

Total No. of Questions : 9]

SEAT No. :

**P6492**

**[5868]-109**

[Total No. of Pages : 4

**First Year Engineering**  
**ENGINEERING MATHEMATICS - II**  
**(2019 Pattern) (Semester - I & III) (107008)**

*Time : 2½ Hours]*

*[Max. Marks : 70*

*Instructions to the candidates:*

- 1) *Q.No. 1 is compulsory.*
- 2) *Solve Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q.7, Q.8, or Q.9.*
- 3) *Neat diagrams must be drawn whenever necessary.*
- 4) *Figures to the right indicate full marks.*
- 5) *Use of electronic pocket calculator is allowed.*
- 6) *Assume suitable data if necessary.*

**Q1)** Write the correct option for the following multiple choice questions.

a)  $\int_0^{\frac{\pi}{2}} \cos^6 x =$  [2]

- |                         |                       |
|-------------------------|-----------------------|
| i) $\frac{5}{16}$       | ii) $\frac{5\pi}{32}$ |
| iii) $\frac{16\pi}{10}$ | iv) $\frac{5\pi}{48}$ |

b) The curve  $y^2(x - a) = x^2(2a - x)$  is [2]

- i) Symmetric about X - axis and net passing through origin
- ii) Symmetric about Y - axis and net passing through origin
- iii) Symmetric about X - axis and passing through origin
- iv) Symmetric about Y - axis and passing through origin

c) The value of double integral  $\int_0^1 \int_0^1 \frac{1}{\sqrt{1-x^2} \sqrt{1-y^2}} dx dy$  is [2]

- |                        |                        |
|------------------------|------------------------|
| i) $\frac{\pi}{2}$     | ii) $\frac{\pi}{2}$    |
| iii) $\frac{\pi^2}{4}$ | iv) $\frac{\pi^2}{16}$ |

**P.T.O.**

d) The Centre (C) and radius (r) of the sphere  $x^2 + y^2 + z^2 - 2y - 4z - 11 = 0$  are [2]

- i)  $C \equiv (0, 1, 2); r = 4$  ii)  $C \equiv (0, -1, -2); r = 2$   
 iii)  $C \equiv (0, 2, 4); r = 4$  iv)  $C \equiv (0, 1, 2); r = 2$

e) The number of loops in the rose curve  $r = a \cos 4\theta$  are [1]

- i) 2 ii) 4  
 iii) 6 iv) 8

f)  $\iint_R dx dy$  represents [1]

- i) Volume ii) Centre of gravity  
 iii) Moment of inertia iv) Area of region R

**Q2)** a) If  $I_n = \int_{\pi/4}^{\pi/2} \cot^n \theta d\theta$  prove that  $I_n = \frac{1}{n-1} - I_{n-2}$ . [5]

b) Show that  $\int_0^1 x^{m-1} (1-x^2)^{n-1} dx = \frac{1}{2} \beta\left(\frac{m}{2}, n\right)$ . [5]

c) Prove that  $\int_0^1 \frac{x^a - 1}{\log x} dx = \log(1+a), a \geq 0$ . [5]

OR

**Q3)** a) If  $I_n = \int_0^{\pi/2} x^n \sin x dx$  then prove that  $I_n = n \left(\frac{\pi}{2}\right)^{n-1} - n(n-1)I_{n-2}$ . [5]

b) Show that  $\int_0^{\infty} e^{-h^2 x^2} dx = \frac{\sqrt{\pi}}{2h}$ . [5]

c) Show that [5]

$$\int_a^b e^{-x^2} dx = \frac{\sqrt{\pi}}{2} [\operatorname{erf}(b) - \operatorname{erf}(a)]$$

OR

- Q4)** a) Trace the curve  $x^2y^2 = a^2(y^2 - x^2)$ . [5]  
 b) Trace the curve  $r = a(1 - \sin \theta)$ . [5]  
 c) Find the whole length of the loop of the curve  $3y^2 = x(x-1)^2$ . [5]

OR

- Q5)** a) Trace the curve  $y^2(2a-x) = x^3$ . [5]  
 b) Trace the curve  $r = a \cos 2\theta$ . [5]  
 c) Trace the curve  $x^{2/3} + y^{2/3} = a^{2/3}$ . [5]

- Q6)** a) Prove that the two spheres  $x^2 + y^2 + z^2 - 2x + 4y - 4z = 0$  and  $x^2 + y^2 + z^2 + 10x + 2z + 10 = 0$  touch each other and find the co-ordinates of the point of contact. [5]

- b) Find the equation of right circular cone whose vertex is  $(1, -1, 2)$ , axis is the line  $\frac{x-1}{2} = \frac{y+1}{1} = \frac{z-2}{2}$  and the semi-vertical angle  $45^\circ$ . [5]

- c) Find the equation of right circular cylinder of radius  $a$  whose axis passes through the origin and makes equal angles with the co-ordinate axes. [5]

OR

- Q7)** a) Show that the plane  $x - 2y - 2z - 7 = 0$  touches the sphere  $x^2 + y^2 + z^2 - 10y - 10z - 31 = 0$ . Also find the point of contact. [5]

- b) Find the equation of right circular cone with vertex at origin, axis the  $Y$ -axis and semi-vertical angle  $30^\circ$ . [5]

- c) Find the equation of right circular cylinder of radius  $\sqrt{6}$  whose axis is the line  $\frac{x}{1} = \frac{y}{-1} = \frac{z}{1}$ . [5]

**Q8) a)** Change the order of integration and evaluate  $\int_0^{\pi} \int_x^{\pi} \frac{\sin y}{y} dx dy$ . [5]

b) Find the area of one loop of  $r = a \sin 2\theta$ . [5]

c) Find the moment of inertia of one loop of the lemniscate  $r^2 = a^2 \cos 2\theta$  about initial line. Given that  $\rho = \frac{2m}{a^2}$ ,  $m$  is the mass of loop of lemniscate. [5]

OR

**Q9) a)** Evaluate  $\int_1^2 \int_0^{2-y} y dx dy$  over the region enclosed by the parabola  $x^2 = y$ , and the line  $y = x + 2$ . [5]

b) Evaluate  $\iiint x^2 yz dx dy dz$ , throughout the volume bounded by the plane  $x = 0, y = 0, z = 0$   $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ . [5]

c) Find the  $y$ -coordinate of the centre of gravity of the area bounded by  $r = a \sin \theta$  and  $r = 2a \sin \theta$ . Given that the area bounded by these curves is  $\frac{3\pi a^2}{4}$ . [5]

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