

IF STATE SPACE IS DISCRETE, X IS
A DISCRETE RANDOM VARIABLE

$$p_X(x) = \mathbb{P}(X = x)$$

NOTATION $X \sim p_X(x)$ \approx

MOMENTS

$$E[X^n] = \sum_{i \in S} i^n p_X(i)$$

\nwarrow STATE SPACE

ZEROth MOMENT

$$E[X^0] = \sum_{i \in S} p_X(i) = 1$$

FIRST MOMENT

$$E[X] = \sum_{i \in S} i p_X(i) = \text{MEAN}$$

SYMBOL μ_X

SECOND MOMENT

$$E[X^2] = \sum_{i \in S} i^2 p_X(i) =$$

$$E[X^2] - \mu_x^2 = E[(X - \mu)^2]$$

= VARIANCE

SYMBOL σ_x^2

$$\sigma_x = \sqrt{\sigma_x^2} \quad \text{STANDARD DEVIATION}$$

FAMOUS DISCRETE RANDOM VARIABLES

• UNIFORM BETWEEN a , AND b

• BERNOLLI $X=0 \quad P(X=0) = p_x(0) = 1-p$
 $X=1 \quad P(X=1) = p_x(1) = p$

$$E[X] = p$$

STOCHASTIC PROCESS OF INDEPENDENT
BERNOLLI TRIALS

$$X_t = \{ \underbrace{000}_{1} \underbrace{11}_{1} \underbrace{000100000}_{n} \dots \}$$

• GEOMETRIC

$$p_x(k) = (1-p)^{k-1} \cdot p \quad k=0, 1, 2, \dots$$

ATTEMPTS IN A SERIES OF
BERNOLLI TRIALS UNTIL FIRST

Success $X=1$

$$E[X] = \frac{1}{p}$$

BINOMIAL \downarrow "CHOOSE"

$$p_X(k) = \binom{n}{k} p^k (1-k)^{n-k}$$

OF successes ($X=1$) IN A
SERIES OF n BERNOULLI TRIALS

RECAP

$$\begin{bmatrix} p_A(t+1) \\ p_B(t+1) \\ p_C(t+1) \end{bmatrix} = \begin{bmatrix} p_{A \rightarrow A} & p_{B \rightarrow A} & p_{C \rightarrow A} \\ p_{A \rightarrow B} & p_{B \rightarrow B} & p_{C \rightarrow B} \\ p_{A \rightarrow C} & p_{B \rightarrow C} & p_{C \rightarrow C} \end{bmatrix} \cdot \begin{bmatrix} p_A(t) \\ p_B(t) \\ p_C(t) \end{bmatrix}$$

\downarrow
 M

\downarrow
COLUMNS MUST
SUM TO UNITY

ASIDE REMEMBER

$$M \vec{p}^{t+1} = \vec{p}^t$$

\uparrow TRANSPOSE!

$$\vec{p}^{t+1} = M \cdot \vec{p}^t$$

PS 2

S	E1	E2	E3	I	END	
0	0	0			0	S
1	0	0	(E3 → E1)		0	E1
0	1	0			0	E2
0	0	1			0	E3
0	0	0			0	I
0	0	0			1	END