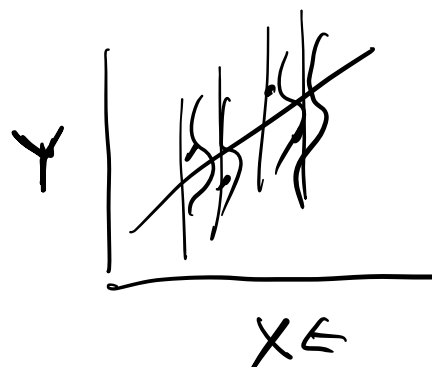IF ERRORS ARE iid NORMAL

MAXIMUM LIKELIHOOD IS
EQUIV TO LEAST SQUARES

IF (IN ADDITION) $L(\beta_i)$ IS LINEAR IN

β_i

EX. $Y = \beta_0 + \beta_1 X + \beta_2 X^2$
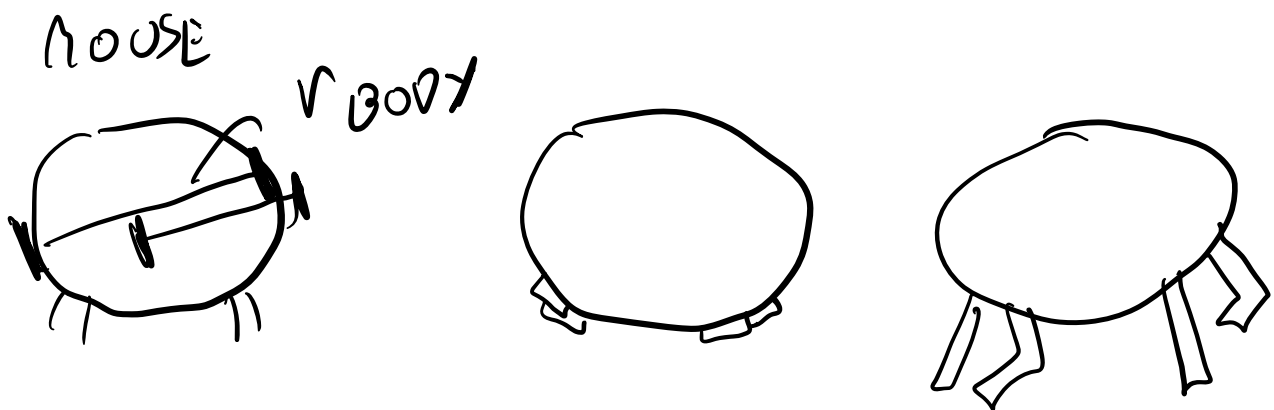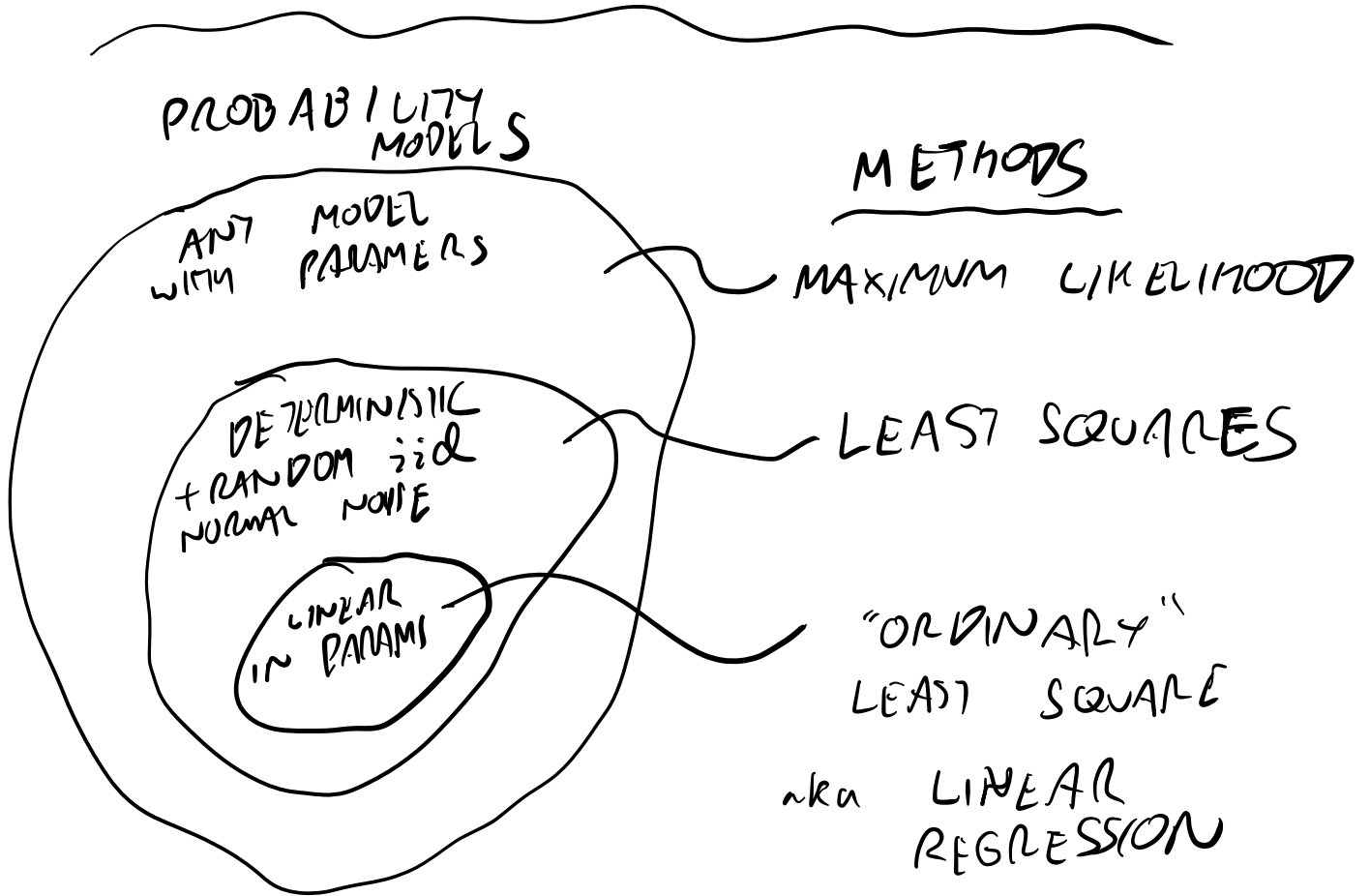
$+ \beta_3 \sqrt{X} + E_i$

↑

NORMAL
iid

THEN

$$\begin{bmatrix} Y_1 \\ \vdots \\ Y_N \end{bmatrix} = \begin{bmatrix} 1 & X_1 & X_1^2 \\ 1 & X & \\ 1 & X & \cdots \\ 1 & X & \\ 1 & X_N & X_N^2 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_3 \end{bmatrix} + \begin{bmatrix} E_1 \\ \vdots \\ E_N \end{bmatrix}$$

⟶ LINEAR ALGEBRA PROBLEM TO

MINIMIZE SSR

PROBABILITY MODELS

ANY MODEL WITH PARAMERS

DETERMINISTIC + RANDOM iid NORMAL NOISE

LINEAR IN PARAMS

METHODS

MAXIMUM LIKELIHOOD

LEAST SQUARES

"ORDINARY" LEAST SQUARE

aka LINEAR REGRESSION

MOUSE ↓ BODY

NOT TO SCALE

DOG, MOUSE, ELEPHANT

ALLOMETRIC SCALING

BONE

r — SIZE

VOLUME & MASS

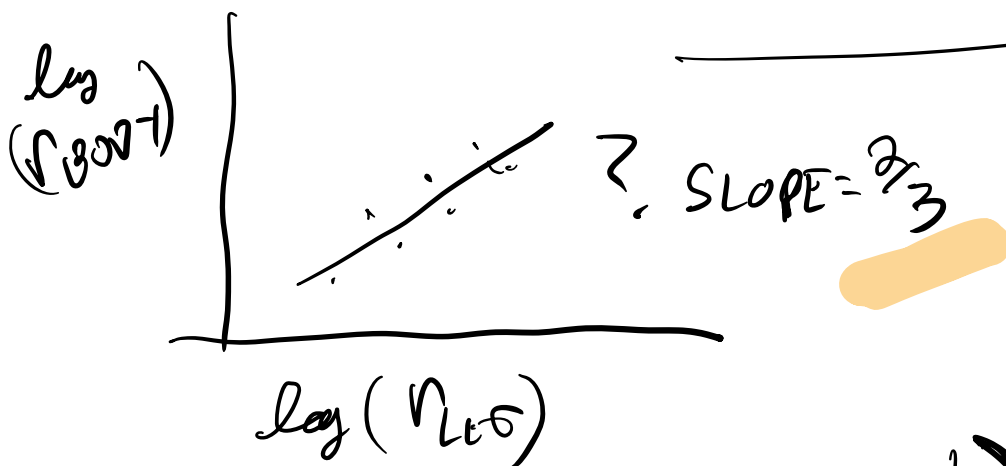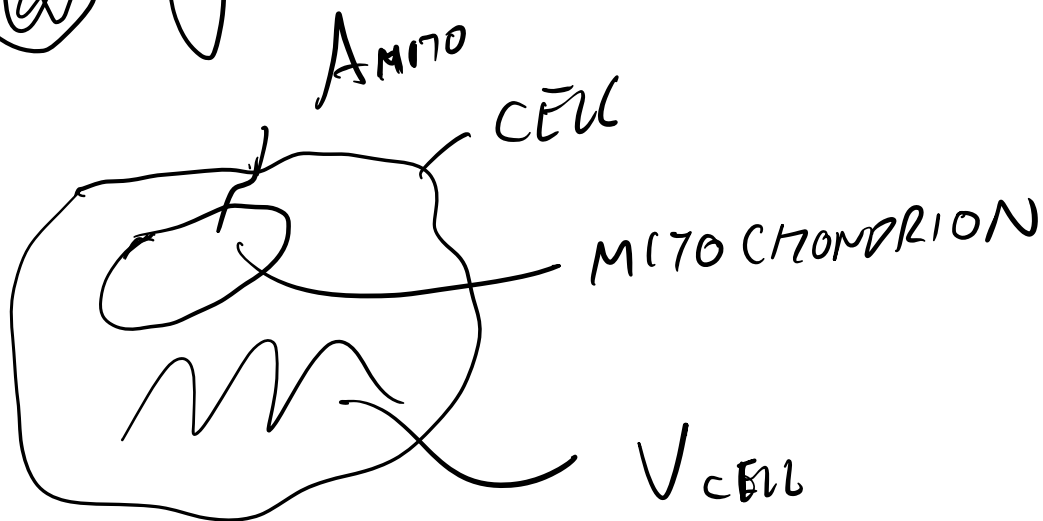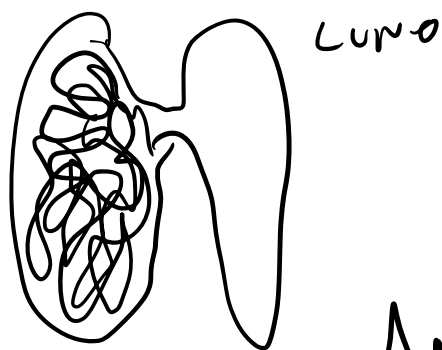$$V \propto r^3_{\text{BODY}}$$

CROSS SECTION

$$A \propto r^2_{\text{LEG}}$$

$$V \propto A$$

$$r^3_{\text{BODY}} \propto r^2_{\text{LEG}}$$

$$r_{\text{BODY}} \propto r^{2/3}_{\text{LEG}}$$

?, SLOPE $= \frac{2}{3}$

LUNG

A_{MITO}

CELL

MITOCHONDRION

V_{CELL}

$$n_{MITO} \propto r_{CELL}^{\alpha} + \epsilon \quad \text{ERRORS}$$

$Y = \log n_{MITO}$ $X = \log n_{CELL}$

$$Y \neq \alpha X$$
$$= \log(r^{\alpha} + \epsilon)$$

SYNTHETIC
DATA $\longrightarrow$ ◯ ← LEARNING

↑
GROUND
TRUTH

$\downarrow$
ESTIMATES

NLS !

$x == y$

$x = y$

$x \leftarrow y$