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Exercise Sheet #1

Exercise 1.

ⓐ Redshift: $z = -1 + \frac{\lambda_{obs}}{\lambda_{emi}}$

Lyman-alpha: $\lambda_{emi} = 1215{,}67\,\text{Å}$ → At rest frame

$\lambda_{obs} = 1213.615\,\text{Å}$ → Observed, measured from Andromeda's spectrum. (From Earth)

$\therefore\ z = -0.00168$ → (We have a blueshift)

Since $|z| \ll 1$ we can take $z \approx \frac{v}{c}$ → $c = 3\times10^5\ \text{km/s}$

Then, $v \approx \cancel{-504}\ \text{km/s} = -504\ \text{km/s}$

ⓑ $D = 780\ \text{kpc} = 2.4\times10^{19}\ \text{km}$; using $v = H_0 \cdot D$, with $H_0 = 71\ \text{km/s/Mpc}$

$= 0.78\ \text{Mpc}$

$v = 55.38\ \text{km/s}$ → Velocity now is possitive. However that is not the case in the result in the velocity measured with the redshift.

The difference between the results is because $v = H_0 \cdot D$ → the Hubble constant arise from the expansion of the universe, which is observed for far away galaxies. Andromeda is a really close galaxy and also from the local group. It's velocity is subject to the dynamics from the local group of galaxies.

ⓒ $v \approx c \cdot z = 3\times10^5\ \text{km/s} \cdot 0.05 = 15\times10^3\ \text{km/s}$.

$$D = \frac{v}{H_0} = \frac{15\times10^3\ \text{km/s}}{71\ \frac{\text{km/s}}{\text{Mpc}}} \Rightarrow \boxed{D = 211{,}26\ \text{Mpc}}$$

(2) $((T/K) \sim 1.5 \cdot 10^{10} \cdot (t/s)^{-1/2})$ Using

(a) This reaction will freeze at 10^{-12} s. At this time, $T \sim 1.5 \times 10^{16}$ K, Because this production is only possible for sufficiently high energy, and because of the inflation, energy and temperature decreased and matter-antimatter were separated before annihilation.

(b) With $t \simeq 1$ s

Here the ratio of photon to baryon is 10^9.

Using $((T/K) \sim 1,5 \times 10^{10} (t/s)^{-\frac{1}{2}}) \to \boxed{T \sim 1,5 \cdot 10^{10} \text{ K}}$

Then for the energy: $E \sim k_b T$

$$E \sim 1,38 \cdot 10^{-23} \cdot 1,5 \cdot 10^{10} \frac{m^2 \, kg \, \cancel{K}}{s^2 \, \cancel{K}} = 2.07 \times 10^{-13} \text{ J}$$
$$= 1292094 \text{ eV}$$
$$\boxed{E \simeq 1,29 \text{ Mev}}$$

(c) Considering the $p + n \rightleftharpoons D + \gamma$ reaction:

Masses:

Proton: $1.6726219 \times 10^{-27}$

Neutron: $1.6749274 \times 10^{-27}$

Deuteron: $3.3435832 \times 10^{-27}$

Photon: energy related to $E = \boxed{\Delta m} c^2$

$$\Delta m = \frac{1.6726219 + 1.6749274}{10^{27}} - \frac{3.3435832}{10^{27}} = \frac{3.9661237}{10^{30}}$$

$E = \Delta m c^2 = 3.569511 \times 10^{-13}$ J $= 2.228$ Mev $\to$ This is the minimum energy needed from a photon to photo-desintegrate deuteron.

$E = h\nu \to h = 6.62607 \times 10^{-34}$ J$\cdot$s

Frecuency: $\nu = \frac{E}{h} = \frac{3.569511 \times 10^{-13} \text{ J}}{6.62607 \times 10^{-34} \text{ J} \cdot s} = 5.38 \times 10^{20}$ Hz

The remaining deuterons will combine themselves with electrons to form deuterium. Thus, the higher the density of matter, then more deuteron will convert into deuterium or Helium.