Math 341 / 650.03 Spring 2021 (3 credits)
Course Syllabus

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Course Homepage	`https://github.com/kapelner/QC_Math_341_Spring_2021`
Slack Homepage	`https://QCMath341Spring2021.slack.com/`
Contact	Write to `@kapelner` on slack
Lecture Time / Loc	Mon and Wed 3:10 – 4:25PM / on zoom
Instructor Office Hours / Loc	Mon and Wed 4:30 - 4:55PM / on zoom, Thurs 10:50-11:15AM / on zoom
TA / TA Office Hours / Loc	Abhinav Patil / TBD week-week / on zoom

Course Overview

MATH 341. Bayesian Modeling. 3 hr.; 3 cr. Prereq.: MATH 241. A review of frequentist methods followed by a survey of statistical modeling using the Bayesian framework: prior distribution design, including Jeffrey's priors; likelihood models; posterior probabilities; hypothesis tests; Bayesian linear regression; Gibbs sampling; Metropolis-Hastings (basic Bayesian computing). Emphasis on real-world applications, including those in finance and applied probability. The goal is to be fluent enough to understand how industry uses Bayesian modeling and computation by the end of the course.

Statistics has historically been taught from the frequentist perspective. Recently, the Bayesian perspective has become popular (1) due to their models' performance on previously intractable problems and the recent availability of inexpensive computational power and (2) it solves many philosophical quandaries in the Frequentist perspective. Further, many scientific journals are moving away from Frequentist p-values and confidence intervals in favor of the Bayesian analogues. It is imperative to learn this perspective as you will see these models in industry and this mode of thinking is becoming mainstream in science at large.

Here are the main topics to be covered:

- Data modeling with parametric families
- Bayes Rule as it applies to parameters
- prior distribution design
- Jeffrey's priors
- likelihood models and maximum likelihood estimators
- posterior probabilities
- Bayesian inference: credibility intervals
- Bayesian inference: p-values for hypothesis tests
- mixture priors
- mixture models
- Newton-Raphson and Expectation-Maximization Algorithms
- basic computing for Bayesian models including Gibbs sampling and Metropolis-Hastings Sampling

The only prerequisite is Math 241 or equivalent. You should be familiar with the following:

- Basic Set Theory
- Counting Methods | permutations and combinations
- Basic Probability Theory | axioms, conditional probability, in/dependence
- Modeling with Discrete Random Variables: Bernoulli, Hypergeometric, Binomial, Pois- son, Geometric, Negative Binomial, Uniform Discrete and others
- Expectation and Variance
- Modeling with Continuous Random Variables: Exponential, Uniform and Normal
- Frequentist Confidence Intervals and Hypothesis Testing for one-sample proportions

We will review the above *throughout the semester* when needed and we will do so rapidly.

This is not your typical mathematics course. This course develops ideas and concepts for helping to make decisions based on randomness and we will do lots of modeling of real-world situations. The course does not dwell on theory nor details of computation but will make use of computation especially using the `R` statistical language.

Course Materials

Textbook: Introduction to Bayesian Statistics by William M. Bolstad First Edition. It can be purchased used on Amazon. This is *recommended*. It is a way to get "another take" on the material. However, most of the material in the class comes from the lecture notes.

Popular Book: We will also be reading the non-fiction novel "The Theory that Would not Die: How Bayes' Rule Cracked the Enigma Code, Hunted Down Russian Submarines, and Emerged Triumphant from Two Centuries of Controversy" by Sharon Bertsch McGrayne which can also be purchased on Amazon. This is *required* — you will have homework questions directly from this book.

Computer Software: We will also be using `R` which is a free, open source statistical programming language and console. You can download it from: `http://cran.mirrors.hoobly.com/`. I do not expect you to do *any* programming. I will be giving you `R` code to run and expect you to interpret the results based on concepts explained during the course.

Calculator: You can use a TI-84, 85, 89 or any calculator which you wish. I strongly suggest you use Wolfram Alpha and its smartphone app.

The 650.x section

You are the students taking this course as part of a masters degree in mathematics. Thus, there will be extra homework problems for you and you will be graded on a separate curve.

Announcements

Course announcements will be made via slack in the #general channel (not on email). I am known to send a few slack messages per week on important issues. If you are not on slack, you will miss all class announcements!!! We will discuss how to sign up to slack in the section on Homework #0 (page 9).

The Use of Slack as a Learning Management System

As the course homepage is updated (e.g. a new homework assignment is posted), you will hear about it in slack. You will also find the video recordings of lectures there. (If there are multiple sections of the class, only one section's lectures will be recorded). Each assignment will have its own channel. You can feel free to discuss things with your fellow students there. If you are asking me a question, you must do so in the `#discussions` channel for a general questions or the assignment-specific channel (e.g. `#HW03`) so other students can see the question and benefit from the answer. If you pm me, I will just ask you to move it to the public channel. Do not be afraid to ask questions. There are many people who will have your same question!

Slack is a wildly successful company that recently got bought by Salesforce because businesses use it. Pretend you are working at one of these businesses: no posting about random stuff; keep things professional!

Class Meetings

There are 28 scheduled meetings. Of these, 23 will be lectures, 2 will be midterm exams which are in class and 2 will be review periods (the meeting before the exams). The exam schedule is given on page 10.

I am canceling Monday, May 17 (the last meeting) due to a Jewish holiday. This meeting would have been a review session for the final exam. We will decide when to have this session during finals week that fits the majority's schedule.

Zoom Policies

Zoom policies: your video must always be on. You can use an appropriate background but it must be a static image. Also: no snap camera or similar nonsense. No chatting on zoom.

Lecture Schedule

Below is a tentative detailed enumeration of the lectures, topics with time estimates below:

Lec 1 [40min] Review of discrete and continuous random variables (rvs), support, probability mass functions (PMFs), cumulative distribution functions (CDFs), probability density functions (PDFs); joint mass functions (JMFs), joint density functions (JDFs), independence, iid multiplication rule; [20min] the Bernoulli rv, parameters, parameter space, degenerate rv; [5min] parametric models; [10min] statistical inference and its three goals

Lec 2 [30min] the likelihood function, the log-likelihood function, example with iid Bernoulli, example with iid Geometric; [20min] the MLE and properties of the MLE; [10min] introduction to frequentist confidence intervals (CIs); [15min] intoduction to hypothesis testing and frequentist retainment regions

Lec 3 [30min] Problems and limitations with frequentist CIs and testing, valid interpretation of frequentist CIs, the frequentist p-value; [35min] review of definition of conditional probability, Bayes Rule, Bayes Theorem; [10min] marginal and conditional PMFs

Lec 4 [10min] Bayes rule for two rvs; [10min] anatomy of the Bayes identity: the likelihood, prior, prior predictive distribution and posterior distribution; [40min] example posterior calculation with discrete parameter space and principle of indifference; [15min] Bayesian point estimation with the maximum a posteriori (MAP) estimate, conditions for equivalence with the MLE

Lec 5 [25min] Proof that Bayesian Inference is iterative in the data; [15min] uniform prior for the bernoulli iid model; [10min] Bayesian point estimation with the posterior median and posterior expectation; [20min] derivation of general beta posterior for

the bernoulli iid model, intro to beta distribution, beta function, gamma function; [5min] point estimation with beta posterior

Lec 6 [10min] all legal shapes of the beta distribution; [35min] the beta-binomial bayesian model; prior parameters (hyperparameters) and posterior parameters, point estimates; [10min] definition of conjugacy, beta-binomial conjugacy; [10min] pseudo-data interpretation of the prior parameters; [10min] shrinkage estimators and the beta-binomial posterior expectation as a shrinkage estimator

Lec 7 [15min] One-sided and two-sided credible regions (CRs); [5min] CR for beta-binomial model; [10min] high density regions; [20min] one-sided and two-sided Hypothesis testing and the decisions to reject or retain the null; [10min] decisions in the Bayesian framework for one-sided hypothesis testing, Bayesian p-values; [15min] beta-binomial examples

Lec 8 [20min] two approaches for two-sided testing in the Bayesian framework; [40min] posterior predictive distribution formula, example for one future observation in the beta-binomial model; [15min] mixture and compound distributions

Lec 9 [65min] the betabinomial distribution as an overdispersed binomial, example with birth data, proof of the general posterior predictive distribution for the beta-binomial model; [10min] Laplace and Haldane priors

Lec 10 [25min] Informative priors for the beta-binomial model, example with baseball batting averages, shrinkage in informative priors, empiral Bayes estimation; [10min] definition of odds, reparameterization of the binomial with odds; [5min] PDF change of variables formula, proof that prior of indifference for binomial probability is not prior of indifference for odds; [15min] Jeffrey's prior specification concept; [10min] PDF/PMF decomposition into kernel and normalization constants; [10min] definition of Fisher information, computation of Fisher information for the binomial distribution

Lec 11 [30min] Definition of Jeffrey's prior, derivation of Jeffrey's prior for the beta-binomial model, verification that it robust to reparameterizations of the binomial model's parameter; [10min] proof of Jeffrey's prior satisfies Jeffrey's prior specification concept; [10min] derivation of Poisson model; [15min] derivation of the Poisson model's conjugate prior via kernel decomposition (the Gamma); [20min] Gamma shapes and properties

Lec 12 [15min] pseudodata interpretation of hyperparameters in the gamma-poisson model; [20min] derivation of the shrinkage point estimator for the gamma-poisson model; [10min] CRs for the gamma-poisson model; [20min] uninformative priors for the gamma-poisson model;

Lec 13 [45min] derivation of the posterior predictive distribution being extended negative binomial in the gamma-poisson model; [15min] kernel decomposition of the normal PDF; [15min] Normal posterior under laplace prior

Lec 14 [75min] derivation of the normal-normal conjugate model, pseudodata interpretation of the hyperparameters, Haldane prior, point estimation in the normal-normal model, Jeffrey's prior derivation, shrinkage estimator

Lec 15 [40min] derivation of the normal posterior predictive distribution for the normal-normal model; [10min] derivation of the inversegamma distribution, properties of the inverse gamma distribution [35min] normal-inversegamma model, laplace prior, pseudodata interpretation of the hyperparameters, haldane prior

Lec 16 [10min] point estimation in the normal-inversegamma model; [15min] Jeffrey's prior derivation for the normal-inversegamma model; [30min] derivation of the Student's T posterior predictive distribution for the normal-inversegamma model; [10min] shrinkage estimation in the normal-inversegamma model

Lec 17 [75min] The two-dimensional normal-inverse-gamma (NIG) distribution, its kernel, its use in bayesian inference for the conjugate NIG-NIG model

Lec 18 [15min] Marginal mean T distribution in the NIG posterior; [15min] Marginal variance inverse-gamma distribution in the NIG posterior; [55min] derivation of the Student's T posterior predictive distribution in the NIG-NIG model

Lec 19 [30min] Sampling from the NIG distribution; [45min] the kernel of the semiconjugate NIG model

Lec 20 [35min] Grid sampling, distribution sampling via kernel grid sampling, disadvantages of grid sampling; [40min] systematic sweep Gibbs sampling, burning the chain, sampling from the semi-conjugate NIG model

Lec 21 [20min] Autocorrelation grid sampling, thinning the chain; [30min] approximate inference with Gibbs samples; [25min] change point detection model

Lec 22 [55min] normal mixture model with data augmentation; [35min] Bayes Factors

Lec 23 [60min] Metropolis algorithm, Metropolis-Hastings algorithm, metropolis-within-Gibbs, transition kernels

Lecture Upload

As many previous students have noted, my handwritten notes are useful to me and not to many others. (Although this has been improved by zoom where I now "type" on the "chalkboard"). I will still be rewarding students for taking notes, scanning them in and sending them to me. You will be rewarded in two ways: (1) if you do this for more than 10 lectures, you will be given the automatic 5 participation points (see grading policy on page 11) for your classroom participation grade and (2) you have the option for me to say your name publicly on the course homepage. Make sure you follow these instructions:

- You have *one week only* from the time of the lecture to provide me lecture notes.

- There must be *one* file and it must be in PDF format only.
- The file must be <2MB. No exceptions. I will tell you to shrink the PDF if not.
- The file must be named `lecxxkapelner.pdf` where you replace `xx` with two digits corresponding to the lecture number i.e. 01, 02, 09, 10, ..., 23 and you replace `kapelner` with your last name in all lowercase letters. If your file is renamed incorrectly, I will tell you to rename it and send it back.
- Vertically oriented (readable without rotating your head 90 degrees.).
- The scan is sharp and not blurry! Blurry submissions will be rejected.
- The lecture must be added as a pull request through github. Follow these steps:
    1. Create your `github.com` account (if you don't have one already).
    2. Go to the course homepage. In the top right corner you'll see a "fork" button. Click that. It will create your own copy of the course's homepage.
    3. Follow the instructions on this site.
    4. If you wish to upload more than one lecture. Do not fork again! But you'll have to pull the current changes from the course homepage. Follow the instructions on this site.
- The pull request must consist of the PDF in the `lectures` folder. Additionally, your full name and the appropriate link must be placed in the README file *after* my link and everyone else's text link. This may take a few tries for you to get this right. But once you do, you'll be a real open source contributor!

If you add your notes, you are (1) agreeing to the MIT license which means someone can freely copy your notes and even make money off of it (and not owe you a cent!) and (2) since github is mirrored, once your upload is on the web, it is there indeliby forever. If you are not comfortable with these two points, do not send me your notes!!

Homework

There will be 7–10 theory homework assignments. Homeworks will be assigned and placed on the course homepage and will usually be due a week later in class. Homework will be **graded** out of 100 with extra credit getting scores possibly > 100. I will be doing the grading and will grade an *arbitrary subset of the assignment* which is determined after the homework is handed in.

During this pandemic, homework must be handed in by emailing it to me as a PDF. You must do one of two things:

- **Print out the homework and handwrite your answers in the alotted space for each question. Then scan your homework as a PDF. There are a ton of good photo-PDF apps for both iPhone and droid.**
- **Open the PDF on your device and use a PDF-editing program to electronically handwrite your answers and save the PDF.**

I will NOT accept homework that is not atop the original rendered homework PDF file. Remember to write your name. There are no regrades during this pandemic semester.

You are encouraged to seek help from me if you have questions. After class and office hours are good times. You are highly recommended to work with each other and help each other. You must, however, submit your own solutions, *with your own write-up* and in *your own words*. There can be no collaboration on the actual *writing*. Failure to comply will result in severe penalties. The university honor code is something I take seriously and I send people to the Dean every semester for violations.

Philosophy of Homework

Homework is the *most* important part of this course.[1] Success in Statistics and Mathematics courses comes from experience in working with and thinking about the concepts. It's kind of like weightlifting; you have to lift weights to build muscles. My job as an instructor is to provide assistance through your zone of proximal development. With me, you can grow more than you can alone. To this effect, homework problems are color coded green for easy, yellow for harder, red for challenging and purple for extra credit. You need to know how to do all the greens by yourself. If you've been to class and took notes, they are a joke. Yellows and reds: feel free to work with others. Only do extra credits if you have already finished the assignment. The "[Optional]" problems are for extra practice — highly recommended for exam study.

Time Spent on Homework

This is a four credit course. Thus, the amount of work outside of the 4hr in-class time per week is 8-12 hours. I will aim for 10hr of homework per week on average. However, doing the homework well is your sole responsibility since I will make sure that by doing the homework you will study and understand the concepts in the lectures and you won't have all that much to do when the exams roll around.

Late Homework Policy

Late homework will be penalized 10 points per business day (Monday–Friday save holidays) for a maximum of five days. *Do not ask for extensions*; just hand in the homework late. After five days, **you can hand it in whenever you want** until May 18 at noon.

[1] In one student's observation, I give a "mind-blowing homework" every week.

As far as I know, this is one of the most lenient and flexible homework policies in college. I realize things come up. Do not abuse this policy; you will fall far, far behind.

LaTeX Homework Bonus Points

Part of good mathematics is its beautiful presentation. Thus, there will be a 1–7 point bonus added to your theory homework grade for typing up your homework using the LaTeX typesetting system based on the elegance of your presentation. The bonus is arbitrarily determined by me.

I recommend using overleaf to write up your homeworks (make sure you upload both the hw#.tex and the preamble.tex file). This has the advantage of (a) not having to install anything on your computer and thus not having to maintain your LaTeX installation (b) allowing easy collaboration with others (c) alway having a backup of your work since it's always on the cloud. If you insist to have LaTeX running on your computer, you can download it for Windows here and for MAC here. For editing and producing PDF's, I recommend TeXworks which can be downloaded here. Please use the LaTeX code provided on the course homepage for each homework assignment.

If you are handing in homework this way, read the comments in the code; there are two lines to comment out and you should replace my name with yours and write your section. The easiest way to use overleaf is to copy the raw text from hwxx.tex and preamble.tex into two new overleaf tex files with the same name. If you are asked to make drawings, you can take a picture of your handwritten drawing and insert them as figures or leave space using the "\vspace" command and draw them in after printing or attach them stapled.

Since this is extra credit, do not ask me for help in setting up your computer with LaTeX in class or in office hours. Also, **never share your LaTeX code with other students** — it is cheating and if you are found I will take it seriously.

Homework Extra Credit

There will be many extra credit questions sprinkled throughout the homeworks. They will be worth a variable number of points arbitrarily assigned based on my perceived difficulty of the exercise. Homework scores in the 140's are not unheard of. They are a good boost to your grade; I once had a student go from a B to and A- based on these bonuses.

Homework #0

To get a perfect grade on this assignment, you must:

(1) email me at kapelner@qc.cuny.edu from the email address you wish to be contacted at for this course (most commonly this is a gmail address),

(2) in the email, you must say "My name is <Your Full Name as appears in the registrar> and I have read, understand and agree to all the material in the course syllabus"

(3) within a few days of receiving your email, I will send you an invite for slack and you must sign up

This assignment is due Wednesday, Feb 3 11:59PM and will receive a grade of 0 or 100 with the usual 10 point penalty for lateness. You cannot get credit until you signup for slack. Point #2 constitutes a contract — you are agreeing to this syllabus.

Examinations

Examinations are solely based on homeworks (which are rooted in the lectures)! If you can do all the green and yellow problems on the homeworks, the exams should not present any challenge. I will *never* give you exam problems on concepts which you have not seen at home on one of the weekly homework assignments. There will be three exams and the schedule is below.

Exam Schedule

- Midterm examination I will be Wednesday, March 3 on zoom during class time with the first review session on the class meeting prior
- Midterm examination II will be Wednesday, April 21 on zoom during class time with a review on the class meeting prior.
- The final examination will be TBD but on zoom with a review usually on the last class meeting of the semester.

Exam Policies and Materials

I allow you to bring any calculator you wish but it cannot be your phone. The only other items allowed are pencil and eraser. I do not recommend using pen but it is allowed. Food is not allowed during exams but beverages are allowed.

I also allow "cheat sheets" on examinations. For both midterms, you are allowed to bring *one* 8.5" × 11" sheet of paper (front and back). Two sheets single sided are not allowed. On this paper you can write anything you would like which you believe will help you on the exam. For the final, you are allowed to bring *three* 8.5" × 11" sheet of paper (front and back). Six sheets single sided are not allowed. I will be handing back the cheat sheets so you can reuse your midterm cheat sheets for the final if you wish.

Cheating on Exams

If I catch you cheating, you can either take a zero on the exam, or you can roll the dice before the University Honor Council who may choose to suspend you.

Missing Exams

There are no make-up exams. If you miss the exam, you get a zero. If you are sick, I need documentation of your visit to a hospital or doctor. Expect me to call the doctor or hospital to verify the legitimacy of your note.

Special Services

If you are a student who takes exams at the special services center, I need to see your blue slip or receive email evidence one week before the exam to make proper arrangements with the center.

Class Participation

This portion of your grade is assessed based on your level of interaction during the course lectures e.g. asking and answering questions. You can be guaranteed a perfect grade by uploading your lectures per the policy in that section on 6. Participation on slack also counts towards this total.

Attendance

Part of your grade will be based on attendance. Since the course is on zoom, I have a full audit of the second each of you enters and exits the "classroom". Your score will be computed automatically as the proportion of seconds you attended class divided by the total number of class seconds.

Grading and Grading Policy

Your course grade will be calculated based on the percentages as follows:

Homework	20%
Midterm Examination I	20%
Midterm Examination II	20%
Final Examination	30%
Class participation	5%
Attendance	5%

The semester is split into three periods :

(a) From the beginning until midterm I. Midterm I covers material during this time..

(b) From midterm I to midterm II. Midterm II covers material in this period only.

(c) From midterm II until the final. The final is cumulative and covers all course material.

Each of the periods is assessed evenly. Thus, each period must count the same towards your grade. Since there is 75% of the grade allotted to exams, there is 25% allotted to each period. Thus, the final is upweighted towards the material covered in the third period. In summary, the final will have 5/35 points $\approx 14\%$ for the first period's material, 5/35 points $\approx 14\%$ for the second period's material and 25/35 points $\approx 71\%$ for the last period's material. A good strategy for the final is to just study the material after Midterm II and minimal studying for the previous material.

The Grade Distribution

As this is a small and advanced class, the class curve will be quite generous. Last year, it was approximately 40% A's and 40% B's. If you do your homework and demonstrate understanding on the exams, you should expect to be rewarded with an A or a B. C's are for those who "dropped out" somewhere mid-semester or who cannot demonstrate basic understanding. I have never given an F in this class. Don't give me a reason to change this tradition!

Checking your grade and class standing

You can always check your grades in real-time using `http://qc.gradesly.com`. You will enter in your QC ID number (or email address used for HW #0). I will provide you with your password by emails upon completion of HW #0.

Auditing

Auditors are welcome in both sections. They are encouraged to do all homework assignments. I will even grade them. Note that the university does not allow auditors to take examinations.