$\emptyset$

CONTINUOUS RANDOM VARIABLES

$X \in S$

EX $S \in [0,1)$
$S \in (-\infty, \infty)$
$S \in [0, \infty)$

HAS UNITS!

$p_X(x)$

PROBABILITY DENSITY

$$\int_S p_X(x)\, dx = 1$$

SUCH THAT

$$\mathbb{P}(A) = \int_A p_X(x)\, dx$$

CUMULATIVE DISTRIBUTION

$$\mathbb{P}(X \le x) = F_X(x)$$

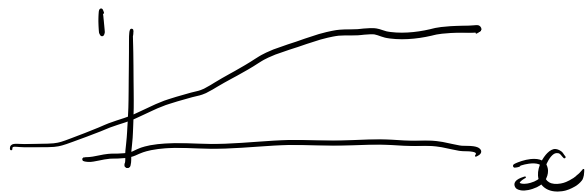
$$F_X(x) = \int^x p_X(\tilde{x})\, d\tilde{x}$$

$$\int_{-\infty}$$

$F_X(x) = 0 \qquad x \to -\infty$

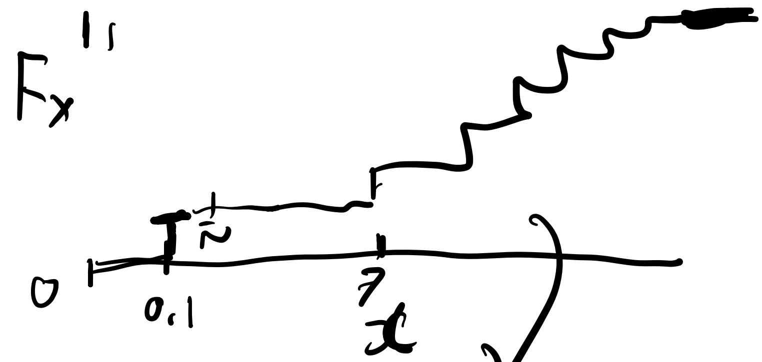
$F_X(x) = 1 \qquad x \to +\infty$

F_X IS NON-DECREASING



$$p_X(x) = \frac{d}{dx} F(x)$$

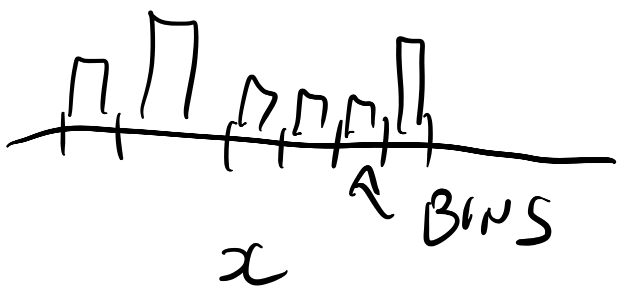
- DATA = $\begin{bmatrix} 0.1 \\ 7 \\ 32 \\ \vdots \end{bmatrix}$



EMPIRICAL

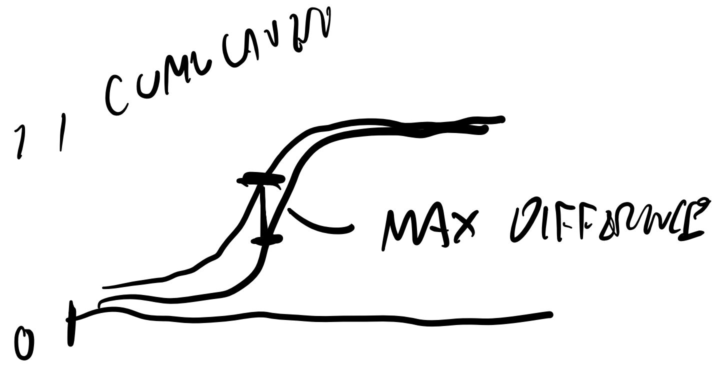
(eCDF)

DATA → DENSITY



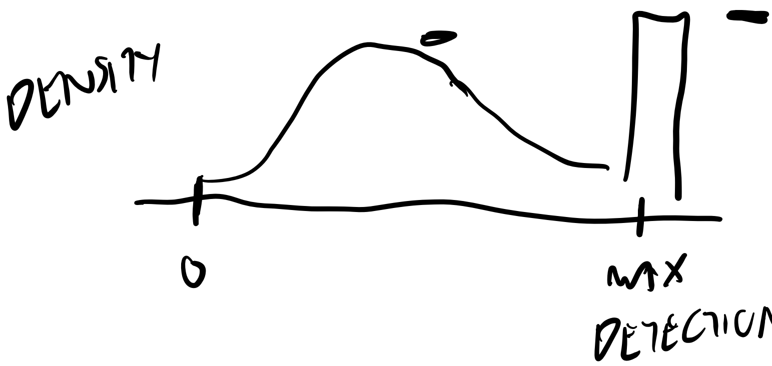
KERNEL

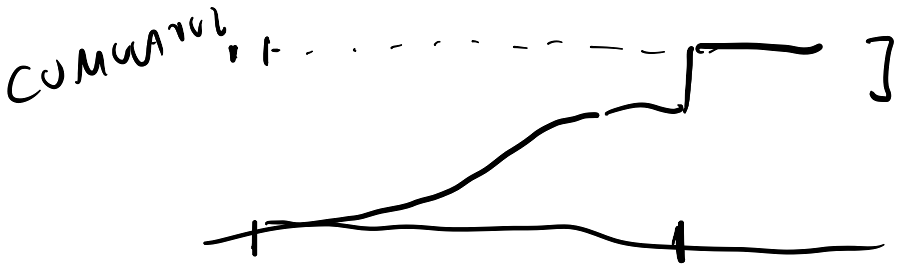
- 



KOLMOGOROV SMIRNOV

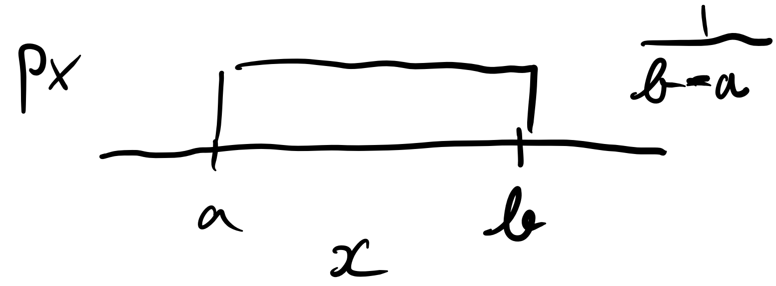
- DATA WITH CONTINUOUS + DISCRETE





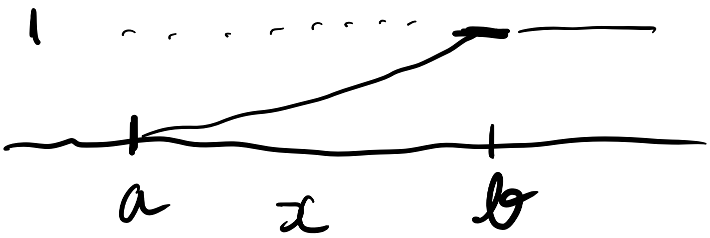
<u>FAMOUS CONTINUOUS RANDOM VARIABLES</u>

- UNIFORM $\quad X \sim \text{unif}(a, b)$

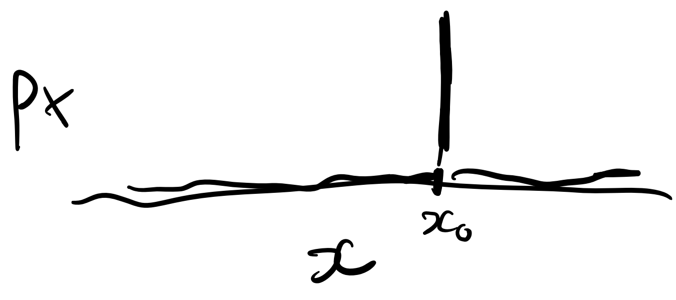


$$p_X(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{ELSE} \end{cases}$$

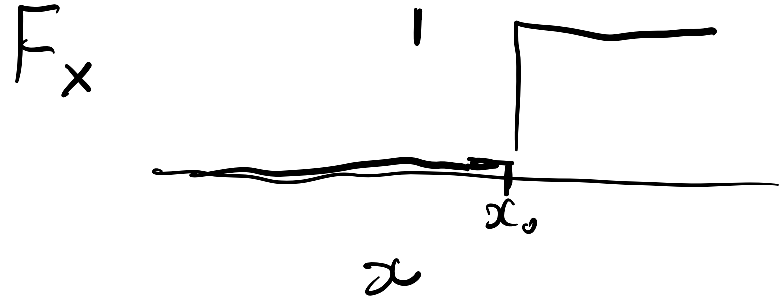
$$F_X(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a < x < b \\ 1 & x \geq b \end{cases}$$



- DELTA DIRAC



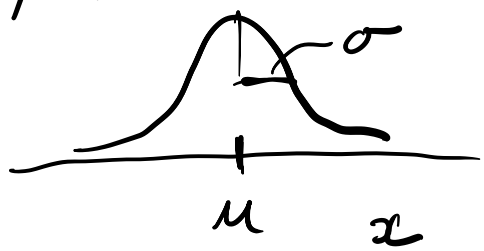
$$p_X(x) = \delta(x - x_0)$$



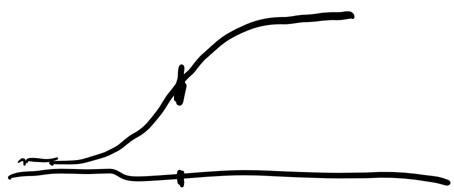
$$F_X(x) = \begin{cases} 0 & x < x_0 \\ 1 & x \geq x_0 \end{cases}$$

- GAUSSIAN $X \sim \text{NORMAL}(\mu, \sigma)$

$$p_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



$F_X(x)$

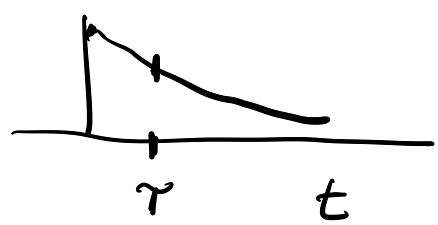


$= \mathrm{erf}$

Z - STANDARD NORMAL   NORMAL(0, 1)

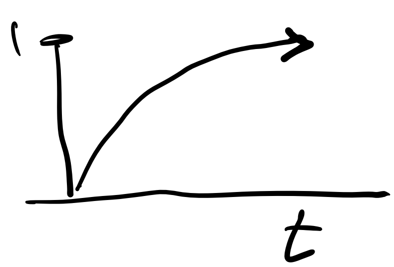
- EXPONENTIAL $T \sim \mathrm{EXP}(\lambda)$

$$p_T(t) = \lambda e^{-\lambda t} \qquad t > 0$$



$$p_T(t) = \frac{1}{\tau} e^{-t/\tau}$$

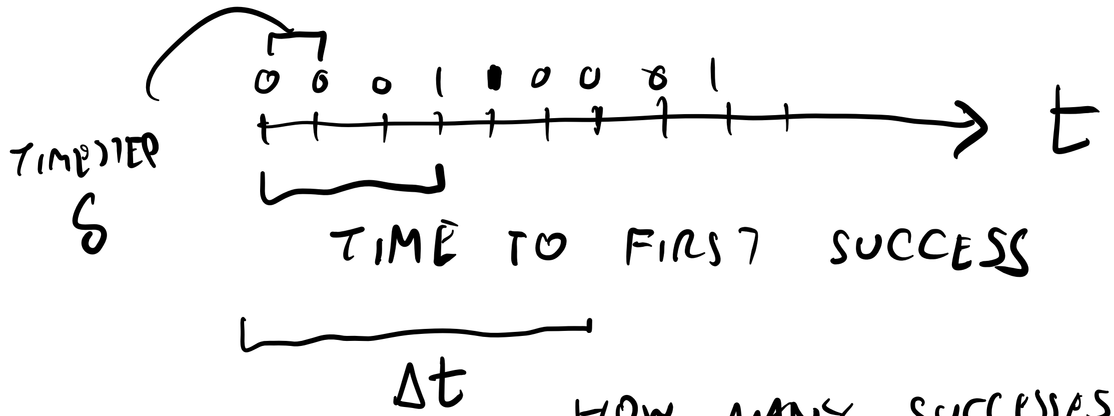
$$F_T(t) = 1 - e^{-t/\tau}$$
$$= 1 - e^{-\lambda t}$$



MEAN $E[T] = \tau = \frac{1}{\lambda}$

POISSON PROCESS

RECALL DISCRETE TIME BERNOULLI SEQUENCE



TIME TO FIRST SUCCESS $T \sim \text{GEOM}$

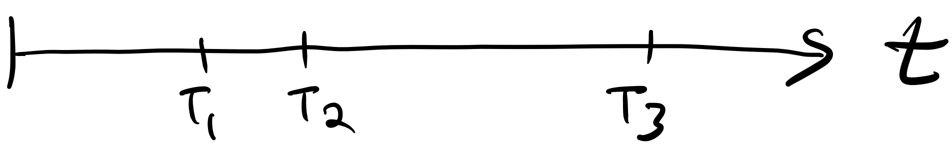
HOW MANY SUCCESSES?

$N \sim$ BINOMIAL

A POISSON PROCESS IS A CONTINUOUS TIME STOCHASTIC PROCESS THAT IS THE LIMIT OF A BERNOULLI TRIAL SEQUENCE WITH TIMESTEP

$\delta \to 0$ AND $p = \lambda\delta$ WITH

$\lambda = \frac{p}{\delta}$ FIXED.



TIME TO FIRST EVENT $T \sim \text{Exp}(\lambda)$

ΔT

HOW MANY EVENTS?

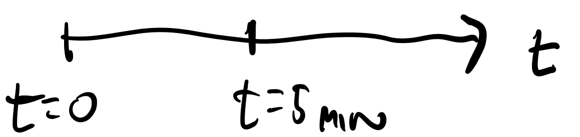
$N \sim$ POISSON

$$p_N(i) = \frac{(\lambda \Delta T)^i e^{-\lambda \Delta T}}{i!}$$

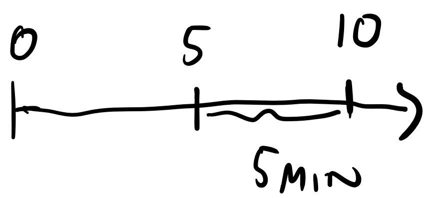
$i \geq 0$

PROPERTIES

- WAITING TIME TO EVENT



$$\mathbb{P}(T_1 < 5\text{MIN}) = 1 - e^{-\lambda \cdot 5\text{MIN}}$$



$$\mathbb{P}(T_1 < 10\text{MIN} \mid T > 5\text{MIN})$$

$$= \frac{\mathbb{P}(T_1 < 10\text{ MIN} \cap T_1 > 5\text{MIN})}{\mathbb{P}(T_1 > 5\text{MIN})}$$

$$= \frac{\int_5^{10} \lambda e^{-\lambda t}\, dt}{1 - (1 - e^{-\lambda 5})} = \ldots$$

$$= 1 - e^{-\lambda \cdot 5\text{MIN}} = \mathbb{P}(T_1 < 5\text{MIN})$$

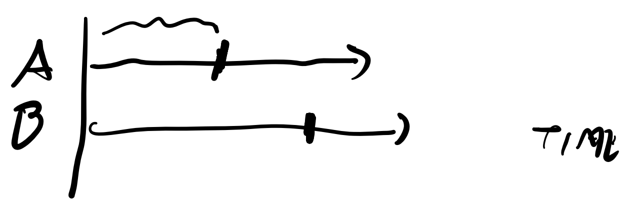
MEMORYLESS PROPERTY

A POISSON PROCESS IS THE UNIQUE MEMORYLESS CONTINUOUS TIME STOCHASTIC PROCESS.

- IF EVENT A IS POISSON WITH RATE λ_A
  EVENT B IS POISSON WITH RATE λ_B

  THEN THE FIRST EVENT IS POISSON WITH RATE $\lambda_A + \lambda_B$

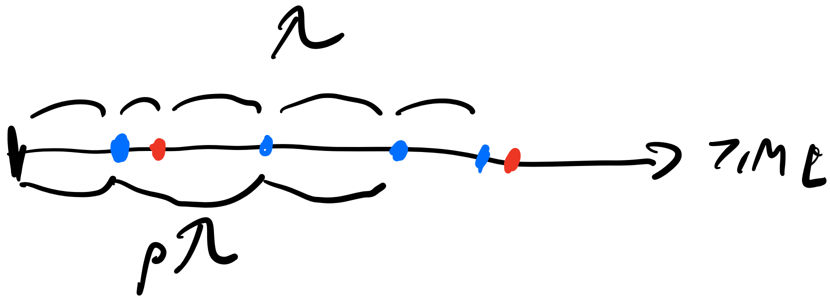
  RACING PROPERTY



- IF A POISSON PROCESS WITH RATE λ HAS TWO TYPES, WITH PROBABILITY p AND $(1-p)$

  THEN "TYPE 1" IS POISSON WITH RATE $p\lambda$
  "TYPE 2" IS POISSON WITH RATE $(1-p)\lambda$

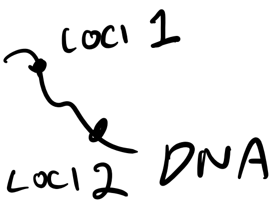
THINNING PROPERTY



MUTATION

$\lambda = 1\ yr^{-1}$

• CASE 1



FIRST MUTATION

$E[T] = ?$

$$E[T] = \frac{1}{\lambda + \lambda} = \frac{1}{2\lambda}$$

• CASE 2

TWO MUTATIONS?