

- A. Many processes, including the spread of an infectious disease through a small community, can be modeled as first-order exponential processes like

$$\frac{dY}{dt} = (R - 1)Y \quad Y(0) = 1$$

where R is a constant. This will either lead to exponential growth or exponential decay.

Assume instead that R is a random variable that is different for each community. Assume it has Gaussian distribution with mean 1 and standard deviation σ ,

$$p_R(r) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(r-1)^2/2\sigma^2}.$$

Find the probability density function $p_Y(y, t)$ of $Y(t)$.

Intuitively, we expect half of the trajectories to grow exponentially, and half of the trajectories to decay exponentially.

Sketch or plot the probability density you found for $p_Y(y, t)$. Does it agree with your intuition?