

Let X be a random variable (rv), let x be a realization from the rv. Let $\text{Supp}[X]$ denote the support of the rv, a subset of the reals. This is a set of all possible values that could be realized thus x belongs to this set.

• Discrete r.v.'s

$$|\text{Supp}[X]| \leq |N| \quad \text{finite or if infinite, it is countably infinite}$$

$$p(x) := P(X=x) \quad \text{probability mass function (PMF)}$$

$$p: \text{Supp}[X] \rightarrow (0,1] \quad \sum_{x \in \text{Supp}[X]} p(x) = 1$$

$$F(x) := P(X \leq x) \quad \text{cumulative distribution function (CDF)}$$

$$F: \mathbb{R} \rightarrow [0,1] \quad \sum_{\{y: y \in \text{Supp}[X] \ \& \ y \leq x\}} p(y) \stackrel{?}{=} \sum_{y=-\infty}^x p(y)$$

• Continuous rvs

$$|\text{Supp}[X]| = |\mathbb{R}| \quad \text{uncountably infinite e.g. } [0,1]$$

The CDF definition remains the same. The PMF doesn't exist. And we define $f(x) := F'(x)$, the probability density function (PDF)

$$P(X \in [a,b]) = F(b) - F(a) = \int_a^b f(x) dx$$

$$\text{Supp}[X] = \{x: f(x) > 0\}$$

$$f: \text{Supp}[X] \rightarrow (0, \infty) \quad , \quad \int_{\text{Supp}[X]} f(x) dx = 1$$

rvs are identified by their CDF/PMF (discrete) or CDF/PDF (contin.)

$$\begin{array}{ll} \text{discrete} \left\{ \begin{array}{l} X \sim \text{Bern}(p) := p^x (1-p)^{1-x} \\ X \sim \text{Binom}(n, p) := \binom{n}{x} p^x (1-p)^{n-x} \end{array} \right. & \begin{array}{l} \text{Supp}[X] = \{0, 1\} \\ \text{Supp}[X] = \{0, 1, \dots, n\} \end{array} \\ \text{cont.} \left\{ \begin{array}{l} X \sim \text{Exp}(\lambda) := \lambda e^{-\lambda x} \\ X \sim N(\mu, \sigma^2) := \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} \end{array} \right. & \begin{array}{l} \text{Supp}[X] = (0, \infty) \\ \text{Supp}[X] = \mathbb{R} \end{array} \end{array}$$

$$\downarrow$$

$$p(1) = p^1 (1-p)^{1-1} = p, \quad p(0) = p^0 (1-p)^{1-0} = 1-p, \quad p\left(\frac{1}{7}\right) = p^{\frac{1}{7}} (1-p)^{\frac{6}{7}}$$

What is p ?? What is n ? What is λ ? What is μ ? sigsq? They are "tuning knobs" which are called parameters. E.g. p controls how often 0's or 1's occur in the Bernoulli rv.

What are the legal values for p ? What are values that "make sense" for the Bernoulli rv? What values respect its support?

$$p \in (0,1) \quad \text{The parameter space for the parameter } p.$$

Why not negative or greater than 1? They're not probabilities. Why not 0 or 1?

$$X \sim \text{Bern}(1) = 1^x (1-1)^{1-x} = p(x) = \text{Deg}(1) := \{1 \text{ w.p. } 1$$

$$p(0) = 1^0 (1-1)^{1-0} = 1 \cdot 0^1 = 0$$

$$p(1) = 1^1 (1-1)^{1-1} = 1 \cdot 0^0 = 1$$

the degenerate rv which is technically a rv but not interesting because it's not "random" it is oxymoronic

As a convention (standard), degenerate parameter values are not considered part of the parameter space.

θ

Let θ denote unknown parameters. And let $\vec{\theta}$ denote multiple unknown parameters. And let Θ denote the parameter space. $\theta \in \Theta$

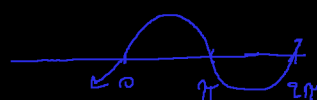
$$X \sim \text{Bern}(\theta), \quad \Theta = (0,1)$$

$$X \sim \text{Binomial}(\theta_2, \theta_1) = \binom{\theta_2}{x} \theta_1^x (1-\theta_1)^{\theta_2-x}, \quad \Theta = (0,1) \times \mathbb{N}$$

$$\mathcal{F}_{\text{Bern}} := \left\{ \theta^x (1-\theta)^{1-x} : \theta \in (0,1) \right\} \quad \text{All possible Bernoulli rvs a "parametric model"}$$

$$\mathcal{F} := \left\{ p(x; \vec{\theta}) : \vec{\theta} \in \Theta \right\} \quad \text{e.g. } f(x; c) = \sin(cx)$$

↑
parameter(s)



$$p(x_1, x_2, \dots, x_n; \theta) \quad \text{joint mass function (JMF)}$$

$$f(x_1, x_2, \dots, x_n; \theta) \quad \text{joint density function (JDF)}$$

$$\begin{array}{l} \text{if } X_1, \dots, X_n \text{ are } \text{ind.} \\ \text{independent} \end{array} \quad \begin{array}{l} \rightarrow p(x_1; \theta) \cdot p(x_2; \theta) \cdot \dots \cdot p(x_n; \theta) \\ \rightarrow f(x_1; \theta) \cdot f(x_2; \theta) \cdot \dots \cdot f(x_n; \theta) \end{array}$$

"multiplication rule"

$$X \sim \text{Deg}(c) := \{c \text{ w.p. } 1, \quad \text{Supp}[X] = \{c\}$$