Population variance : $\sigma^2 = \mathbb{E}\left[(X - E(X))^2\right]$

Sample Variance : $s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (y_i - \bar{y})^2$

Note: The reason we divide by $n-1$ in s^2 & not n is because this makes s^2 an unbiased estimator i.e. $\mathbb{E}[s^2] = \sigma^2$. This means that, on average, the sample variance is accurate.

Next, we unpack the formula for s^2 & try to arrive at the form that appears in exercise D:

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i^2 - 2y_i\bar{y} + \bar{y}^2)$$

$$= \frac{1}{n-1}\left(\sum_{i=1}^{n} y_i^2 - 2\bar{y}\sum_{i=1}^{n} y_i + \sum_{i=1}^{n}\bar{y}^2\right)$$

* Note: $\bar{y} = \sum_{i=1}^{n} y_i / n \Rightarrow n\bar{y} = \sum_{i=1}^{n} y_i$

$$= \frac{1}{n-1}\left(\sum_{i=1}^{n} y_i^2 - 2\bar{y}^2 n + n\bar{y}^2\right)$$

$$= \frac{1}{n-1}\left(\sum_{i=1}^{n} y_i^2 - n\bar{y}^2\right)$$

$$= \frac{1}{n-1}\left(\sum_{i=1}^{n} y_i^2 - n\left(\sum_{i=1}^{n} y_i/n\right)^2\right)$$

$$= \frac{1}{n-1}\left(\sum_{i=1}^{n} y_i^2 - \frac{n}{n^2}\left(\sum_{i=1}^{n} y_i\right)^2\right)$$

$$= \frac{1}{n-1}\left(\sum_{i=1}^{n} y_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} y_i\right)^2\right)$$

Next, let $C_{T,j} = y_i$ $= \frac{1}{n-1}\left(\sum_{i=1}^{n} C_{T,j} - \frac{1}{n}\left(\sum_{i=1}^{n} C_{T,j}\right)^2\right)$.

To find the sample standard deviation or $\sqrt{s^2} = s$, take square root.

$$s = \sqrt{\frac{\sum_{i=1}^{n} C_{T,j}^2 - \frac{1}{n}\left(\sum_{i=1}^{n} C_{T,j}\right)^2}{n-1}} \Rightarrow e^{-rT}\sqrt{\frac{\sum_{i=1}^{n} C_{T,j}^2 - \frac{1}{n}\left(\sum_{i=1}^{n} C_{T,j}\right)^2}{n-1}}$$

Discounted sample S.D. $= e^{-rT}s$