Homework Assignment

Computer Graphics

Assignment 4

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Contents

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1 Exercise 1

1.1 Task 1

$$R_{90} = \begin{bmatrix} cos(\alpha) & -sin(\alpha) & 0 \\ sin(\alpha) & cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} cos(90) & -sin(90) & 0 \\ sin(90) & cos(90) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

1.2 Task 2

Represent $p_1 = (1,1)^T$ and $p = (1.5, 2.5)^T$ and $u = p - p_1 = (0.5, 1.5)^T$ in homogeneous coordinates:

$$p_1 = (1,1,1)^T, \quad p = (1.5, 2.5, 1)^T, \quad u = (0.5, 1.5, 0)^T$$

Rotate the points and the vector:

$$R_{90} \cdot p_1 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \quad p_{1_{90}} = (-1, 1)^T$$

$$R_{90} \cdot p = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1.5 \\ 2.5 \\ 1 \end{bmatrix} = \begin{bmatrix} -2.5 \\ 1.5 \\ 1 \end{bmatrix} \quad p_{90} = (-2.5, 1.5)^T$$

$$R_{90} \cdot u = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.5 \\ 1.5 \\ 0 \end{bmatrix} = \begin{bmatrix} -1.5 \\ 0.5 \\ 0 \end{bmatrix} \quad u_{90} = (-1.5, 0.5)^T$$

Translate the rotated points and vector:

$$T \cdot p_{1_{90}} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \quad p_1' = (0, -1)^T$$

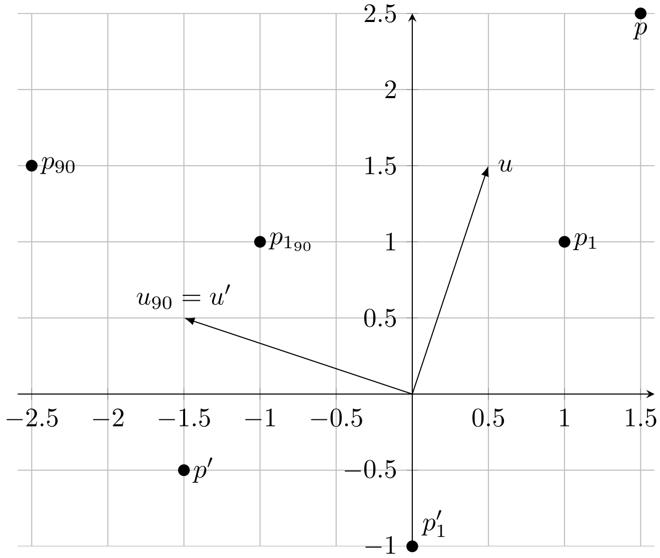
$$T \cdot p_{90} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2.5 \\ 1.5 \\ 1 \end{bmatrix} = \begin{bmatrix} -1.5 \\ -0.5 \\ 1 \end{bmatrix} \quad p' = (-1.5, -0.5)^T$$

$$T \cdot u_{90} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1.5 \\ 0.5 \\ 0 \end{bmatrix} = \begin{bmatrix} -1.5 \\ 0.5 \\ 0 \end{bmatrix} \quad u' = (-1.5, 0.5)^T$$

Verify that $u' = p' - p'$:

$$p' - p_1' = \begin{bmatrix} -1.5 \\ -0.5 \end{bmatrix} - \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -1.5 \\ 0.5 \end{bmatrix} = u'$$

Since u is a vector, it is not affected by the translation. Thus, $u' = u$.



1.3 Task 3

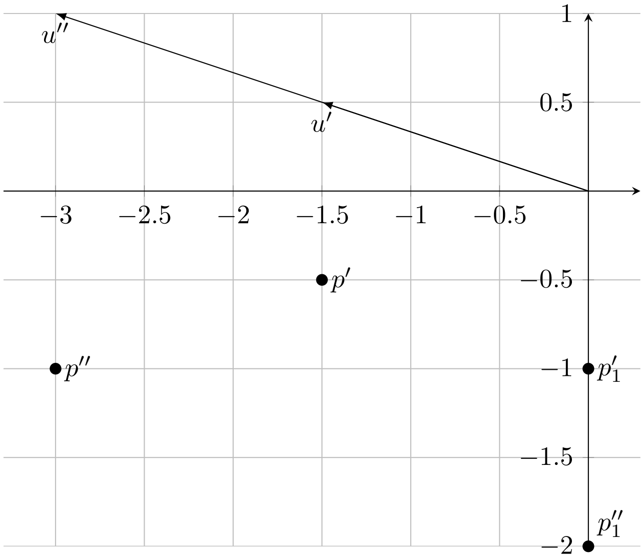
Scaling matrix:

$$S = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S \cdot p'_1 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix} \quad p''_1 = (0, -2)^T$$

$$S \cdot p' = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1.5 \\ -0.5 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \\ 1 \end{bmatrix} \quad p'' = (-3, -1)^T$$

$$S \cdot u' = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1.5 \\ 0.5 \\ 0 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix} \quad u'' = (-3, 1)^T$$



1.4 Task 4

Compute inverse matrix of S:

$$S^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2 Exercise 2

$$p_1 = (6, 0, 4), \quad p_2 = (2, 0, 0), \quad p_3 = (2, 4, 4)$$
$$p = (4, 1, 3)$$

Normal of triangle $[p_1, p_2, p_3]$:

$$n = (p_2 - p_1) \times (p_3 - p_1) = \left(\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 6 \\ 0 \\ 4 \end{pmatrix} \right) \times \left(\begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix} - \begin{pmatrix} 6 \\ 0 \\ 4 \end{pmatrix} \right) = \begin{pmatrix} -4 \\ 0 \\ -4 \end{pmatrix} \times \begin{pmatrix} -4 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 16 \\ 16 \\ -16 \end{pmatrix}$$

Normals of the sub-triangles:

$$n_1 = (p_3 - p) \times (p_1 - p) = \left(\begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix} \right) \times \left(\begin{pmatrix} 6 \\ 0 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix} \right) = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ -4 \end{pmatrix}$$

$$n_2 = (p_1 - p) \times (p_2 - p) = \left(\begin{pmatrix} 6 \\ 0 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix} \right) \times \left(\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix} \right) = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \times \begin{pmatrix} -2 \\ -1 \\ -3 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ -4 \end{pmatrix}$$

$$n_3 = (p_2 - p) \times (p_3 - p) = \left(\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix} \right) \times \left(\begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix} \right) = \begin{pmatrix} -2 \\ -1 \\ -3 \end{pmatrix} \times \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 8 \\ 8 \\ -8 \end{pmatrix}$$

Then compute the signs between the obtained sub-triangles normals with the normal of the triangle:

$$sign(\langle n_1, n \rangle) = sign\left(\left\langle \begin{pmatrix} 4 \\ 4 \\ -4 \end{pmatrix}, \begin{pmatrix} 16 \\ 16 \\ -16 \end{pmatrix} \right\rangle\right) = sign(64 + 64 + 64) = sign(192) = +1$$

$$sign(\langle n_2, n \rangle) = sign\left(\left\langle \begin{pmatrix} 4 \\ 4 \\ -4 \end{pmatrix}, \begin{pmatrix} 16 \\ 16 \\ -16 \end{pmatrix} \right\rangle\right) = sign(64 + 64 + 64) = sign(192) = +1$$

$$sign(\langle n_3, n \rangle) = sign\left(\left\langle \begin{pmatrix} 8 \\ 8 \\ -8 \end{pmatrix}, \begin{pmatrix} 16 \\ 16 \\ -16 \end{pmatrix} \right\rangle\right) = sign(128 + 128 + 128) = sign(384) = +1$$

Since every sign is positive, the point is inside the triangle.

3 Exercise 3

4 Exercise 4

5 Bonus exercise 5