

Total No. of Questions : 8]

SEAT No. :

PC2835

[6352]-59

[Total No. of Pages :4

S.E. (Information Technology)

DISCRETE MATHEMATICS

(2019 Pattern) (Semester- III) (214441)

Time : 2½ Hours]

[Max. Marks : 70

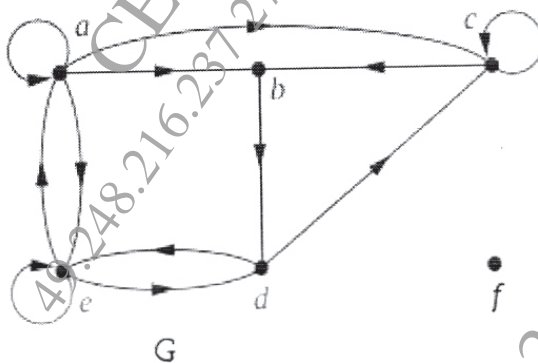
Instructions to the candidates:

- 1) Answer Q.1 or Q.2, Q.3 or Q.4, Q.5 or Q.6, Q.7 or Q.8.
- 2) Neat diagrams must be drawn wherever necessary.
- 3) Figures to the right indicate full marks.
- 4) Assume suitable data, if necessary.

Q1) a) What is a planar graph? Draw planar embedding of K_4 and $K_{3,2}$. [6]

b) For the following set of weights construct an optimal binary prefix tree. Find the weight of the optimal tree 2, 3, 5, 7, 9, 13. [6]

c) Find the in-degree and out-degree of each vertex in the graph G with directed edges shown in Figure. Show that the sum of In-degree and Out-degree is the same in the graph. [6]



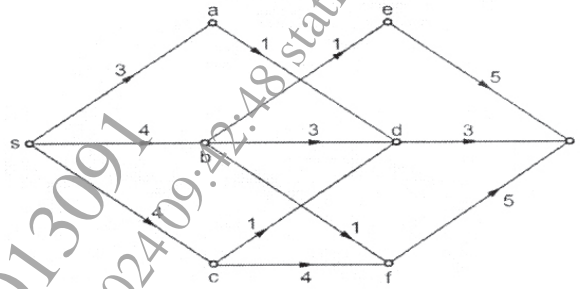
OR

Q2) a) Define the following terms [6]

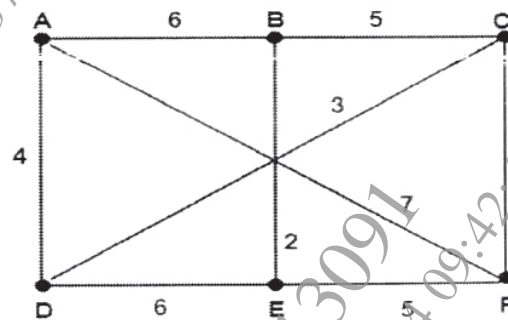
- i) Bipartite Graph
- ii) Hamiltonian Path
- iii) Eulerian Circuit

P.T.O.

- b) Use the labeling procedure to find a maximum flow in the transport network. Shown in the following fig. [6]



- c) Determine the minimum spanning tree of the weighted graph shown in fig using Kruskal's Algorithm: [6]

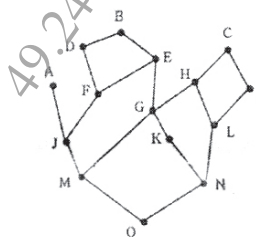


- Q3) a) Solve the following recurrence relation. [6]

$$x_n = 6x_{n-1} - 9x_{n-2} \text{ where } x_0 = 0 \text{ and } x_1 = 3$$

- b) Given $f(x) = 2x + 3$ and $g(x) = 3x - 2$. Find $f(4)$, $g(5)$, $g \circ f(x)$ and $f \circ g(x)$. [6]

- c) Give answers of the following questions w.r.t. Given POSET? [5]



What is the upper bound of B?

What is the minimal element?

How many maximal elements are there?

What is the upper bound of M and N?

OR

2

- Q4)** a) Define POSET. Draw Hasse Diagram for relation R defined over set [6]
 $A = \{1, 2, 3, 4, 6, 8, 12\}$ $R = \{(x, y) \mid x \text{ divides } y\}$
- b) What is an equivalence relation? Explain with an example. [6]
- c) Find the minimum number of students in a class such that four of them are born in the same month? (Use PigeonHole Principle) [5]

- Q5)** a) Using Binary Expansion method solve the following (Show stepwise answer) $7^{50} \bmod 13$. [6]
- b) Find the Euler's totient function of the following numbers [6]
- 77
 - 75
 - 50
- c) Using Chinese Remainder Theorem, find the value of P using following data [6]
- $$P \equiv 3 \pmod{7}$$
- $$P \equiv 6 \pmod{11}$$

OR

- Q6)** a) Find the multiplicative inverse of 13 mod 31 using Extended Euclidean Algorithm. [6]
- b) Compute GCD of the following numbers using Euclidean Algorithm [6]
- GCD (1250, 900)
 - GCD (456, 165)
- c) Explain Fermat's Little Theorem. Find $3^{31} \bmod 7$ using Fermat's Little Theorem [6]

- Q7)** a) Let P be the set of all matrices of the form $\begin{bmatrix} x & y \\ y & x \end{bmatrix}$ where x is a non-zero rational number. * is the matrix multiplication defined over P. Show that it is a group. [6]
- b) Explain Integral Domain with an example [6]
- c) Let $S = \{1, 2, 3, 6, 12\}$, where $a * b$ is defined as LCM (a, b) over set S. Determine whether it is a semigroup, group, or Abelian Group or neither. [5]

OR

Q8) a) Prove that $Z = \{1, 5, 7, 11\}$ is an Abelian group under multiplication mod 12. [6]

b) Let I be the set of all positive integers. For each of the following determine $*$ is a commutative operation or not: [6]

i) $a * b = \max(a, b)$

ii) $a * b = (a + b)/2$

iii) $a * b = a.b$ (Multiplication)

iv) $a * b = \text{power}(a, b)$

v) $a * b = 2a/b$

vi) $a * b = 2a + b$

c) Find the minimum distance of an encoding function [5]

$e: B^2 \rightarrow B^5$ given as: $e(0, 0) = 00000$, $e(0, 1) = 10011$, $e(1, 0) = 01110$, $e(1, 1) = 11111$.

