

Total No. of Questions : 8]

**PC2835**

[6352]-59

SEAT No. :

[Total No. of Pages : 4

**S.E. (Information Technology)**  
**DISCRETE MATHEMATICS**  
**(2019 Pattern) (Semester- III) (214441)**

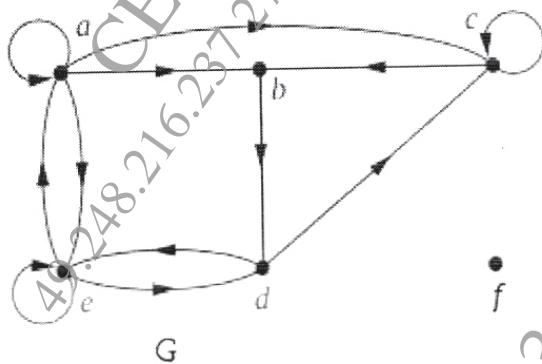
*Time : 2½ Hours]*

*[Max. Marks : 70*

*Instructions to the candidates:*

- 1) Answer Q.1 or Q.2, Q.3 or Q.4, Q.5 or Q.6, Q.7 or Q.8.
- 2) Neat diagrams must be drawn wherever necessary.
- 3) Figures to the right indicate full marks.
- 4) Assume suitable data, if necessary.

- Q1)** a) What is a planar graph? Draw planar embedding of K<sub>4</sub> and K<sub>3,2</sub>. [6]
- b) For the following set of weights construct an optimal binary prefix tree.  
Find the weight of the optimal tree 2, 3, 5, 7, 9, 13. [6]
- c) Find the in-degree and out-degree of each vertex in the graph G with directed edges shown in Figure. Show that the sum of In-degree and Out-degree is the same in the graph. [6]

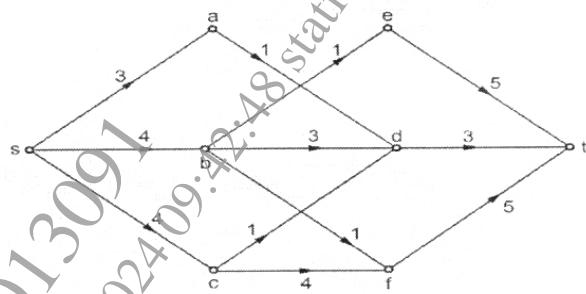


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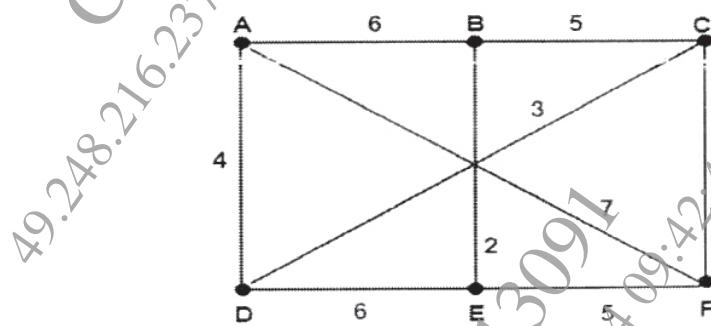
- Q2)** a) Define the following terms [6]
- i) Bipartite Graph
  - ii) Hamiltonian Path
  - iii) Eulerian Circuit

P.T.O.

- b) Use the labeling procedure to find a maximum flow in the transport network. Shown in the following fig. [6]



- c) Determine the minimum spanning tree of the weighted graph shown in fig using Kruskal's Algorithm: [6]

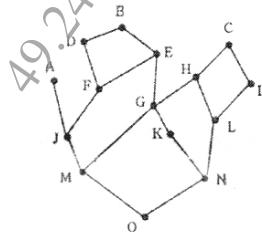


- Q3)** a) Solve the following recurrence relation. [6]

$$x_n = 6x_{n-1} - 9x_{n-2} \text{ where } x_0 = 0 \text{ and } x_1 = 3$$

- b) Given  $f(x) = 2x + 3$  and  $g(x) = 3x - 2$ . Find  $f(4)$ ,  $g(5)$ ,  $gof(x)$  and  $fog(x)$ . [6]

- c) Give answers of the following questions w.r.t. Given POSET? [5]



What is the upper bound of B?

What is the minimal element?

How many maximal elements are there?

What is the upper bound of M and N?

OR

- Q4)** a) Define POSET. Draw Hasse Diagram for relation R defined over set [6]  
 $A = \{1, 2, 3, 4, 6, 8, 12\}$        $R = \{(x,y) \mid x \text{ divides } y\}$  [6]
- b) What is an equivalence relation? Explain with an example. [6]
- c) Find the minimum number of students in a class such that four of them are born in the same month? (Use Pigeon Hole Principle) [5]

- Q5)** a) Using Binary Expansion method solve the following (Show stepwise answer)  $7^{\wedge} 50 \bmod 13$ . [6]
- b) Find the Euler's totient function of the following numbers [6]
- i) 77
  - ii) 75
  - iii) 50
- c) Using Chinese Remainder Theorem, find the value of P using following data [6]
- $$P \equiv 3 \pmod{7}$$
- $$P \equiv 6 \pmod{11}$$

OR

- Q6)** a) Find the multiplicative inverse of 13 mod 31 using Extended Euclidean Algorithm. [6]
- b) Compute GCD of the following numbers using Euclidean Algorithm [6]
- i)  $\text{GCD}(1250, 900)$
  - ii)  $\text{GCD}(456, 165)$
- c) Explain Fermat's Little Theorem. Find  $3^{\wedge} 31 \bmod 7$  using Fermat's Little Theorem [6]

- Q7)** a) Let P be the set of all matrices of the form  $\begin{bmatrix} x & x \\ x & x \end{bmatrix}$  where x is a non-zero rational number. \* is the matrix multiplication defined over P. Show that it is a group. [6]
- b) Explain Integral Domain with an example [6]
- c) Let  $S = \{1, 2, 3, 6, 12\}$ , where  $a * b$  is defined as LCM (a,b) over set S. Determine whether it is a semigroup, group, or Abelian Group or neither. [5]

OR

**Q8) a)** Prove that  $Z = \{1, 5, 7, 11\}$  is an Abelian group under multiplication mod 12. [6]

**b)** Let I be the set of all positive integers. For each of the following determine  $*$  is a commutative operation or not: [6]

i)  $a * b = \max(a, b)$

ii)  $a * b = (a + b)/2$

iii)  $a * b = a.b$  (Multiplication)

iv)  $a * b = \text{power}(a, b)$

v)  $a * b = 2a/b$

vi)  $a * b = 2a + b$

**c)** Find the minimum distance of an encoding function [5]

$e: B^2 \rightarrow B^5$  given as:  $e(0, 0) = 00000$ ,  $e(0, 0) = 10011$ ,  $e(0, 0) = 01110$ ,  $(0, 0) = 11111$ .