Problem 1

(75 points)

a)* Determine the z-transform of the following finite-duration signal

$$x[n] = \left\{ 1, \quad 0, \quad \underset{\uparrow}{2}, \quad 0, \quad 1 \right\}.$$

Also mention the ROC and sketch the time-domain plot of $x[n]$.

b)* Verify the z-transform for

$$x[n] = [\alpha^n \cos\omega_0 n]\, u[n] \overset{z}{\longleftrightarrow} X(z) = \frac{1 - [\alpha\cos\omega_0]\, z^{-1}}{1 - [2\alpha\cos\omega_0]\, z^{-1} + \alpha^2 z^{-2}}, \text{ ROC: } |z| > |\alpha|.$$

c)* Verify the z-transform for

$$x[n] = [\alpha^n \sin \omega_0 n]\, u[n] \overset{z}{\longleftrightarrow} X(z) = \frac{[\alpha \sin \omega_0]\, z^{-1}}{1 - [2\alpha \cos \omega_0]\, z^{-1} + \alpha^2 z^{-2}}, \text{ ROC: } |z| > |\alpha|.$$

d)* Determine the z-transform of the following signal

$$x[n] = \left(\frac{1}{2}\right)^n u[n] + (2)^n u[-n-1]$$

Also mention the ROC and sketch the time-domain plot of $x[n]$.

e)* Determine the z-transform of the following signal

$$x[n] = (2)^n u[n] + \left(\frac{1}{2}\right)^n u[-n-1]$$

Also mention the ROC.

f)* Using the linearity property, determine the z-transform of the following signal

$$x[n] = \{2(0.8^n) + 4(0.5^n)\} u[n].$$

Also mention the ROC.

g)* Use the linearity and time shifting property to determine the z-transform of the signal

$$x[n] = u[n] - u[n-3].$$

Sketch $x[n]$ and also mention the ROC.

h)* Use the linearity and time shifting property to determine the z-transform of the signal

$$x[n] = \begin{cases} 0 & n < 0 \\ \cos \frac{\pi}{4} n & 0 \leq n \leq 7 \\ 0 & n > 7. \end{cases}$$

Sketch $x[n]$ and also mention the ROC.

i)* Using the **exponential property**, determine the z-transform of the following signal

$$x[n] = [r^n \sin \omega_0 n]\, u[n]$$

j)* Using **exponential property**, determine the z-transform of the following signal and simplify as far as possible.

$$x[n] = \cos \pi n \{u[n] - u[n-3]\}$$

k)* Using the **exponential property**, determine the z-transform of the following signal and simplyfy as far as possible

$$x[n] = \cos \frac{\pi}{4} n \left\{ \underset{\uparrow}{2}, \quad 3, \quad 1, \quad 4 \right\}$$

l)* Using the **differentiation property**, determine the z-transform of the following signal

$$y[n] = n\alpha^n u[n]$$

and mention the ROC.

and mention the ROC.

m)* Using the **differentiation property**, determine the z-transform of the following signals

$$y[n] = -n\alpha^n u[-n-1]$$

.

n)* Using the differentiation property, determine the z-transform of the following signals

$$y[n] = (n\alpha^n \cos \omega_0 n)\, u[n]$$

and mention the ROC.

o)* Compute the convolution $x[n]$ of the signals

$$\begin{aligned} x_1[n] &= \left\{ \underset{\uparrow}{2} \quad 3 \quad 1 \quad 4 \right\} \\ x_2[n] &= \left\{ \underset{\uparrow}{1} \quad 1 \right\}. \end{aligned}$$

p)* Compute the convolution $x[n]$ of the signals

$$x_1[n] = u[n]$$
$$x_2[n] = u[n].$$

Hint: For the inverse z-transform, use

$$n\,u[n] \longleftrightarrow -z\frac{\partial}{\partial z}\left(\frac{1}{1-z^{-1}}\right) = \frac{z^{-1}}{(1-z^{-1})^2}, \text{ROC: } |z| > 1$$

with time shifting property.

q)* Compute the correlation $x[n]$ of the signal

$$x_1[n] = \left\{ \underset{\uparrow}{1} \quad 1 \quad -1 \quad 1 \right\}$$
$$x_2[n] = \left\{ \underset{\uparrow}{0} \quad 1 \quad 1 \quad -1 \quad 1 \right\}$$

r) Compute the correlation $x[n]$ of the signal

$$x_1[n] = \left(\frac{1}{2}\right)^n u[n]$$
$$x_2[n] = \left(\frac{1}{2}\right)^n u[n]\,.$$

Hint: For the inverse z-transform, you need the result of Exercise d).