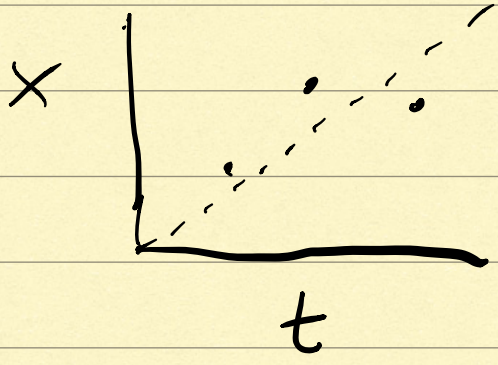
$$p(X;\theta) = L(\theta;X)$$

EX $\frac{dX}{dt} = \beta_1$, $X(0) = \beta_0$

$$X(t) = \beta_0 + \beta_1 t$$



$L = 0$ FOR ANY CHOICE OF β_0, β_1

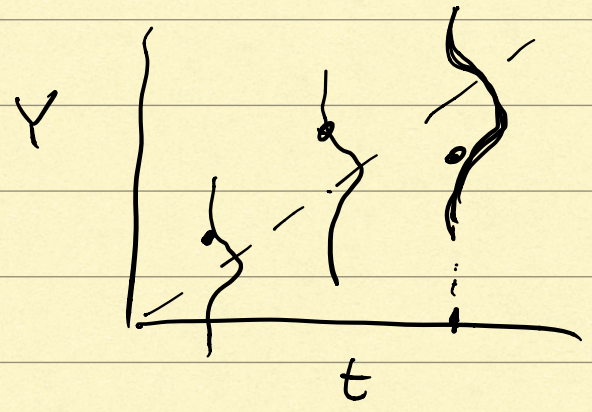
EX $\frac{dX}{dt} = \beta_1$ $X(0) = \beta_0$

$$Y(t) = X(t) + E$$

$E \sim \text{NORMAL}(0, \sigma)$

$$L(\beta_0, \beta_1, \sigma; X_i)$$

$$= \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\left(Y_i - (\beta_0 + \beta_1 t)\right)^2}{2\sigma^2} \right)$$



$$\log L = (2\pi\sigma^2)^{\frac{n}{2}} - \frac{1}{2\sigma^2} \sum \left(Y_i - (\beta_0 + \beta_1 t) \right)^2$$

$$\left.\begin{aligned} &\frac{\partial \log L}{\partial \beta_0} = 0 \\ &\frac{\partial \log L}{\partial \beta_1} = 0 \\ &\frac{\partial \log L}{\partial \sigma} = 0 \end{aligned}\right\} \quad \text{SOLVE FOR } \{\beta_0, \beta_1, \sigma\}$$

...

MAXIMIZED WHEN THE QUANTITY

$\sum \left(Y_i - (\beta_0 + \beta_1 t)\right)^2$ IS MINIMIZED

SUM OF SQUARED ERRORS (SSE) OR SUM OF SQUARED RESIDUALS (SSR)

IF THE LIKELIHOOD IS LINEAR IN β_i, THEN...

$$\text{E.g.} \quad Y = \beta_0 + \beta_1 X + \beta_2 \sqrt{X} + \epsilon$$

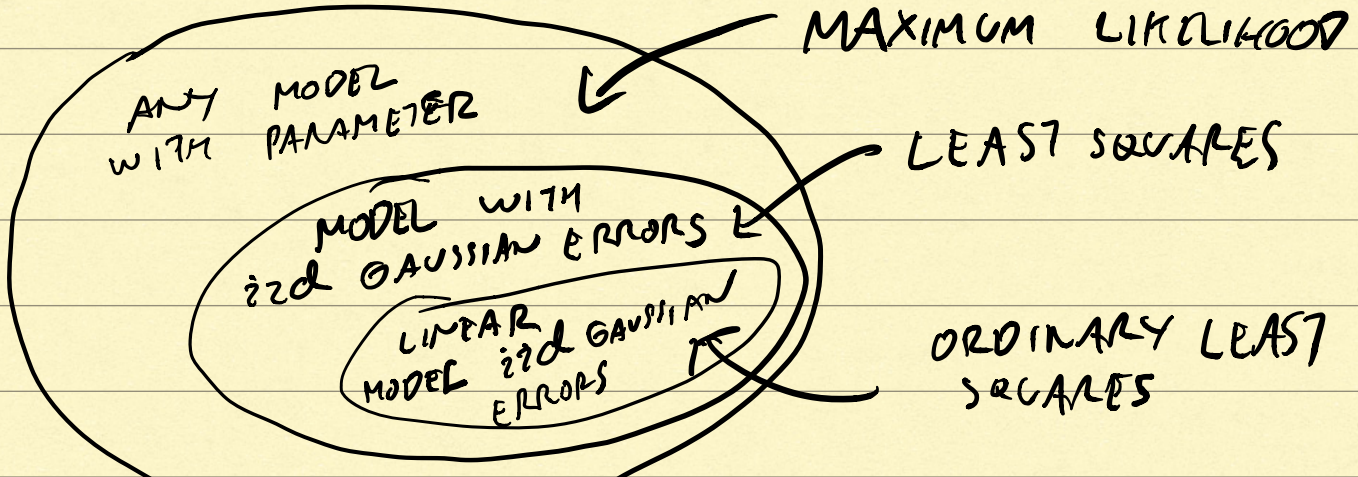
WE CAN WRITE MATRIX FORM

$$\begin{bmatrix} Y_0 \\ \vdots \\ Y_N \end{bmatrix} = \begin{bmatrix} 1 & X_1 & \sqrt{X_1} \\ 1 & X_2 & \sqrt{X_2} \\ 1 & X_3 & \sqrt{X_3} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{bmatrix}$$

DEFINE $\vec{Y} = \underline{\underline{X}} \cdot \vec{\beta} + \vec{\epsilon}$

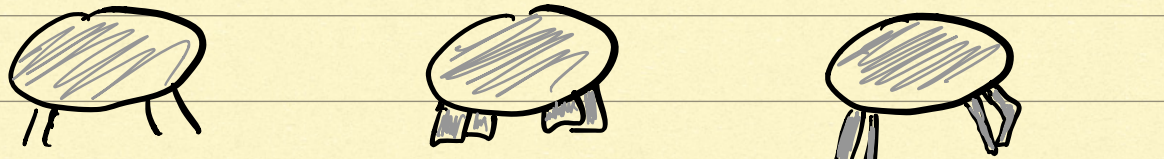
$$SSR = \vec{\epsilon}^T \cdot \vec{\epsilon}$$

$$\frac{\partial SSR}{\partial \vec{\beta}} = 0 \quad \ldots \quad \left(\underline{\underline{X}}^T \underline{\underline{X}}\right) \vec{\beta} = \underline{\underline{X}}^T \cdot \vec{Y}$$



LINEAR REGRESSION

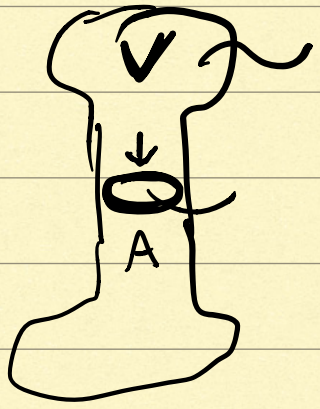
MOUSE ELEPHANT DOG



NOT TO SCALE

3 ANIMAL BODIES & LEGS

ALLOMETRIC SCALING

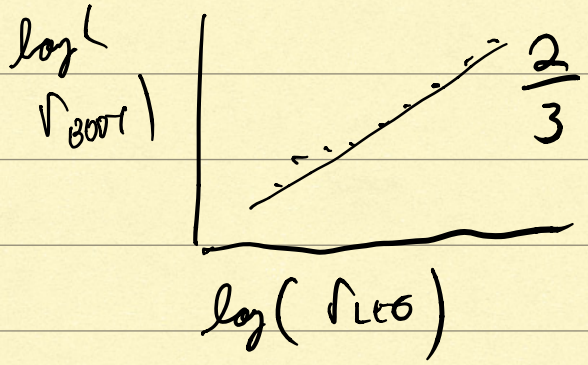


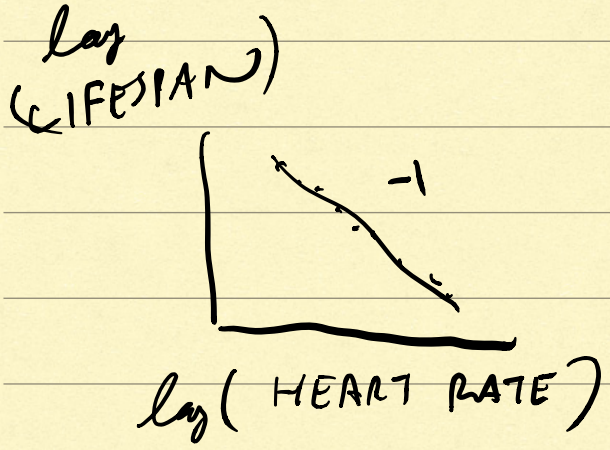
$V \sim r_{BODY}^3$

$A \sim r_{LEG}^2$

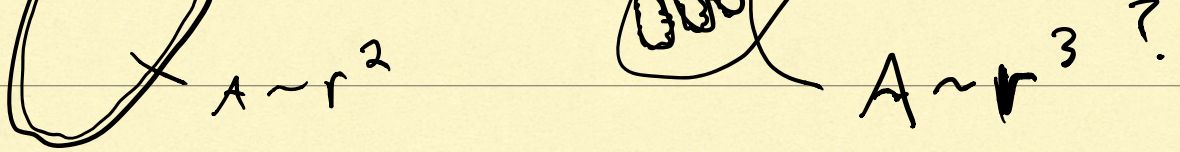
$r_{BODY}^3 \propto r_{LEG}^2$

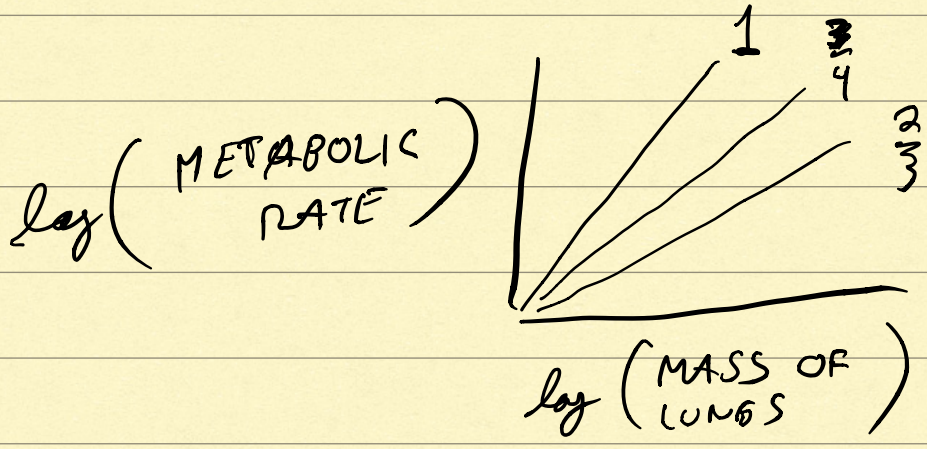
$r_{BODY} \propto r_{LEG}^{\frac{2}{3}}$





EX





EX

CELL VOLUME V_{CELL}

MITOCHONDRIA VOLUME V_{MITO}

$$Y = y_0 X^{\alpha} + \epsilon$$

↖ iid GAUSSIAN NOISE WITH STDEV σ

NOT THE SAME AS

$$\ln Y = \ln y_0 + \alpha \ln X + \epsilon$$