Università della Svizzera italiana | Faculty of Informatics | Institute of Computing CI

Numerical Computing **2022**

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Solution for Project 6

Due date: Wednesday, December 21, 2022, 11:59 PM

Numerical Computing 2022 — Submission Instructions
(Please, notice that following instructions are mandatory: submissions that don't comply with, won't be considered)

- Assignments must be submitted to iCorsi (i.e. in electronic format).
- Provide both executable package and sources (e.g. C/C++ files, Julia). If you are using libraries, please add them in the file. Sources must be organized in directories called:
  Project_number_lastname_firstname
  and the file must be called:
  project_number_lastname_firstname.zip
  project_number_lastname_firstname.pdf
- The TAs will grade your project by reviewing your project write-up, and looking at the implementation you attempted, and benchmarking your code's performance.
- You are allowed to discuss all questions with anyone you like; however: (i) your submission must list anyone you discussed problems with and (ii) you must write up your submission independently.

The purpose of this project is to implement the Simplex Method to find the solution of linear programs, involving both the minimisation and the maximisation of the objective function.

Contents

1. Graphical Solution of Linear Programming Problems [20 points]

1.1 Minimization problem

$$\begin{aligned} \min\ z &= 4x + y \\ \textbf{s.t}\ x + 2y &\le 40 \\ x + y &\ge 30 \\ 2x + 3y &\ge 72 \\ x, y &\ge 0 \end{aligned}$$

Let us plot the inequalities on the Cartesian plane:

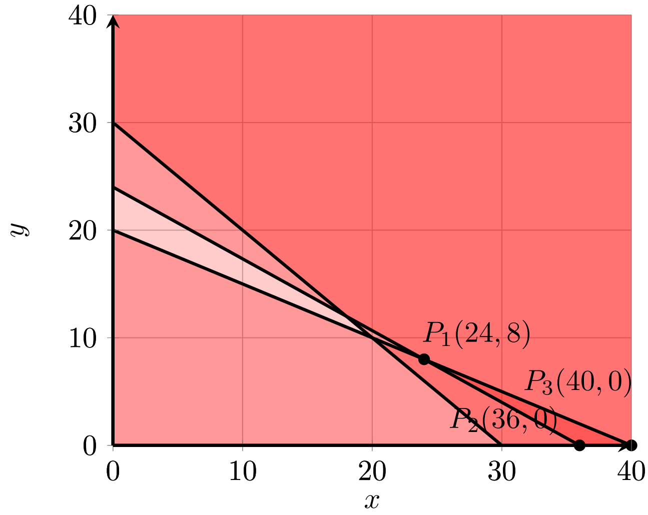

Figure 1: Feasibile region for the Minimization problem

We notice three intersection vertices:
$x = 24 \quad x = 36, y = 0 \quad x = 40, y = 0$
The point with $x = 24$ is the intersection of inequalities $x + 2y \le 40$ and $2x + 3y \ge 72$. Let's find the y component of the intersection point:
$24 + 2y = 40 \Rightarrow y = 8$.

We now need to calculate the objective function value for each vertex:
$p_1 = (24, 8) \quad p_2 = (36, 0) \quad p3 = (40, 0)$

$$\begin{aligned} z_1 &= 4(24) + 8 = 104 \\ z_2 &= 4(36) + 10 = 144 \\ z_3 &= 4(40) + 0 = 160 \end{aligned}$$

Therefore $\min z = z_1 = 104$

1.2 Tailor maximisation problem

The tailor in our problem sells two types of trousers, which we will identify as x for the first type of trousers and y for the second type of trousers.
The manufacturing cost of trouser x equals $m_x = 25$ and for the case of trouser y equals $m_y = 40$.
The retail price for trouser x equals 85 and for trouser y equals 110.
We denote the net profit for each trouser as n_T where T is the type of trouser. We can infer that $n_x = 85 - 25 = 60$ and $n_y = 110 - 40 = 70$.

Note: in the text of the exercise it is written *The tailor estimates a total monthly demand of 265 trousers.* I interpreted it such that the tailor does not expect more than 365 trousers, i.e $x + y \leq 265$. Were it to be intepreted as if the tailor was to expect at least 265 trousers, it would been have modeled with $x + y \geq 265$.
Let us model the objective function aswell as the constraints:

$$
\begin{aligned}
\max\ z &= 60x + 70y \\
\textbf{s.t}\ x + y &\leq 265 \\
25x + 40y &\leq 7000 \\
x, y &\geq 0, z \geq 0
\end{aligned}
$$

That is, the tailor wants to *maximize* the net profit from selling both types of trousers, expects to sell not more than 265 trousers, and spend less that 7000 in raw materials.
The non-negativity constraints are intuitive as it is not possible to sell a negative amount of trousers.
Let's solve the systems of inequalities:

$$
\begin{cases} y = 0 \\ x + y = 265 \end{cases}
$$

$$
\begin{cases} y = 0 \\ x = 265 \end{cases}
$$

$$\Rightarrow P_1(265, 0)$$

$$
\begin{cases} x + y = 265 \\ 25x + 40y = 7000 \end{cases}
$$

$$
\begin{cases} x = 265 - y \\ 6625 - 25y + 40y = 7000 \end{cases}
$$

$$
\begin{cases} x = 265 - y \\ y = 25 \end{cases}
$$

$$\Rightarrow P_2(240, 25)$$

$$
\begin{cases} x = 0 \\ 40y = 7000 \end{cases}
$$

$$\Rightarrow P_3(0, 175)$$

Let's plot the inequalities on the Cartesian Plane:

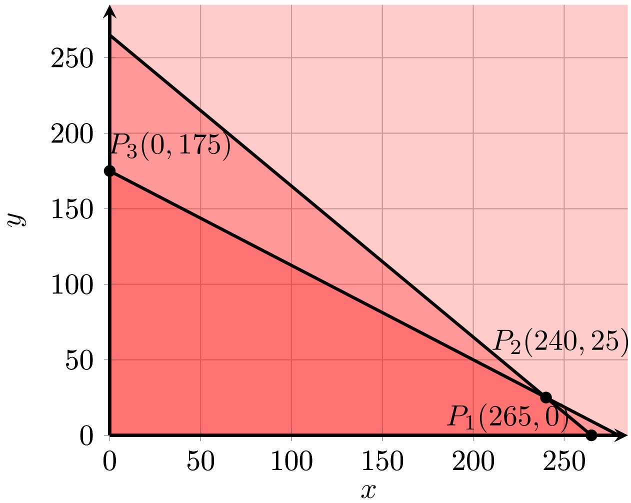

Figure 2: Feasible region for the Tailor maximisation problem

2. Implementation of the Simplex Method [30 points]

The code has been successfully implemented in Julia.
When running the provided tests, all of them pass without incurring in the maximum iterations upper bound.
Please checkout the source files in the `src/` folder, as it contains my implementation.

3. Applications to Real-Life Example: Cargo Aircraft [25 points]

3.1 Modelling the linear program

Before modelling the linear problem we need to define the variable t:

$$t_{xn} \text{where } x \text{ is the tonnes allocated from cargo } C_x \text{ and } n \text{ is the target Storage compartment } S_n$$

There t_14 are the tonnes from Cargo 1 ie C_1 put into Storage compartment 4 ie S_r.

$$\begin{aligned}\max z = {} & (135 * 1 * t_{11}) + (135 * 1.1 * t_{12}) \\ & + (200 * 1 * t_{21}) + (200 * 1.1 * t_{22}) \\ & + (200 * 1.2 * t_{23}) + (200 * 1.3 * t_{24}) \\ & + (410 * 1 * t_{31}) + (410 * 1.1 * t_{32}) \\ & + (410 * 1.2 * t_{33}) + (410 * 1.3 * t_{34}) \\ & + (520 * 1 * t_{41}) + (520 * 1.1 * t_{42}) \\ & + (520 * 1.2 * t_{43}) + (520 * 1.3 * t_{44})\end{aligned}$$

The objective function expresses the need to maximize the amount of profit from properly splitting the cargo in the available Storage compartments from the various Cargo containers.

$$\begin{aligned}\textbf{s.t. } & \textbf{Compartment weight capacity constraints} \\ & t_{11} + t_{21} + t_{31} + t_{45} \leq 18 \\ & t_{12} + t_{22} + t_{32} + t_{46} \leq 32 \\ & t_{13} + t_{23} + t_{33} + t_{47} \leq 25 \\ & t_{14} + t_{24} + t_{34} + t_{48} \leq 17\end{aligned}$$

Here we model the constraints coming from all the cargo loaded into the Storage compartment, not having to exceed the weight limit of the Storage compartment itself.

Compartment volume capacity constraints

$$320 * t_{11} + 510 * t_{21} + 630 * t_{31} + 125 * t_{45} \leq 11930$$
$$320 * t_{12} + 510 * t_{22} + 630 * t_{32} + 125 * t_{46} \leq 22552$$
$$320 * t_{13} + 510 * t_{23} + 630 * t_{33} + 125 * t_{47} \leq 11209$$
$$320 * t_{14} + 510 * t_{24} + 630 * t_{34} + 125 * t_{48} \leq 5870$$

Here we model the constraints coming from all the cargo loaded into the Storage compartment, not having to exceed the volume limit of the Storage compartment itself.

Cargo volume availability constraints

$$(t_{11} + t_{12} + t_{13} + t_{14}) \leq 16$$
$$(t_{21} + t_{22} + t_{23} + t_{24}) \leq 32$$
$$(t_{31} + t_{32} + t_{33} + t_{34}) \leq 40$$
$$(t_{41} + t_{42} + t_{43} + t_{44}) \leq 28$$

Here we model the maximum amount of cargo inside each container. Thus, we can't split more cargo than there actually is.

Non-negativity constraints

$$(t_{11} + t_{12} + t_{13} + t_{14}) \geq 0$$
$$(t_{21} + t_{22} + t_{23} + t_{24}) \geq 0$$
$$(t_{31} + t_{32} + t_{33} + t_{34}) \geq 0$$
$$(t_{41} + t_{42} + t_{43} + t_{44}) \geq 0$$

Obviously the non-negativity constraints come from the impossibility of allocating a negative cargo weight.

3.1. Solving the linear problem using Julia

Let us model the problem in Julia for solving it:

Important: Coefficient matrix A has been omitted from the report as it wouldnt have fit the page. Please check it out inside file `src/exercise2.jl`. The type has been set to "max" as this is a maximisation problem. Matrix A holds all the coefficients for the constraints, as modelled in the subsection before. Vector h contains all the right-hand sides of the constrain inequalities. Vector c contains all the coefficients of the objective function. Vector *sign* contains the information on whether each inequality sign has to be flipped. As this is a maximisation problem, and every explicit constraints inequality in our constraint is $\leq$, we have to flip all of them.

4. Cycling and Degeneracy [10 points]