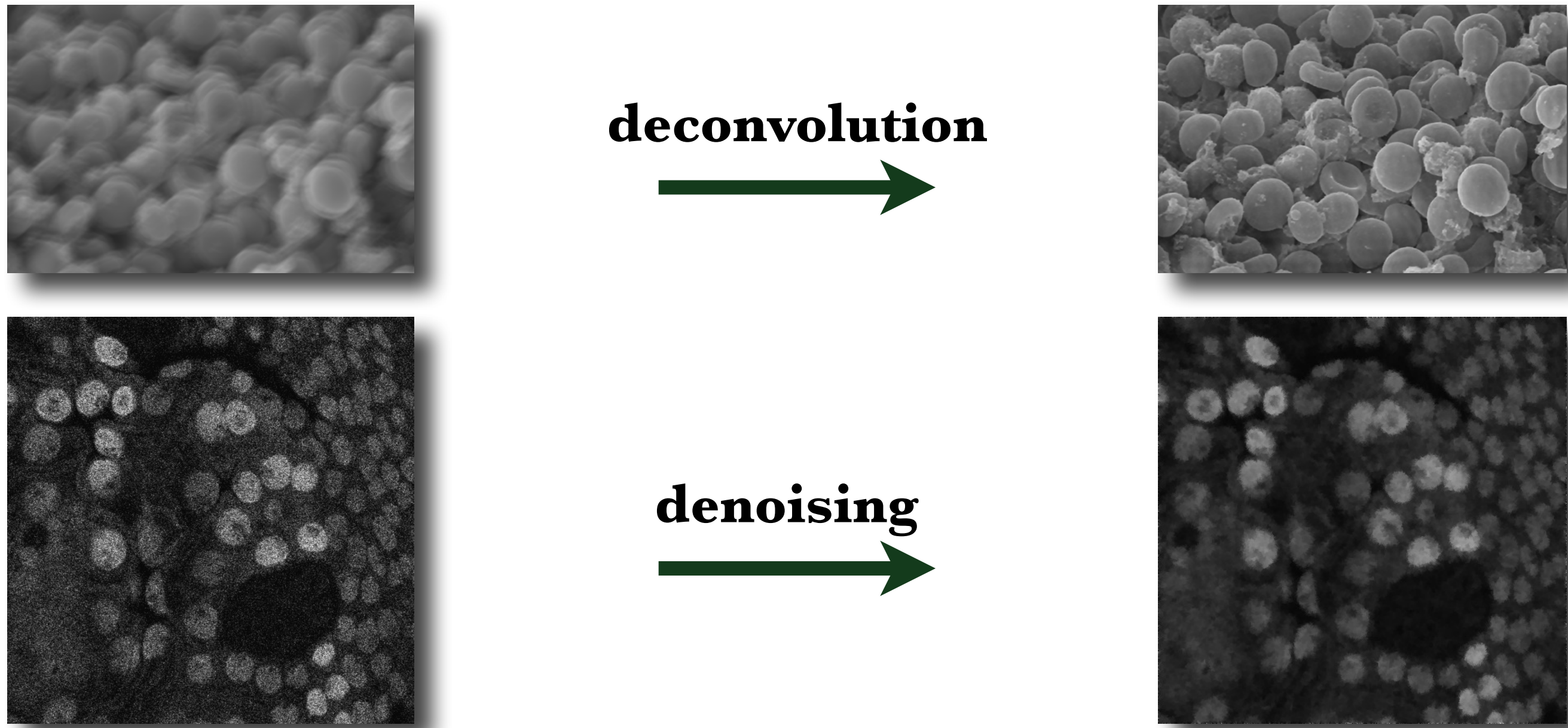


# Bernstein Filter: a new solver for mean curvature regularized models

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## Introduction :

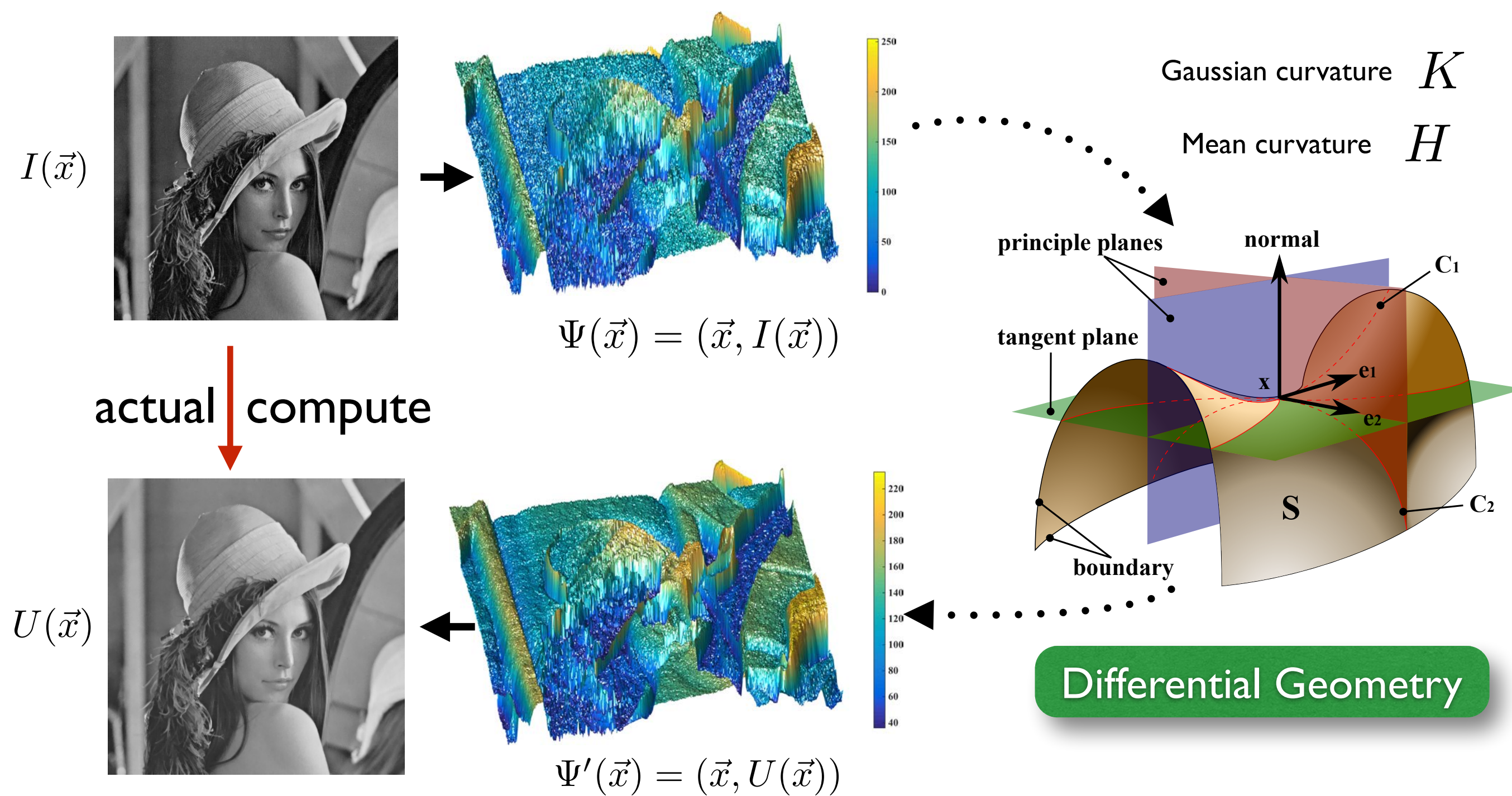
Signal processing problems are usually ill-posed, where a prior or regularization is needed.



Mean Curvature is a good prior

## Mean Curvature Regularization:

Images are surfaces:



Mean curvature is:

$$H(U(\vec{x})) = \frac{(1 + U_y^2)U_{xx} - 2U_xU_yU_{xy} + (1 + U_x^2)U_{yy}}{2(1 + U_x^2 + U_y^2)^{\frac{3}{2}}}$$

## Contribution:

We prove that:

mean curvature is a **CONVEX** term

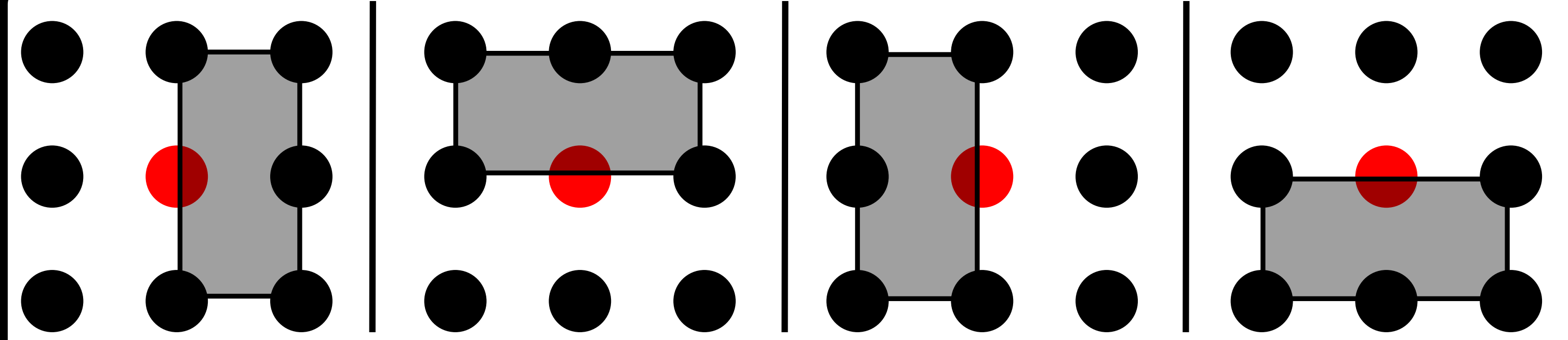
We show that:

Bernstein Filter is **fast** and effective

## Convexity:

According to **Bernstein Theorem**, minimizing mean curvature is assuming that the signal is piece-wise linear. Based on this linearity, we can prove that mean curvature regularization term is convex.

## Bernstein Filter:



Impulsing the linearity on the four half-windows (above) by Least Square Regression, we get the Bernstein Filter:

### Algorithm 1 Bernstein Filter

**Require:** IterationNum,  $I(x_i, y_j)$

$U^0(x_i, y_j) = I(x_i, y_j), t = 0$

**while**  $t < \text{IterationNum}$  **do**

**for**  $i=2:M-1, j=2:N-1$  **do**

$d_1 = \frac{1}{2} [U^t(x_{i-1}, y_j) + U^t(x_{i+1}, y_j)] - U^t(x_i, y_j)$

$d_2 = \frac{1}{2} [U^t(x_i, y_{j-1}) + U^t(x_i, y_{j+1})] - U^t(x_i, y_j)$

    find  $d_m$  such that  $|d_m| = \min_{k=1,2} \{|d_k|\}$

$U^{t+1}(x_i, y_j) = U^t(x_i, y_j) + d_m$

**end for**

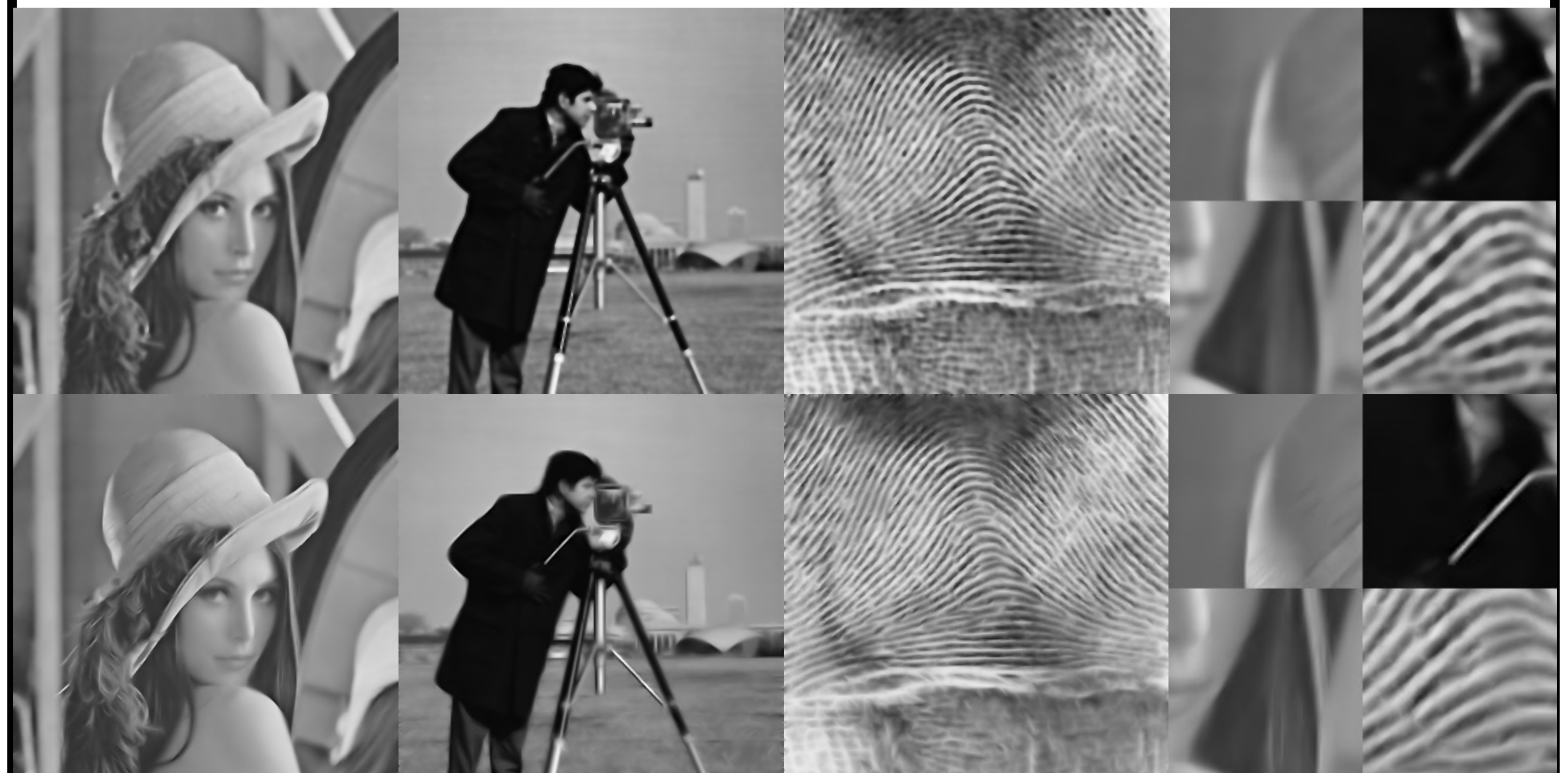
$t = t + 1$

**end while**

**Ensure:**  $U(x_i, y_j)$

## Experiments:

Results from Multi Grid Solver(first row) and Bernstein Filter(second row) are similar because both solve the same variational model. However, our filter is much faster.



**Two or three orders of magnitude FASTER!**

solver (language)	Multigrid (Matlab)	Our filter (Matlab)	Our filter (C++)
Lena	183	1.1	0.025
Cameraman	648	1.1	0.025
Fingerprint	587	1.1	0.025

Table 1: time in seconds on  $512 \times 512$  images. Our filter runs 30 iterations.

Contact and  software:

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<https://github.com/YuanhaoGong/CurvatureFilter>



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Yuanhao Gong et al. A natural scene gradient distribution prior and its application in light microscopy image processing, *IEEE J-STSP*, 2016.

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