

- A. Many processes, including the spread of an infectious disease through a small community, can be modeled as first-order exponential processes like

$$\frac{dY}{dt} = (R - 1)Y \quad Y(0) = 1$$

where  $R$  is a constant. This will either lead to exponential growth or exponential decay.

Assume instead that  $R$  is a random variable that is different for each community. Assume it has Gaussian distribution with mean 1 and standard deviation  $\sigma$ ,

$$p_R(r) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(r-1)^2/2\sigma^2}.$$

- (a) Find the probability density function  $p_Y(y, t)$  of  $Y(t)$ .

Intuitively, we expect half of the trajectories to grow exponentially, and half of the trajectories to decay exponentially.

- (b) Sketch or plot the probability density you found for  $p_Y(y, t)$ .
- (c) Is it true that half the trajectories remain above the initial condition  $Y = 1$ , and half remain below the initial condition  $Y = 1$ ?