Università della Svizzera italiana | Faculty of Informatics | Institute of Computing CI

Numerical Computing **2022**

Student: FULL NAME Discussed with: FULL NAME

Solution for Project 2

Due date: Wednesday, 26 October 2022, 23:59 AM

Numerical Computing 2022 — Submission Instructions
(Please, notice that following instructions are mandatory: submissions that don't comply with, won't be considered)

- Assignments must be submitted to iCorsi (i.e. in electronic format).
- Provide both executable package and sources (e.g. C/C++ files, Julia). If you are using libraries, please add them in the file. Sources must be organized in directories called:
  Project_number_lastname_firstname
  and the file must be called:
  project_number_lastname_firstname.zip
  project_number_lastname_firstname.pdf
- The TAs will grade your project by reviewing your project write-up, and looking at the implementation you attempted, and benchmarking your code's performance.
- You are allowed to discuss all questions with anyone you like; however: (i) your submission must list anyone you discussed problems with and (ii) you must write up your submission independently.

The purpose of this assignment[1] is to learn the importance of sparse linear algebra algorithms to solve fundamental questions in social network analyses. We will use the coauthor graph from the Householder Meeting and the social network of friendships from Zachary's karate club [1]. These two graphs are one of the first examples where matrix methods were used in computational social network analyses.

Social Networks [Total: 85 points + 15 points for report quality]

1. The Reverse Cuthill McKee Ordering [10 points]

2. Sparse Matrix Factorization [20 points]

a)

$$A_{ij} = \begin{cases} 1 & \text{if } i = 1 \text{ or } j = 1 \text{ or } j = n \text{ and } i \neq j \\ n + i - 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$$

[1]This document is originally based on a blog from Cleve Moler, who wrote a fantastic blog post about the Lake Arrowhead graph, and John Gilbert, who initially created the coauthor graph from the 1993 Householder Meeting. You can find more information at `http://blogs.mathworks.com/cleve/2013/06/10/lake-arrowhead-coauthor-graph/`. Most of this assignment is derived from this archived work.

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 11 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 12 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 13 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 14 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 15 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 16 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 17 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 18 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Number of non-zero elements: 44;

b)

A matrix $A \in R^{n \times n}$ will have $n \times n$ entries. The non-zero entries are:

- n entries in the first row
- n entries in the last row
- n entries in the first column
- n entries in the last column
- n entries in the diagonal

So $5n$ minus the entries that are counted twice or more:

- A_{11} is counted in the first row, the first column and in the diagonal
- A_{nn} is counted in the last row, the last column and in the diagonal
- A_{1n} is counted in the first row and the last column
- A_{n1} is counted in the last row and the first column

So the total number of non-zero entries is: $5n - 2 - 2 - 1 - 1 = 5n - 6$.

d)

e)

3. Degree Centrality [5 points]

4. The Connectivity of the Coauthors [5 points]

5. PageRank of the Coauthor Graph [5 points]

6. Zachary's karate club: social network of friendships between 34 members [40 points]

References

[1] The social network of a karate club at a US university, M. E. J. Newman and M. Girvan, Phys. Rev. E 69,026113 (2004) pp. 219-229.