Assignment 1: Floating Point Computation and Direct Linear System Solvers

MT 3X03: Scientific Computation

Due at 11:59 PM on Friday, October 24

Fall 2025

Use of Generative AI Policy

If you use it, treat generative AI as you would a search engine: you may use it to answer general queries about scientific computing, but any specific component of a solution or lines of code must be cited (see the syllabus for citation guidelines).

Submission Guidelines

Submit the following files on Avenue to Learn:

1. A PDF called `<FIRSTNAME>_<LASTNAME>_a1.pdf` (e.g., `matthew_giamou_a1.pdf`) containing all of your plots and written answers to mathematical and discussion questions (no need to include Julia code here). Show all steps in your solutions.
2. A file called `<FIRSTNAME>_<LASTNAME>_a1.jl` containing all of your Julia code solutions (we will run this with autograding scripts, so be sure to test it carefully). Use the `a1_template.jl` file we have provided as boilerplate: it contains function signatures for you to implement. Include all helper functions you implement as part of your solution in this file. **Do not use any additional `import` or `using` statements beyond what is provided in `a1_template.jl` - these will be detected and you will receive a grade of zero.**

The PDF can be any combination of typed/scanned/handwritten, so long as it is legible (you will receive a grade of zero on any section that cannot be easily understood). **Do not** submit the plotting scripts or test script provided to you. Read the syllabus to find the MSAF policy for this assignment.

This is an individual assignment. All submitted work must be your own, or appropriately cited from scholarly references. Submitting all or part of someone else's solution is an academic offence.

Problems

There are 7 problems for a total of 50 points.

Floating Point Computation

This section contains questions that you must answer with pen-and-paper mathematical analysis. You may find it useful to check your work in this section with small Julia programs, but there is no need to submit any code (i.e., Problems 1-3 will be graded based solely on your answers in `<FIRSTNAME>_<LASTNAME>_a1.pdf`.

Problem 1 (6 points)**:** Suppose you approximate e^x by its truncated Maclaurin series. For $x = 0.5$, derive how many terms of the series are needed to achieve (absolute) truncation error of less than 10^{-10}.

Problem 2 (9 points)**:** Give an example in base-10 floating point arithmetic (with precision t of your choosing) where

a) (6 points) $(a + b) + c \neq a + (b + c)$,

b) (3 points) $(a * b) * c \neq a * (b * c)$.

Direct Linear System Solvers

This section involves implementing multiple algorithms for solving square linear systems of the form $Ax = b$. Problems 4-7 will involve implementations in Julia, while Problem 8 is a mathematical analysis question. **Do not use the backslash operator or the `inv` function provided by the `LinearAlgebra` library for any of your solutions to problems in this section (you will receive a grade of zero).**

Problem 3 (5 points)**:** Implement Gaussian elimination **without** pivoting in the `gaussian_elimination` function signature. This will be tested by a script similar to `test_a1.jl`.

Problem 4 (5 points)**:** Implement backward and forward substitution in the `backward_substitution` and `forward_substitution` function signatures. These will be tested by a script similar to `test_a1.jl`.

Problem 5 (10 points)**:** Implement Gaussian elimination **with partial pivoting** via the LU-decomposition in the `lu_partial_pivoting` function signature. This will be tested by a script similar to `test_a1.jl`.

Problem 6 (5 points)**:** Implement the `lu_solve` function signature that uses the output of `lu_partial_pivoting` to solve a linear system. This will be tested in conjunction with `lu_partial_pivoting` by a script similar to `test_a1.jl`.

Problem 7 (10 points)**:** After implementing the solutions to Problems 4-7, run `plot_error.jl` after changing the `include` statement to open your solution file. Examine the four plots that this script creates and include them in your solution PDF along with your answers to the following questions:

a) (5 points) Examine the scatter plots comparing the norm of the residual and the norm of the absolute solution error as a function of the condition number of the system matrix A. Can you see any trends? How do the residual and the absolute solution error differ? How does pivoting affect the performance of Gaussian elimination? Provide a mathematical explanation for what you observe.

b) (5 points) Examine the plots comparing the mean norm of the residual and the mean norm of the absolute solution error as a function of the diagonal scaling factor $\alpha > 0$ in the modified system matrix $A + \alpha I$. Can you see any trends? How do the residual and the absolute solution error differ? How does pivoting affect the performance of Gaussian elimination? Provide a mathematical explanation for what you observe.