Adaptive and Array Signal Processing
Homework 07

This problem intends to compare the resolution capabilities of the MUSIC algorithm, MVDR beamformer and classical Fourier based periodogram when applied to an azimuth angle-of-arrival estimation task. We consider a linear array consisting of $M = 12$ uniformly spaced antenna elements. Three equally powered, uncorrelated, plane wavefronts are impinging at the array. We have $N = 100$ snapshots available and the Signal-to-Noise ratio is SNR $=$ 10dB (white gaussian noise, uncorrelated with the signals). The transmitted signals are Q-PSK modulated and have unit power, i.e. they take on the four values:

$$\left\{ \frac{1}{\sqrt{2}}(1+\mathrm{j}), \frac{1}{\sqrt{2}}(1-\mathrm{j}), \frac{1}{\sqrt{2}}(-1+\mathrm{j}), \frac{1}{\sqrt{2}}(-1-\mathrm{j}) \right\}.$$

1. Use *MATLAB* or *OCTAVE* to plot the power spectra as a function of the spatial frequency μ, normalized to the so called *standard beamwidth*

   $$\mu_B = \frac{2\pi}{M}$$

   for the following spatial separations

   - $\mu_1 = -2\mu_B$, $\mu_2 = 0$, $\mu_3 = 2\mu_B$ (two beamwidth separation)
   - $\mu_1 = -\mu_B$, $\mu_2 = 0$, $\mu_3 = \mu_B$ (one beamwidth separation)
   - $\mu_1 = -0.5\mu_B$, $\mu_2 = 0$, $\mu_3 = 0.5\mu_B$ (one half beamwidth separation)
   - $\mu_1 = -0.1\mu_B$, $\mu_2 = 0$, $\mu_3 = 0.1\mu_B$ (one tenth beamwidth separation)

   for

   - The MVDR spectrum, $S_{\mathrm{MVDR}}(\mu) = \frac{1}{\mathbf{a}^H(\mu)\mathbf{R}_{xx}^{-1}\mathbf{a}(\mu)}$
   - The Fourier spectrum, $S_{\mathrm{PER}}(\mu) = \frac{1}{N \cdot M^2} \sum_{n=1}^{N} \left| \sum_{k=0}^{M-1} x_{k+1}(t_n) e^{-j\mu k} \right|^2$
   - the MUSIC spectrum, $S_{\mathrm{MUSIC}}(\mu) = \frac{\mathbf{a}^H(\mu)\mathbf{a}(\mu)}{\mathbf{a}^H(\mu)\mathbf{U}_o\mathbf{U}_o^H\mathbf{a}(\mu)}$

2. Repeat the above problem with an SNR $= 20$dB.

Hints:

- If the spatial frequencies of the impinging wavefronts are packed into a $d \times 1$ column vector mu, then the array output for N snapshots can be calculated in MATLAB like

```
X = exp(i*([0:M-1]')*mu')*(sign(randn(d,N))+i*sign(randn(d,N)))/(sqrt(2)) +
    sqrt(d)*(randn(M,N)+i*randn(M,N))/(sqrt(2)*10^(SNR/20));
```

- Plot the power spectra in $-3\mu_B \leq \mu \leq 3\mu_B$ using about 200 equally spaced points.
- Help on OCTAVE available at: http://www.gnu.org/software/octave/

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The main code of the program is the following:

```
clear all

M = 12;
N = 100;
d = 3;
mu_b = 2*pi/M;

figure(1)
title('Two beamwidth separation, SNR=10 dB')
power_spectra(M,N,d,2  ,mu_b,10);
figure(2)
title('One beamwidth separation, SNR=10 dB')
power_spectra(M,N,d,1  ,mu_b,10);
figure(3)
title('One half beamwidth separation, SNR=10 dB')
power_spectra(M,N,d,0.5,mu_b,10);
figure(4)
title('One tenth beamwidth separation, SNR=10 dB')
power_spectra(M,N,d,0.1,mu_b,10);
figure(5)
title('Two beamwidth separation, SNR=20 dB')
power_spectra(M,N,d,2  ,mu_b,20);
figure(6)
title('One beamwidth separation, SNR=20 dB')
power_spectra(M,N,d,1  ,mu_b,20);
figure(7)
title('One half beamwidth separation, SNR=20 dB')
power_spectra(M,N,d,0.5,mu_b,20);
figure(8)
title('One tenth beamwidth separation, SNR=20 dB')
power_spectra(M,N,d,0.1,mu_b,20);
```

It calls the function called power_spectra, which is listed below:

```
function power_spectra(M,N,d,b,mu_b,SNR)

mu = [-b*mu_b; 0; b*mu_b];
X = exp(i*([0:M-1]')*mu')*(sign(randn(d,N))+i*sign(randn(d,N)))/(sqrt(2)) + 
sqrt(d)*(randn(M,N)+i*randn(M,N))/(sqrt(2)*10^(SNR/20));
Rxx_est = X*X'/N;
[U,D] = SortedEig(Rxx_est);
U0 = U(:,d+1:M);

mu = linspace(-3*mu_b, 3*mu_b, 200)';
k = 0:M-1;
n = 1:N;
for l = 1:length(mu)
    a = exp(i*([0:M-1]')*mu(l));
    S_MVDR(l) = 1/(a'*inv(Rxx_est)*a);
```

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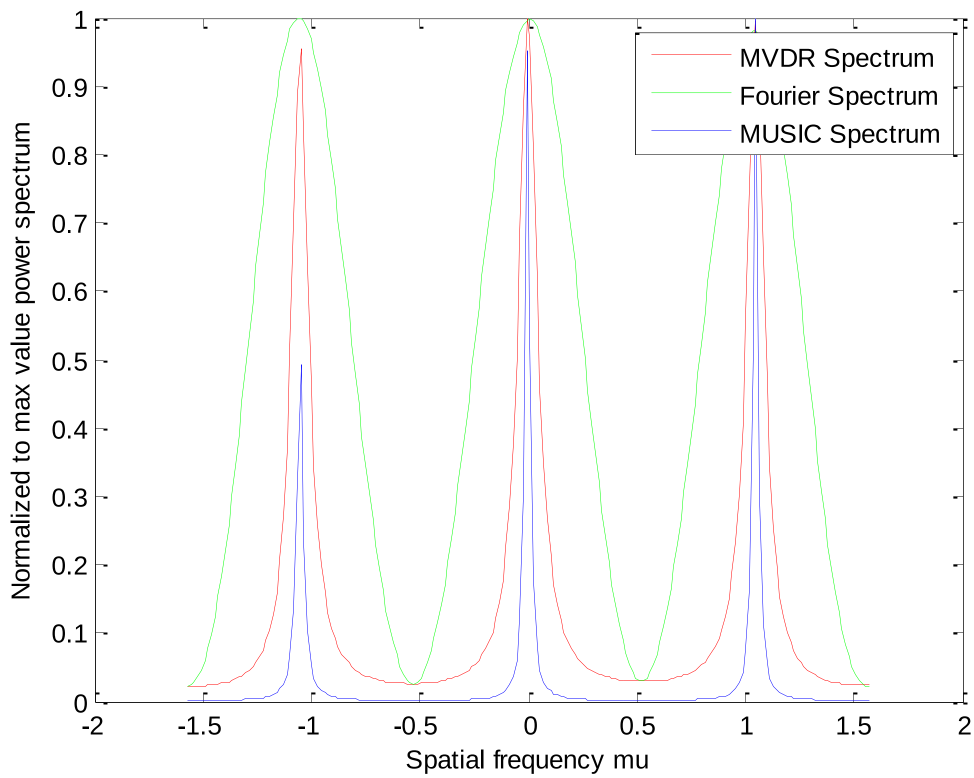
```
    S_PER(l) = norm(abs(a'*X))^2/(N*M^2);
    S_MUSIC(l) = (norm(a)/norm(a'*U0))^2;
end

plot(mu, S_MVDR/max(S_MVDR), 'r')
hold on
plot(mu, S_PER/max(S_PER), 'g')
hold on
plot(mu, S_MUSIC/max(S_MUSIC), 'b')
legend('MVDR Spectrum','Fourier Spectrum', 'MUSIC Spectrum')
xlabel('Spatial frequency mu')
ylabel('Normalized to max value power spectrum')
```



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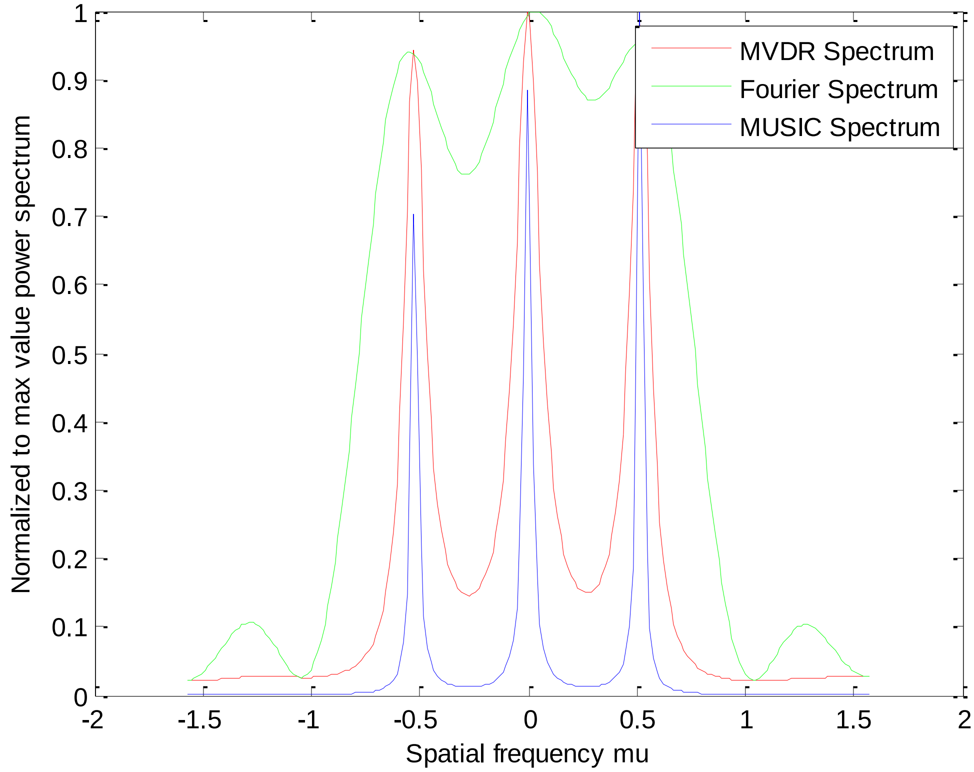

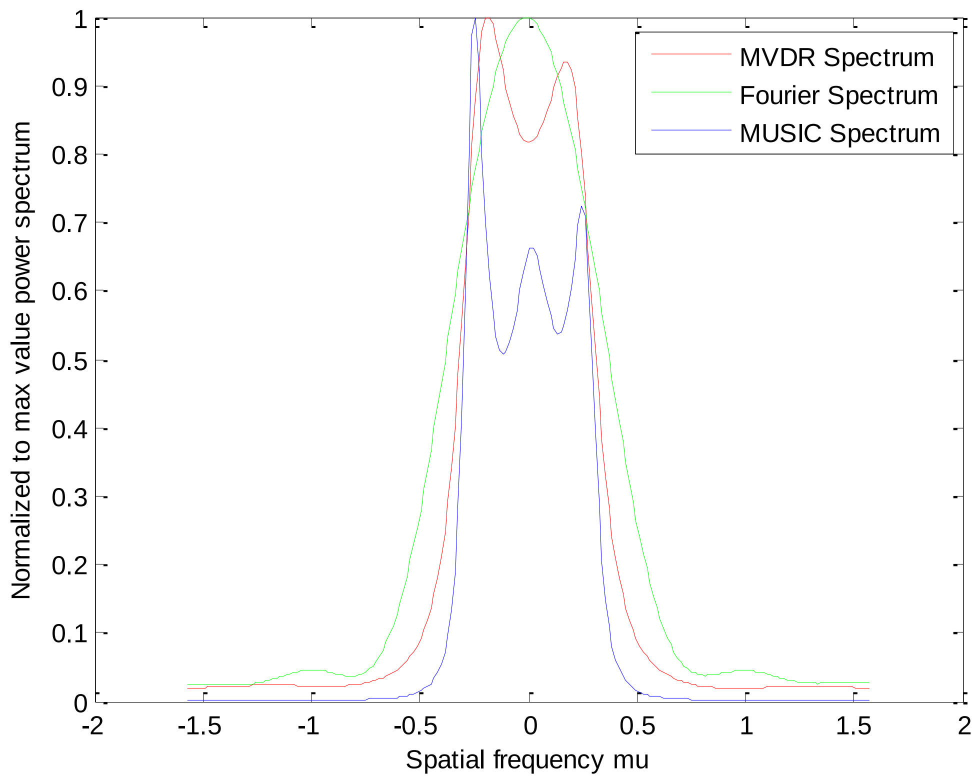

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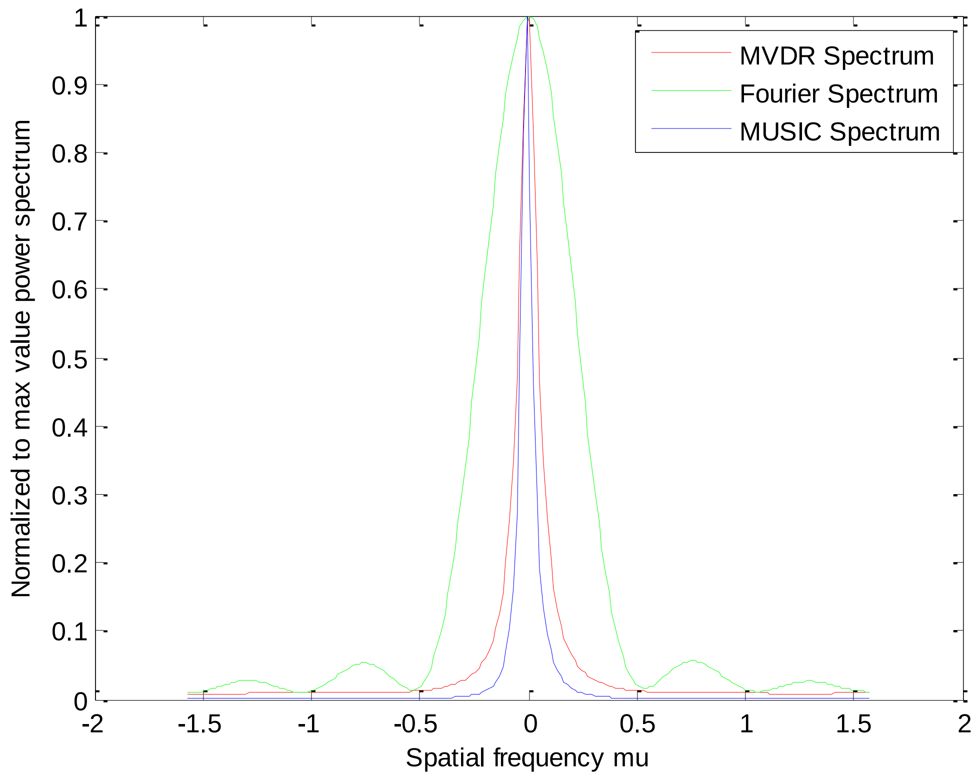

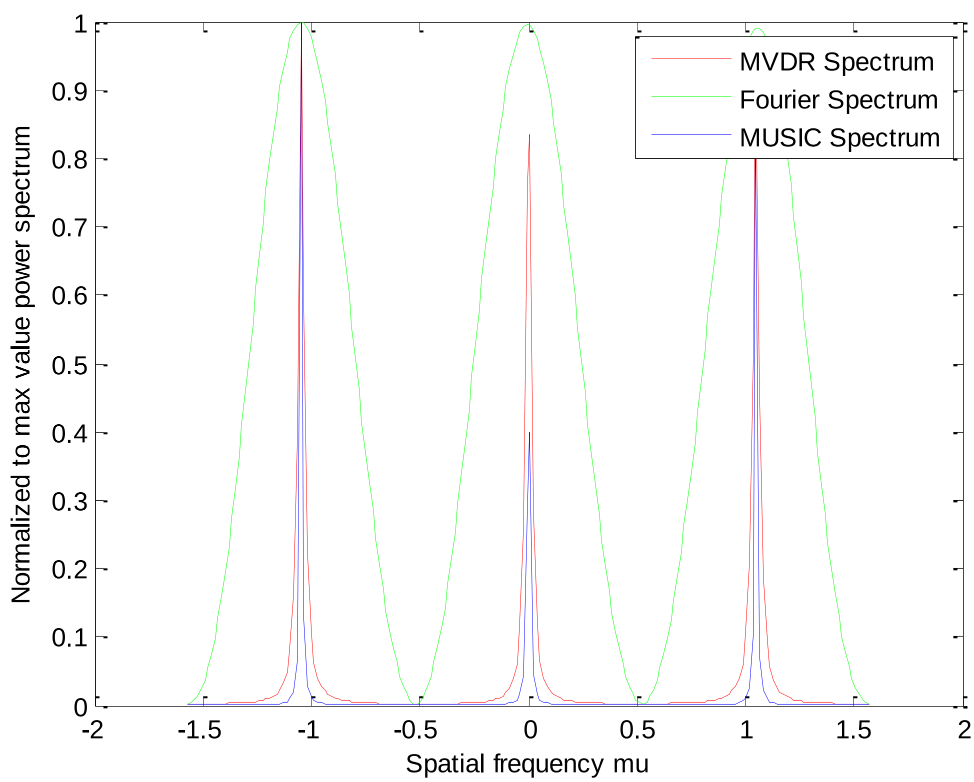

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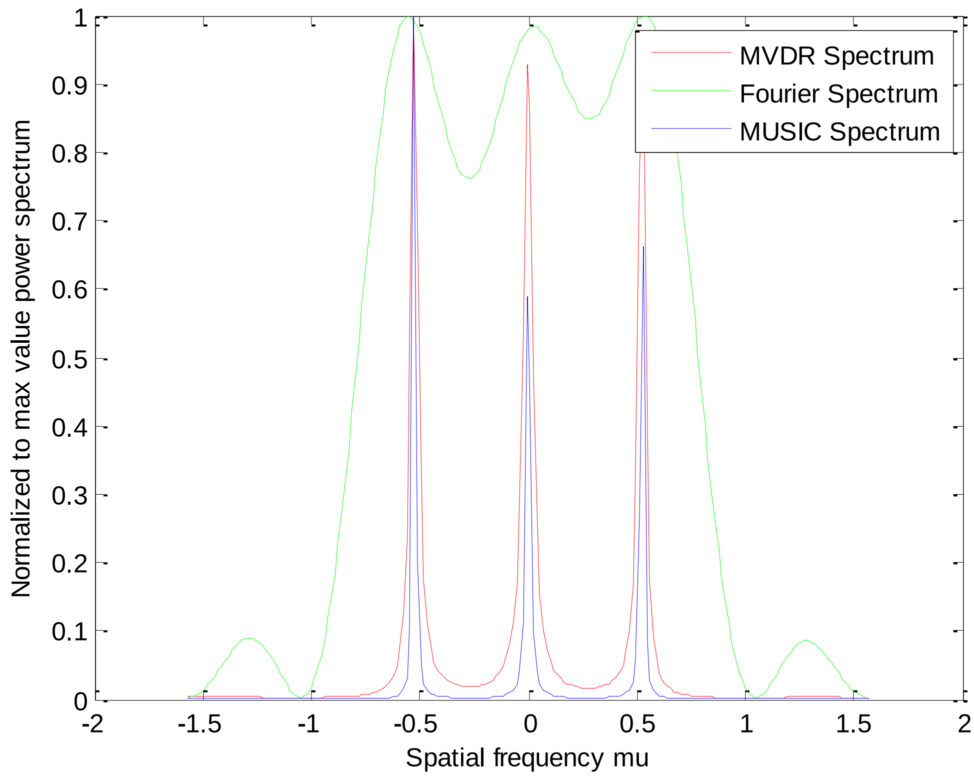

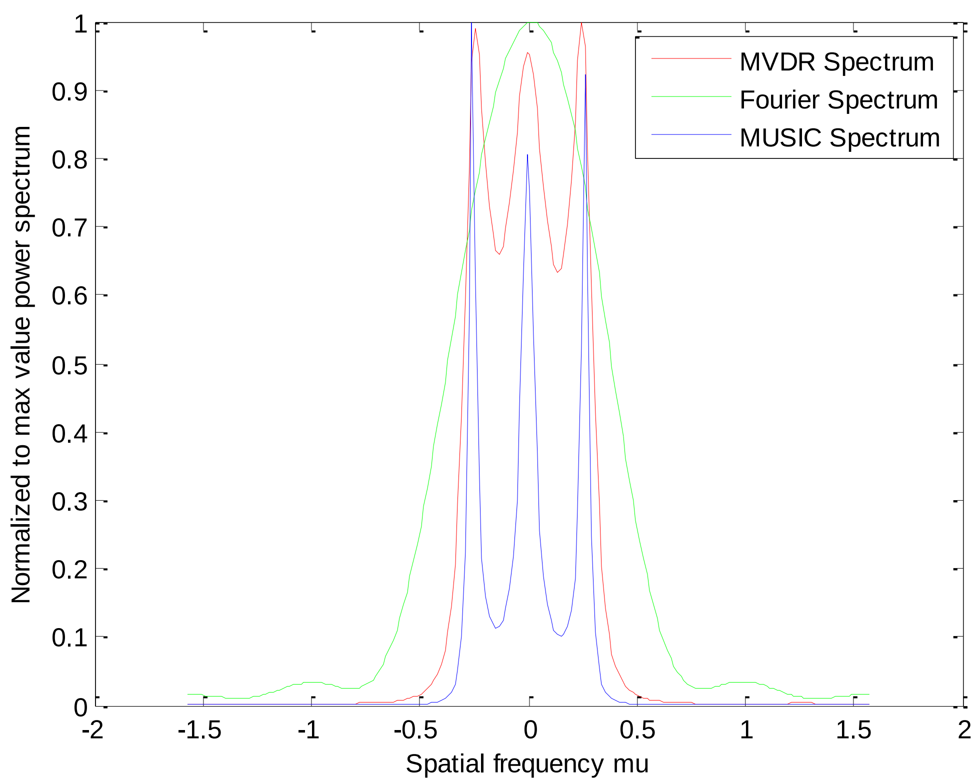

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