Total No. of Questions : 8]

SEAT No. :

PD-4095

[Total No. of Pages : 4

[6402]-55

S.E. (I.T)

DISCRETE MATHEMATICS

(2019 Pattern) (Semester - III) (214441)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Attempt Q.1 or Q.2, Q.3 or Q.4, Q.5 or Q.6, Q.7 or Q.8 .*
2) *Figures to the right indicate full marks.*
3) *Draw neat figures wherever necessary.*
4) *Use of scientific calculator is allowed.*
5) *Assume suitable data, if necessary.*

Q1) a) Determine the number of edges in a graph with 8 nodes, 3 of degree 2,4 of degree 3 and 2 of degree 4. Draw one such graphs. **[6]**

b) Construct an optimal tree for the weights 8,9,10, 11, 13, 15, and 22. Find the weight of the optimal tree. **[6]**

c) Find the chromatic number with the help of graph coloring for: **[6]**

i) K6 (complete graph with 6 vertices)

ii) Any complete bipartite graph.

iii) C7 (cyclic graph with 7 vertices).

OR

Q2) a) Define with graph: **[6]**

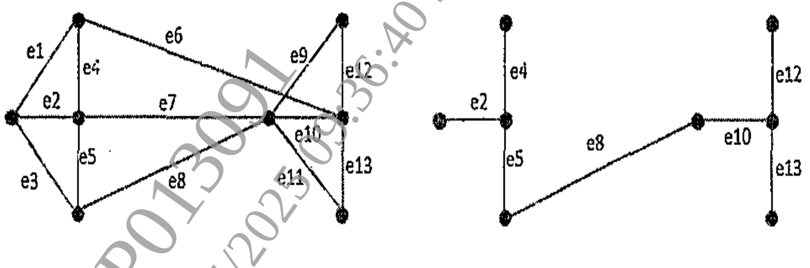
i) Complete Graph

ii) Regular Graph

iii) Bipartite Graph

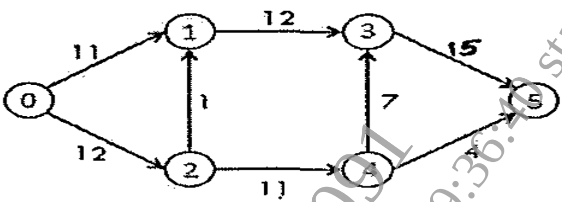
P.T.O.

b) Determine the Fundamental system of Cutsets for the following Graph G with respect to the given spanning tree T. **[6]**



Graph G Spanning Tree T

c) Using labelling procedure, find the max flow for the following transport network. **[6]**



Q3) a) Consider the following relations on {1, 2, 3, 4}: **[6]**

- R1 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)},
- R2 = {(1, 1), (1, 2), (2, 1)},
- R3 = {(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)},
- R4 = {(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)},
- R5 = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)},
- R6 = {(3, 4), (4, 3)}.

Which of these relations are reflexive or irreflexive or neither?

b) Solve the following recurrence relation **[6]**

$ar+7a_{r-1}+10a_{r-2} = 2^r$ where, a1=3, a2=6

c) Functions, f, g & h are defined on the set X= {1,2, 3} as

f = {(1, 2), (2,3), (3, 1)}

g = {(1, 3), (2, 1), (3, 2)}

h = {(1, 2), (2, 1), (3, 3)}

i. Find hog and goh. Are they equals?

ii. Find hogof and gohof. **[5]**

OR

Q4) a) Consider these relations on the set of integers:

$R1=\{(a, b) \mid a \leq b\}$,

$R2 = \{(a, b) \mid a>b\}$,

$R3 = \{(a, b) \mid a = b \text{ or } a = -b\}$,

$R4=\{(a,b) \mid a = b\}$,

$R5=\{(a,b) \mid a=b+1\}$,

$R6=\{(a,b) \mid a+b \leq 3\}$

Which of these relations contain each of the pairs (1, 1), (1, 2), (2, 1), (1, –1), and (2, 2)? **[6]**

b) Let R be a relation on set A= (0, 1, 2,3,4). Which ordered pairs are in the relation R represented by the matrix? M_R is as given below. List the ordered pair to find the reflexive closure and symmetric closure. **[6]**

0	1	1	0	0
1	0	0	1	1
0	1	1	1	1
0	1	1	0	0
0	0	1	0	1

c) Let A = {2, 3, 4, 5, 6} where R_1 and R_2 be the relation on A such that $R_1 = \{(a, b) \mid a - b = 2\}$ and $R_2=\{(a, b) \mid a+1 = b \text{ or } a = 2b\}$. Find R_1R_2, R_2R_1, $R_1R_2R_1$ also verify $(R_1R_2)^c = R_2^cR_1^c$ **[5]**

Q5) a) Using Euclidean Algorithm find GCD of 189 & 462. **[4]**

b) Using the Chinese Remainder Theorem find the value of X such that:

X = 1 mod 3

X = 2 mod 5

X = 9 mod 11 **[10]**

c) Determine quotient and remainder for the following:

i. 97/11

ii. -97/11 **[4]**

OR

Q6) a) Find the Euler's Totient function of the following numbers:

i. 10

ii. 100

iii. 1024 **[6]**

b) Find the multiplicative inverse of 35 mod 96 using Extended Euclidean Algorithm. **[6]**

c) Using Euler's Theorem and Binary expansion method solve the following (Show step-wise answer) 19^155 mod 55. **[6]**

Q7) a) Find the hamming distance between x and y

- x=1101010 y=1010000
- x=0111110 y=0111011
- x=00101001 y=10101011
- x=11000010 y=00100101 **[4]**

b) Let P be the set of all matrices of the form [[x x],[xx]] where x is a non-zero rational number. * is the matrix multiplication defined over P. + is an addition operation. Show that (P, +, *) is a Commutative Ring. **[10]**

c) Define Abelian Group considering all five properties. **[3]**

OR

Q8) a) Show that the (2,5) encoding function e:B2→B5 defined by e(00)=00000, e(10)=10101,e(01)=01110, e(11)=11011 is a group code. **[6]**

b) Let Z_n = {0, 1,2, ..., n-1}. Let ⊕ a binary operation on Z_n such that for a and b in Z_n

a⊕b= a+b if a + b < n = a + b – n if a + b > = n **[6]**

c) Let, *f* and *g* be two permutations on a set X = {1, 2,3,4,5, 6}. Find the product of *f* and *g* and also find the cycles in *f* and *g*. **[5]**

$$f = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 6 & 5 & 1 & 4 & 2 \end{bmatrix} \quad g = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 5 & 3 & 4 & 2 \end{bmatrix}$$

❖❖❖