Uncertainty treatment of discrete elements in Maxwell equations' resolution using the FDTD method.

Alberto Prados Pérez

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Alberto Prados Pérez

Memoria del **Trabajo Fin de Máster**.
Máster en Física y Matemáticas (FisyMat)
University of Granada.

Tutored by:

Dr. Luis Manuel Díaz Angulo

Contents

English Abstract

Analysis of subcell phenomenology in the Yee algorithm using the FDTD method...

1 Introduction to the FDTD method.

The numerical FDTD method ("finite difference time domain") enables the equations in partial derivatives resolution. As a first step, this method will be applied to one of the simplest equations, the one dimensional wave equation:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}. \tag{1.1}$$

The analytic solution of this equation is known and it is just the combination of two functions F and G in the following way:

$$u(x,t) = F(x+ct) + G(x-ct). \tag{1.2}$$

Now that we know this, finite differences will be computed. First, the following function will be expanded as a Taylor series, $u(x, t_n)$, about the point x_i to point $x_i+\Delta x$, keeping time fixed [Allen Taflove,Susan C.Hagness]:

$$u(x_i+\Delta x)|_{t_n} = u|_{x_i,t_n} + \Delta x \frac{\partial u}{\partial x}\bigg|_{x_i,\,t_n} + \frac{(\Delta x)^2}{2}\frac{\partial^2 u}{\partial x^2}\bigg|_{x_i,\,t_n} + \frac{(\Delta x)^3}{3!}\frac{\partial^3 u}{\partial x^3}\bigg|_{x_i,\,t_n} + \frac{(\Delta x)^4}{4!}\frac{\partial^4 u}{\partial x^4}\bigg|_{\xi_1,\,t_n}, \tag{1.3}$$

where the last term is the error term and point ξ_1 is located in the interval $(x_i, x_i+\Delta x)$. If series expansion is carried about point $x_i - \Delta x$, and both expresions are added, it remains:

$$u(x_i+\Delta x)\bigg|_{t_n} + u(x_i-\Delta x)\bigg|_{t_n} = 2u|_{x_i,t_n} + (\Delta x)^2\frac{\partial^2 u}{\partial x^2}\bigg|_{x_i,\,t_n} + O\left[(\Delta x)^2\right], \tag{1.4}$$

the last term being the error...

2 Maxwell's equations and the Yee algorithm.

Maxwell's equations in their general form are written in three dimensions as...

2.1 One dimensional Maxwell's equations.

One dimensional Maxwell's equations, in free space, are reduced to:

$$\frac{\partial E_x}{\partial t} = -\frac{1}{\epsilon_0}\frac{\partial H_y}{\partial z}, \tag{2.1}$$

$$\frac{\partial H_y}{\partial t} = -\frac{1}{\mu_0}\frac{\partial E_x}{\partial z}. \tag{2.2}$$

These equations are those of a plane wave propagating in the z direction with the electric field in the x direction and the magnetic field oriented in the y direction. Using the central difference approximation in the spatial and time derivatives, the following form of the equations is obtanied:

$$\frac{E_x^{n+\frac{1}{2}}(k) - E_x^{n-\frac{1}{2}}(k)}{\Delta t} = -\frac{1}{\epsilon_0}\frac{H_y^n\left(k+\frac{1}{2}\right) - H_y^n\left(k-\frac{1}{2}\right)}{\Delta x} \tag{2.3}$$

$$\frac{H_y^{n+1}\left(k+\frac{1}{2}\right) - H_y^n\left(k+\frac{1}{2}\right)}{\Delta t} = -\frac{1}{\mu_0}\frac{E_x^{n+\frac{1}{2}}(k+1) - E_x^{n+\frac{1}{2}}(k)}{\Delta x}. \tag{2.4}$$

...

2.1.1 Propagation in a dielectric medium.

2.1.2 Propagation in a lossy dielectric medium.

3 Stability in the FDTD method.

Courant condition.

4 Source types (Hard source/Soft Source).

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