Time Duration: 3Hr

Total Marks: 80

N.B.:1) Question no.1 is compulsory.
2) Attempt any three questions from Q.2to Q.6.
3) Use of statistical tables permitted.
4) Figures to the right indicate full marks.

Maximum
Marks

Q1. a) Evaluate $\int_C |z|dz$, where C is the left half of unit circle $|z| = 1$ from z= -i to z= i. **[5]**

b) If A=$\begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$, then find the eigen values of $4A^{-1} + 3A + 2I$. **[5]**

c) If the tangent of the angle made by the line of regression of y on x is 0.6 and $\sigma_y = 2\sigma_x$, find the correlation coefficient between x and y. **[5]**

d) Construct the dual of the following L.P.P. **[5]**

Minimise $z = x_2 + 3x_3$
Subject to $\qquad 2x_1 + x_2 \leq 3$
$\qquad\qquad x_1 + 2x_2 + 6x_3 \geq 5$
$\qquad\qquad -x_1 + x_2 + 2x_3 = 2$
$\qquad\qquad x_1, x_2, x_3 \geq 0$

Q2. a) Evaluate $\int_C \frac{e^{zz}}{(z+1)^4} dz$, where c is the circle $|z-1|=3$. **[6]**

b) Show that the matrix $A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$ is derogatory. **[6]**

c) For a normal variate with mean 2.5and standard deviation 3.5, find the probability that
(i) $2 \leq X \leq 4.5$, (ii) $-1.5 \leq X \leq 5.3$. **[8]**

Q3. a) The daily consumption of electric power is a random variable X with probability **[6]**

distribution function $f(x) = \begin{cases} kxe^{-\frac{x}{3}} &, x > 0 \\ 0 &, x \leq 0 \end{cases}$

Find the value of k, the expectation of k and the probability that on a given day the

electric consumption is more than expected value.

b) Solve the following L.P.P. by simplex method **[6]**
Maximise $z = 4x_1 + 10x_2$
Subject to $2x_1 + x_2 \leq 10$
$\qquad\qquad 2x_1 + 5x_2 \leq 20$
$\qquad\qquad 2x_1 + 3x_2 \leq 18$
$\qquad\qquad x_1, x_2 \geq 0$

c) Expand $f(z) = \frac{2}{(z-1)(z-2)}$ in the regions (i) $|z| < 1$ $(ii) 1 < |z| < 2 (iii) |z| > 2$. **[8]**

Q4. a) The incidence of an occupational disease in an industry is such that the workers have **[6]**
20% chance of suffering from it. What is the probability that out of 6 workers chosen
at random 4 or more will be suffering from the disease?

b) Calculate the coefficient of correlation between X and Y from the following data. **[6]**

X	3	5	4	6	2
Y	3	4	5	2	6

c) Show that the matrix $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is diagonalizable. Find the transforming **[8]**

matrix M and the diagonal form D.

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Q5.a) Can it be concluded that the average life- span of an Indian is more than 70 years, if a random sample of 100 Indians has an average life span of 71.8 years with standard deviation 8.9 years? [6]

b) Evaluate $\int_0^{2\pi} \frac{d\theta}{3+2\cos\theta}$, using Cauchy's residue theorem. [6]

c) Using the Kuhn – Tucker conditions, solve the following N.L.P.P. [8]

Maximise $z = x_1{}^2 + x_2{}^2$
Subject to $x_1 + x_2 - 4 \leq 0$
$2x_1 + x_2 - 5 \leq 0$
$x_1, x_2 \geq 0$

Q6.a) A die was thrown 132 times and the following frequencies were observed. [6]

No obtained	1	2	3	4	5	6	Total
Frequency	15	20	25	15	29	28	132

Test the hypothesis that the die is unbiased.

b) Two independent samples of sizes 8 and 7 gave the following results. [6]

Sample 1	19	17	15	21	16	18	16	14
Sample 2	15	14	15	19	15	18	16	

Is the difference between sample means significant?

b) Using Penalty (Big-M) method solve the following L.P.P. [8]
Maximise $z = 3x_1 - x_2$
Subject to $2 x_1 + x_2 \leq 2$
$x_1 + 3x_2 \geq 3$
$x_2 \leq 4$
$x_1, x_2 \geq 0$

ALL THE BEST!