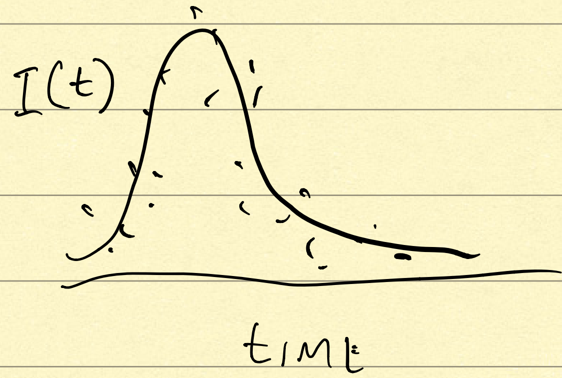
Hi EVERYONE!

WELCOME TO MATH 227C!

## MATH 227C

CONTEXT

$$\frac{d}{dt}\begin{bmatrix} S \\ E \\ I \\ R \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix}$$




COMPLEXITY: TOO COMPLEX?

TOO SIMPLE?

VARIANCE-BIAS TRADEFF

COMPLEXITY - SIMPLICITY

FIT A MODEL TO DATA, OR LEARN A MODEL FROM DATA

MAXIMUM LIKELIHOOD

BAYESIAN POSTERIORS - METROPOLIS HASTINGS


TIME TO ZERO

MEAN FIRST PASSAGE TIME

## AXIOMS OF PROBABILITY

X - RANDOM VARIABLE

$\mathcal{X}$ - STATE SPACE / SAMPLE SPACE

eg FLIP A COIN $\{H, T\}$

$\{1, 2, 3, 4\}$

$(0, +\infty)$

ELEMENTS ARE CALLED EVENTS

CAN BE COMBINED:

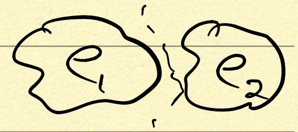
$e_1 \cup e_2$ UNION "OR"

$e_1 \cap e_2$ INTERSECT "AND"

$S \setminus e_1$ COMPLEMENT "NOT"

X HAS A PROBABILITY FUNCTION

$\mathbb{P}(e)$

- $0 \leq \mathbb{P}(e)$
- $\mathbb{P}(\text{SAMPLE SPACE}) = 1$
- IF $e_1 \cap e_2 = \text{NOTHING}$ THEN 

$$\mathbb{P}(e_1 \cup e_2) = \mathbb{P}(e_1) + \mathbb{P}(e_2)$$

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$\Rightarrow$ $\mathbb{P}(e) \leq 1$

$\mathbb{P}(\text{NOTHING}) = 0$

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FAIR DIE SAMPLE SPACE $\{1, 2, 3, 4, 5, 6\}$

$e_A = \{\text{EVEN}\} \Rightarrow \mathbb{P}(e_A) = 3/6$

$e_B = \{<3\} \Rightarrow \mathbb{P}(e_B) = 2/6$

NOTE $\mathbb{P}(e_A \cup e_B) \neq \mathbb{P}(e_A) + \mathbb{P}(e_B)$

$$\frac{4}{6} \neq \frac{3}{6} + \frac{2}{6}$$

## CONDITIONAL PROBABILITY OF A GIVEN B

$$\mathbb{P}(A \mid B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}$$

TRY $\mathbb{P}(e_A \mid e_B) = \frac{1/6}{2/6} = \frac{1}{2}$

TWO EVENTS ARE INDEPENDENT IF

$$\mathbb{P}(A \cap B) = \mathbb{P}(A) \cdot \mathbb{P}(B)$$

THEN $\mathbb{P}(A \mid B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} = \frac{\mathbb{P}(A)\cdot\mathbb{P}(B)}{\mathbb{P}(B)}$

$= \mathbb{P}(A)$

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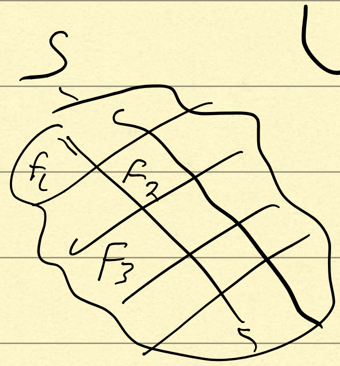
SUPPOSE SAMPLE SPACE $S$ CAN BE SPLIT UP INTO $F_1, F_2 \ldots F_N$

$\bigcup F_i = S$, $\quad F_i \cap F_j$ = EMPTY




PARTITION

$$\mathbb{P}(e) = \mathbb{P}(e \cap F_1) + \mathbb{P}(e \cap F_2) + \dots + \mathbb{P}(e \cap F_n)$$

$$= \mathbb{P}(e \mid F_1)\mathbb{P}(F_1) + \dots$$

$$\mathbb{P}(e \mid F_n)\mathbb{P}(F_1)$$

LAW OF TOTAL PROBABILITY

$$\mathbb{P}(F_j \mid e) = \frac{\mathbb{P}(F_j \cap e)}{P(e)}$$

$$= \frac{\mathbb{P}(e \mid F_j)\mathbb{P}(F_j)}{P(e)}$$

$$= \frac{\mathbb{P}(e \mid F_j)\mathbb{P}(F_j)}{\mathbb{P}(e \mid F_1)\mathbb{P}(F_1) + \dots \mathbb{P}(e \mid F_n)\mathbb{P}(F_n)}$$

BAYES!

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

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EX. HEADLINE: 54% HOSPITALIZATION

$< 50$ yrs OLD

$$P(Y \mid H)$$

HOSPITALIZATION RATE : 0.1 <50yrs

0.2 >50yrs

P(H|Y)

P(Y) = 0.70

PS1B