ALBERT R.

I $\quad 4x_1 + 4x_2 - 7x_3 = 12$
$\quad 10x_2 + 5x_3 = 10$
$\quad 3x_1 + 3x_2 + x_3 = 9$

$$A = \begin{pmatrix} 4 & 4 & -7 \\ 0 & 10 & 5 \\ 3 & 3 & 1 \end{pmatrix} \quad b = \begin{pmatrix} 12 \\ 10 \\ 9 \end{pmatrix}$$

$+2$
1
0

<u>Verificare</u>
$+8 + 4 = 12$ ✓
$10 + 0 = 10$ ✓
$6 + 3 + 0 = 9$ ✓

Desc QR cu Householder

Pas 1 $\quad P = I_3 - \frac{1}{\beta} u u^T$

$$\sigma = a_{11}^2 + a_{21}^2 + a_{31}^2 = 4^2 + 0^2 + 3^2 = 16 + 9 = 25$$

$$k = \pm\sqrt{\sigma} \qquad k = -\text{semn}(a_{11}) \cdot 5$$

$$\boxed{k = -5}$$

$$\beta = \cancel{25 - (-5)}\ \sigma - k \cdot a_{11} = 25 - (-5) \cdot 4 = 25 + 20 = 45$$

$$u = \begin{pmatrix} a_{11} - k \\ a_{21} \\ a_{31} \end{pmatrix} = \begin{pmatrix} 4+5 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 9 \\ 0 \\ 3 \end{pmatrix}$$

$$u u^T = \begin{pmatrix} 9 \\ 0 \\ 3 \end{pmatrix} \begin{pmatrix} 9 & 0 & 3 \end{pmatrix} = \begin{pmatrix} 81 & 0 & 27 \\ 0 & 0 & 0 \\ 27 & 0 & 9 \end{pmatrix}$$

$$P_1 = P = I_3 - \frac{1}{\beta} u u^T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} \frac{81}{45} & 0 & \frac{27}{45} \\ 0 & 0 & 0 \\ \frac{27}{45} & 0 & \frac{9}{45} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} \frac{9}{5} & 0 & \frac{3}{5} \\ 0 & 0 & 0 \\ \frac{3}{5} & 0 & \frac{1}{5} \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{4}{5} & 0 & -\frac{3}{5} \\ 0 & 1 & 0 \\ -\frac{3}{5} & 0 & \frac{4}{5} \end{pmatrix}$$

$$A = P_1 \cdot A = \begin{pmatrix} -\frac{4}{5} & 0 & -\frac{3}{5} \\ 0 & 1 & 0 \\ -\frac{3}{5} & 0 & \frac{4}{5} \end{pmatrix} \begin{pmatrix} 4 & 4 & -7 \\ 0 & 10 & 5 \\ 3 & 3 & 1 \end{pmatrix} = \begin{pmatrix} -\frac{16}{5} - \frac{9}{5} & -\frac{16}{5} - \frac{9}{5} & +\frac{28}{5} - \frac{3}{5} \\ 0 & 10 & 5 \\ -\frac{12}{5} + \frac{12}{5} & -\frac{12}{5} + \frac{12}{5} & +\frac{21}{5} + \frac{4}{5} \end{pmatrix}$$

$$= \begin{pmatrix} -5 & -5 & 5 \\ 0 & 10 & 5 \\ 0 & 0 & 5 \end{pmatrix}$$ are col 1 în formă sup Δ

Alg ne spune să ne oprim dacă și col 2 e în formă Δ sup

ALBERT R.

$P = I - \frac{1}{\beta} u u^T$

$\sigma = 10^2 + 0^2 = a_{22}^2 + a_{32}^2 = 100$

$k = = \pm\sqrt{\sigma} \quad k = -\text{semn}(a_{22}) \cdot 10 = -10$

$\beta = 100 + 10 \cdot 10 = 200$

$u = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad uu^T = O_m$

$P = I_m = I_3 - O_3 \Rightarrow P_2 = I_m$

$A = P_2 A = I_m A = A$

$R = \begin{pmatrix} -5 & -5 & 5 \\ 0 & 10 & 5 \\ 0 & 0 & 5 \end{pmatrix} \quad Q = \underset{P_1}{\begin{pmatrix} -\frac{4}{5} & 0 & -\frac{3}{5} \\ 0 & 1 & 0 \\ -\frac{3}{5} & 0 & \frac{4}{5} \end{pmatrix}} \cdot \underset{P_2}{I_m} = \begin{pmatrix} -\frac{4}{5} & 0 & -\frac{3}{5} \\ 0 & 1 & 0 \\ -\frac{3}{5} & 0 & \frac{4}{5} \end{pmatrix}$

$Q^T \cdot / \; QRx = b \quad$ (Q ortogonală $\Rightarrow Q^{-1} = Q^T$

$Rx = Q^T b$

$Q^T = \begin{pmatrix} -\frac{4}{5} & 0 & -\frac{3}{5} \\ 0 & 1 & 0 \\ -\frac{3}{5} & 0 & \frac{4}{5} \end{pmatrix}$ simetr

$$\begin{aligned} -5x_1 - 5x_2 + 5x_3 &= -15 \\ 10x_2 + 5x_3 &= 10 \\ 5x_3 &= 0 \end{aligned}$$

$Q^T b = \begin{pmatrix} -\frac{4}{5} & 0 & -\frac{3}{5} \\ 0 & 1 & 0 \\ -\frac{3}{5} & 0 & \frac{4}{5} \end{pmatrix} \begin{pmatrix} 12 \\ 10 \\ 9 \end{pmatrix} = \begin{pmatrix} -\frac{48}{5} - \frac{27}{5} \\ 10 \\ -\frac{36}{5} + \frac{36}{5} \end{pmatrix} = \begin{pmatrix} -\frac{75}{5} \\ 10 \\ 0 \end{pmatrix} = \begin{pmatrix} -15 \\ 10 \\ 0 \end{pmatrix}$

Met substituției inverse

$x_3 = \frac{0}{5} = 0$

$x_2 = \frac{10 - 5 \cdot 0}{10} = 1$

$x_1 = \frac{-15 - 5 \cdot 0 + 5 \cdot 1}{-5} = \frac{10}{-5} = -2$

$x = \begin{pmatrix} +2 \\ 1 \\ 0 \end{pmatrix}$ ✓

II. ALBERT R.

x	-1	1	2	3
f	-4	2	11	32

$x_0=-1 \quad x_1=1 \quad x_2=2 \quad x_3=3$

$y_0=-4 \quad y_1=2 \quad y_2=11 \quad y_3=32$

$$L_{m3}(x)=y_0+[x_0x_1]f(x-x_0)+\dots+[x_0x_1\dots x_m]f(x-x_0)(x-x_1)\dots(x-x_{m-1})$$

<u>schema lui Aitken</u>

		Pas 1	Pas 2	Pas 3
-1	-4			
1	2	3		
2	11	9	2	
3	32	21	12	$\frac{5}{2}$

i, offset $=k$

$[x_i]f=y_i \Rightarrow dd[0,0]=-4$

~~$[x_0x_1]=$~~

$dd[1,0]=2$

$dd[2,0]=11$

$dd[3,0]=32$

$$dd[0,1]=\frac{dd[1,0]-dd[0,0]}{x_1-x_0}=\frac{2-(-4)}{1-(-1)}=\frac{6}{2}=3=[x_0x_1]$$

$$dd[1,1]=\frac{dd[2,0]-dd[1,0]}{x_2-x_1}=\frac{11-2}{2-1}=9=[x_1x_2]$$

$$dd[2,1]=\frac{dd[3,0]-dd[2,0]}{x_3-x_2}=\frac{32-11}{3-2}=21=[x_2x_3]$$

$$dd[0,2]=\frac{dd[1,1]-dd[0,1]}{x_2-x_0}=\frac{9-3}{2-(-1)}=\frac{6}{3}=2=[x_0x_1x_2]$$

$$dd[1,2]=\frac{dd[2,1]-dd[1,1]}{x_3-x_1}=\frac{21-9}{3-2}=12=[x_1x_2x_3]$$

$$dd[0,3]=\frac{dd[1,2]-dd[0,2]}{x_3-x_0}=\frac{12-2}{3-(-1)}=\frac{10}{4}=\frac{5}{2}=[x_0x_1x_2x_3]$$

$$L_{m3}(x)=-4+3(x-(-1))+2(x-(-1))(x-1)+\frac{5}{2}(x-(-1))(x-1)(x-2)$$

$$L_{m3}(\bar{x})=L_{m3}(0)=-4+3(0+1)+2(0+1)(0-1)+\frac{5}{2}(0+1)(0-1)(0-2)$$

$$=-4+3-2+\frac{5}{\cancel{2}}\cdot\cancel{2}=-6+8=\boxed{2}$$

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III ALBERT R.

A pătratică nesimetrică

$(Ax,x)_{\mathbb{R}^n} < 0 \;,\; \forall x \neq 0 \;,\; x \in (\mathbb{R}^n)$

$$x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \qquad A = \begin{pmatrix} a_{11} & a_{12} \dots & a_{1n} \\ \vdots & & \\ a_{n1} & a_{n2} \dots & a_{nn} \end{pmatrix}$$

$$Ax = \begin{pmatrix} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \\ \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n \end{pmatrix}$$

$$(Ax,x) = x^T \cdot Ax = (x_1\ x_2 \dots x_n) \cdot Ax$$
$$= (a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n)x_1 +$$
$$(a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n)x_2 +$$
$$\vdots$$
$$(a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n)x_n < 0 \;,\; \forall x_1$$
$$= a_{11}x_1^2 + (a_{21} + a_{31} + \dots + a_{n1})x_1 +$$
$$a_{22}x_2^2 + (a_{12} + a_{32} + \dots + a_{n2})x_2 +$$
$$\dots$$
$$a_{nn}x_3^2 + (a_{1n} + a_{2n} + \dots + a_{n-1\,n})x_n =$$

în cel mai dacă $\sum_{i \neq 1}^{n} a_{i1} > 0 \;\&\&\; x_1 > 0 \Rightarrow a_{11} = \dfrac{-\sum_{i \neq 1}^{n} a_{i1}}{x_1}$

$(Ax,x) = (x, A^T x) < 0$ A este nesimetric $= A^T \neq A$

$(A^T x)^T \cdot x = x^T A \cdot x = (Ax,x)$ ✓

$$\cancel{A = \begin{pmatrix} n & 1 & 1 \dots & 1 \\ 1 & n & 1 \dots & 1 \\ & & \vdots & \\ 1 & 1 \dots & & n \end{pmatrix}} \quad \cancel{\begin{matrix} -n \cdot x_1^2 + (n-1) \cdot x_1 + \\ -n \cdot x_2^2 + (n-1)x_2 + \\ -n x_n^2 + (n-1)x_n \end{matrix}}$$

$$A = \begin{pmatrix} -1 & 1 \dots 0 & -1 \\ -1 & -1 & \dots & 0 \\ 0 & 0 & -1 \dots & 0 \\ 1 & 0 \dots & 0 & -1 \end{pmatrix} \Rightarrow (Ax,x) = \begin{matrix} -1x_1^2 + 0 \\ -x_2^2 + 0 \\ -x_3^2 + 0 \\ \dots \\ -x_n^2 + 0 \end{matrix}$$

$x \in \mathbb{R}^n \Rightarrow$ x_i nr reale

$\cancel{\boxed{A = -I_n}}$ e simetrică

$$\boxed{A = -I_n + \begin{pmatrix} 0 & 1 & 0 & \dots \\ -1 & 0 & 0 & \dots \\ 0 & 0 & 0 & \dots \\ 0 & 0 & \dots & \end{pmatrix}}$$

pt a nu fi simetrică să dar sumele rămână 0

$\Rightarrow x_i^2 > 0$

$\Rightarrow -x_i^2 < 0$

$\Rightarrow (Ax,x) < 0$

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