Homework Assignment

Computer Graphics

Assignment 9

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Contents

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1 Exercise 4

A triangle formed by points $A = (0, 0, 0, 1)$, $B = (\frac{1}{2}, 0, 0, 1)$, $C = (0, \frac{1}{\sqrt{2}}, 0, 1)$ is rendered with the following model-view-projection matrix (i.e., the product of projection, view, and model matrix):

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & -\frac{1}{4} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We want to determine the percentage of the rendered image which is covered by the triangle.
The vertex A will be projected to:

$$A_p = MA = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & -\frac{1}{4} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{4} \\ 1 \end{bmatrix}$$

The vertex B will be projected to:

$$B_p = MB = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & -\frac{1}{4} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ 0 \\ -\frac{1}{4} \\ 1 \end{bmatrix}$$

The vertex C will be projected to:

$$C_p = MC = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & -\frac{1}{4} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{2} \\ \frac{1}{4} \\ 1 \end{bmatrix}$$

After the projection transformation the coordinates are mapped to a unit cube: $[-1, 1] \times [-1, 1] \times [-1, 1]$.
To convert them from normalized coordinates (NC) to screen coordinates (SC):

- x is mapped linearly from $[-1, 1]$ to $[0, w]$;
- y is mapped linearly from $[-1, 1]$ to $[0, h]$;
- z is mapped linearly from $[-1, 1]$ to $[0, 1]$.

All the vertices are inside the box so they will all be rendered on the screen. The z-value is not needed for this exercise so we can discard it. It is used to determine which vertex is closed to the camera when there are multiple objects in the scene. If we set the screen width $w = 100$ and the height $h = 100$:

$$A_p = \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{4} \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{4} \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 50 \\ 50 \end{bmatrix} = a$$

$$B_p = \begin{bmatrix} \frac{1}{2} \\ 0 \\ -\frac{1}{4} \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{1}{2} \\ 0 \\ -\frac{1}{4} \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{1}{2} \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 75 \\ 50 \end{bmatrix} = b$$

$$C_p = \begin{bmatrix} 0 \\ \frac{1}{2} \\ \frac{1}{4} \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ \frac{1}{2} \\ \frac{1}{4} \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ \frac{1}{2} \end{bmatrix} \Rightarrow \begin{bmatrix} 50 \\ 75 \end{bmatrix} = c$$

For the exercise scope we could avoid mapping the points to real screen coordinates, but we just wanted to show until the end the real process.
Now that we have the three vertices in screen coordinates we can compute the area of the triangle:

$$\begin{aligned} Area_{triangle} &= \left| \frac{a_x b_y + b_x c_y + c_x a_y - a_y b_x - b_y c_x - c_y a_x}{2} \right| \\ &= \left| \frac{50 \cdot 50 + 75 \cdot 75 + 50 \cdot 50 - 50 \cdot 75 - 50 \cdot 50 - 75 \cdot 50}{2} \right| \\ &= 312.5 \end{aligned}$$

The area of the screen is $w \cdot h = 100 \cdot 100 = 10000$.
Thus the percentage of rendered image covered by the triangle is $312.5/10000 = 3.125\%$