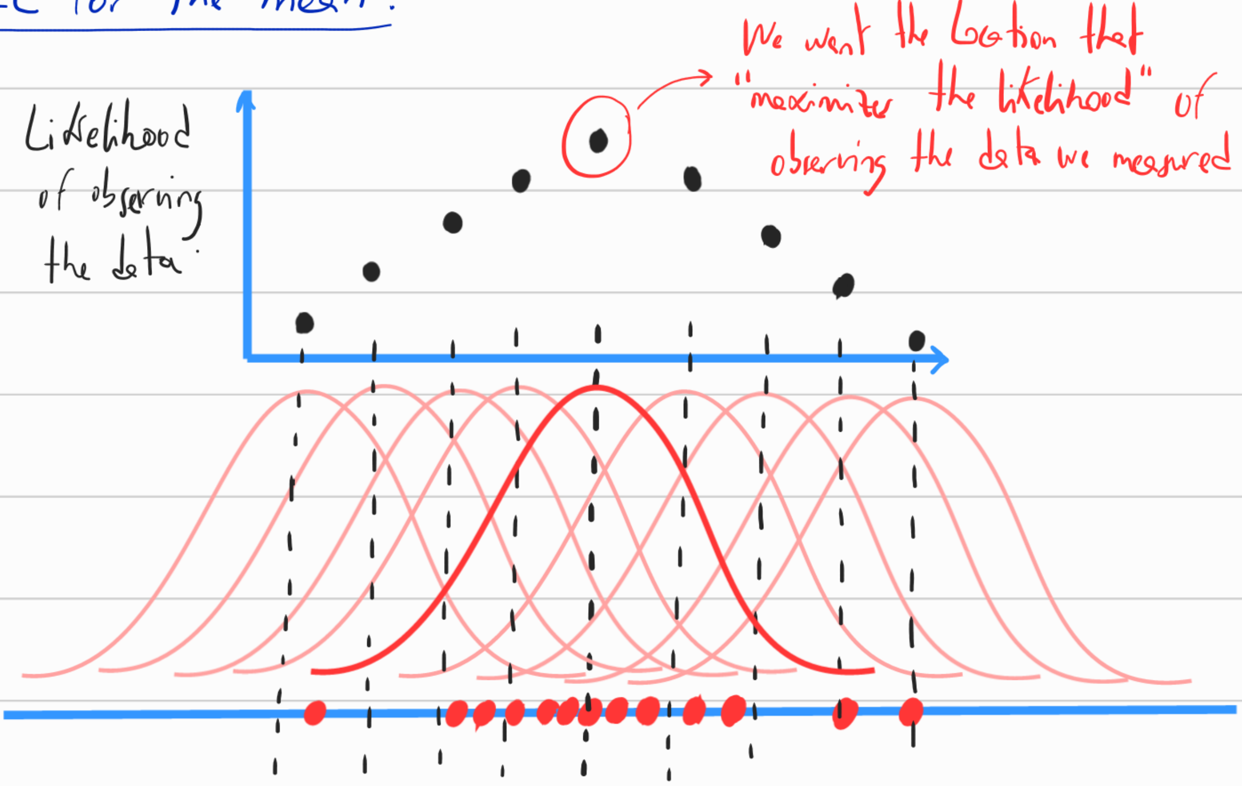
Maximum Likelihood

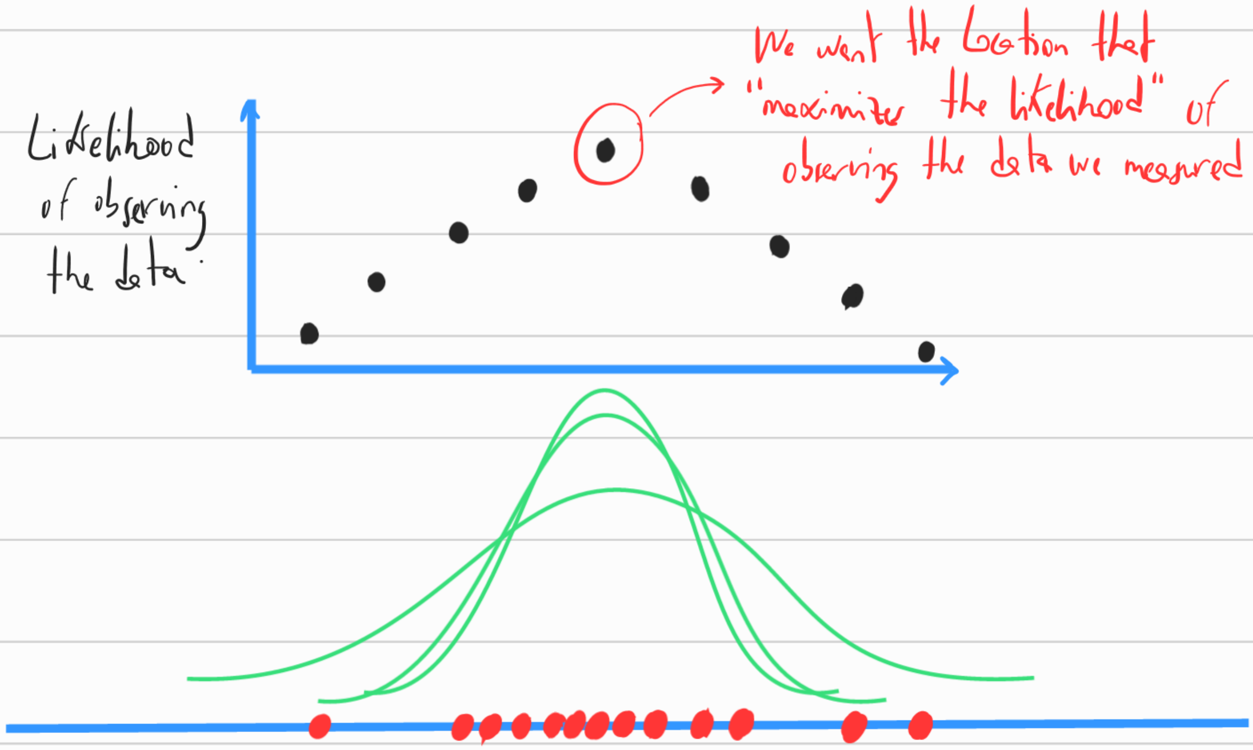
Goal → Find the optimal way to fit a distribution to the data

MLE → Maximum Likelihood Estimations

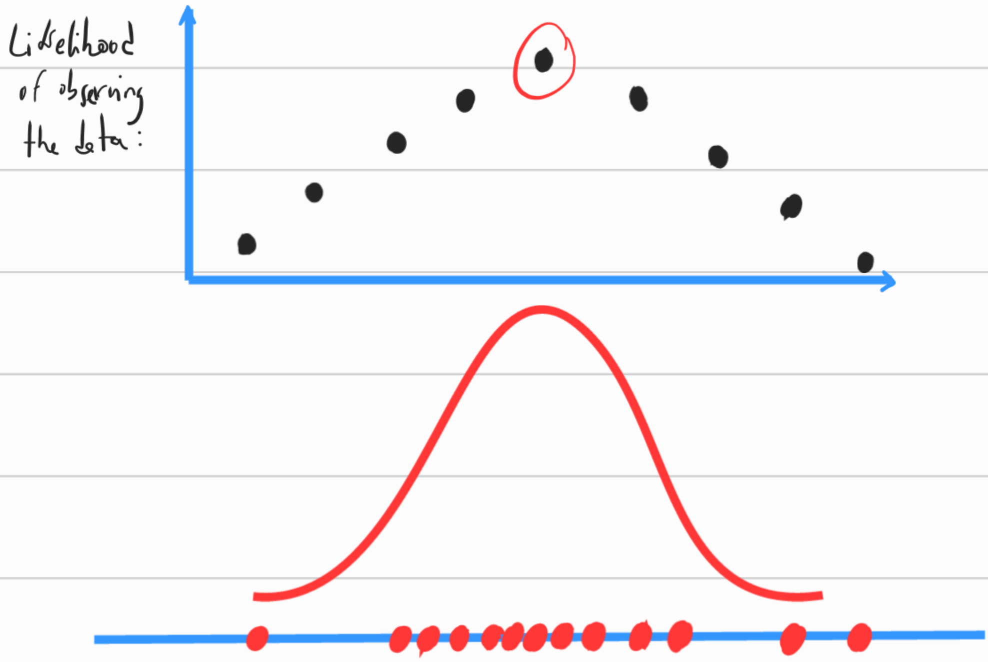
MLE for the mean:



MLE for the Standard Deviation:



Normal distribution that has been "fit" to the data by using MLE,



$$P(X_1 = x_1, X_2 = x_2, \dots, X_n = x_n) = \text{Likelihood} = L$$

$$\left.\frac{\partial L}{\partial P}\right|_{\hat{P}_{MLE}} = 0 \longrightarrow \left.\frac{\partial \log L}{\partial P}\right|_{\hat{P}_{MLE}} = 0$$

<u>Example:</u> $f(x;\lambda) = \frac{1}{24} \cdot \lambda^5 \cdot x^4 \cdot \exp(-\lambda x)$ where $\lambda > 0$, $x > 0$

$$L = \prod_{i=1}^{n} \frac{1}{24} \cdot \lambda^5 \cdot x_i^4 \cdot \exp(-\lambda \cdot x_i) = \left(\frac{1}{24}\right)^n \cdot \lambda^{5n} \left(\prod_{i=1}^{n} x_i^4\right) \cdot \exp\left(-\lambda \cdot \sum_{i=1}^{n} x_i\right)$$

$$\log L = n \cdot \log\left(\frac{1}{24}\right) + 5n \cdot \log\lambda + 4 \cdot \sum_{i=1}^{n} \log x_i - \lambda \cdot \sum_{i=1}^{n} x_i$$

$$\lambda_{MLE} \rightarrow \left.\frac{\partial L}{\partial \lambda}\right|_{\lambda_{MLE}} = 0 \rightarrow \left.\frac{\partial \log L}{\partial \lambda}\right|_{\lambda_{MLE}} = 0$$

$$\frac{\partial \log L}{\partial \lambda} = 0 + 5n \cdot \frac{1}{\lambda} + 0 - \sum_{i=1}^{n} x_i = \frac{5n}{\lambda} - \sum_{i=1}^{n} x_i$$

$$\left.\frac{\partial \log L}{\partial \lambda}\right|_{\lambda_{MLE}} = 0 = \frac{5n}{\lambda_{MLE}} - \sum_{i=1}^{n} x_i \longrightarrow \boxed{\lambda_{MLE} = \frac{5n}{\sum_{i=1}^{n} x_i}}$$