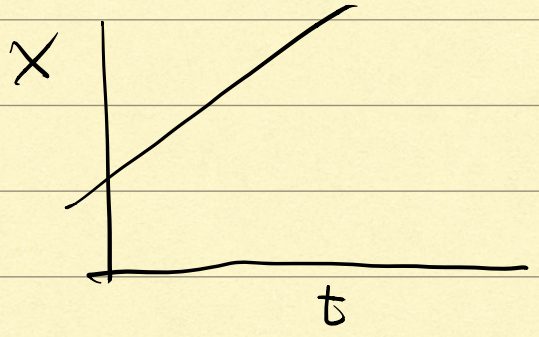
LIKELIHOOD $L(\theta) = p(x; \theta)$ (x: DATA, θ: PARAMETER)

STRATEGY

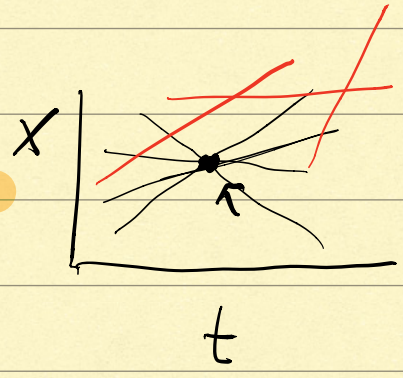
PICK PARAMETERS THAT MAXIMIZE $L(\theta)$

EX. $\frac{dx}{dt} = \beta_1 \qquad x(0) = \beta_0$

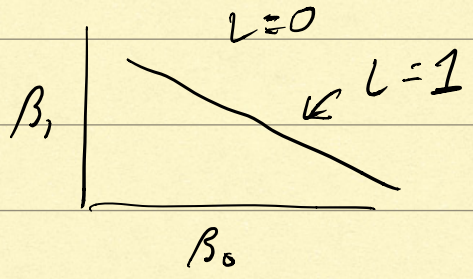
$x(t) = \beta_0 + \beta_1 t$



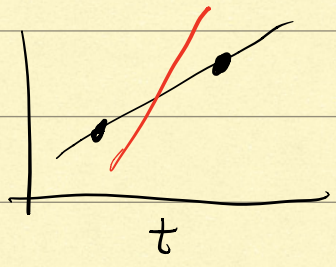
1 DATA POINT

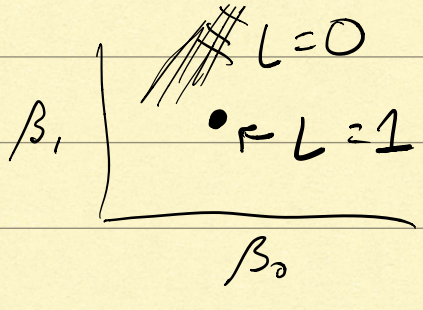


LIKELIHOOD

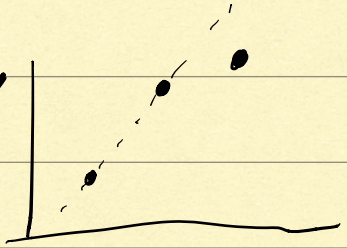


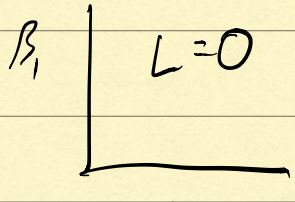
2 DATA POINT





3 DATA POINT





t β_0

NEW MODEL $\frac{dX}{dt} = \beta_1$, $X(0) = \beta_0$

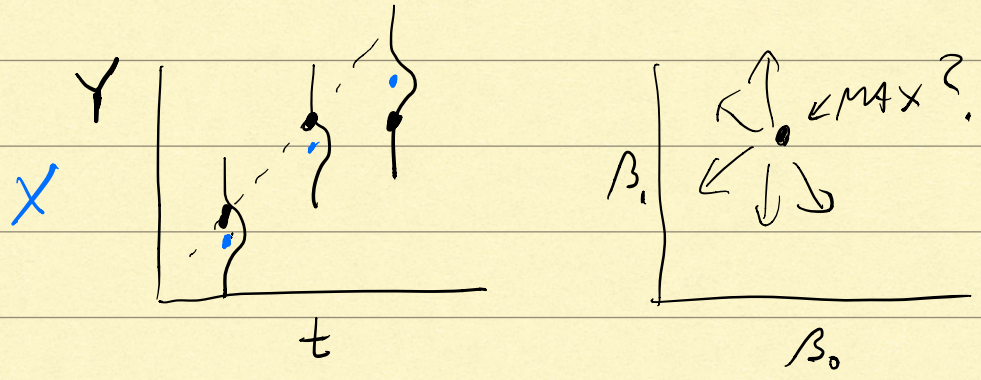
$X(t) = \beta_0 + \beta_1 t$

$Y = X + E_i$

E_i NOISE RANDOM VARIABLE

$E_i \sim \text{NORMAL}(0, \sigma) \rightarrow p_E(e) =$

σ VARIANCE



$L(\beta_0, \beta_1, \sigma) =$

$$\prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(- \frac{(Y_i - (\beta_0 + \beta_1 t))^2}{2\sigma^2} \right)$$

$\frac{\partial L}{\partial \beta_0} = 0$, $\frac{\partial L}{\partial \beta_1} = 0$, $\frac{\partial L}{\partial \sigma} = 0$ SOLVE FOR $(\beta_0, \beta_1, \sigma)$

... L IS MAXIMIZED WHEN

$$\sum_{i=1}^{N} \left(Y_i - (\beta_0 + \beta_1 t) \right)^2$$

IS MINIMIZED

SUM OF SQUARED ERRORS (SSE)

SUM OF SQUARED RESIDUALS (SSR)

MAXIMUM LIKELIHOOD (IF ERRORS ARE NORMALized) IS EQUIVALENT TO LEAST SQUARES.

IF $L(\beta_i)$ IS LINEAR IN β_i

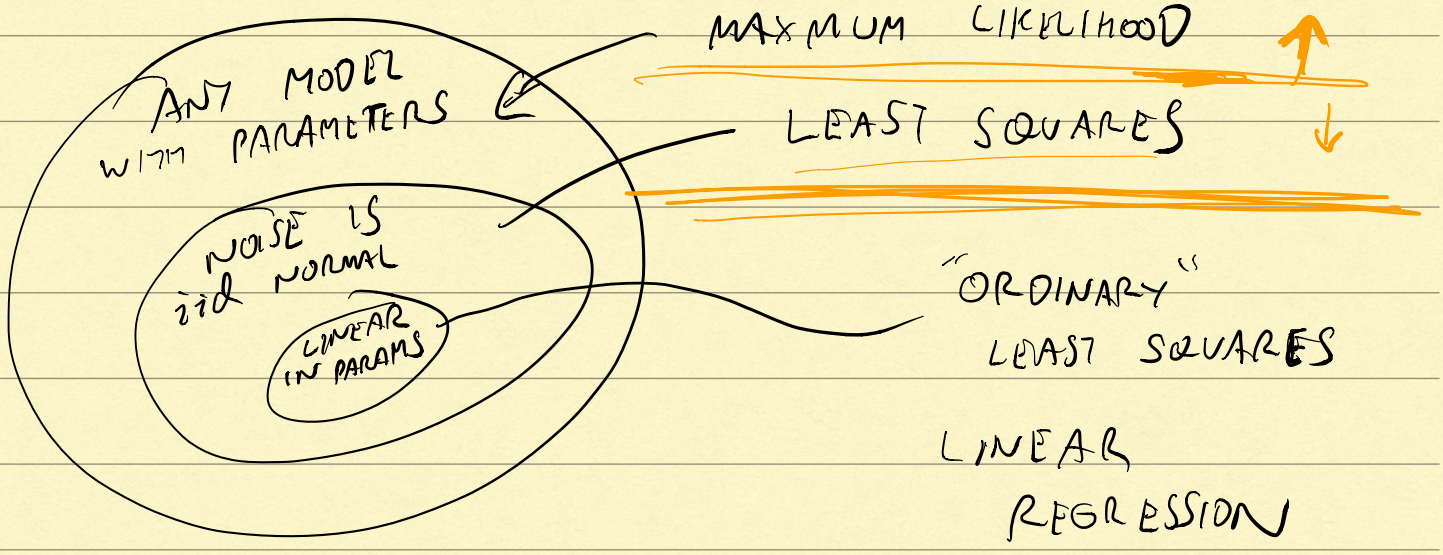
EX $Y = \beta_0 + \beta_1 X + \beta_2 \sqrt{X} + E_i$

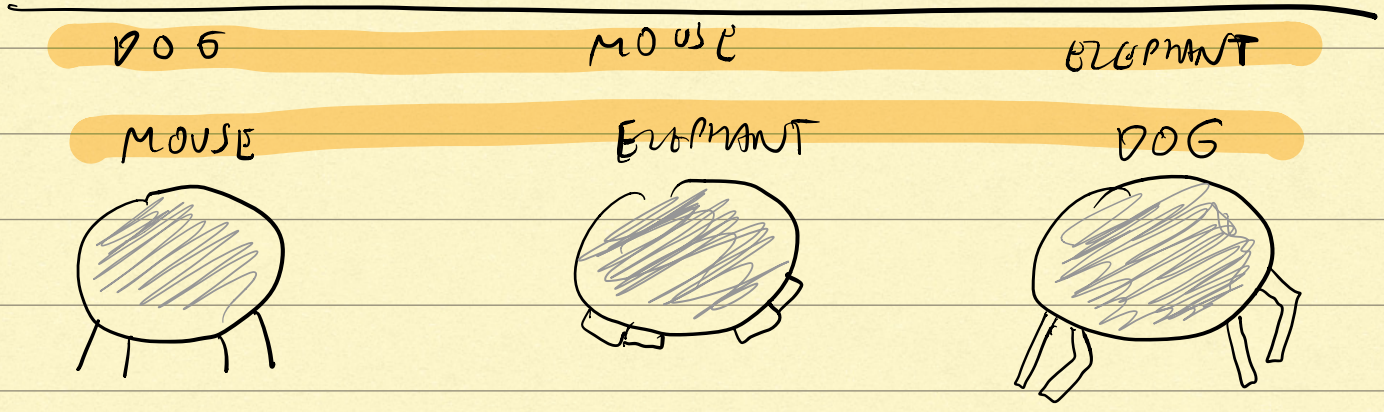
THEN
$$\begin{bmatrix} Y_0 \\ \vdots \\ Y_N \end{bmatrix} = \begin{bmatrix} 1 & X_0 & \sqrt{X_0} \\ 1 & \vdots & \vdots \\ 1 & X_N & \sqrt{X_N} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix} + \begin{bmatrix} E_0 \\ \vdots \\ E_N \end{bmatrix}$$

THEN $\vec{Y} = \underline{\underline{X}}\,\vec{\beta} + \vec{\epsilon}$ (MATRIX)

$SSR = \vec{\epsilon}^{\,T}\vec{\epsilon}$

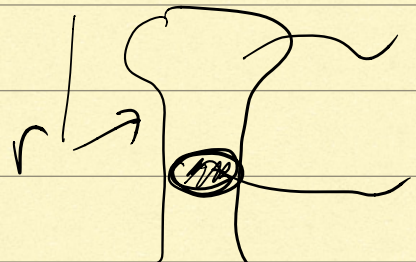
$$\frac{\partial SSR}{\partial \vec{\beta}} = 0 \Rightarrow \ldots \quad (\underline{X}^T \underline{X})\vec{\beta} = \underline{X}^T \cdot \vec{Y}$$





NOT TO SCALE

ALLOMETRIC SCALING



VOLUME $V \sim r_{BODY}^3$

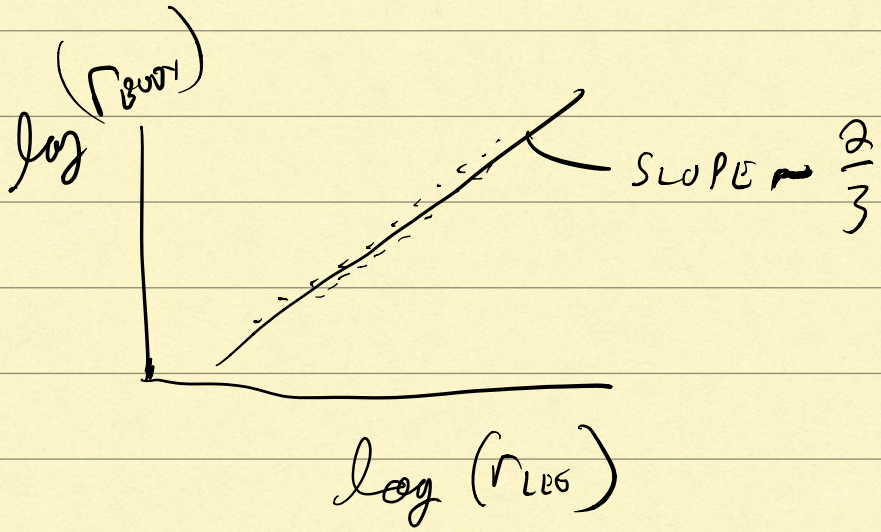
CROSS SECTION AREA $A \sim r_{LEG}^2$

BONE

$V \propto A$

$r_{BODY}^3 \propto r_{LEG}^2$

$r_{BODY} \propto r_{LEG}^{2/3}$



EX