

Homework Assignment

Computer Graphics

Assignment 1

Albert Cerfeda
Alessandro Gobbetti

Contents

| | | |
|---|------------|---|
| 1 | Exercise 1 | 1 |
| 2 | Exercise 2 | 2 |



Università
della
Svizzera
italiana

Faculty of
Informatics

23.09.2022

Università della Svizzera italiana
Faculty of Informatics
Switzerland

1 Exercise 1

$$x = (\sqrt{2}, 1, 0)^T$$

$$y = (1, 1, 1)^T$$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ -1 & -3 & -3 \end{pmatrix}$$

Task 1

$$\langle x, y \rangle = \cos \alpha \|x\| \|y\|$$

$$\Rightarrow \cos \alpha = \frac{\langle x, y \rangle}{\|x\| \|y\|}$$

$$\langle x, y \rangle = \sqrt{2} + 1$$

$$\begin{aligned} \cos \alpha &= \frac{\sqrt{2} + 1}{\sqrt{3} * \sqrt{3}} \\ &= \frac{\sqrt{2} + 1}{3} \end{aligned}$$

Task 2

$$\hat{z} = \frac{x \times y}{\|z\|}$$

$$z_1 = \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1$$

$$z_2 = - \begin{vmatrix} x_1 & y_1 \\ x_3 & y_3 \end{vmatrix} = - \begin{vmatrix} \sqrt{2} & 1 \\ 0 & 1 \end{vmatrix} = -\sqrt{2}$$

$$z_3 = \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} = \begin{vmatrix} \sqrt{2} & 1 \\ 1 & 1 \end{vmatrix} = \sqrt{2} - 1$$

Thus:

$$\begin{aligned} \hat{z} &= \frac{(1 \quad -\sqrt{2} \quad \sqrt{2} - 1)^T}{\sqrt{\langle z, z \rangle}} \\ &= \frac{(1 \quad -\sqrt{2} \quad \sqrt{2} - 1)^T}{\sqrt{3 + 2 + 1 - 2\sqrt{2}}} \\ &= \frac{(1 \quad -\sqrt{2} \quad \sqrt{2} - 1)^T}{\sqrt{6 - 2\sqrt{2}}} \end{aligned}$$

Task 3

$$\begin{aligned} u &= Az \\ &= \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ -1 & -3 & -3 \end{pmatrix} \cdot \frac{(1 \quad -\sqrt{2} \quad \sqrt{2}-1)^T}{\sqrt{6-2\sqrt{2}}} \end{aligned}$$

$$\begin{aligned} u &= \frac{1}{\sqrt{6-2\sqrt{2}}} \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ -1 & -3 & -3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -\sqrt{2} \\ \sqrt{2}-1 \end{pmatrix} \\ &= \frac{1}{\sqrt{6-2\sqrt{2}}} \begin{pmatrix} 1 \cdot 1 + 1 \cdot (-\sqrt{2}) + 1 \cdot (\sqrt{2}-1) \\ 2 \cdot 1 + 2 \cdot (-\sqrt{2}) + 1 \cdot (\sqrt{2}-1) \\ -1 \cdot 1 - 3 \cdot (-\sqrt{2}) - 3 \cdot (\sqrt{2}-1) \end{pmatrix} \\ &= \frac{1}{\sqrt{6-2\sqrt{2}}} \begin{pmatrix} 1 - \sqrt{2} + \sqrt{2} - 1 \\ 2 - 2\sqrt{2} + \sqrt{2} - 1 \\ -1 + 3\sqrt{2} - 3\sqrt{2} + 3 \end{pmatrix} \\ &= \frac{1}{\sqrt{6-2\sqrt{2}}} \begin{pmatrix} 0 \\ 1 - \sqrt{2} \\ 2 \end{pmatrix} \end{aligned}$$

2 Exercise 2

$$c = (1 \quad 1 \quad 1)^T$$

$$\|l\| = \sqrt{\langle l, l \rangle} = \sqrt{3}$$

$$\begin{aligned} \alpha &= 180 - 90 - \arccos \frac{r}{\|l\|} \\ &= 90 - \arccos \frac{\frac{\sqrt{2}}{2}}{\sqrt{3}} \\ &= 90 - \arccos \frac{\sqrt{2}}{2\sqrt{3}} \\ &= 90 - 65.91^\circ = 24.09^\circ \end{aligned}$$