Chromatic Dispersion Compensation Using Complex-Valued All-Pass Filter

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***Abstract*—We propose a new optimization framework to compensate chromatic dispersion by complex-valued infinite impulse response (IIR) all-pass filter. The design of the IIR all-pass filter is based on minimizing the mean square error (MSE) in group-delay and phase cost functions. Necessary conditions are derived and incorporated in a multi-step optimization framework to ensure the stability of the resulting IIR filter. It is shown that IIR filter achieves similar or slightly better performance compared to its finite impulse response (FIR) counterpart. Moreover, IIR filtering requires significantly less number of taps to compensate the same CD channel compared to FIR filtering.**

I. Introduction

Various digital signal processing (DSP) algorithms to optical communication have been investigated to improve the transmission performance by mitigating fiber optic impairments like polarization mode dispersion (PMD) and chromatic dispersion (CD). PMD has a time varying transfer function whereas CD is static in nature and changes the phase of input signal, i.e., it has an all-pass behavior. As presented in [1], for a symbol rate of T, a $\frac{T}{2}$ tap delay finite impulse response (FIR) filter may be used to remove the effects of CD. The number of FIR filter taps required grows linearly with increasing dispersion and fiber length. The efficient way to implement linear convolution is using frequency domain equalization (FDE) technique by fast Fourier transform (FFT). FDE is very attractive because it has much lower complexity in comparison to the time domain equalization (TDE) when filter taps are very large [2]. But when filter taps are small (short transmission distance in this case) then it is advantageous to use TDE. In [3], G. Goldfarb and G. Li suggested dispersion compensation (DC) using TDE technique based on infinite impulse response (IIR) all-pass filter. They showed that IIR filtering achieves similar performance with substantially reduced number of operations (multiplications). Their methodology requires Hilbert transformer and time reversal operation to design real-coefficients IIR filter separately for real and imaginary part of the transmitted signals.

Several methods [4],[5],[6],[7] are available in the literature for the design of digital all-pass filter to match a desired phase response. In [8], J.S. Abel and J.O. Smith presented the method of all-pass filter design scheme based on the desired group-delay function which overcomes the following two major shortcomings of the existing methods

1) numerical difficulties and precision for high order all-pass filter design.
2) a priori selection of filter order rather than determined by the algorithm.

Although their design methodology is quite simple and fast but it doesn't take into account the desired phase behavior. In this paper, we will incorporate the desired group delay as well as desired phase behavior in a multi-step multi-objective optimization framework and show that their is significant improvement compared to [8] in terms of bit error rate (BER) for low order all-pass filter design.

The main contribution of this paper is the presentation of an optimization framework to design a stable complex-valued IIR all-pass filter for CD equalization. Our design methodology doesn't require the use of Hilber transformer and time reversal operation, thereofore, complexity is less compared to [3]. Moreover, our technique has better performance for low and moderate order all-pass filter design methods compared to [8]. Although our framework is explained for CD equalization but our method is quite general and can be applied to any channel for phase equalization. This paper is organized as follows. Section II introduces the channel transfer function for CD. Section III describes the equalizer design based on IIR filtering and derives necessary conditions to determine filter order and stability. Section IV formulate the task of finding the coefficients of the filter in terms of a constraint optimization problem based on two design criterion. Section V explains our framework to solve the constraint optimization problem. Simulation results comparing FIR and IIR filtering are presented in Section VI. Conclusions are given in Section VII.

II. Channel Model

The low-pass equivalent model of CD channel of a single mode fiber of length L can be written as

$$H_{\mathrm{CD}}(\Omega) = \exp\left(-j \cdot \frac{\lambda_0^2}{4\pi c} \cdot D \cdot L \cdot \Omega^2\right) , \tag{1}$$

where Ω, λ_0, D and c are baseband radial frequency, operating wavelength, fiber dispersion parameter and speed of light, respectively. If we sample the signal by sampling frequency B Hz, then the equivalent model in discrete domain can be represented as

$$H_{\mathrm{CD}}(\omega) = \exp(-j \cdot \alpha \cdot \omega^2) , \tag{2}$$

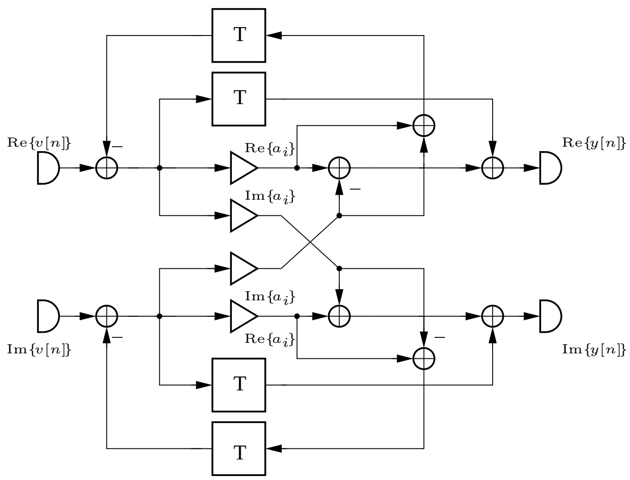

Fig. 1. Efficient implementation of 1st order IIR all-pass section.

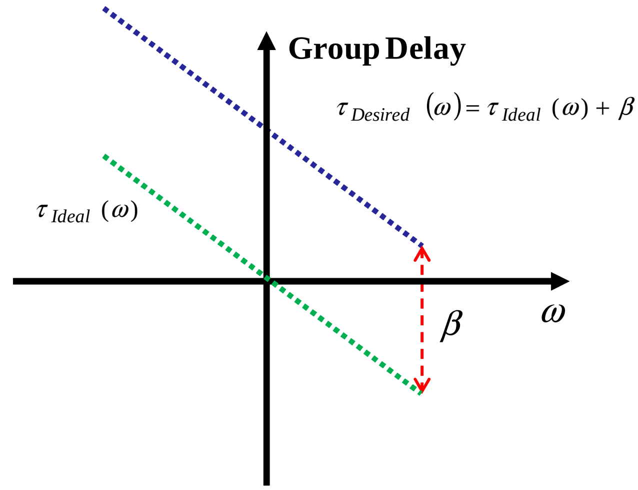

Fig. 2. Desired group delay.

where $\alpha = \lambda_0^2 \cdot B^2 \cdot D \cdot L / (4\pi c)$ and $\omega \in [-\pi, \pi)$. It is clear from (2) that the CD channel has an all-pass characteristic, i.e., it only changes the phase of the input signal.

III. Equalizer Design

The transfer function of an ideal equalizer to compensate CD channel is obtained by taking the inverse of (2)

$$G_{\text{Ideal}}(\omega) = \exp(+j \cdot \alpha \cdot \omega^2)\ . \tag{3}$$

Since the CD channel exhibits an all-pass behavior, therefore, the natural choice to equalize it by the cascade of N_{IIR} first order IIR all-pass sections of the form

$$\begin{aligned} G_{\text{IIR}}(z) &= \prod_{i=1}^{N_{\text{IIR}}} \frac{-a_i^* + z^{-1}}{1 - a_i \cdot z^{-1}} \\ &= \prod_{i=1}^{N_{\text{IIR}}} \frac{-\rho_i e^{-j\theta_i} + z^{-1}}{1 - \rho_i e^{j\theta_i} \cdot z^{-1}}\ , \end{aligned} \tag{4}$$

where ρ_i and θ_i are the radius and angle of the i^{th} pole location in the complex z-plane. Each first order section of complex valued IIR all-pass filter can be efficiently realized by four real multiplications as shown in Fig. 1. Before presenting our framework to find the coefficients of IIR all-pass filter, two important questions relating to stability and filter order selection need to be addressed. The answers to both the questions are given based on group delay characteristic.

The group delay of an ideal equalizer is obtained by taking the negative gradient of the phase response with respect to ω in (3),

$$\tau_{\text{Ideal}}(\omega) = -\frac{\partial}{\partial \omega} \arg\{G_{\text{Ideal}}(\omega)\} = -2 \cdot \alpha \cdot \omega\ . \tag{5}$$

The phase response of an IIR all-pass filter is [9]

$$\begin{aligned} \phi_{\text{IIR}}(\omega) &= \sum_{i=1}^{N_{\text{IIR}}} \phi_{\text{IIR}_i}(\omega) \\ &= \sum_{i=1}^{N_{\text{IIR}}} \left[-\omega - 2 \cdot \arctan\left(\frac{\rho_i \cdot \sin(\omega - \theta_i)}{1 - \rho_i \cdot \cos(\omega - \theta_i)} \right) \right]\ . \end{aligned} \tag{6}$$

The total group delay of (4) is given by the expression

$$\begin{aligned} \tau_{\text{IIR}}(\omega) &= \sum_{i=1}^{N_{\text{IIR}}} \tau_{\text{IIR}_i}(\omega) \\ &= \sum_{i=1}^{N_{\text{IIR}}} \frac{1 - \rho_i^2}{1 + \rho_i^2 - 2 \cdot \rho_i \cdot \cos(\omega - \theta_i)}\ . \end{aligned} \tag{7}$$

For a stable IIR filter, all poles must lie inside the unit circle. But it was shown in [8] that approximating the negative group delay using Eq.(7) implies that $\rho_i > 1$. Therefore, a constant integer factor β is added in $\tau_{\text{Ideal}}(\omega)$ to make the desired group delay a positive function. Desired group delay and phase response take the form

$$\tau_{\text{Desired}}(\omega) = -2 \cdot \alpha \cdot \omega + \beta\ , \tag{8}$$

$$\phi_{\text{Desired}}(\omega) = \alpha \cdot \omega^2 - \beta \cdot \omega - \phi_0\ , \tag{9}$$

where $\beta = \lceil 2 \cdot \alpha \cdot \pi \rceil$. In section V, a method will be given to calculate the phase correction term ϕ_0 which is also an integration constant.

The order of all-pass filter is chosen using the argument of area under the desired group delay curve. It states that the area under the group delay function of an all-pass filter should match that of the desired group delay, i.e.,

$$\int_{-\pi}^{\pi} \tau_{\text{IIR}}(\omega) \cdot d\omega = \int_{-\pi}^{\pi} \tau_{\text{Desired}}(\omega) \cdot d\omega. \tag{10}$$

Since the group delay is the negative derivative of the phase response with respect to frequency, its integral around the unit circle is simply the negative phase accumulated during one traversal of the unit circle (c.f. (6)),

$$\begin{aligned}\int_{-\pi}^{\pi} \tau_{\mathrm{IIR}_i}(\omega) \cdot d\omega &= \phi_{\mathrm{IIR}_i}(-\pi) - \phi_{\mathrm{IIR}_i}(\pi) \\ &= 2\pi. \end{aligned} \tag{11}$$

Therefore, total area contributed by N_{IIR} sections is

$$\int_{-\pi}^{\pi} \tau_{\mathrm{IIR}}(\omega) \cdot d\omega = 2\pi \cdot N_{\mathrm{IIR}} \ . \tag{12}$$

Area under the desried group delay curve is given by

$$\int_{-\pi}^{\pi} \tau_{\mathrm{Desired}}(\omega) \cdot d\omega = 2\pi \cdot \beta. \tag{13}$$

Comparing (12) and (13), the total number of all-pass stages are given by

$$N_{\mathrm{IIR}} = \left\lceil \left(\frac{\lambda_0^2}{2c}\right) \cdot D \cdot B^2 \cdot L \right\rceil \ . \tag{14}$$

In [10], number of taps to equalize the same CD channel with FIR filter is derived as

$$N_{\mathrm{FIR}} \approx 2 \left\lfloor \left(\frac{\lambda_0^2}{2c}\right) \cdot D \cdot B^2 \cdot L \right\rfloor + 1 \tag{15}$$

which has almost twice the complexity compared to an IIR all-pass equalizer, i.e., $N_{\mathrm{FIR}} \approx 2 \cdot N_{\mathrm{IIR}}$.

IV. Design Criteria

The objective is to design IIR all-pass equalizer whose phase response matches the desired phase response of (9). Therefore, product of an all-pass equalizer and the CD channel has to be ideally

$$G_{\mathrm{IIR}}(\omega) \cdot H_{\mathrm{CD}}(\omega) = e^{-j(\phi_0 + \beta \cdot \omega)} \ . \tag{16}$$

The mean square error of the transfer function containing the phase information is defined as

$$\mathrm{MSE}_{\text{trans. phase}} = \int_{-\pi}^{\pi} \mid G_{\mathrm{IIR}}(\omega) \cdot H_{\mathrm{CD}}(\omega) - e^{-j(\phi_0 + \beta \cdot \omega)} \mid^2 d\omega \ . \tag{17}$$

The coefficients of IIR all-pass equalizer are found by solving the following cost function

$$\Psi_{\text{trans. phase}} = \min_{\rho_i, \theta_i, \phi_0} \mathrm{MSE}_{\text{trans. phase}} \text{ s.t. } \rho_i < 1, \ i = 1, 2, \ldots, N_{\mathrm{IIR}} \ . \tag{18}$$

The optimization problem in (18) is non-convex and non-linear so we can only solve it by non-linear optimization techniques. But usually such solvers require good initial guess of the solution and it may stuck into the local minima if the initial solution is far from the global minima. In order to overcome this problem, we first minimize the mean square error in group delay $\mathrm{MSE}_{\mathrm{GD}}$ metric by using group delay cost function Ψ_{GD}, i.e.,

$$\Psi_{\mathrm{GD}} = \min_{\rho_i, \theta_i} \mathrm{MSE}_{\mathrm{GD}} \text{ s.t. } \rho_i < 1, \ i = 1, 2, \ldots, N_{\mathrm{IIR}} \tag{19}$$

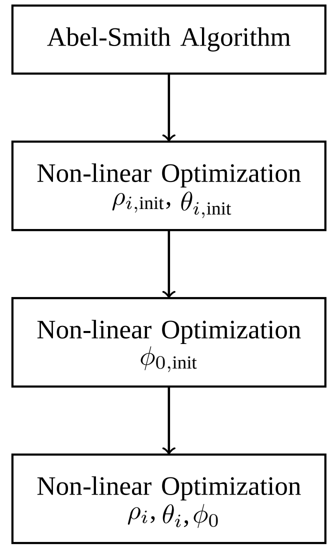

Fig. 3. Block diagram of optimization framework.

where

$$\mathrm{MSE}_{\mathrm{GD}} = \int_{-\pi}^{\pi} \mid \tau_{\mathrm{Desired}}(\omega) - \sum_{i=1}^{N_{\mathrm{IIR}}} \tau_{\mathrm{IIR}_i}(\omega) \mid^2 d\omega \ . \tag{20}$$

Although (19) is also non-linear but the initial solution can easily found by the Abel-Smith algorithm which can be further optmimized to reach the sub-optimal solution. This sub-optimal result then provides good starting solution to solve optimzation problem (18).

V. Optimization Framework

The flow chart of our framework for solving (18) is shown in Fig. 3. The initial estimate of radii and angles are found by the Abel-Smith algorithm which is then refined by a non-linear solver to find the solution of (19). The solution of (19) is used as an initial starting guess to find the final optimal solution of (18). Any gradient based non-linear solver can be used to solve optimization problem (18) and (19) respectively. In the following, we will provide details of all the steps in our optimization framework.

A. Abel-Smith Algorithm

In [8], J.S. Abel and J.O. Smith described a method to extract filter coefficients for an all-pass design with an arbitrary group delay. The design procedure is as follow:

1) Divide $\tau_{\mathrm{Desired}}(\omega)$ into 2π-area frequency bands, as illustrated in Fig. 4.
2) Fit a first-order (complex) all-pass section $G_{\mathrm{IIR}_i}(\omega)$ to each band as described below.
3) Cascade the first-order sections to form the all-pass filter,

$$G_{\mathrm{IIR}}(\omega) = \prod_{i=1}^{N_{\mathrm{IIR}}} G_{\mathrm{IIR}_i}(\omega) \ .$$

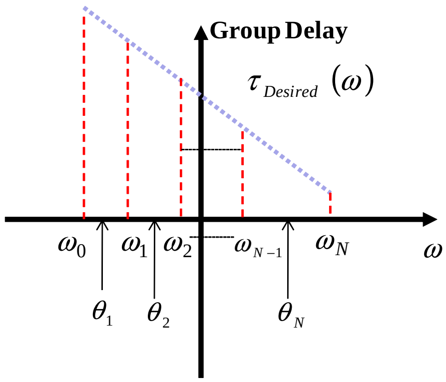

Fig. 4. Segmentation of $\tau_{\text{Desired}}(\omega)$ into 2π area bands.

The pole frequency is taken to be the band midpoint,

$$\theta_i = \frac{\omega_{i-1} + \omega_i}{2}. \tag{21}$$

The expression for pole radius is derived as,

$$\rho_i = \mu_i - \sqrt{\mu_i^2 - 1}\ , \tag{22}$$

where

$$\mu_i = \frac{1 - \zeta \cdot \cos(\Delta_i)}{1 - \zeta}, \qquad \Delta_i = \frac{\omega_i - \omega_{i-1}}{2},$$

and ζ is taken from the interval $\begin{bmatrix} 0.75 & 0.85 \end{bmatrix}$.

B. Non-linear Optimization, Ψ_{GD}

The solution provided by the Abel-Smith algorithm is improved by finding a solution to (19) using a non-linear solver.

C. Non-linear Optimization, Ψ_{ϕ_0}

Till this point, we have only estimates of the radii and angles. In order to get the intial phase correction term ϕ_0, the following uncostrained optimization problem is solved

$$\Psi_{\phi_0} = \min_{\phi_0} \text{MSE}_{\text{trans. phase}}\ . \tag{23}$$

D. Non-linear Optimization, $\Psi_{trans.\ phase}$

The final step of our framework solves (18) using a non-linear optimization solver with initial estimate of radii, angles and phase rotation provided by the first three steps of the optimization framework.

In next section, we will compare performance comparison in terms of BER for chromatic dispersion compensation between IIR and FIR equalizer.

VI. Simulation Results

A 28 GBaud QPSK transmission with digital coherent receiver applying two-fold oversampling with $B = 56$ GS/s is used to verify our technique for CD equalization. System parameters are $\lambda_0 = 1550$ nm, $D = 16$ ps/nm/km and $L = 23$ km. Moreover, Mach-Zehnder modulator (MZM) is used as a pulse shaper at the transmitter side. Also optical and electrical filter with cut-off frequencies 17.5 GHz and 19.1 GHz respectively are used at the receiver side.

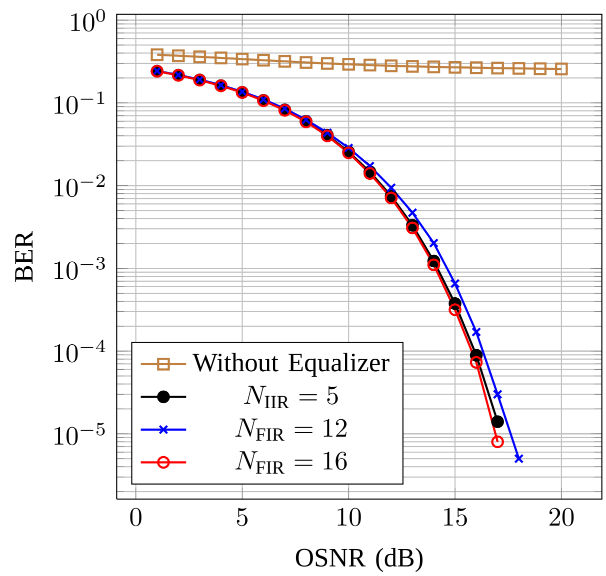

Fig. 5. BER VS OSNR.

Substituting system parameter values in (14) and (15), number of taps of IIR all-pass and FIR equalizer turns out to be $N_{\text{IIR}} = 5$ and $N_{\text{FIR}} = 9$ respectively. The coefficients of IIR all-pass equalizer are computed based on our optimization framework. We obtain $\phi_0 = -2.4189$ and the following set of coefficients:

	1	2	3	4	5
ρ_i	0.8162	0.7402	0.6759	0.6060	0.51101
θ_i	-2.7448	-2.0700	-1.2960	-0.3768	0.8132

Since IIR all-pass equalizer needs one extra complex multiplication for the phase correction term ϕ_0, therefore, we will select $N_{\text{FIR}} = 12$ for the fair comparison that complexity of FIR equalizer is doubled compared to IIR all-pass equalizer. Coefficients of FIR equalizer are calculated by discretizing ω in (3) into N_{FIR} points and taking inverse discrete Fourier transform.

For performance analysis, the optical signal to noise ratio (OSNR) at a bit error ratio (BER) of 10^{-3} is chosen as a figure of merit. Fig. 5 compares the BER performance of an IIR all-pass equalizer with the FIR equalizer as a function of OSNR. It is evident from the figure that the performance of an IIR all-pass equalizer is slightly better than the FIR one with half the complexity. Moreover, we increase the number of FIR taps such that performance of both the filters are same and FIR filter taps turn out to be 16, i.e., $N_{\text{FIR}} = 16$. In other words, we can say that the IIR all-pass equalizer compensates the CD channel with the same performance compared to its FIR counterpart with slightly less than half the number of FIR taps.

We will also compare the performance of our method and

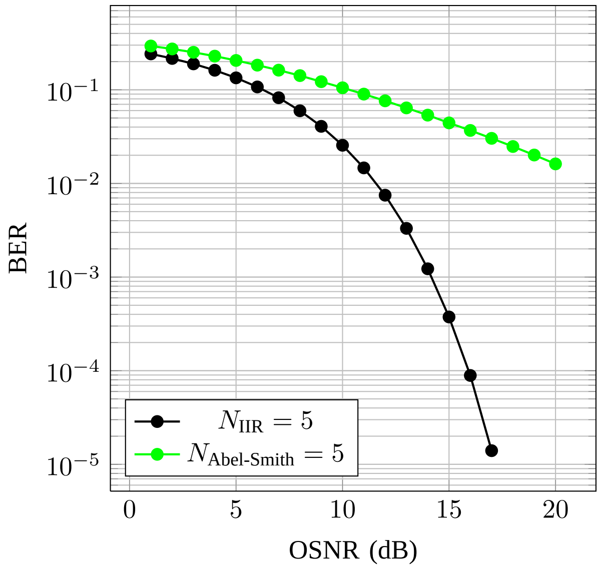

Fig. 6. BER VS OSNR.

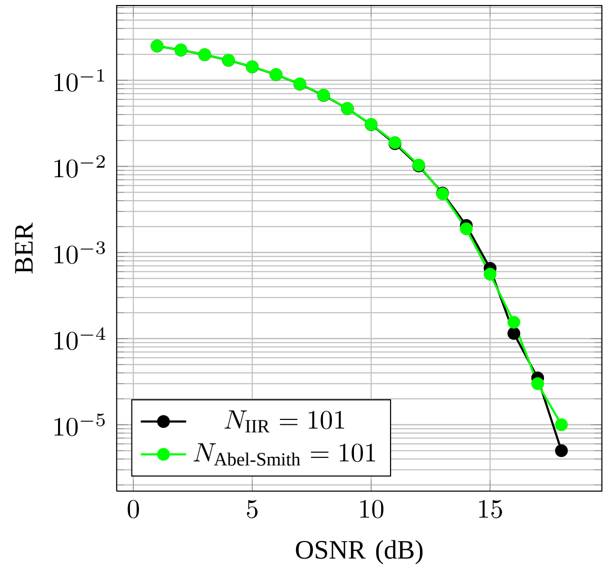

Fig. 7. BER VS OSNR.

Abel-Smith method [8] for both low and very high all-pass filter order. Figs. 6 and 7 shows the BER performance for 5^{th} order and 101^{th} order all-pass filter corresponding to fiber length of 23 km and 502 km respectively. The results show that our optimization framework has significant better performance at low filter order compared to Abel-Smith algorithm. The reasoning behind the better performance at low filter order comes from the fact that we are including the desired phase constraint into consideration in the optimization framework. On the other hand, both the algorithms at high filter order have the same performance although our framework is much more computational expensive compared to Abel-Smith algorithm. The approximation of Abel-Smith algorithm in finding the coefficients of the filter based on desired group delay behavior get better for high filter order so there is no improvement in incorporating the phase constraint.

VII. Conclusion

In this paper, we presented a framework to equalize CD channel with complex-valued IIR all-pass filter. We derived necessary conditions based on group delay characteristic to select minimum filter order and stability of resulting IIR all-pass filter. An optimization framework is proposed which finds the coefficients IIR all-pass filter using two step approach. In the first step, a solution to group delay cost function using (19) is found by minimizing the mean square error in group delay metric. In the second and final step, the coefficients are obtained by minimizing the mean square error of the transfer function in phase metric using (18). For performance evaluation, BER metric is used and it can be seen from the simulation results that IIR filtering has slight better performance in comparison to FIR filtering. The benefit obtained by our approach is twofold. Firstly, it reduces the tap count by a factor of almost 50% compared to FIR filtering for the same performance. Secondly, it eliminates the use of Hilbert transformer [3] whose complexity is not negligible at shorter transmission distances. Moreover, the performance of our method is also better compared to Abel-Smith algorithm for low filter order. It is important to mention that we explained our framework in the context of approximating a phase response of a CD equalizer with IIR filter but the same framework can be applied to approximate any arbitrary phase response.

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