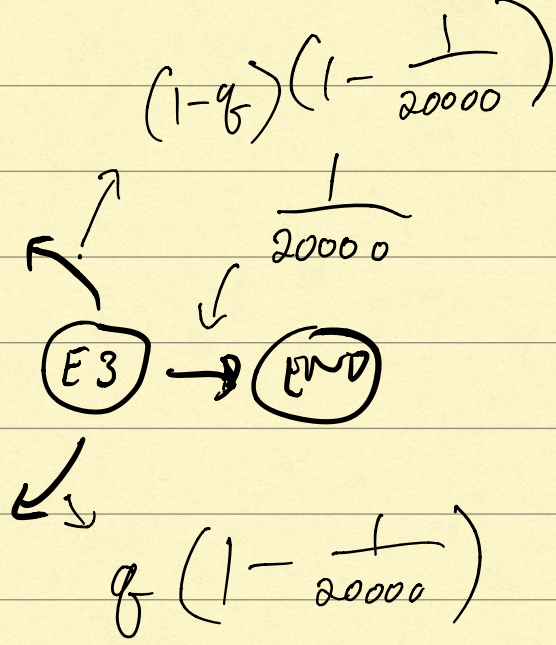
Hi AGAIN!

PS2



MEMORYLESS PROPERTY

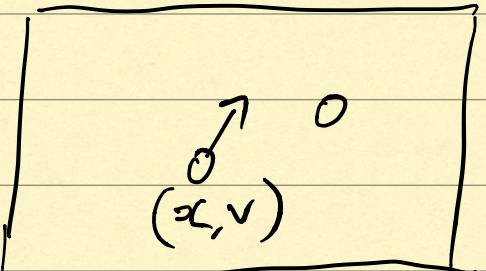
$$\mathbb{P}(X_{n+1} \mid X_n, X_{n-1}, X_{n-2} \ldots) = \mathbb{P}(X_{n+1} \mid X_n)$$

EX. 3-STATE SYSTEM THAT DEPENDS ON LAST 2 STATES

$$\mathbb{P}(X_{n+1} \mid X_n, X_{n-1})$$

- REDEFINE STATE AS $\{X_n, X_{n-1}\}$
  $\Rightarrow$ 9-STATE SYSTEM

BILLIARD TABLE



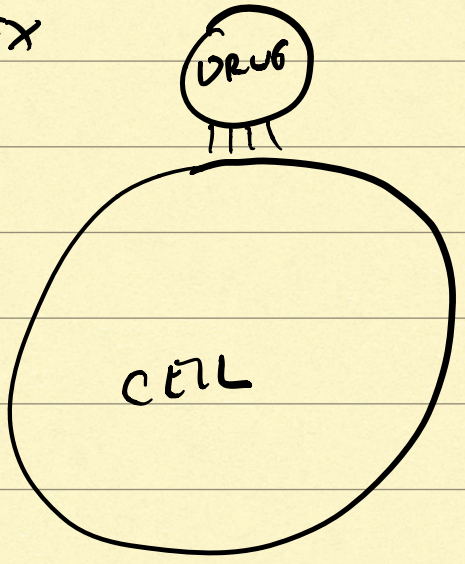
EX. COIN FLIPS

H, T, H, H, H, T, T, T, H

→ TIME

MEAN FIRST PASSAGE TIMES

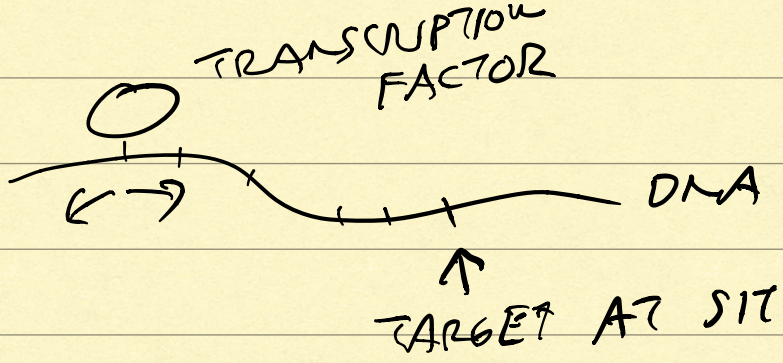
EX



IF CURRENTLY 3 "LEGS" ATTACHED, HOW LONG UNTIL 0 LEGS?

3 → 2 → 1 → 2 → 3 → 2 → 1 → 0

EX



FOR A MARKOV CHAIN

$$\vec{p}_{i+1} = M \cdot \vec{p}_i \qquad \vec{p}_0 = \begin{bmatrix} \\ \\ \end{bmatrix}$$

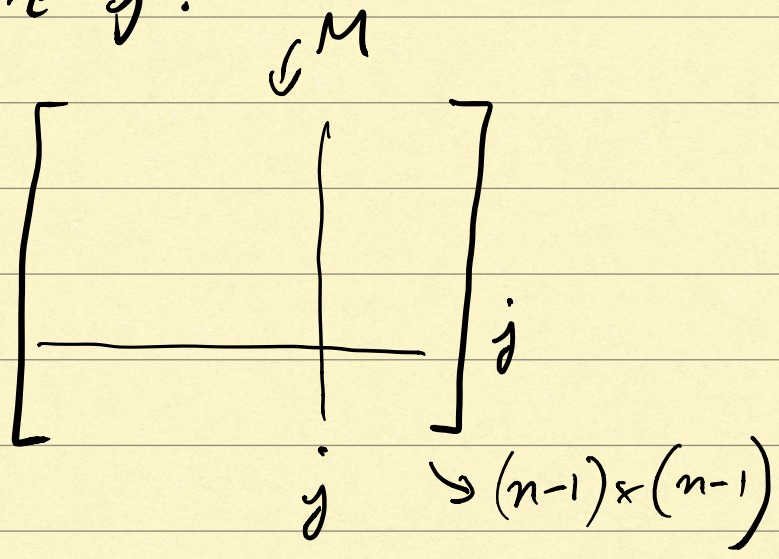
(STATES)

$n \times n$ (n STATES)

ON AVERAGE, HOW LONG FROM STATE k TO FIRST VISIT TO STATE j?

THEOREM:

LET $M_{-j} =$



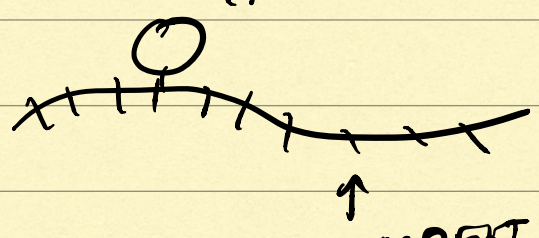
LET T_{kj} BE THE MEAN TIME FROM k TO j

$$\vec{T}_j = \begin{bmatrix} T_{1j} \\ T_{2j} \\ \vdots \\ T_{nj} \end{bmatrix}_{(n-1)}$$

THEN $\begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \end{bmatrix} = (M_{-j} - I) \cdot \vec{T}_j$

(I: IDENTITY MATRIX)

EX



$j = 5$

$$\vec{\rho}_{n+1} = \begin{bmatrix} 0.6 & 0.2 & & & 0.2 \\ 0.2 & 0.6 & 0.2 & & \\ & 0.2 & 0.6 & 0.2 & \\ & & 0.2 & 0.6 & 0.2 \\ 0.2 & & & 0.2 & 0.6 \end{bmatrix} \cdot \vec{\rho}_n$$

$$T_{kj} = E[T_j \mid X_0 = k]$$

$$E[T_j \mid X_0 = k] = \left(E[T_j \mid X_0 = k-1] + 1\right) \rho_{k-1}$$
$$\left(E[T_j \mid X_0 = k] + 1\right) \rho_k$$
$$\left(E[T_j \mid X_0 = k+1] + 1\right) \rho_{k+1}$$

$$T_{kj} = \left(T_{k-1,j} + 1\right) 0.2 +$$
$$\left(T_{k,j} + 1\right) 0.6 +$$
$$\left(T_{k+1,j} + 1\right) 0.2$$

$$-1 = 0.2 T_{k-1,j} + (0.6 - 1) T_{k,j} + 0.2 T_{k+1,j}$$

$$\Rightarrow \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix} = (M_{-j} - I) \cdot \vec{T}_j$$

CONTINUOUS RANDOM VARIABLES

X - SAMPLE SPACE THAT IS CONTINUOUS

EX $S = [0,1]$
$S = (-\infty, \infty)$

PROBABILITY DENSITY FUNCTION $p_x(x)$

SUCH THAT $\mathbb{P}(A) = \int_{x \in A} p_x(x)\,dx$

A: EVENT

$p_x(x)$ HAS UNITS

CUMULATIVE DISTRIBUTION

$$\mathbb{P}(X \leq x) = F_x(x)$$

FROM AXIOMS $\int_{x \in S} p_x(x)\,dx = 1$

S: SPACE

$F_x(x) \to 0$ AS $x \to -\infty$

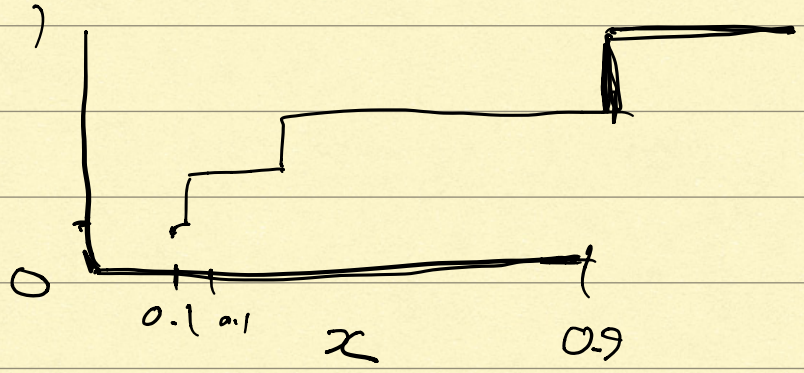
$F_x(x) \to 1$ AS $x \to +\infty$

$F_x(x)$ IS NON-DECREASING

$$p_x(x) = \frac{d}{dx} F_x(x), \qquad F_x(x) = \int_{-\infty}^{x} p_x(\tilde{x})\, d\tilde{x}$$

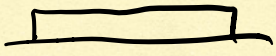
IN PRAISE OF CUMULATIVE DISTRIBUTIONS

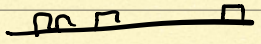
DATA $X =$ 0.1, 0.11, 0.2, 0.9



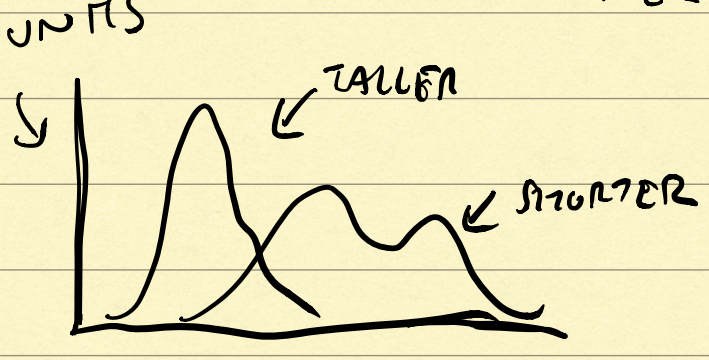
TO GET DENSITY,

BINNING → HISTOGRAM

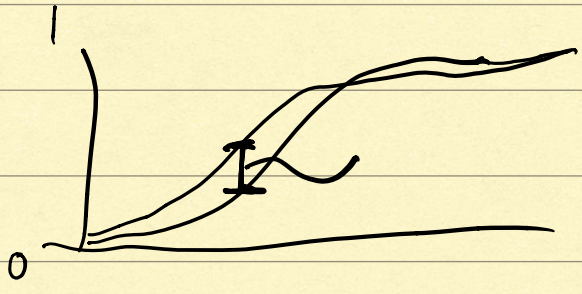




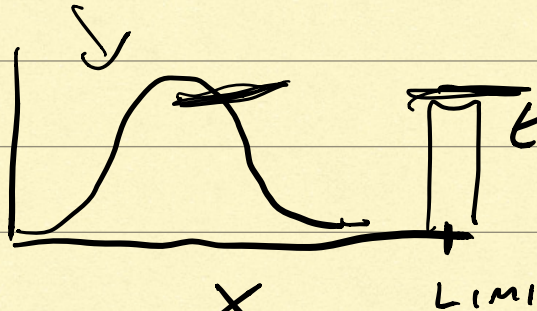
OR KERNEL SMOOTHING



DENSITY



CUMULATIVES



CUMULATIVE LIMIT