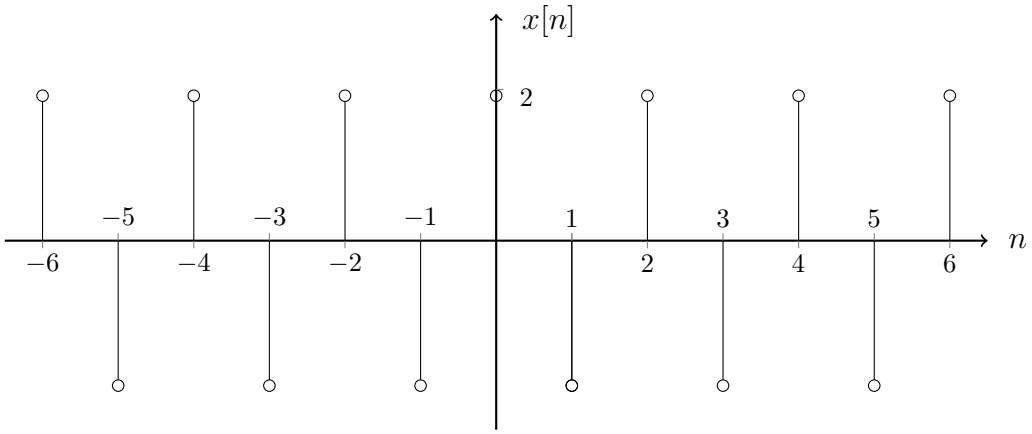
Problem 1

(29 points)

Given is the following signal $x[n]$



a)* Show the graphical representation of the signal $y\,[n] = x\,[Mn/L]$, where $M = 2$ and $L = 3$. What is sampling frequency of after up-sampling and downsampling operations if the original sampling frequency is 8 KHz.

b)* Show the graphical representation of the signal $y\,[n] = x\,[n] + x\,[n-1]\,x\,[2n]$.

c)* Sketch the block diagram representation of the discrete-time signal described by the input-output relation

$$y[n] = \frac{1}{4}y[n-2] + \frac{1}{2}x[n+1], \tag{1}$$

where $x[n]$ is the input and the $y[n]$ is the output of the system.

d)* Determine whether the system $y[n] = x[n]\cos(n)$ is time-invariant or time-variant

e)* Determine whether the system $y[n] = x[-n]$ is time-invariant or time-variant

f)* Determine whether the system $y[n] = x[2n]$ is time-invariant or time-variant

g)* Determine if the system described by the relation

$$y[n] = x[n] + y[n-1]$$

is linear or non-linear.

h)* Determine if the system described by the relation

$$y[n] = \text{Re}\left\{e^{j\frac{\pi}{2}} x[n]\right\}$$

where $x[n] = x_R[n] + jx_I[n]$ is linear or non-linear.

i)* What is the dimension of the matrix $\mathbf{H}$ in

$$\begin{bmatrix} y[0] \\ y[1] \\ y[2] \\ \vdots \\ \vdots \\ y[M+K] \end{bmatrix} = \underbrace{\begin{bmatrix} h[0] & 0 & \cdots & 0 \\ h[1] & h[0] & \cdots & 0 \\ \vdots & h[1] & \ddots & \vdots \\ h[M] & \vdots & \ddots & h[0] \\ 0 & h[M] & \ddots & h[1] \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h[M] \end{bmatrix}}_{\mathbf{H}} \begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[K] \end{bmatrix} ?$$

Let

$$x[n] = \{1, 3, 5, 3, 1\}$$
$$h[n] = \{1, 2, 1\}$$

j)* Find $y[n] = x[n] * h[n]$