

Problem 1

(75 points)

a)* Determine the z -transform of the following finite-duration signal

$$x[n] = \left\{ \begin{array}{ccccc} 1, & 0, & 2, & 0, & 1 \end{array} \right\}.$$

Also mention the ROC and sketch the time-domain plot of $x[n]$.b)* Verify the z -transform for

$$x[n] = [\alpha^n \cos \omega_0 n] u[n] \xleftrightarrow{Z} X(z) = \frac{1 - [\alpha \cos \omega_0] z^{-1}}{1 - [2\alpha \cos \omega_0] z^{-1} + \alpha^2 z^{-2}}, \text{ ROC: } |z| > |\alpha|.$$



c)* Verify the z -transform for

$$x[n] = [\alpha^n \sin \omega_0 n] u[n] \xleftrightarrow{z} X(z) = \frac{[\alpha \sin \omega_0] z^{-1}}{1 - [2\alpha \cos \omega_0] z^{-1} + \alpha^2 z^{-2}}, \text{ ROC: } |z| > |\alpha|.$$



d)* Determine the z -transform of the following signal

$$x[n] = \left(\frac{1}{2}\right)^n u[n] + (2)^n u[-n-1]$$

Also mention the ROC and sketch the time-domain plot of $x[n]$.

e)* Determine the z -transform of the following signal

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$$x[n] = (2)^n u[n] + \left(\frac{1}{2}\right)^n u[-n-1]$$

Also mention the ROC.

f)* Using the linearity property, determine the z -transform of the following signal

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$$x[n] = \{2(0.8^n) + 4(0.5^n)\} u[n].$$

Also mention the ROC.

g)* Use the linearity and time shifting property to determine the z -transform of the signal

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$$x[n] = u[n] - u[n-3].$$

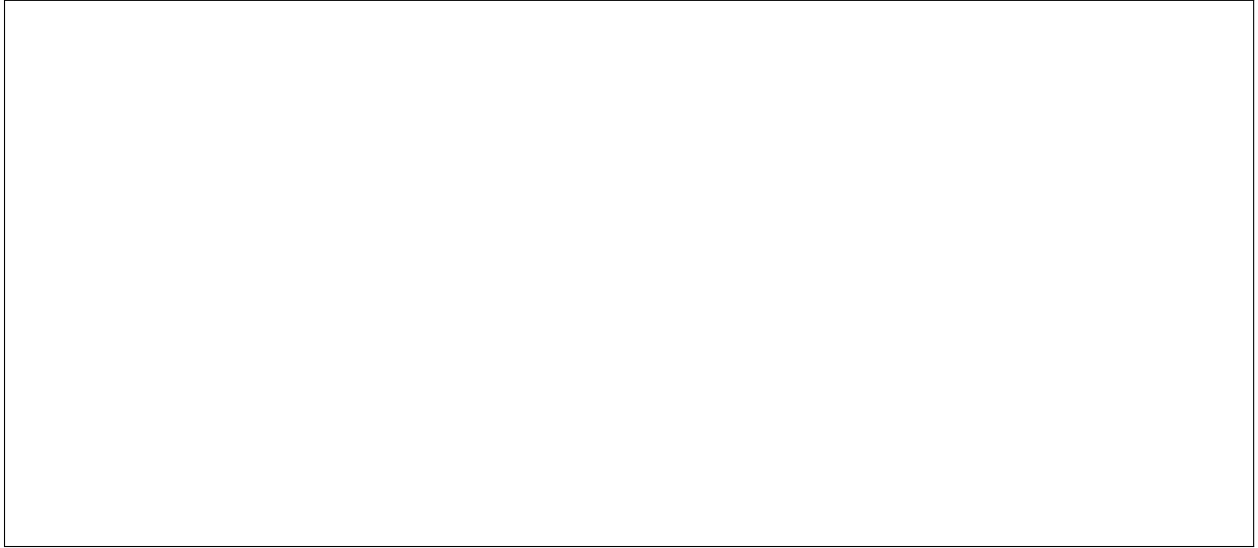
Sketch $x[n]$ and also mention the ROC.

h)* Use the linearity and time shifting property to determine the z -transform of the signal



$$x[n] = \begin{cases} 0 & n < 0 \\ \cos \frac{\pi}{4}n & 0 \leq n \leq 7 \\ 0 & n > 7. \end{cases}$$

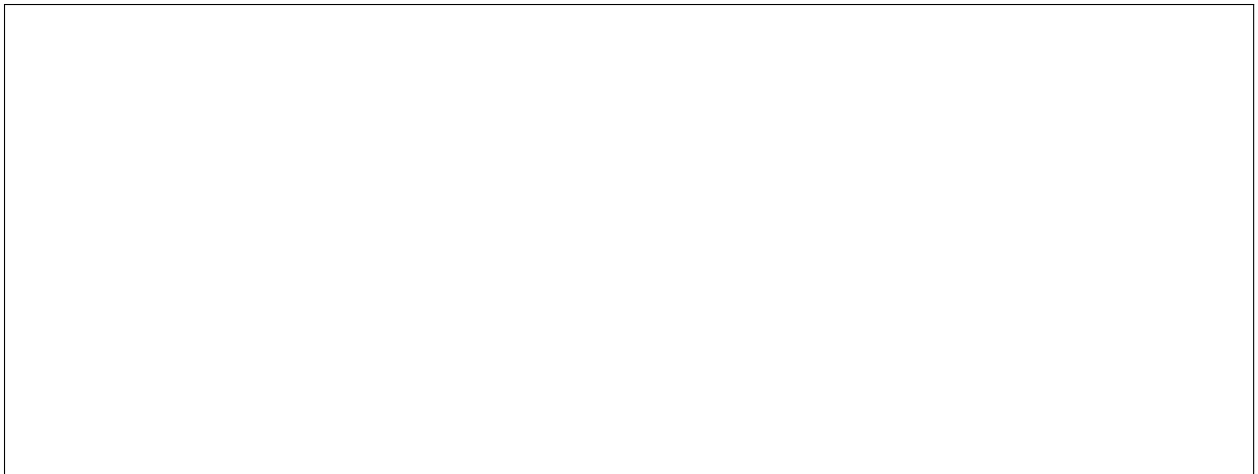
Sketch $x[n]$ and also mention the ROC.



i)* Using the **exponential property**, determine the z -transform of the following signal



$$x[n] = [r^n \sin \omega_0 n] u[n]$$



j)* Using **exponential property**, determine the z -transform of the following signal and simplify as far as possible.



$$x[n] = \cos \pi n \{u[n] - u[n-3]\}$$

k)* Using the **exponential property**, determine the z -transform of the following signal and simplify as far as possible



$$x[n] = \cos \frac{\pi}{4} n \left\{ \underset{\uparrow}{2}, \quad 3, \quad 1, \quad 4 \right\}$$

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l)* Using the **differentiation property**, determine the z -transform of the following signal

$$y[n] = n\alpha^n u[n]$$

and mention the ROC.

and mention the ROC.

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m)* Using the **differentiation property**, determine the z -transform of the following signals

$$y[n] = -n\alpha^n u[-n-1]$$

.

n)* Using the differentiation property, determine the z -transform of the following signals



$$y[n] = (n\alpha^n \cos \omega_0 n) u[n]$$

and mention the ROC.

o)* Compute the convolution $x[n]$ of the signals



$$x_1[n] = \left\{ \underset{\uparrow}{2} \quad 3 \quad 1 \quad 4 \right\}$$

$$x_2[n] = \left\{ \underset{\uparrow}{1} \quad 1 \right\}.$$

☐ p)* Compute the convolution $x[n]$ of the signals

$$x_1[n] = u[n]$$

$$x_2[n] = u[n].$$

Hint: For the inverse z -transform, use

$$n u[n] \longleftrightarrow -z \frac{\partial}{\partial z} \left(\frac{1}{1 - z^{-1}} \right) = \frac{z^{-1}}{(1 - z^{-1})^2}, \text{ROC: } |z| > 1$$

with time shifting property.

☐ q)* Compute the correlation $x[n]$ of the signal

$$x_1[n] = \left\{ \begin{matrix} 1 & 1 & -1 & 1 \\ \uparrow & & & \end{matrix} \right\}$$

$$x_2[n] = \left\{ \begin{matrix} 0 & 1 & 1 & -1 & 1 \\ \uparrow & & & & \end{matrix} \right\}$$

r) Compute the correlation $x[n]$ of the signal



$$x_1[n] = \left(\frac{1}{2}\right)^n u[n]$$
$$x_2[n] = \left(\frac{1}{2}\right)^n u[n].$$

Hint: For the inverse z -transform, you need the result of Exercise d).

