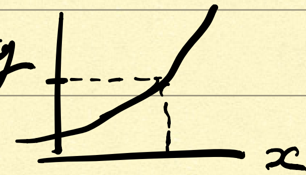
$X \sim p_X(x)$ AND $Y = g(X)$ THEN

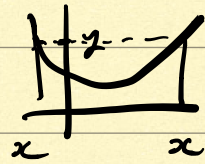
$$p_Y(y) = \sum_k p_X(g_k^{-1}(y)) \cdot \left(\left| \frac{dg}{dx_k} \right| \right)^{-1}$$

WHERE $\{x_k\}$ IS THE PRE-IMAGE OF y

IF $g(x)$ IS MONOTONIC



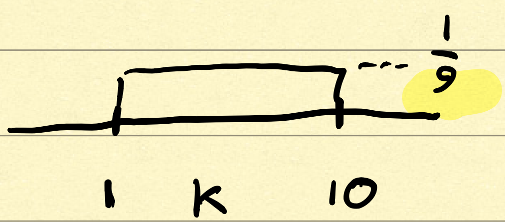
(NOT



)

THEN $p_Y(y) = p_X(g^{-1}(y)) \left| \frac{dg}{dx} \right|^{-1}$ ← $x = g^{-1}(y)$

EX



$p_K(k)$

$K \sim$ UNIFORM $[1, 10]$

K - RATE

T - MEAN TIME

$T = \frac{1}{K}$

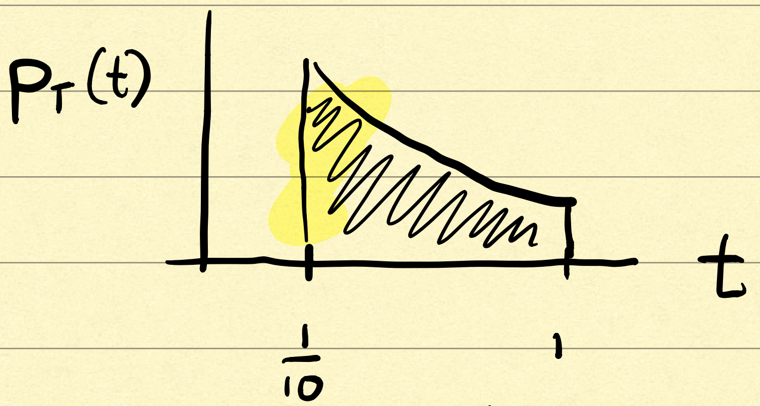
WHAT IS $p_T(t)$?

$$g(k) = \frac{1}{k} \qquad g^{-1}(t) = \frac{1}{t}$$

$$\frac{dg}{dk} = \frac{-1}{k^2}$$

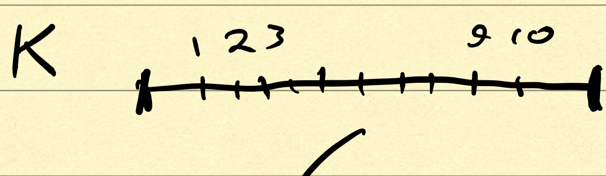
$$\left.\frac{dg}{dk}\right|_{k=g^{-1}(t)} = -t^2$$

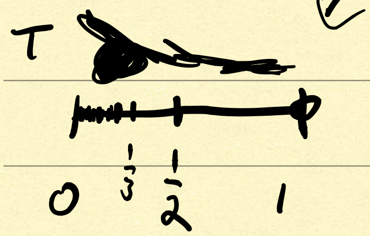
$$p_T(t) = \begin{cases} \frac{1}{9} \cdot t^{-2} & \frac{1}{10} < t < 1 \\ 0 & \text{ELSE} \end{cases}$$



CHECK : $\displaystyle\int_{\frac{1}{10}}^{1} p_T(t)\,dt = \int_{\frac{1}{10}}^{1} \frac{1}{9} t^{-2} dt = -\frac{1}{9}\frac{1}{t}\Bigg|_{t=\frac{1}{10}}^{t=1}$

$$= \frac{1}{9}(10-1) = 1 \quad ✓$$





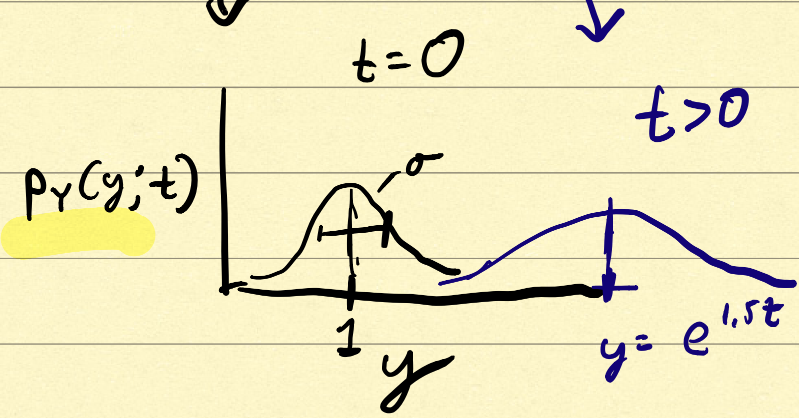
EX HETEROGENITY / PARAMETRIC NOISE

$$\frac{dY}{dt} = (R-1)Y \qquad R = 2.5$$

$$Y(0) = A \qquad A \sim p_A(a)$$

$$p_A(a) = \frac{1}{\sqrt{2\pi}\,\sigma_0}\, e^{-\frac{(a-1)^2}{2\sigma_0^2}}$$

$$Y(t) = A e^{+1.5t}$$



$$Y = g(A) = g(A;t)$$

$$g^{-1}(y) = \frac{y}{e^{+1.5t}} = y e^{-1.5t}$$

$$\frac{dg}{da} = e^{1.5t}$$

$$\left.\frac{dg}{da}\right|_{a=g^{-1}(y)} = e^{1.5t} \qquad p_X(g^{-1}(y)) \left|\frac{dg}{da}\right|^{-1}$$

$$p_Y(y;t) = \frac{1}{\sqrt{2\pi}\,\sigma_0}\exp\left(-\frac{(ye^{-1.5t}-1)^2}{2\sigma_0^2}\right)e^{-1.5t}$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma_0 e^{+1.5t}}\exp\left(-\frac{(y-e^{+1.5t})^2}{2(\sigma_0 e^{+1.5t})^2}\right)$$