Total No. of Questions : 8]

SEAT No. :

PB3651

[Total No. of Pages : 3

[6261]-59

S.E. (Information Technology Engg.)

DISCRETE MATHEMATICS

(2019 Pattern) (Semester - III) (214441)

Time : 2½ Hours] ***[Max. Marks : 70***

Instructions to the candidates:

1) *Answer Q.1 or Q.2, Q.3 or Q.4, Q.5 or Q.6, Q.7 or Q.8.*
2) *Neat diagram must be drawn wherever necessary.*
3) *Figures to the right indicate full marks.*
4) *Assume suitable data, if necessary.*

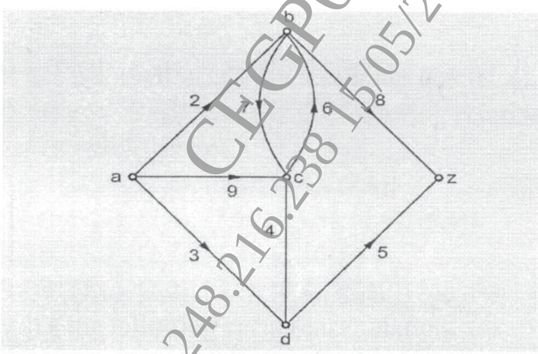
Q1) a) Give an example of the following graphs **[6]**

i) Eulerian but not Hamiltonian

ii) Hamiltonian but not Eulerian

iii) Eulerian as well as Hamiltonian

b) Using the labeling procedure, find the maximum flow in the following transport network. **[6]**



c) What is the Prefix Code? Which of the following codes are prefix codes? Justify your answer. **[6]**

i) a: 101, e: 11, t: 001, s: 011, n: 010

ii) a: 010, e: 11, t: 011, s: 1011, n: 1001, i: 10101

OR

Q2) a) A connected planar graph has nine vertices with degree **[6]**

2, 2, 2, 3, 3, 3, 4, 4, 5

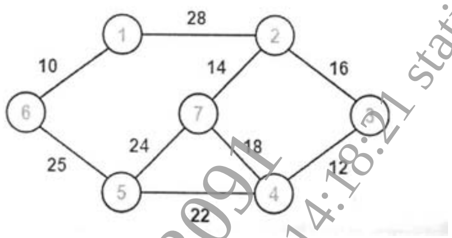
i) Find number of edges

ii) Find number of faces

iii) Construct such a graph

P.T.O.

b) Find the minimum spanning tree and weight of it for the given graph using Prim's algorithm. **[6]**



c) Suppose that someone starts a chain letter. Each person who receives the letter is asked to send it on to four other people. Some people do this, but others do not send any letters. How many people have seen the letter, including the first person, if no one receives more than one letter and if the chain letter ends after there have been 100 people who read it but did not send it out? How many people sent out the letter? **[6]**

Q3) a) What is Function? Given a relation R = {(1,4), (2,2), (3,10), (4,8), (5,6)} and check whether the following relations R1, R2, R3 & R4 are functions or not. **[6]**

R1 = {(1,4), (2,4), (3,4), (4,4), (5,4)}

R2 = {(1,2), (2,4), (2,10), (3,8), (4,6), (5,4)}

R3 = {(1,6), (2,2), (4,4), (5,10)}

R4 = {(1,6), (2,2), (3,2), (4,4), (5,10)}

b) Solve the following recurrence relation. **[6]**

$a_n = 5a_{n-1} - 6a_{n-2}$ where $a_0 = 2$ and $a_1 = 5$

c) Find the minimum number of students in a class such that five of them are born in the same month? (Use PigeonHole Principle). **[5]**

OR

Q4) a) Find the transitive closure by using Warshall's algorithm for the given relation as: **[6]**

R = {(1,1), (1,4), (2,1), (2,2), (3,3,), (4,4)}

b) Define POSET. Draw Hasse Diagram for relation R defined over set A. A = {1, 2, 3, 4, 6, 8, 12} R = {(x,y)| x divides y}. **[6]**

c) Given f(x) = 2x + 3 and g(x) = 3x – 2. Find f(7), gof(x) and fog(x). **[5]**

Q5) a) Which of the following congruences is true? Justify your answer. **[6]**

i) $5127 \equiv 1297 \pmod{20}$

ii) $577 \equiv 7188 \pmod{11}$

iii) $1492 \equiv 717 \pmod{31}$

b) Compute GCD of the following numbers using Euclidean Algorithm **[6]**

i) GCD(745, 1250)

ii) GCD(485, 1551)

c) Using Chinese Remainder Theorem, find the value of P using following data **[6]**

$P \equiv 4 \pmod{5}$

$P \equiv 5 \pmod{7}$

OR

Q6) a) Find multiplicative inverse of 5 mod 31 using Extended Euclidean Algorithm. **[6]**

b) Find totient function of the following numbers **[6]**

i) 77

ii) 75

iii) 50

c) What is a Mersenne prime number? Which of the following numbers is the Mersenne Prime number? 19, 31, 1023, 63,7. **[6]**

Q7) a) Let S{1,2,3,6,12}, where a*b is defined as LCM (a,b) over set S. Determine whether it is a semigroup, group, or Abelian Group or neither. **[6]**

b) Consider the set A={1,3,5,7,9.......} i.e. a set of odd positive integers. Determine whether A is closed under: **[6]**

i) a*b = a + b

ii) a*b = 2a – b

iii) a*b = a.b (Multiplication)

iv) a*b = power (a,b)

v) a*b = max(a,b)

vi) a*b = GCD(a,b)

c) Prove that Z={1,5,7,11} is an Abelian group under multiplication mod 12. **[5]**

OR

Q8) a) Show that the set of all positive rational numbers forms an abelian group under the composition * defined by a * b = (ab)/2. **[6]**

b) Explain Field with an example. **[6]**

c) Prove that Hamming Distance d(x,y) = 0 iff x = y where x and y are codewords. **[5]**