Thanks to social distancing and vaccine rollout, UCI has entered Phase III Research, and biologists are allowed back in their labs.

In the old days, it used to rain in California.

Suppose a biologist possesses 4 umbrellas which he employs in going from his home to his lab and vice versa. If he is at home (lab) at the beginning (end) of his day and it is raining, then he will take an umbrella with him to the lab (home), provided there is one to be taken. If it is not raining, then he never takes an umbrella. Assume that, independent of the past, it rains at the beginning (end) of a day with probability $p = 0.1$.

The biologist currently has 2 umbrellas at home and 2 at the lab. How long, on average, until he gets wet?

Hint preamble: I suggest you think and discuss how you would approach this problem before looking at the hint. What are the states of the Markov chain you would define?

Hint: Define a Markov chain with 6 states, where state i is having $i-1$ umbrellas with him (i.e., it is the number of umbrellas at home if he is at home, and the number in the lab if he is at the lab). The sixth state is the one in which he had zero umbrellas with him, and it rained, and therefore he gets wet. Write down the Markov transition matrix for this Markov chain.