

COMPSCI/SFWRENG 2FA3
Discrete Mathematics with Applications II
Winter 2021

Assignment 6

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Assignment 6 consists of two problems. You must write your solutions to the problems using LaTeX.

Please submit Assignment 6 as two files, `Assignment_6_YourMacID.tex` and `Assignment_6_YourMacID.pdf`, to the Assignment 6 folder on Avenue under Assessments/Assignments. *YourMacID* must be your personal MacID (written without capitalization). The `Assignment_6_YourMacID.tex` file is a copy of the LaTeX source file for this assignment (`Assignment_6.tex` found on Avenue under Contents/Assignments) with your solution entered after each problem. The `Assignment_6_YourMacID.pdf` is the PDF output produced by executing

```
pdflatex Assignment_6_YourMacID
```

This assignment is due **Sunday, March 14, 2021 before midnight**. You are allow to submit the assignment multiple times, but only the last submission will be marked. **Late submissions and files that are not named exactly as specified above will not be accepted!** It is suggested that you submit your preliminary `Assignment_6_YourMacID.tex` and `Assignment_6_YourMacID.pdf` files well before the deadline so that your mark is not zero if, e.g., your computer fails at 11:50 PM on March 14.

Although you are allowed to receive help from the instructional staff and other students, your submission must be your own work. Copying will be treated as academic dishonesty! If any of the ideas used in your submission were obtained from other students or sources outside of the lectures and tutorials, you must acknowledge where or from whom these ideas were obtained.

Background

For any three regular expressions α , β , and γ , the following properties hold:

1. Commutativity of union: $\alpha + \beta = \beta + \alpha$
2. Associativity of union: $(\alpha + \beta) + \gamma = \alpha + (\beta + \gamma)$
3. Associativity of concatenation: $(\alpha\beta)\gamma = \alpha(\beta\gamma)$
4. Distribution of union: $\alpha(\beta + \gamma) = \alpha\beta + \alpha\gamma$ and $(\beta + \gamma)\alpha = \beta\alpha + \gamma\alpha$.
5. \emptyset is the identity element for union: $\alpha + \emptyset = \alpha$.
6. \emptyset is the annihilator element for concatenation: $\alpha\emptyset = \emptyset\alpha = \emptyset$.
7. ϵ is the identity element for concatenation: $\alpha\epsilon = \alpha$.
8. Idempotence of Kleene star: $\alpha^{**} = \alpha^*$.
9. Some additional properties of Kleene star:
 - a. $\alpha^* + \epsilon = \alpha^*$
 - b. $\alpha^* + \alpha^* = \alpha^*$
 - c. $\alpha^*\alpha^* = \alpha^*$
 - d. $\alpha\alpha^* + \epsilon = \alpha^*$
 - e. $\alpha^*\alpha + \epsilon = \alpha^*$

Problems

1. The regular expression $\alpha = (0110 + 01)(10)^*$ can be written in a more simplified way.
 - a. **[10 points]** By reasoning based on $L(\alpha)$, define a shorter regular expression equal to α . Explain your reasoning.

$L(\alpha)$ can be simplified as $(01)(10)^*$. Because $\alpha = (0110 + 01)(10)^*$ defines a set of strings that begin with 0110 or 01 and the rest of string is consist of concatenation of 10, $(10)^n$ ($n \geq 0$). The string that begin with 0110 can be regarded that they begin with 01, and the 10 right after the 01 can be regarded that it is combined with the $(10)^n$ while $n = n + 1$. In this way, all the strings that begin with 0110 can be expressed as strings that begin with 01. For example, 01101010 can be expressed as $(0110)(10)^2$ or $(01)(10)^3$.

- b. **[10 points]** Using the properties explained above, prove that the expression you came up with in (a), is indeed equal to α .

Proof $(0110 + 01)(10)^* = (01)(10)^*$

$$\begin{aligned} & (0110 + 01)(10)^* && \langle \text{LHS} \rangle \\ &= (0110 + 01\epsilon)(10)^* && \langle \epsilon \text{ is the identity element for concatenation} \rangle \\ &= (01)(10 + \epsilon)(10)^* && \langle \text{Distribution of union} \rangle \\ &= (01)((10 + \epsilon)(10)^*) && \langle \text{Associativity of concatenation} \rangle \\ &= (01)((10)(10)^* + \epsilon(10)^*) && \langle \text{Distribution of union} \rangle \\ &= (01)((10)(10)^* + (10)^*) && \langle \epsilon \text{ is the identity element for concatenation} \rangle \\ &= (01)((10)(10)^* + \epsilon + (10)^*) && \langle \alpha^* + \epsilon = \alpha^* \rangle \\ &= (01)((10)^* + (10)^*) && \langle \alpha\alpha^* + \epsilon = \alpha^* \rangle \\ &= (01)(10)^* && \langle \alpha^* + \alpha^* = \alpha^* \rangle \end{aligned}$$

Qiang Gao, gaoq20, 3/14/2021