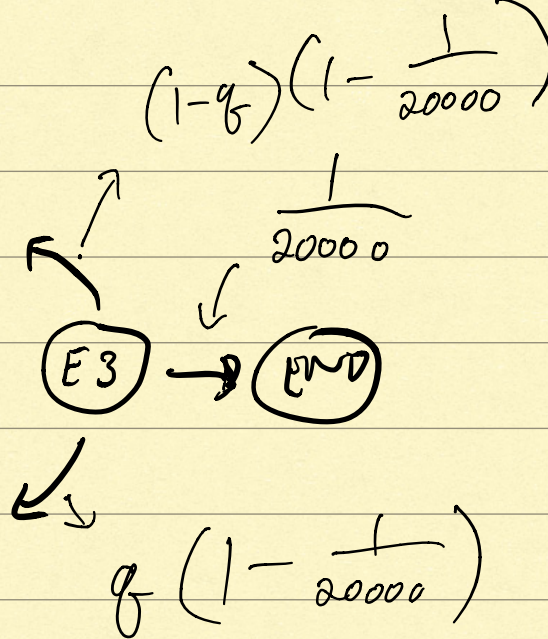


Hi AGAIN!

PS2



MEMORYLESS PROPERTY

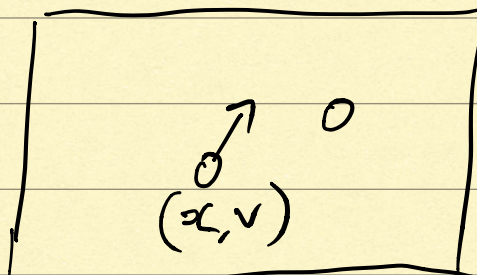
$$P(X_{n+1} | X_n, X_{n-1}, X_{n-2}, \dots) = \underline{P(X_{n+1} | X_n)}$$

EX. 3-STATE SYSTEM THAT DEPENDS ON LAST 2 STATES

$$P(X_{n+1} | X_n, X_{n-1})$$

- REDEFINE STATE AS  $\{X_n, X_{n-1}\}$   
 $\Rightarrow$  9-STATE SYSTEM

BILLIARD TABLE





EX. COIN FLIPS

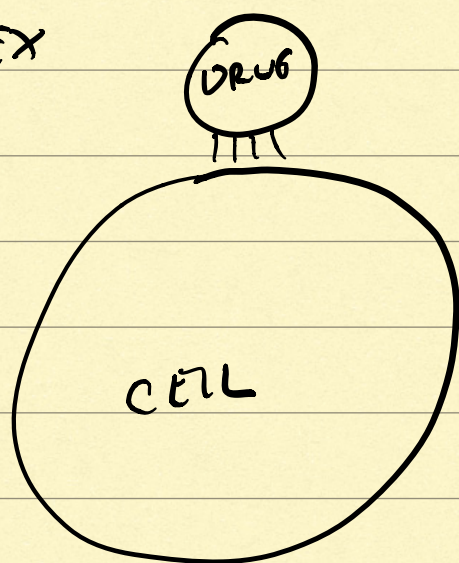


H, T, H, H, H, T, T, T, H

→ TIME

MEAN FIRST PASSAGE TIMES

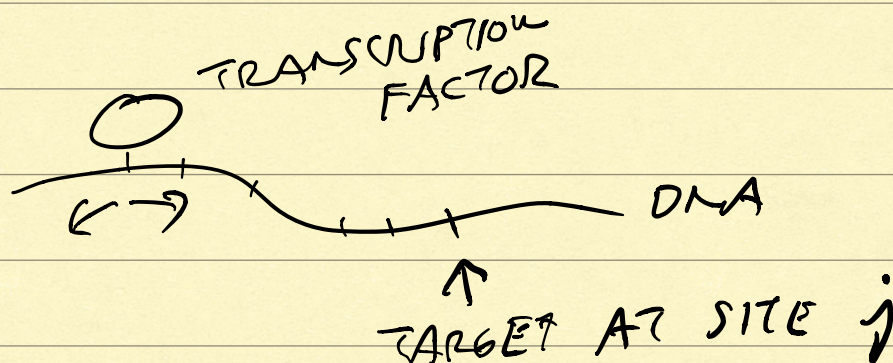
EX



IF CURRENTLY 3 "LEGS" ATTACHED,  
HOW LONG UNTIL 0 LEGS?

3 → 2 → 1 → 2 → 3 → 2 → 1 → 0

EX



FOR A MARKOV CHAIN

$$\vec{p}_{i+1} = M \cdot \vec{p}_i$$

(states)

$$\vec{p}_0 = \begin{bmatrix} \phantom{0} \\ \phantom{0} \end{bmatrix}$$



ON AVERAGE, HOW LONG FROM STATE  $k$  TO FIRST VISIT TO STATE  $j$ ?  $M$

LEMMA:

LET  $M_{-j} = \begin{bmatrix} & & \\ & & \\ \hline & & \end{bmatrix}$   $\begin{matrix} \uparrow M \\ j \\ \hookrightarrow (n-1) \times (n-1) \end{matrix}$

$$\vec{T}_j = \begin{bmatrix} T_{1j} \\ T_{2j} \\ \vdots \\ T_{nj} \end{bmatrix}_{(n-1)}$$

IDENTITY  
MATRIX

THEM  $\begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \end{bmatrix} = (M_j - I) \cdot \vec{T}_j$

↑ TARGET STATE  $j = 5$



$$\vec{p}_{n+1} = \begin{bmatrix} 0.6 & 0.2 & & & 0.2 \\ 0.2 & 0.6 & 0.2 & & \\ & & 0.2 & 0.6 & 0.2 \\ & & & 0.2 & 0.6 & 0.2 \\ 0.2 & & & & 0.2 & 0.6 \end{bmatrix} \cdot \vec{p}_n$$

$$T_{kj} = E[T_j | X_0 = k]$$

$$E[T_j | X_0 = k] = \begin{pmatrix} E[T_j | X_0 = k-1] + 1 \\ E[T_j | X_0 = k] + 1 \\ E[T_j | X_0 = k+1] + 1 \end{pmatrix} \begin{pmatrix} p_{k-1} \\ p_k \\ p_{k+1} \end{pmatrix}$$

$$T_{kj} = (T_{k-1,j} + 1) \cdot 0.2 + (T_{k,j} + 1) \cdot 0.6 + (T_{k+1,j} + 1) \cdot 0.2$$

$$-1 = 0.2 T_{k-1,j} + (0.6 - 1) T_{k,j} + 0.2 T_{k+1,j}$$

$$\Rightarrow \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix} = (M_j - I) \cdot \vec{T}_j$$


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# CONTINUOUS RANDOM VARIABLES

$X$  - SAMPLE SPACE THAT IS CONTINUOUS

$$\text{EX } S = [0, 1] \\ S = (-\infty, \infty)$$

PROBABILITY DENSITY FUNCTION  $p_X(x)$

SUCH THAT  $\mathbb{P}(A) = \int_{x \in A} p_X(x) dx$

$\uparrow$   
EVENT

$\uparrow$   
 $p_X(x)$  HAS UNITS

CUMULATIVE DISTRIBUTION

$$\mathbb{P}(X \leq x) = F_X(x)$$

FROM AXIOMS  $\int_{x \in S} p_X(x) dx = 1$

$\uparrow$   
SPACE

$$F_X(x) \rightarrow 0 \quad \text{AS } x \rightarrow -\infty$$

$$F_X(x) \rightarrow 1 \quad \text{AS } x \rightarrow +\infty$$

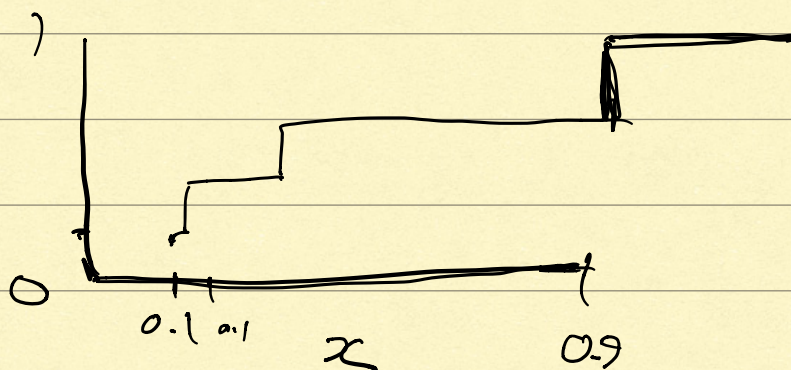


$F_X(x)$  is NON-DECREASING

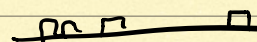
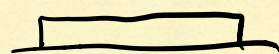
$$p_X(x) = \frac{d}{dx} F_X(x), \quad F_X(x) = \int_{-\infty}^x p_X(\tilde{x}) d\tilde{x}$$

IN PRAISE OF CUMULATIVE DISTRIBUTIONS

DATA 0.1  
 $X =$  0.11  
 0.2  
 0.9

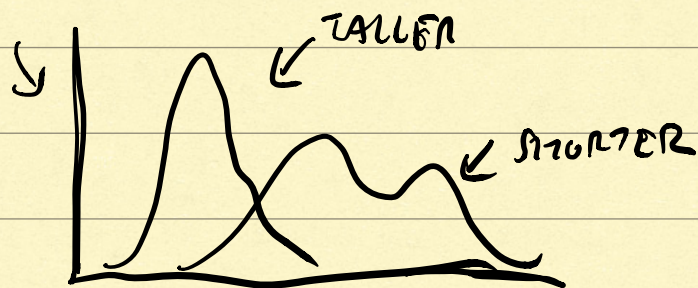


TO GET DENSITY,  
 BINNING  $\rightarrow$  HISTOGRAM

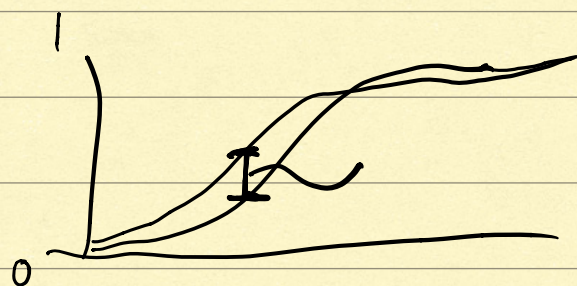


JNPS

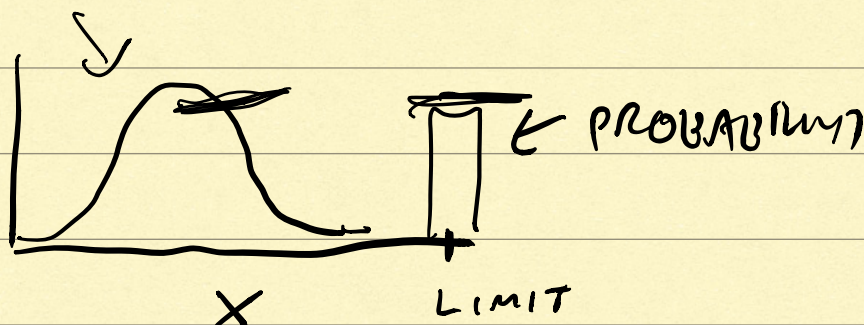
OR KERNEL SMOOTHING



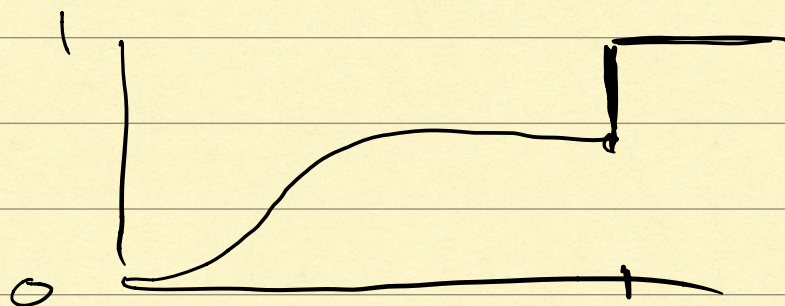
DENSITY



CUMULATIVE S







CUMULATIVE LIMIT

