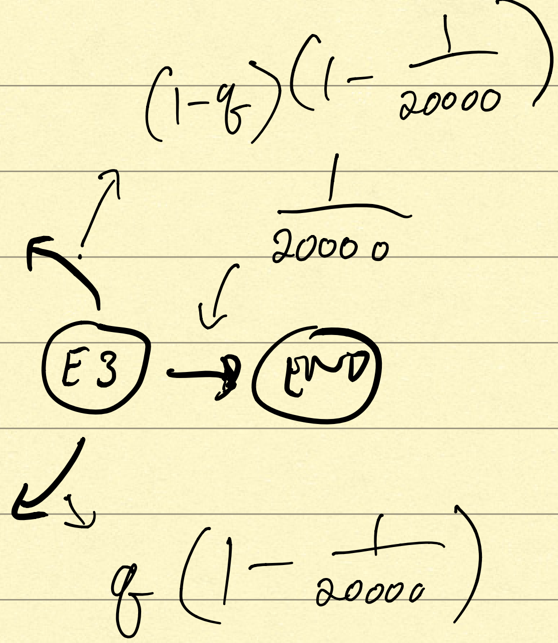
Hi AGAIN!

PS2



MEMORYLESS PROPERTY

$$\mathbb{P}(X_{n+1} \mid X_n, X_{n-1}, X_{n-2} \ldots) = \underline{\mathbb{P}(X_{n+1} \mid X_n)}$$

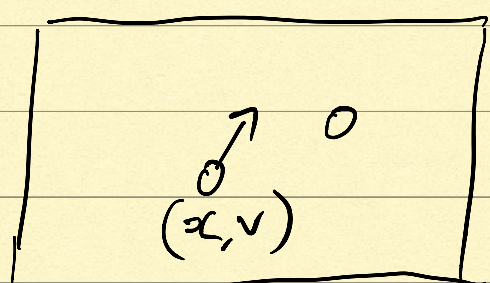
EX. 3-STATE SYSTEM THAT DEPENDS ON LAST 2 STATES

$$\mathbb{P}(X_{n+1} \mid X_n, X_{n-1})$$

- REDEFINE STATE AS $\{X_n, X_{n-1}\}$

$\Rightarrow$ 9-STATE SYSTEM

BILLARD TABLE



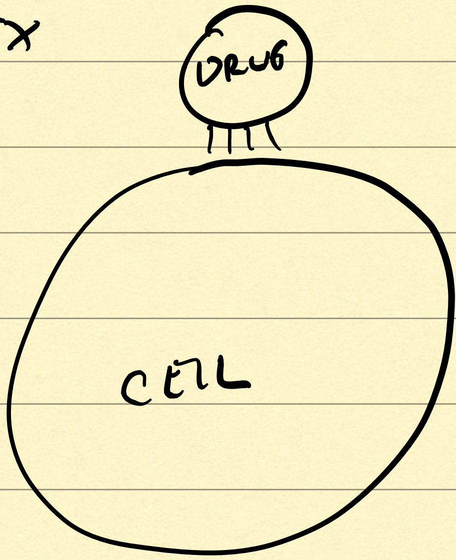
EX. COIN FLIPS

H, T, H, H, H, T, T, T, H

$\rightarrow$ TIME

MEAN FIRST PASSAGE TIMES

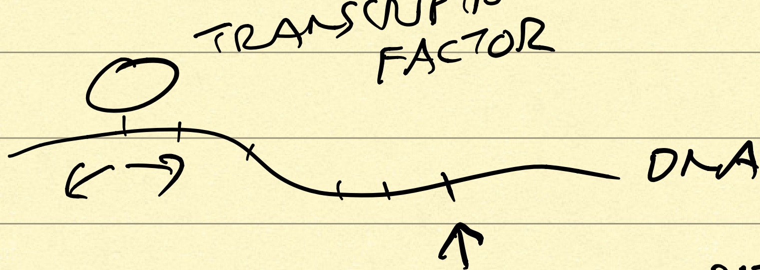
EX



IF CURRENTLY 3 "LEGS" ATTACHED, HOW LONG UNTIL 0 LEGS?

$3 \to 2 \to 1 \to 2 \to 3 \to 2 \to 1 \to 0$

EX



TARGET AT SITE j

FOR A MARKOV CHAIN

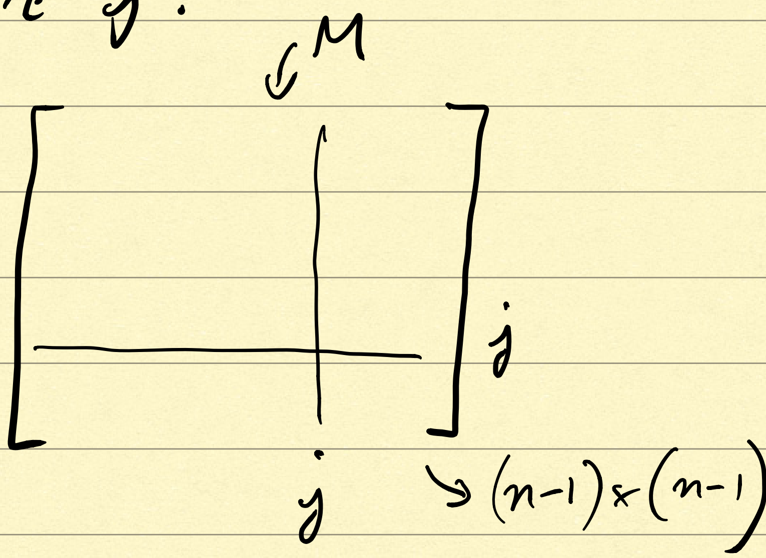
$$\vec{p}_{i+1} = M \cdot \vec{p}_i \qquad \vec{p}_0 = [\ ]$$

$n \times n$ (n STATES)

ON AVERAGE, HOW LONG FROM STATE k TO FIRST VISIT TO STATE j?

THEOREM:

LET $M_{-j} =$



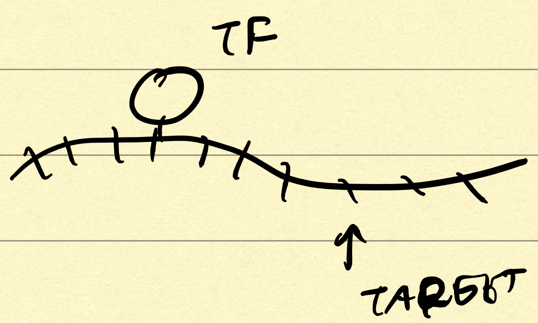
LET T_{kj} BE THE MEAN TIME FROM k TO j

$$\vec{T}_j = \begin{bmatrix} T_{1j} \\ T_{2j} \\ \vdots \\ T_{nj} \end{bmatrix}_{(n-1)}$$

THEN

$$\begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \end{bmatrix} = (M_{-j} - \underset{\text{IDENTITY MATRIX}}{I}) \cdot \vec{T}_j$$

EX



$$\vec{p}_{n+1} = \begin{bmatrix} 0.6 & 0.2 & & & 0.2 \\ 0.2 & 0.6 & 0.2 & & \\ & 0.2 & 0.6 & 0.2 & \\ & & 0.2 & 0.6 & 0.2 \\ 0.2 & & & 0.2 & 0.6 \end{bmatrix} \cdot \vec{p}_n$$

$$T_{kj} = E[T_j \mid X_0 = k]$$

$$E[T_j \mid X_0 = k] = (E[T_j \mid X_0 = k-1] + 1)\, p_{k-1}$$
$$(E[T_j \mid X_0 = k] + 1)\, p_k$$
$$(E[T_j \mid X_0 = k+1] + 1)\, p_{k+1}$$

$$T_{kj} = (T_{k-1,j} + 1)\, 0.2 +$$
$$(T_{k,j} + 1)\, 0.6 +$$
$$(T_{k+1,j} + 1)\, 0.2$$

$$-1 = 0.2 T_{k-1,j} + (0.6 - 1)\, T_{k,j} + 0.2 T_{k+1,j}$$

$$\Rightarrow \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix} = (M_{-j} - I) \cdot \vec{T}_j$$