In the name of GOD

Assignment 5

Narges Vali

401102791

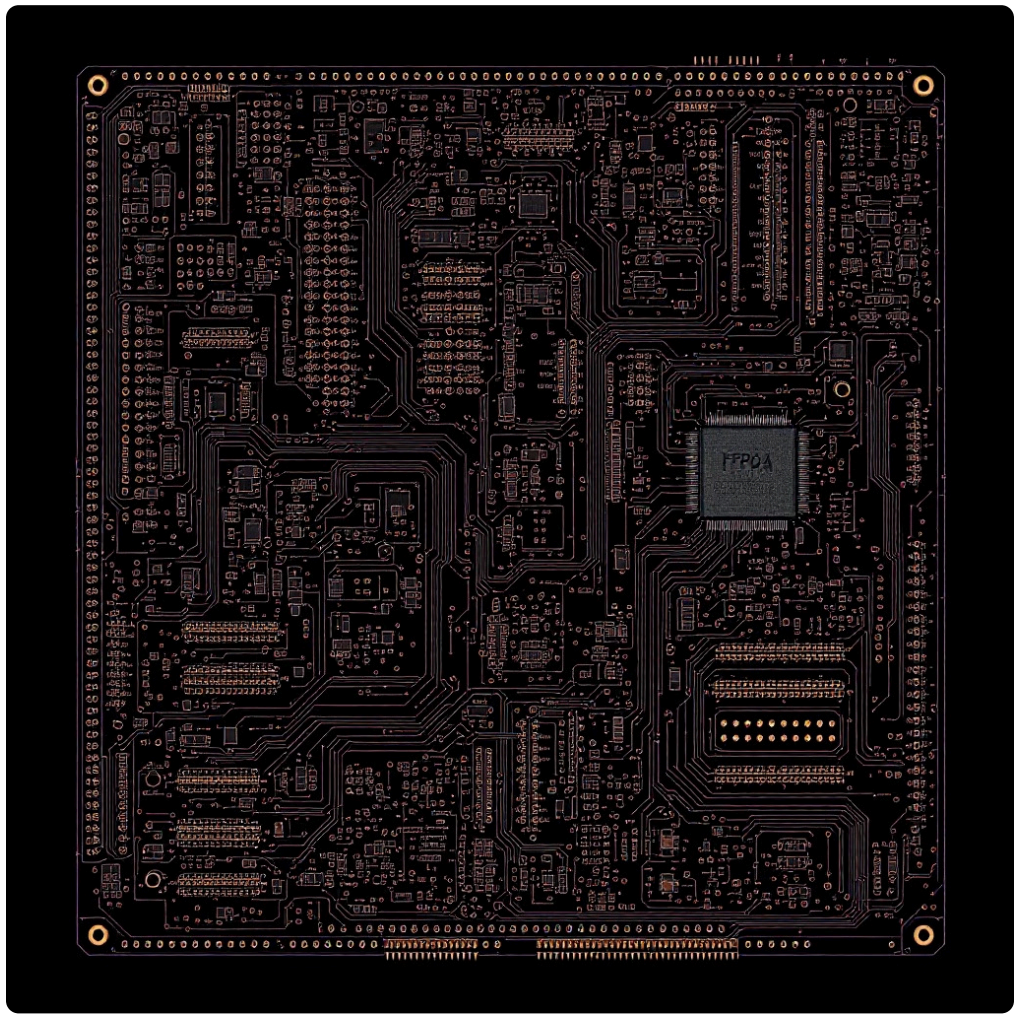
1. Logic Function Minimization using ROBDD and Quine–McCluskey Algorithm

2. ROBDD Construction for a Given Combinational Circuit with Different Variable Orders

3. Reduction of OBDT to ROBDD by Merging Isomorphic Subgraphs

4. Variable Ordering Impact on ROBDD Size and Optimal Order Determination

5. Function Realizability Check using SAT Solver and Characteristic Equations



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Question1 - Logic Function Minimization using NOR2D and Quine-McCluskey Algorithm

The objective of this task is to minimize the following Boolean function using two approaches:

$$f = \sum_{4}(0, 1, 2, 3, 4, 6, 8, 10, 11, 15)$$

1. Using Reduced Ordered Binary Decision Diagrams (ROBDDs)
2. Using the Quine-McCluskey Algorithm

Part A – ROBDD Construction and Analysis

In this first part, we analyzed the function by constructing the Binary Decision Diagram (BDD) and its reduced form (ROBDD) for the variable ordering $a > b > c > d$.

Order 1: a > b > c > d

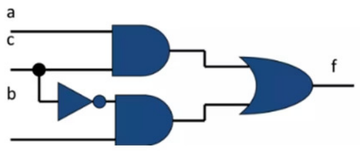
DRAWIO:

root node
f
a
b
c
d
1
0
1
0
1
0

Question 2 . ROBDD Construction for a Given Combinational Circuit with Different Variable Orders

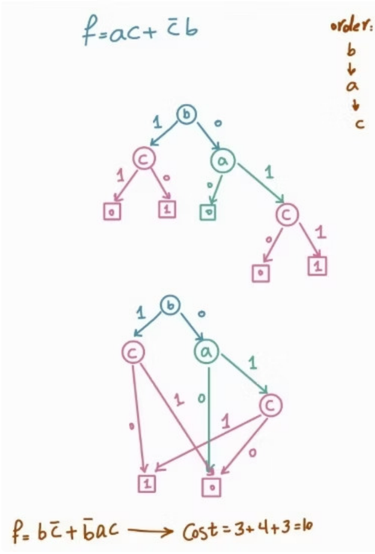
Part 1 – ROBDD Construction for Different Orders

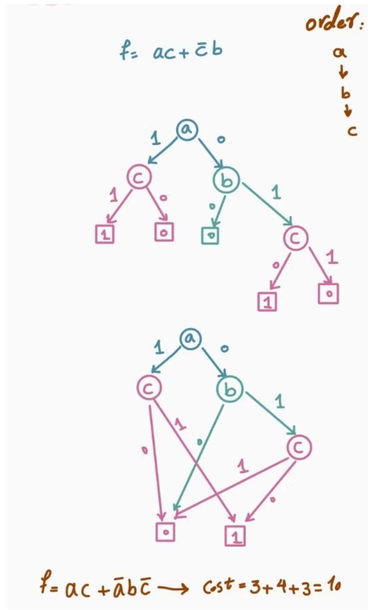
The Boolean function under consideration is:



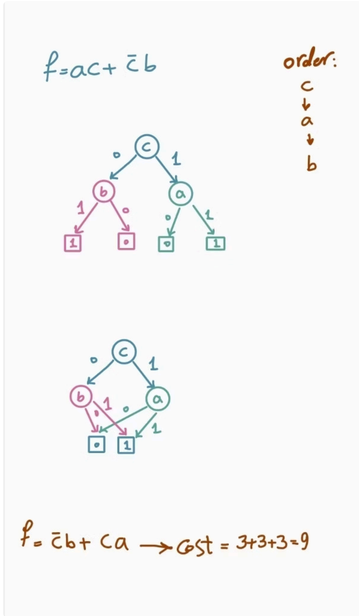
$$f(a,b,c) = ac + \bar{c}b$$

Using the specified variable orders, I constructed the **OBDD** and reduced them to obtain the corresponding **ROBDDs**. The total **cost** was computed for each case.





I also considered an **additional variable order** of my own to compare with the given ones. Interestingly, this custom order resulted in a **lower cost**.



Part 2 – ROBDD Cost Comparison

Variable Order	ROBDD Cost	Number of Non-terminal Nodes
a → b → c	10	4 nodes
b → a → c	10	4 nodes
c → a → b	**9**	**3 nodes**

The custom ordering c → a → b yielded the lowest cost, demonstrating that choosing the right variable order can lead to a more compact and efficient ROBDD.

Part 3 – Importance of Variable Ordering in Circuit Design

The selection of variable ordering plays a **crucial role** in the cost and efficiency of ROBDD-based circuit implementations. Here's why:

- Different orderings affect how often subgraphs can be merged or eliminated during reduction.
- A poor ordering may lead to redundant branches and an increased number of nodes, resulting in higher hardware or memory costs.
- A good ordering enables more **sharing of sub-graphs**, leading to a more compact and **cost-effective implementation**.

In digital design, especially in logic synthesis, minimizing the size of the ROBDD can directly reduce **gate count, power consumption,** and **overall complexity** of the resulting circuit. Therefore, selecting an optimal or near-optimal variable order is often a key step in design optimization.

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Question 3. Reduction of OBDT to ROBDD by Merging Isomorphic Subgraphs

The goal of this section is to reduce an **Ordered Binary Decision Tree (OBDT)** to its minimized form: a **Reduced Ordered Binary Decision Diagram (ROBDD)** by eliminating redundant and duplicate nodes.

Explanation of Reduction Process

The initial OBDT shown in the diagram follows the variable order:

$x < y < z$

Each internal node represents a decision based on a variable, and branches correspond to binary outcomes (0 or 1). In the full OBDT:

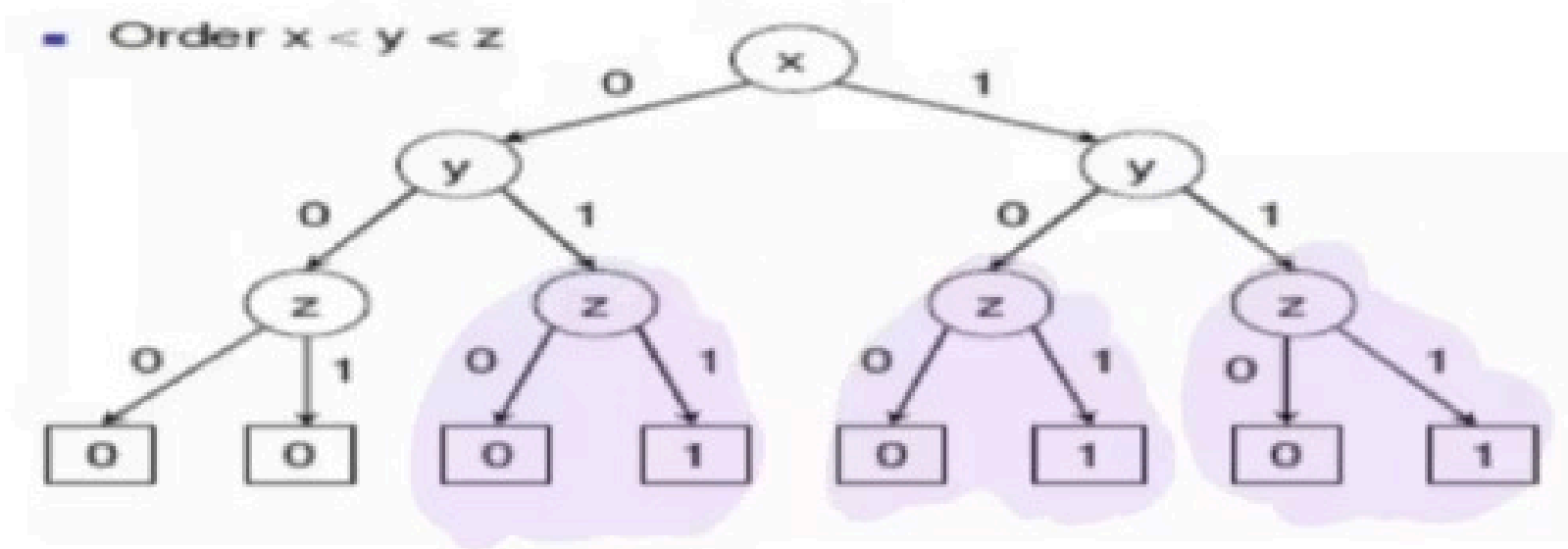
- The subgraphs rooted at each instance of variable `z` are structurally identical.
- This creates **redundancy** that can be eliminated.

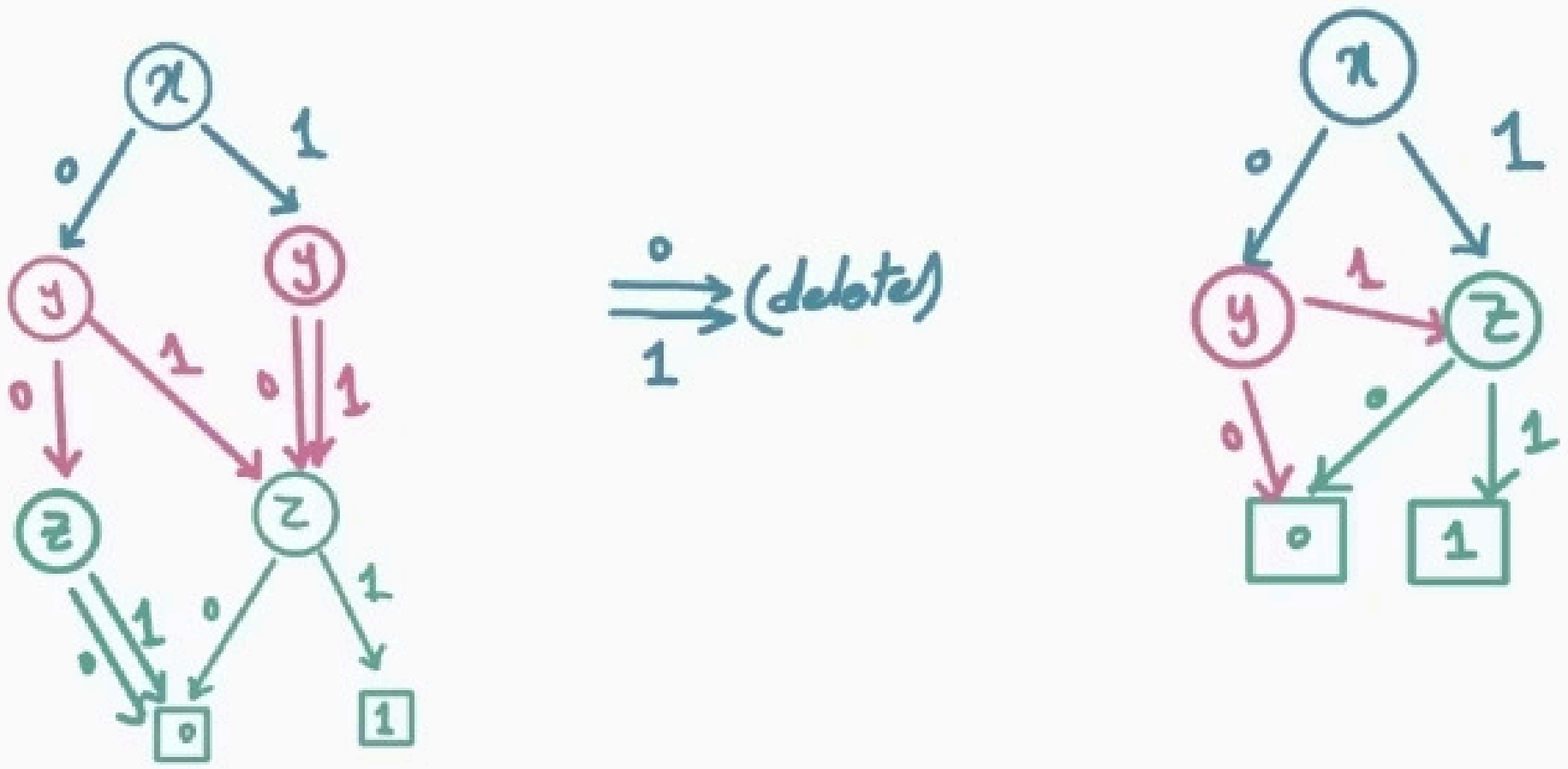
The reduction process consists of two key steps:

1. **Merging Isomorphic Subgraphs**: Identical subgraphs (e.g., both `z` subtrees producing `0` and `1`) are merged into a single instance to avoid duplication.
2. **Removing Redundant Nodes**: If a node's both branches point to the same child, the node is unnecessary and can be bypassed.

In the given example:

- All `z` subgraphs are the same.
- After merging them and eliminating unnecessary nodes, the final ROBDD contains **only one instance of each distinct subfunction**, resulting in a more compact representation.





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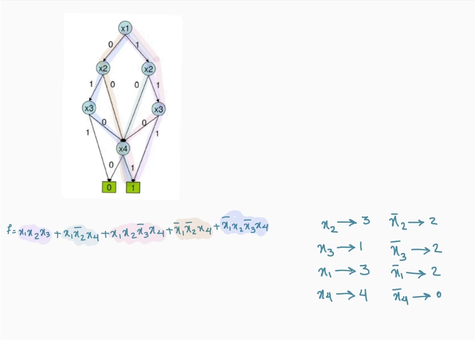
Question 4. Variable Ordering Impact on ROBDD Size and Optimal Order Determination

In this question, the effect of variable ordering on the size of the Reduced Ordered Binary Decision Diagram (ROBDD) was studied for the following Boolean function:



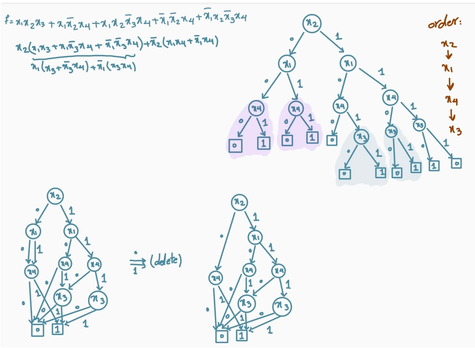
$$f(x_1, x_2, x_3, x_4) = x_1x_2x_3 + x_1\overline{x_2}x_4 + x_1x_2\overline{x_3}x_4 + \overline{x_1}\overline{x_2}x_4 + \overline{x_1}x_2x_3x_4$$

Method

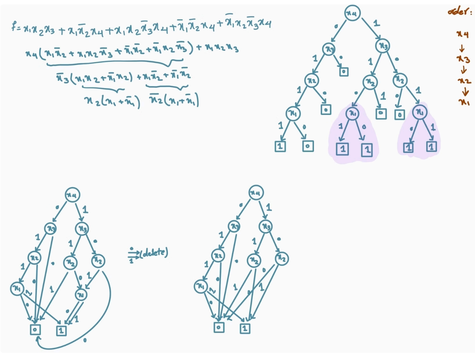
I implemented the ROBDD construction process using multiple different variable orderings and compared the final ROBDDs both visually and by node count.

The ROBDDs were drawn manually for the following variable orders:

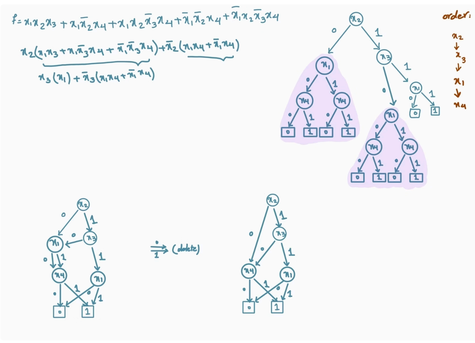
- $x_2 \to x_1 \to x_4 \to x_3$



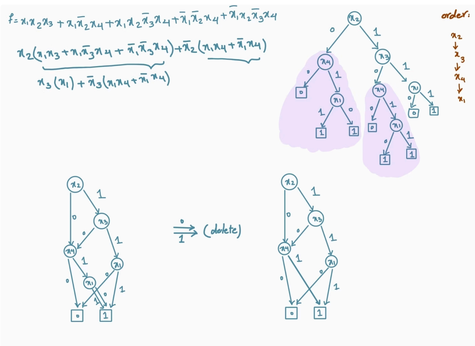
- $x_4 \to x_3 \to x_2 \to x_1$



- $x_2 \to x_3 \to x_1 \to x_4$



- $x_2 \to x_3 \to x_4 \to x_1$



Each diagram was reduced by:

Results

Variable Order	ROBDD Node Count
$x_2 \to x_1 \to x_4 \to x_3$	7 nodes
$x_4 \to x_3 \to x_2 \to x_1$	7 nodes
$x_2 \to x_3 \to x_1 \to x_4$	**4 nodes** ✅
$x_2 \to x_3 \to x_4 \to x_1$	**4 nodes** ✅

Why Do These Differences Occur?

The ROBDD structure is heavily influenced by the **positioning of the most discriminative variables**:

- When variables that **separate decision paths early** are placed first, **common substructures are exposed earlier**, allowing more reduction.
- Poor orderings delay decision-making, leading to **duplicate subtrees** and less sharing.
- In this case, placing x_3 and x_4 earlier revealed symmetric patterns that allowed merging more paths.

Thus, the **right variable order can dramatically reduce the size and cost** of the resulting circuit.

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Question 5. Function Realizability Check using SAT Solver and Characteristic Equations

In this question, I verified whether two Boolean functions can be mapped onto a given logic structure using a SAT-based approach. The structure includes:

- A 4-to-1 multiplexer controlled by S1, S0
- Configuration bits B0, B1, B2, B3
- An input X
- Internal node Z and output G

The SAT formulation checks if there exists a valid assignment for B0 to B3 such that the function behaves as expected for all input combinations.

$$\begin{aligned} &(B_0 + S_1 + S_0 + z)(\overline{B_0} + S_1 + S_0 + z)(B_1 + S_1 + \overline{S_0} + z)(\overline{B_1} + S_1 + \overline{S_0} + z) \\ F = &(B_1 + \overline{S_1} + S_0 + z)(\overline{B_1} + \overline{S_1} + S_0 + z)(B_2 + \overline{S_1} + \overline{S_0} + z)(\overline{B_2} + \overline{S_1} + \overline{S_0} + z) \\ &(\overline{x} + \overline{z} + G)(x + G)(x + \overline{G}) \end{aligned}$$

Function 1:

$$f_1 = X\,S_1\,S_0 + X\,S_1\overline{S_0} + X\overline{S_1}\,\overline{S_0}$$

I manually analyzed and wrote out the SAT clauses for all input combinations. The evaluations for each case are listed below:

Input Cases:

1. $X = 0, S_1 = 0, S_0 = 0$

$$f_{\overline{X}\,\overline{S_1}\,\overline{S_0}}(G) = (B_0 + \cancel{S_1} + \cancel{S_0} + \overline{z})(\overline{B_0} + \cancel{S_1} + \cancel{S_0} + \overline{z})(B_1 + \cancel{S_1} + \cancel{\overline{S_0}} + \overline{z})(\overline{B_1} + \cancel{S_1} + \cancel{\overline{S_0}} + \overline{z})$$
$$(B_2 + \cancel{\overline{S_1}} + \cancel{S_0} + \overline{z})(\overline{B_2} + \cancel{\overline{S_1}} + \cancel{S_0} + \overline{z})(\overline{x} + \overline{z} + G)(x + \overline{G})$$

$$\Rightarrow (x=0, S_1=0, S_0=0) \Rightarrow \quad G=0 \Rightarrow \overline{z}=0 \Rightarrow B_0=0 \;;\; B_1 \text{ don't care} \;;\; B_2 \text{ don't care}$$
$$\Rightarrow \overline{z}=1 \Rightarrow B_0=1 \;;\; B_1 \text{ don't care} \;;\; B_2 \text{ don't care}$$

$$z = B_0 = \text{don't care} \;;\; B_1 = \text{don't care} \;;\; B_2 = \text{don't care}$$

2. $X = 0, S_1 = 0, S_0 = 1$

$$f_{\overline{X}\,\overline{S_1}\,S_0}(G) = (1)(1)(B_1 + \overline{z})(\overline{B_1} + \overline{z})(1)(1)(1)(1)(B_2 + \overline{z})(\overline{B_2} + \overline{z})(x + \overline{G})$$

$$\Rightarrow (x=0, S_1=0, S_0=1) \Rightarrow \quad G=0 \Rightarrow \overline{z}=0 \Rightarrow B_1=0 \;;\; B_2 \text{ don't care} \;;\; B_1 \text{ don't care}$$
$$\Rightarrow \overline{z}=1 \Rightarrow B_1=1 \;;\; B_2 \text{ don't care} \;;\; B_1 \text{ don't care}$$

$$z = B_1 = \text{don't care} \;;\; B_0 = \text{don't care} \;;\; B_2 = \text{don't care}$$

3. $X = 0, S_1 = 1, S_0 = 0$

$$f_{\overline{X}\,S_1\,\overline{S_0}}(G) = (1)(1)(1)(1)(\overline{B_2} + \cancel{\overline{S_1}} + \cancel{S_0} + \overline{z})(1)(1)(1)(1)(B_3 + \overline{z})(x + \overline{G})$$

$$\Rightarrow (x=0, S_1=1, S_0=0) \Rightarrow \quad G=0 \Rightarrow \overline{z}=0 \Rightarrow B_2=0 \;;\; B_3 \text{ don't care} \;;\; B_2 \text{ don't care}$$
$$\Rightarrow \overline{z}=1 \Rightarrow B_2=1 \;;\; B_3 \text{ don't care} \;;\; B_2 \text{ don't care}$$

$$z = \overline{B_2} = \text{don't care} \;;\; B_3 = \text{don't care} \;;\; B_2 = \text{don't care}$$

4. $X = 0, S_1 = 1, S_0 = 1$

$$f_{\overline{X}\,S_1\,S_0}(G) = (1)(1)(1)(1)(1)(1)(\overline{B_3} + \cancel{\overline{S_1}} + \cancel{\overline{S_0}} + \overline{z})(\overline{B_3} + \cancel{\overline{S_1}} + \cancel{\overline{S_0}} + \overline{z})(1)(1)$$

$$\Rightarrow (x=0, S_1=1, S_0=1) \Rightarrow \quad G=0 \Rightarrow \overline{z}=0 \Rightarrow B_3=0 \;;\; B_3 \text{ don't care} \;;\; B_2 \text{ don't care}$$
$$\Rightarrow \overline{z}=1 \Rightarrow B_3=1 \;;\; B_3 \text{ don't care} \;;\; B_2 \text{ don't care}$$

$$z = B_3 = \text{don't care} \;;\; B_2 = \text{don't care} \;;\; B_3 = \text{don't care}$$

5. $X = 1, S_1 = 0, S_0 = 0$

$$f_{X\,\overline{S_1}\,\overline{S_0}}(G) = (B_0 + \cancel{\overline{S_1}} + \cancel{\overline{S_0}} + \overline{z})(\overline{B_0} + \cancel{\overline{S_1}} + \cancel{\overline{S_0}} + \overline{z})(1)(1)(1)(1)(1)(1)(\overline{x} + \overline{z} + G)(\overline{x} + \overline{G})(1)$$

$$\Rightarrow (x=1, S_1=0, S_0=0) \Rightarrow \quad G=1 \Rightarrow \overline{z}=1 \Rightarrow B_0=1 \;;\; B_0 \text{ don't care} \;;\; B_1 \text{ don't care}$$

$$z = B_0 = \text{don't care} \;;\; B_1 = \text{don't care} \;;\; B_2 = \text{don't care}$$

6. $X = 1, S_1 = 0, S_0 = 1$

$$f_{X\,\overline{S_1}\,S_0}(G) = (1)(1)(1)(1)(B_1 + \cancel{\overline{S_1}} + \cancel{S_0} + \overline{z})(\overline{B_1} + \cancel{\overline{S_1}} + \cancel{S_0} + \overline{z})(1)(1)(1)(1)(\overline{x} + \overline{z} + G)(\overline{x} + \overline{G})(1)$$

$$\Rightarrow (x=1, S_1=0, S_0=1) \Rightarrow \quad G=1 \Rightarrow \overline{z}=0 \Rightarrow B_1=0 \;;\; B_1 \text{ don't care} \;;\; B_2 \text{ don't care}$$
$$\Rightarrow \overline{z}=1 \Rightarrow B_1=1 \;;\; B_1 \text{ don't care} \;;\; B_2 \text{ don't care}$$

$$z = B_1 = \text{don't care} \;;\; B_0 = \text{don't care} \;;\; B_2 = \text{don't care}$$

7. $X = 1, S_1 = 1, S_0 = 0$

$$f_{X\,S_1\,\overline{S_0}}(G) = (1)(1)(1)(1)(\overline{B_2} + \cancel{\overline{S_1}} + \cancel{S_0} + \overline{z})(\overline{B_2} + \cancel{\overline{S_1}} + \cancel{S_0} + \overline{z})(1)(1)(\overline{x} + \overline{z} + G)(\overline{x} + \overline{G})(1)$$

$$\Rightarrow (x=1, S_1=1, S_0=0) \Rightarrow \quad G=1 \Rightarrow \overline{z}=1 \Rightarrow B_2=0 \;;\; B_2 \text{ don't care} \;;\; B_3 \text{ don't care}$$

$$\underbrace{z = 1 \;,\; B_2 = 0}_{z = \overline{B_2} = 1} \;;\; B_3 = \text{don't care} \;;\; B_2 = \text{don't care}$$

8. $X = 1, S_1 = 1, S_0 = 1$

$$f_{X\,S_1\,S_0}(G) = (1)(1)(1)(1)(1)(1)(1)(1)(\overline{B_3} + \cancel{\overline{S_1}} + \cancel{\overline{S_0}} + \overline{z})(\overline{B_3} + \cancel{\overline{S_1}} + \cancel{\overline{S_0}} + \overline{z})(\overline{x} + \overline{z} + G)(\overline{x} + \overline{G})(1)$$

$$\Rightarrow (x=1, S_1=1, S_0=1) \Rightarrow \quad G=1 \Rightarrow \overline{z}=1 \Rightarrow B_3=1 \;;\; B_3 \text{ don't care} \;;\; B_2 \text{ don't care}$$

$$\underbrace{z = 1 \;,\; B_3 = 1}_{z = \overline{B_3} = 1} \;;\; B_2 = \text{don't care} \;;\; B_3 = \text{don't care}$$

From this full evaluation, I found that the SAT constraints are satisfiable with the following configuration:

$$\boxed{B_0 = 1, \quad B_1 = 0, \quad B_2 = 1}$$

All other bits are **don't care**. So, f1 is **SAT**.

Function 2:

$$f_2 = X\,S_1\,S_0$$

I repeated the SAT analysis for this simpler function. The relevant input combinations checked include:

Input Cases:

1. $X = 0, S_1 = 0, S_0 = 0$

$$f_{\overline{X}\,\overline{S_1}\,\overline{S_0}}(G) = (B_0 + \cancel{\overline{S_1}} + \cancel{\overline{S_0}} + \overline{z})(\overline{B_0} + \cancel{\overline{S_1}} + \cancel{\overline{S_0}} + \overline{z})(B_1 + \cancel{\overline{S_1}} + \cancel{\overline{S_0}} + \overline{z})(\overline{B_1} + \cancel{\overline{S_1}} + \cancel{\overline{S_0}} + \overline{z})$$
$$(B_2 + \cancel{\overline{S_1}} + \cancel{S_0} + \overline{z})(\overline{B_2} + \cancel{\overline{S_1}} + \cancel{S_0} + \overline{z})(\overline{x} + \overline{z} + G)(\overline{x} + \overline{G})$$

$$\Rightarrow (x=0, S_1=0, S_0=0) \Rightarrow \quad G=0 \Rightarrow \overline{z}=0 \Rightarrow B_0=0 \;;\; B_1 \text{ don't care} \;;\; B_2 \text{ don't care}$$
$$\Rightarrow \overline{z}=1 \Rightarrow B_0=1 \;;\; B_1 \text{ don't care} \;;\; B_2 \text{ don't care}$$

$$z = B_0 = \text{don't care} \;;\; B_1 = \text{don't care} \;;\; B_2 = \text{don't care}$$

2. $X = 0, S_1 = 0, S_0 = 1$

$$f_{\overline{X}\,\overline{S_1}\,S_0}(G) = (1)(1)(1)(1)(B_1 + \overline{z})(\overline{B_1} + \overline{z})(1)(1)(1)(1)(1)(\overline{x} + \overline{z})(x + \overline{G})$$

$$\Rightarrow (x=0, S_1=0, S_0=1) \Rightarrow \quad G=0 \Rightarrow \overline{z}=0 \Rightarrow B_1=0 \;;\; B_0 \text{ don't care} \;;\; B_2 \text{ don't care}$$
$$\Rightarrow \overline{z}=1 \Rightarrow B_1=1 \;;\; B_0 \text{ don't care} \;;\; B_2 \text{ don't care}$$

$$z = B_1 = \text{don't care} \;;\; B_0 = \text{don't care} \;;\; B_2 = \text{don't care}$$

3. $X = 0, S_1 = 1, S_0 = 0$

$$f_{\overline{X}\,S_1\,\overline{S_0}}(G) = (1)(1)(1)(1)(\overline{B_3} + \cancel{\overline{S_1}} + \cancel{S_0} + \overline{z})(1)(1)(1)(1)(B_3 + \overline{z})(x + \overline{G})$$

$$\Rightarrow (x=0, S_1=1, S_0=0) \Rightarrow \quad G=0 \Rightarrow \overline{z}=0 \Rightarrow B_3=0 \;;\; B_3 \text{ don't care} \;;\; B_2 \text{ don't care}$$
$$\Rightarrow \overline{z}=1 \Rightarrow B_3=1 \;;\; B_3 \text{ don't care} \;;\; B_2 \text{ don't care}$$

$$z = \overline{B_3} = \text{don't care} \;;\; B_2 = \text{don't care} \;;\; B_3 = \text{don't care}$$

4. $X = 0, S_1 = 1, S_0 = 1$

$$f_{\overline{X}\,S_1\,S_0}(G) = (1)(1)(1)(1)(1)(1)(1)(1)(\overline{B_3} + \cancel{\overline{S_1}} + \cancel{\overline{S_0}} + \overline{z})(\overline{B_3} + \cancel{\overline{S_1}} + \cancel{\overline{S_0}} + \overline{z})(1)(1)$$

$$\Rightarrow (x=0, S_1=1, S_0=1) \Rightarrow \quad G=0 \Rightarrow \overline{z}=0 \Rightarrow B_3=0 \;;\; B_3 \text{ don't care} \;;\; B_2 \text{ don't care}$$
$$\Rightarrow \overline{z}=1 \Rightarrow B_3=1 \;;\; B_3 \text{ don't care} \;;\; B_2 \text{ don't care}$$

$$z = B_3 = \text{don't care} \;;\; B_2 = \text{don't care} \;;\; B_3 = \text{don't care}$$

5. $X = 1, S_1 = 0, S_0 = 0$

$$f_{X\,\overline{S_1}\,\overline{S_0}}(G) = (B_0 + \cancel{\overline{S_1}} + \cancel{\overline{S_0}} + \overline{z})(\overline{B_0} + \cancel{\overline{S_1}} + \cancel{\overline{S_0}} + \overline{z})(1)(1)(1)(1)(1)(1)(\overline{x} + \overline{z} + G)(\overline{x} + \overline{G})(1)$$

$$\Rightarrow (x=1, S_1=0, S_0=0) \Rightarrow \quad G=0 \Rightarrow \overline{z}=0 \Rightarrow B_0=0 \;;\; B_0 \text{ don't care} \;;\; B_1 \text{ don't care}$$

$$\underbrace{z = 0 \;,\; B_0 = 0}_{z = \overline{B_0} = 0} \;;\; B_1 = \text{don't care} \;;\; B_2 = \text{don't care}$$

6. $X = 1, S_1 = 0, S_0 = 1$

$$f_{X\,\overline{S_1}\,S_0}(G) = (1)(1)(1)(B_1 + \cancel{\overline{S_1}} + \cancel{S_0} + \overline{z})(\overline{B_1} + \cancel{\overline{S_1}} + \cancel{S_0} + \overline{z})(1)(1)(1)(1)(\overline{x} + \overline{z} + G)(\overline{x} + \overline{G})(1)$$

$$\Rightarrow (x=1, S_1=0, S_0=1) \Rightarrow \quad G=0 \Rightarrow \overline{z}=0 \Rightarrow B_1=0 \;;\; B_0 \text{ don't care} \;;\; B_1 \text{ don't care}$$

$$\underbrace{z = 0 \;,\; B_1 = 0}_{z = \overline{B_1} = 0} \;;\; B_0 = \text{don't care} \;;\; B_2 = \text{don't care}$$

7. $X = 1, S_1 = 1, S_0 = 0$

$$f_{X\,S_1\,\overline{S_0}}(G) = (1)(1)(1)(1)(\overline{B_2} + \cancel{\overline{S_1}} + \cancel{S_0} + \overline{z})(\overline{B_2} + \cancel{\overline{S_1}} + \cancel{S_0} + \overline{z})(1)(1)(\overline{x} + \overline{z} + G)(\overline{x} + \overline{G})(1)$$

$$\Rightarrow (x=1, S_1=1, S_0=0) \Rightarrow \quad G=0 \Rightarrow \overline{z}=0 \Rightarrow B_2=0 \;;\; B_2 \text{ don't care} \;;\; B_3 \text{ don't care}$$

$$\underbrace{z = 0 \;,\; B_2 = 0}_{z = \overline{B_2} = 0} \;;\; B_3 = \text{don't care} \;;\; B_2 = \text{don't care}$$

8. $X = 1, S_1 = 1, S_0 = 1$

$$f_{X\,S_1\,S_0}(G) = (1)(1)(1)(1)(1)(1)(1)(1)(\overline{B_3} + \cancel{\overline{S_1}} + \cancel{\overline{S_0}} + \overline{z})(\overline{B_3} + \cancel{\overline{S_1}} + \cancel{\overline{S_0}} + \overline{z})(\overline{x} + \overline{z} + G)(\overline{x} + \overline{G})(1)$$

$$\Rightarrow (x=1, S_1=1, S_0=1) \Rightarrow \quad G=1 \Rightarrow \overline{z}=1 \Rightarrow B_3=1 \;;\; B_3 \text{ don't care} \;;\; B_2 \text{ don't care}$$

$$\underbrace{z = 1 \;,\; B_3 = 1}_{z = \overline{B_3} = 1} \;;\; B_2 = \text{don't care} \;;\; B_3 = \text{don't care}$$

So, in the same SAT encoding, **B1 must be both 0 and 1**, which is impossible.

🔴 This leads to **conflicting clauses:**

- Clause E must be satisfied when B1=0
- Clause F must be satisfied when B1=1

These two cannot be simultaneously satisfied.
✅ Thus,
❌ **f2 is UNSAT.**

Structural Limitation Analysis

In this structure:

- Only **4 configuration bits** B0 to B3 are used
- Each bit is mapped to a specific address via S1,S0
- There is **no dynamic reconfiguration** — meaning one Bi must work for all possible inputs that select it

This becomes problematic when:

- The same configuration index (e.g., B1) is required to behave differently under different inputs (as in f2)
- This creates contradictions in the SAT clauses and makes certain functions **inexpressible**

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End of Report ✅

Thank you!

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