}$:

In [16]:
```
l = [[-Bl]*(M+2)]*(N+1)
u = [[-Bu]*(M+2)]*(N+1)
```

We can compute the $d_{i,j+1}$ elements:

In [17]:
```
for i in range(1,N+1):
    for j in range(M+1):
        d[i][j+1] = 1+r*Delta_tau+Bl+Bu+Delta_tau/v*(lambda_n**Y*g2_lambda_n_Delta_x[i]+lambda_
p**Y*g2_lambda_p_Delta_x[N-i])
```

We define the $R_{i,j}$ elements. To do so, we divide the calculation into 5 terms that we sum at the end.

In [18]:
```
def R(i,j):
    sum_1 = 0
    sum_2 = 0
    sum_3 = 0
    sum_4 = 0
    term_5 = 0

    for k in range(1, i):
        sum_1 += lambda_n**Y*(w[i-k][j]-w[i][j]-k*(w[i-k-1][j]-w[i-k][j]))*(g2_lambda_n_Delta_x
[k]-g2_lambda_n_Delta_x[k+1])

    for k in range(1, i):
        sum_2 += (w[i-k-1][j]-w[i-k][j])/(lambda_n**(1-Y)*Delta_x)*(g1_lambda_n_Delta_x[k]-g1_l
ambda_n_Delta_x[k+1])

    for k in range(1, N-i):
        sum_3 += lambda_p**Y*(w[i+k][j]-w[i][j]-k*(w[i+k+1][j]-w[i+k][j]))*(g2_lambda_p_Delta_x
[k]-g2_lambda_p_Delta_x[k+1])

    for k in range(1, N-i):
        sum_4 += (w[i+k+1][j]-w[i+k][j])/(lambda_p**(1-Y)*Delta_x)*(g1_lambda_p_Delta_x[k]-g1_l
ambda_p_Delta_x[k+1])

    xi = x_min + i*Delta_x
    term_5 = K*lambda_n**Y*g2_lambda_n_Delta_x[i]-np.exp(xi)*(lambda_n+1)**Y*g2_lambda_n_plus_D
elta_x[i]

    return sum_1+sum_2+sum_3+sum_4+term_5
```

Accoding to leture 5, on slide 28, we have the following difference equation below. We assume that we know at a time τ_j, $w_{i,j}$. We thus solve a linear system to find $w_{i,j+1}$ for all i. We choose m iterations to updates the values of w at each cycle.

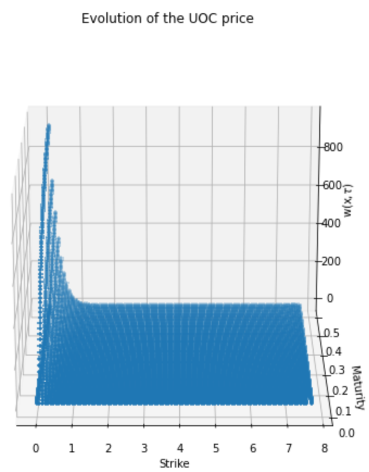
In [19]:
```
start_time = time.time()

# Number of iterations

m = 5
for itr in range(m):
```

```
    for j in range(0,M):
        for i in range(1,N):
            w[i][j+1] = (w[i][j]+Delta_tau/v*R(i,j)-l[i][j+1]* w[i-1][j+1]-u[i][j+1]* w[i+1][j+
1])/d[i][j+1]

elapsed_time = time.time() - start_time

print('The execution time was %0.7f seconds' % elapsed_time)
```

```
The execution time was 6.2168341 seconds
```

We plot the results.

In [20]:
```
fig = plt.figure(figsize = (8, 8))
ax = plt.axes(projection = '3d')

m = '+'

ax.scatter(np.array(array_x), np.array(array_t), np.array(w), marker=m)

ax.view_init(20, 270)

ax.set_xlabel('Strike')
ax.set_ylabel('Maturity')
ax.set_zlabel(r'w(x,$\tau$)')
plt.title('Evolution of the UOC price')

fig.savefig('plot.png', bbox_inches='tight')

plt.show()
```



Let us find the UOC option premium at S0 and maturity T = 0.5. To do so we use an interpolation function. We find:

In [21]:
```
UOC = np.interp(np.log(S0), np.array(array_x)[:,M], w[:,M])
print(UOC)
```

```
1.7449607702117078
```

By using an online calculator of the Black Scholes (BS) model, we find a value of 3.97. As we are using a different model, it is normal that we don't get the exact same value but we will see if we can get closer to this value.

To improve our result we could increase the discretization N/M or reduce the interval $\left[x_{\min}, x_{\max}\right]$ by taking a bigger value for $x_{\min}$. By doing so, for the same discretization N, we have a better resolution.

Let us take $S_{\min} = K/2$ (before we had $S_{\min} = 1$) with $N = 100$ and $M = 100$

So let us reinitialize our model and run it again for $S_{\min} = K/2$.

In [22]:
```
S_max = B
S_min = K/2
x_max = np.log(S_max)
x_min = np.log(S_min)

N = 100
M = 100

Delta_x = (x_max-x_min)/N
Delta_tau = (T-0)/M

w = np.zeros((N+1, M+1))
l = np.zeros((N+1, M+2))
d = np.zeros((N+1, M+2))
u = np.zeros((N+1, M+2))
R = np.zeros((N+1, M+1))

array_x = np.zeros((N+1, M+1))
array_t = np.zeros((N+1, M+1))

for i in range(N+1):
    for j in range(M+1):
        array_x[i][j] = x_min + i*Delta_x
        array_t[i][j] = j*Delta_tau
```

```python
for i in range(N+1):
    x = x_min + i*Delta_x
    w[i][0] = max((np.exp(x)-K),0)

for i in range(M+1):
    w[0][i] = 0
    w[N][i] = 0

lambda_p = (theta**2/sig**4+2/(sig**2*v))**(1/2)-theta/sig**2
lambda_n = (theta**2/sig**4+2/(sig**2*v))**(1/2)+theta/sig**2

g1_lambda_n_Delta_x = [0]
g1_lambda_p_Delta_x = [0]
g2_lambda_n_Delta_x = [0]
g2_lambda_p_Delta_x = [0]
g2_lambda_n_plus_Delta_x = [0]
g2_lambda_p_minus_Delta_x = [0]

for k in range(1, N+1):
    g1_lambda_n_Delta_x.append(g1(k*lambda_n*Delta_x))
    g1_lambda_p_Delta_x.append(g1(k*lambda_p*Delta_x))
    g2_lambda_n_Delta_x.append(g2(k*lambda_n*Delta_x))
    g2_lambda_p_Delta_x.append(g2(k*lambda_p*Delta_x))
    g2_lambda_n_plus_Delta_x.append(g2(k*(lambda_n+1)*Delta_x))
    g2_lambda_p_minus_Delta_x.append(g2(k*(lambda_p-1)*Delta_x))

Bl = sigma2(Delta_x)*Delta_tau/(2*Delta_x**2)-(r-q+omega(Delta_x)-1/2*sigma2(Delta_x))*Delta_ta
u/(2*Delta_x)
Bu = sigma2(Delta_x)*Delta_tau/(2*Delta_x**2)+(r-q+omega(Delta_x)-1/2*sigma2(Delta_x))*Delta_ta
u/(2*Delta_x)

l = [[-Bl]*(M+2)]*(N+1)
u = [[-Bu]*(M+2)]*(N+1)

for i in range(1,N+1):
    for j in range(M+1):
        d[i][j+1] = 1+r*Delta_tau+Bl+Bu+Delta_tau/v*(lambda_n**Y*g2_lambda_n_Delta_x[i]+lambda_
p**Y*g2_lambda_p_Delta_x[N-i])
```

In [23]:
```python
def R(i,j):
    sum_1 = 0
    sum_2 = 0
    sum_3 = 0
    sum_4 = 0
    term_5 = 0

    for k in range(1, i):
        sum_1 += lambda_n**Y*(w[i-k][j]-w[i][j]-k*(w[i-k-1][j]-w[i-k][j]))*(g2_lambda_n_Delta_x
[k]-g2_lambda_n_Delta_x[k+1])

    for k in range(1, i):
        sum_2 += (w[i-k-1][j]-w[i-k][j])/(lambda_n**(1-Y)*Delta_x)*(g1_lambda_n_Delta_x[k]-g1_l
ambda_n_Delta_x[k+1])

    for k in range(1, N-i):
        sum_3 += lambda_p**Y*(w[i+k][j]-w[i][j]-k*(w[i+k+1][j]-w[i+k][j]))*(g2_lambda_p_Delta_x
[k]-g2_lambda_p_Delta_x[k+1])

    for k in range(1, N-i):
        sum_4 += (w[i+k+1][j]-w[i+k][j])/(lambda_p**(1-Y)*Delta_x)*(g1_lambda_p_Delta_x[k]-g1_l
ambda_p_Delta_x[k+1])

    xi = x_min + i*Delta_x
    term_5 = K*lambda_n**Y*g2_lambda_n_Delta_x[i]-np.exp(xi)*(lambda_n+1)**Y*g2_lambda_n_plus_D
elta_x[i]

    return sum_1+sum_2+sum_3+sum_4+term_5
```

We calculate the new w function.

In [24]:
```python
start_time = time.time()

# Number of iterations

m = 5
for itr in range(m):
    for j in range(0,M):
        for i in range(1,N):
            w[i][j+1] = (w[i][j]+Delta_tau/v*R(i,j)-l[i][j+1]* w[i-1][j+1]-u[i][j+1]* w[i+1][j+
1])/d[i][j+1]

elapsed_time = time.time() - start_time

print('The execution time was %0.7f seconds' % elapsed_time)
```

```
The execution time was 6.2010911 seconds
```

We obtain the following result:

In [25]:
```python
UOC = np.interp(np.log(S0), np.array(array_x)[:,M], w[:,M])
print(UOC)
```

```
4.139740590658877
```

We are thus closer to the result obtain with with the BS model. Thus the choice of the parameters and the resolution are very important.

To solve this problem, we could also use another method. For example by using the tridiagonal solver.

Let us write a code using this solver and compare the result with what we obtained before.

First we define the tridiagonal solver function.

In [26]:
```
def diagSolver(l, d, u, B, n):
    l_ = np.zeros(n+1)
    d_ = np.zeros(n+1)
    u_ = np.zeros(n+1)
    for i in range(1,n+1):
        l_[i] = l[i]
        d_[i] = d[i]
        u_[i] = u[i]
    for i in range(n-1, 0, -1):
        x = u_[i]/d_[i+1]
        d_[i] = d_[i] - x * l_[i+1]
        B[i] = B[i] - x * B[i+1]
    B[1] = B[1]/d_[1]
    i_start = 1
    for i in range(i_start+1, n+1):
        B[i] = (B[i] - l_[i] * B[i-1])/d_[i]
```

Now let us solve our problem.

We define the discretization parameters.

In [27]:
```
N = 200
M = 20
```

We pre-calculate some values as before to speed-up the calculation.

In [28]:
```
g1_lambda_n_Delta_x = [0]
g1_lambda_p_Delta_x = [0]
g2_lambda_n_Delta_x = [0]
g2_lambda_p_Delta_x = [0]
g2_lambda_n_plus_Delta_x = [0]
g2_lambda_p_minus_Delta_x = [0]

for k in range(1, N+1):
    g1_lambda_n_Delta_x.append(g1(k*lambda_n*Delta_x))
    g1_lambda_p_Delta_x.append(g1(k*lambda_p*Delta_x))
    g2_lambda_n_Delta_x.append(g2(k*lambda_n*Delta_x))
    g2_lambda_p_Delta_x.append(g2(k*lambda_p*Delta_x))
    g2_lambda_n_plus_Delta_x.append(g2(k*(lambda_n+1)*Delta_x))
    g2_lambda_p_minus_Delta_x.append(g2(k*(lambda_p-1)*Delta_x))
```

We initialize the variables.

In [29]:
```
x = np.zeros(N+1)
W = np.zeros(N+1)

A = np.zeros(M)
r_ = np.zeros(M)
q_ = np.zeros(M)

L = np.zeros(N)
U = np.zeros(N)
D = np.zeros(N)
B = np.zeros(N)

B_n = Delta_tau*(1-np.exp(-lambda_n*Delta_x))/(v*lambda_n*Delta_x)
B_p = Delta_tau*(1-np.exp(-lambda_p*Delta_x))/(v*lambda_p*Delta_x)
```

We put the initial and boundary conditions.

In [30]:
```
for i in range(N+1):
    x[i] = np.log(S_min)+i*Delta_x
    if i == N:
        W[i] = 0
    else:
        if np.exp(x[i]) > K:
            W[i] = np.exp(x[i])-K
        else:
            W[i] = 0
```

We have the main loop.

In [31]:
```
start_time = time.time()

for j in range(M-1, -1, -1):
    q_[j] = q
    r_[j] = r
    A[j] = (r_[j]-q_[j]+np.log(1-theta*v-sig*sig*v/2)/v)*Delta_tau/Delta_x
    for i in range(1, N):
        B[i] = W[i]
    for i in range(1, N):
        if i == N-1:
            L[i] = A[j] - B_n
            D[i] = 1 + r_[j]*Delta_tau + B_p + B_n + Delta_tau*((lambda_n**Y*g2(i*Delta_x*lambd
a_n)+lambda_p**Y*g2((N-i)*Delta_x*lambda_p)))/v
            for k in range(1, i):
                B[i] += lambda_n**Y*(W[i-k]-W[i]-k*(W[i-k-1]-W[i-k]))*(g2_lambda_n_Delta_x[k]-g
2_lambda_n_Delta_x[k+1])
                B[i] += (W[i-k-1]-W[i-k])/(lambda_n**(1-Y)*Delta_x)*(g2_lambda_n_Delta_x[k]-g2_
lambda_n_Delta_x[k+1])
        elif i == 1:
            D[i] = 1 + r_[j]*Delta_tau + B_p + B_n + Delta_tau*((lambda_n**Y*g2(i*Delta_x*lambd
a_n)+lambda_p**Y*g2((N-i)*Delta_x*lambda_p)))/v
            U[i] = -A[j] - B_p
            for k in range(1, N-i):
                B[i] += lambda_p**Y*(W[i+k]-W[i]-k*(W[i+k+1]-W[i+k]))*(g1_lambda_p_Delta_x[k]-g
```

```
1_lambda_p_Delta_x[k+1])
                B[i] += (W[i+k+1]-W[i+k])/(lambda_p**(1-Y)*Delta_x)*(g1_lambda_p_Delta_x[k]-g1_
lambda_p_Delta_x[k+1])
        else:
            L[i] =  A[j] - B_n
            D[i] =  1 + r_[j]*Delta_tau + B_p + B_n + Delta_tau*((lambda_n**Y*g2(i*Delta_x*lamb
da_n)+lambda_p**Y*g2((N-i)*Delta_x*lambda_p)))/v
            U[i] = -A[j] - B_p
            for k in range(1, N-i):
                B[i] += lambda_p**Y*(W[i+k]-W[i]-k*(W[i+k+1]-W[i+k]))*(g2_lambda_p_Delta_x[k]-g
2_lambda_p_Delta_x[k+1])
                B[i] += (W[i+k+1]-W[i+k])/(lambda_p**(1-Y)*Delta_x)*(g1_lambda_p_Delta_x[k]-g1_
lambda_p_Delta_x[k+1])
            for k in range(1, i):
                B[i] += lambda_n**Y*(W[i-k]-W[i]-k*(W[i-k-1]-W[i-k]))*(g2_lambda_n_Delta_x[k]-g
2_lambda_n_Delta_x[k+1])
                B[i] += (W[i-k-1]-W[i-k])/(lambda_n**(1-Y)*Delta_x)*(g1_lambda_n_Delta_x[k]-g1_
lambda_n_Delta_x[k+1])
        B[i] += K*lambda_n**Y*g2(i*Delta_x*lambda_n) -np.exp(x[i])*(lambda_n+1)**Y*g2(i*Delta_x
*(lambda_n+1))
    diagSolver(L,D,U,W,N-1)

elapsed_time = time.time() - start_time
print('The execution time was %0.7f seconds' % elapsed_time)
```

```
The execution time was 1.4833007 seconds
```

Now let us calculate the option premium.

In [32]:
```
UOC = np.interp(np.log(S0), x, W)
print("The UOC premium for strike %s the option premium is %6.4f" % (2000,UOC))
```

```
The UOC premium for strike 2000 the option premium is 3.2507
```