

Break-Ground:

Derivatives of products are tricky

Here we see a dialogue where young mathematicians talk about derivatives of products and products of derivatives.

Check out this dialogue between two calculus students (based on a true story):

Devyn: Hey Riley, remember the sum rule for derivatives?

Riley: You know I do.

Devyn: What do you think that the “product rule” will be?

Riley: Let’s give this a spin:

$$\frac{d}{dx}(f(x) \cdot g(x)) = f'(x) \cdot g'(x)?$$

Devyn: Hmmm, let’s give this theory an acid test. Let’s try

$$f(x) = x^2 + 1 \quad \text{and} \quad g(x) = x^3 - 3x$$

Now

$$\begin{aligned} f'(x)g'(x) &= (2x)(3x^2 - 3) \\ &= 6x^3 - 6x. \end{aligned}$$

Riley: On the other hand,

$$\begin{aligned} f(x)g(x) &= (x^2 + 1)(x^3 - 3x) \\ &= x^5 - 3x^3 + x^3 - 3x \\ &= x^5 - 2x^3 - 3x. \end{aligned}$$

Devyn: And so,

$$\frac{d}{dx}(f(x) \cdot g(x)) = 5x^4 - 6x^2 - 3.$$

Riley: Wow. Hmmm. It looks like our guess was incorrect.

Devyn: I’ve got a feeling that the so-called “product rule” might be a bit tricky.

Learning outcomes: Explain why the product rule is not given by multiplying the derivatives of the products. Apply the sum rule repeatedly to find the derivative of a product. Relate the sum rule, the constant multiple rule, and the product rule.

Derivatives of products are tricky

Problem 1 Above, our intrepid young mathematicians guess that the “product rule” might be:

$$\frac{d}{dx}(f(x) \cdot g(x)) = f'(x) \cdot g'(x)?$$

Does this **ever** hold true?

Free Response:

Problem 2 Our mathematicians mentioned the sum rule at the beginning of their conversation. The sum rule can be used on certain functions that look like products. Find five examples of such functions, and take their derivatives. What other rule could be used in this case?

Free Response:

Xarma Boost 3 Write down at least **five** questions for this lecture. After you have your questions, label them as “Level 1,” “Level 2,” or “Level 3” where:

Level 1 Means you know the answer, or know exactly how to do this problem.

Level 2 Means you think you know how to do the problem.

Level 3 Means you have no idea how to do the problem.

Free Response:
