IF STATE SPACE IS DISCRETE, X IS A DISCRETE RANDOM VARIABLE

$$p_X(x) = \mathbb{P}(X = x)$$

NOTATION $X \sim p_X(x)$

MOMENTS

$$E[X^n] = \sum_{i \in S} i^n p_X(i)$$

STATE SPACE

ZEROTH MOMENT

$$E[X^0] = \sum_{i \in S} p_X(i) = 1$$

FIRST MOMENT

$$E[X] = \sum_{i \in S} i \, p_X(i) = \text{MEAN}$$

SYMBOL μ_X

SECOND MOMENT

$$E[X^2] = \sum_{i \in S} i^2 p_X(i) =$$

$$E[X^2] - \mu_x^2 = E[(x-\mu)^2]$$

$= \text{VARIANCE}$

SYMBOL σ_x^2

$\sigma_x = \sqrt{\sigma_x^2}$ STANDARD DEVIATION

FAMOUS DISCRETE RANDOM VARIABLES

- UNIFORM BETWEEN a, AND b

- BERNOULLI

$X=0 \quad \mathbb{P}(X=0) = p_X(0) = 1-p$

$X=1 \quad \mathbb{P}(X=1) = p_X(1) = p$

$$E[X] = p$$

STOCHASTIC PROCESS OF INDEPENDENT BERNOULLI TRIALS

$$X_t = \{\underbrace{0\ 0\ 0\ 1}_{1}\ 1\ 0\ 0\ 0\ 1\ 0\ 0\ \overbrace{0\ 0\ 0 \ldots}^{n}\}$$

1

- GEOMETRIC

$$p_X(k) = (1-p)^{k-1} \cdot p \qquad k = 0, 1, 2 \ldots$$

ATTEMPTS IN A SERIES OF BERNOULLI TRIALS UNTIL FIRST

SUCCESS $X=1$

$$E[X] = \frac{1}{p}$$

- BINOMIAL

"CHOOSE"

$$p_X(k) = \binom{n}{k} p^k (1-k)^{n-k}$$

$\boxed{\text{\# OF SUCCESSES } (X=1) \text{ IN A SERIES OF } n \text{ BERNOULLI TRIALS}}$

RECAP

$$\begin{bmatrix} p_A(t+1) \\ p_B(t+1) \\ p_C(t+1) \end{bmatrix} = \begin{bmatrix} p_{A\to A} & p_{B\to A} & p_{C\to A} \\ p_{A\to B} & p_{B\to B} & p_{C\to B} \\ p_{A\to C} & p_{B\to C} & p_{C\to C} \end{bmatrix} \cdot \begin{bmatrix} p_A(t) \\ p_B(t) \\ p_C(t) \end{bmatrix}$$

$\underline{M}$

COLUMNS MUST SUM TO UNITY

ASIDE REDNER

$$M \vec{p}^{\,t+1} = \vec{p}^{\,t}$$

TRANSPOSE!

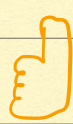
$$\vec{p}^{\,t+1} = M \cdot \vec{p}^{\,t}$$

PS2

$$
\begin{array}{ccccccc}
S & E1 & E2 & E3 & I & END & \\
\left[\begin{array}{cccccc}
0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & \boxed{\frac{20000}{20011}} & \boxed{} & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & \boxed{\frac{10}{20011}} & \boxed{} & 0 \\
0 & 0 & 0 & \boxed{\frac{1}{20011}} & 0 & 1
\end{array}\right] & & & & & & \begin{array}{c} S \\ E1 \\ E2 \\ E3 \\ I \\ END \end{array}
\end{array}
$$

$$
\begin{bmatrix} 0 \\ \frac{19989}{20000} \\ 0 \\ 0 \\ \frac{10}{20000} \\ \frac{1}{20000} \end{bmatrix}
$$

$$
\begin{bmatrix} 0 \\ \frac{19999}{20010} \\ 0 \\ 0 \\ ? \\ ? \end{bmatrix}
$$

 2x