Grafică tema 6

Cap 3 Transformări geometrice

t6p1.cpp

- glMatrixMode(GL_PROJECTION) } stive de matrici
  GL_MODELVIEW

GL_PROJECTION - creat pt a crea un volum de vizualizare (ce se vede pe ecran)

GL_MODELVIEW - facem transformări: translatăm rotim etc

Pt o stivă oarecare avem urm operații

glLoadIdentity (se plasează în vârful stivei mat I_4)
glPushMatrix (duplică mat din vf stivei)
gl ~~Push~~ PopMatrix } salvare/restaurare de contexte

$C.X \to C.C.X$

$C.X \to X$ (atenție la erori)

glMulMatrix(M) $C.X \to CM.X$
glTranslatex
glScalex
glRotatex } similare cu glMulMatrix doar că creează M automat

$(\text{unghi}, \underbrace{-,-,-}_{\text{punct/vector}})$ $(10,0,0,1)$ - rotație în jurul axei Z.

Exemplu
glLoadIdentity(); I_4
glRotated(20,1,0,0) $I_4 \cdot R_X(20°)$
glRotated(-20,0,1,0) $I_4 \cdot R_X(20°) \cdot R_Y(-20°)$
DisplayAxe()

Axele NU sunt invariante

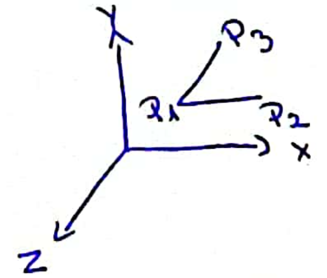
```
case 'z':
    glClear(GL_COLOR_BUFFER_BIT);
    DisplayZ();
    DisplayAxe();
    DisplayObiect();
```

Axele sunt invariante

```
case 'z':
    glClear( =//= );
    DisplayAxe();
    glPushMatrix();
        DisplayZ();
        DisplayObiect();
    glPopMatrix();
```

5.7

2.



depinde de cadran
- în curs e prez cadran I

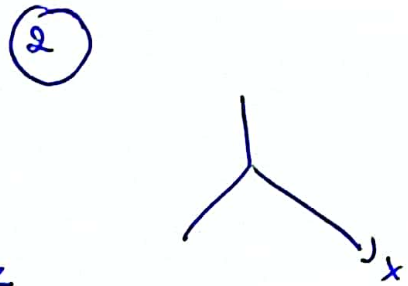
Ordinea transformărilor este inversă

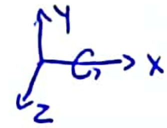
glOrtho - nu mă ating încă de el

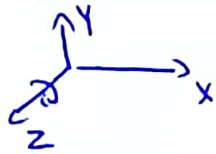
1. În 3D nu sunt 8 cadrane?
2. Δ trebuie desenat oriunde, chiar dacă nu apare pe ecran?

① Translatăm punctul P_1 în origine

②  Rotație Oy a.i. $P_1' P_2' \in Oyz$



③  Rotație Ox a.i. $P_1'', P_2'' \in Oz$

4  Rotație Oz a.i. $P_1''' P_3''' \in Oyz$

① $T(-x_1, -y_1, z_1) \Rightarrow P_1' = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad P_2' = \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \\ z_2 - z_1 \\ 1 \end{pmatrix} \quad P_3' = \begin{pmatrix} x_3 - x_1 \\ y_3 - y_1 \\ z_3 - z_1 \\ 1 \end{pmatrix}$

② Fie $Q(x_2', 0, z_2')$ proiecția punctului P_2' pe Oxz

$D_1 = |P_1'Q| = \sqrt{x_2'^2 + z_2'^2} = \sqrt{(x_2 - x_1)^2 + (z_2 - z_1)^2}$

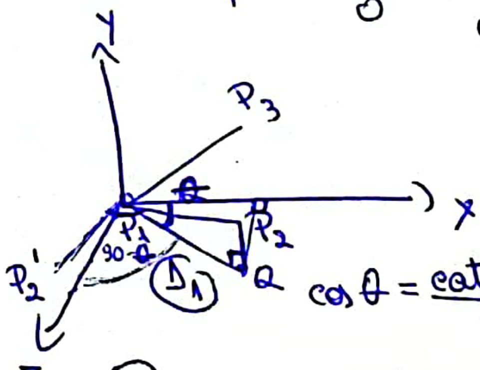
Rotim în jurul ~~lu~~ Axei Oy a.i. $P_2'' \in Oyz$
$Q \in Oz$ axă

Fie θ unghiul $\angle(QP_1', Ox)$

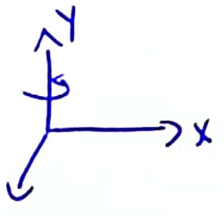
Rotim $-(90 - \theta)$ grade : $R_y(\theta - 90) = \begin{pmatrix} \cos(\theta - 90) & 0 & \sin(\theta - 90) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta - 90) & 0 & \cos(\theta - 90) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$\frac{z_2 - z_1}{D_1}$ $\quad$ $\frac{x_2 - x_1}{D_1}$

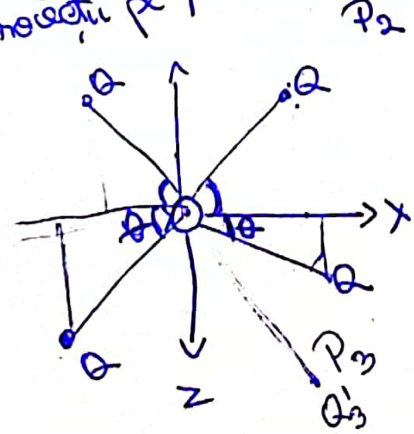
$P_2'' = \begin{pmatrix} 0 \\ y_2 - y_1 \\ D_1 \\ 1 \end{pmatrix} \quad P_2'' \in Oyz$



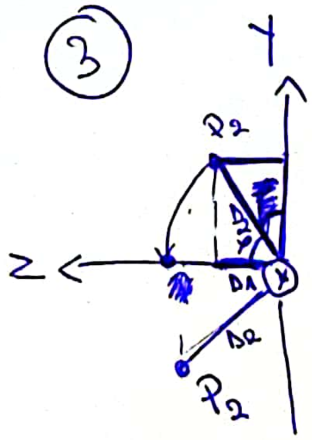
$\cos\theta = \frac{\text{cateta opusă}}{\text{ipotenuză}}$



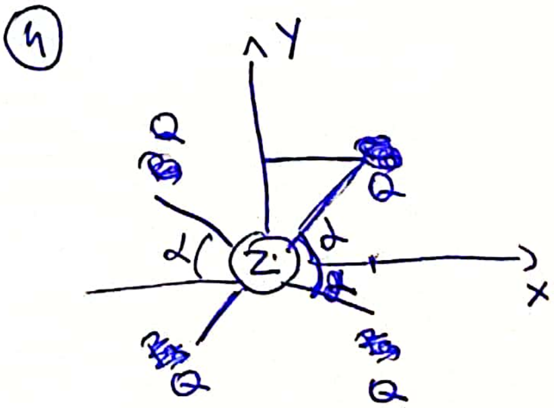
② Proiecții pe y intacte

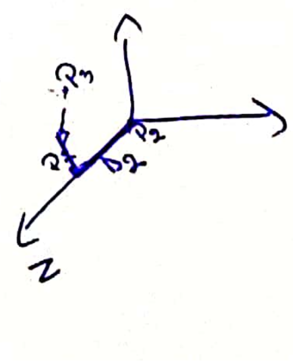


③



④



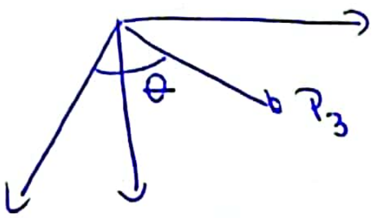


3.

3

$$\begin{pmatrix} \frac{z_2-z_1}{D_1} & 0 & \frac{-x_2-x_1}{D_1} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{x_2-x_1}{D_1} & 0 & \frac{z_2-z_1}{D_1} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

x, z



T

P_3 $\begin{pmatrix} x_3-x_1 \\ y_3-x_1 \\ z_3-z_1 \\ 1 \end{pmatrix}$

$$\begin{pmatrix} \cancel{x_3-x_1} \\ \cancel{y_3-x_1} \\ \cancel{z_3-z_1} \end{pmatrix} \begin{pmatrix} \frac{z_2-z_1}{D_1} & 0 & -\frac{x_2-x_1}{D_1} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{x_2-x_1}{D_1} & 0 & \frac{z_2-z_1}{D_1} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_3-x_1 \\ y_3-y_1 \\ z_3-z_1 \\ 1 \end{pmatrix}$$

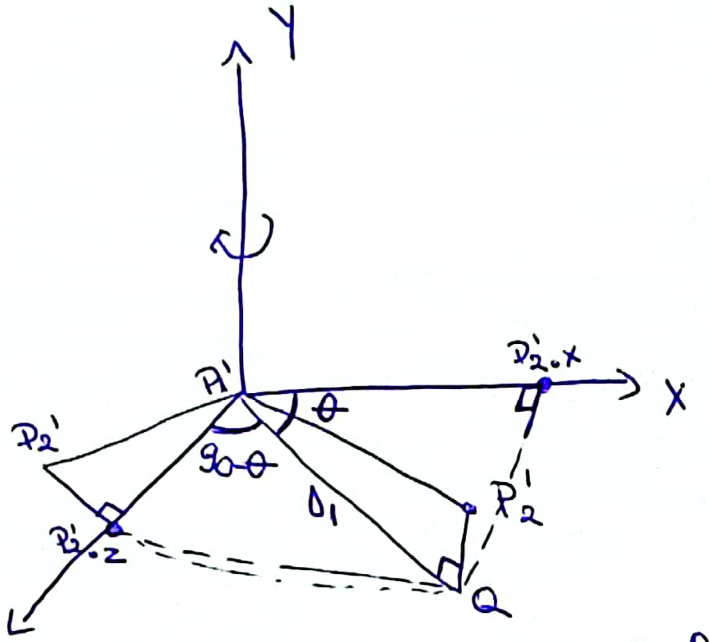
$$\begin{pmatrix} \frac{(z_2-z_1)\cdot(x_3-x_1)}{D_1} - \frac{(z_3-z_1)\cdot(x_2-x_1)}{D_1} \\ y_3-y_1 \\ \frac{x_2-x_1\cdot(x_3-x_1)}{D_1} + \frac{(z_2-z_1)\cdot(z_3-z_1)}{D_1} \\ 1 \end{pmatrix}$$

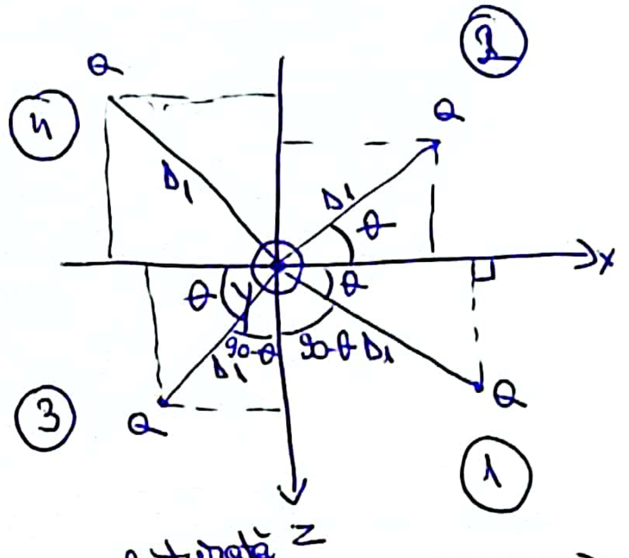
$$
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \frac{D_1}{D_2} & -\frac{y_2-y_1}{D_2} & 0 \\
0 & \frac{y_2-y_1}{D_2} & \frac{D_1}{D_2} & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
\frac{(z_2-z_1)(x_3-x_1)-(z_3-z_1)(x_2-x_1)}{D_1} \\
y_3-y_1 \\
\frac{(x_2-x_1)(x_3-x_1)+(z_2-z_1)(z_3-z_1)}{D_1} \\
1
\end{pmatrix}
$$

$$
\begin{array}{c}
X \\ Y \\ \\ \\
\end{array}
\begin{pmatrix}
\frac{(z_2-z_1)(x_3-x_1)-(z_3-z_1)(x_2-x_1)}{D_1} \\
\frac{(y_3-y_1)\cdot D_1}{D_2} - \frac{(y_2-y_1)((x_2-x_1)(x_3-x_1)+(z_2-z_1)(z_3-z_1))}{D_1 D_2} \\
\frac{(y_2-y_1)(y_3-y_1)}{D_2} + \frac{D_1}{D_2} \cdot \\
1
\end{pmatrix}
$$

I Translatăm p1 în origine $T(-x_1,-y_1,z_1) \Rightarrow P_1' = \begin{pmatrix}0\\0\\0\end{pmatrix}$ $P_2' = \begin{pmatrix}x_2-x_1\\y_2-y_1\\z_2-z_1\end{pmatrix}$ $P_3' = \begin{pmatrix}x_3-x_1\\y_3-y_1\\z_3-z_1\end{pmatrix}$

II Rotație Oy a.i. $P_1'P_2' \in Oyz$





$Q(x_2', 0, z_2')$

$D_1 = |P_1'Q| = \sqrt{x_2'^2 + z_2'^2}$
$= \sqrt{(x_2-x_1)^2 + (z_2-z_1)^2}$

$\cos\theta = \frac{\text{cat } \cancel{\text{opusă}} \text{ alăturată}}{\text{ip}} = \frac{P2.x - P1.x\ (=P_2'.x)}{D_1}$

$\theta = \text{acos}(\cos\theta)$

Rotația se face (unghi, 0, 1, 0)

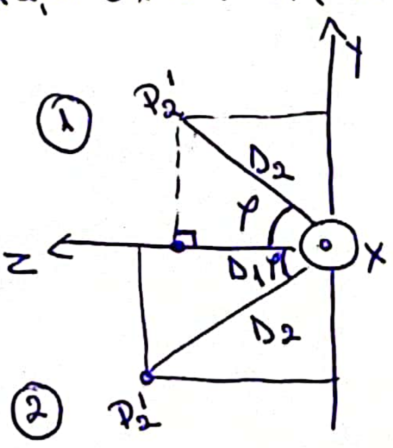
1 unghi $= -(90° - \cancel{\ \ } \theta)$ pt a fi invers trigonometric

2 unghi $= -(90° + \theta)$

3 unghi $= 90° - \theta$

4 unghi $= 90° + \theta$

III Rotație Ox a.i. $P_1'', P_2'' \in Oz$



$D_2 = \sqrt{(P_2'.y)^2 + D_1^2} = \sqrt{(P2.x - P1.x)^2 + (P2.z - P1.z)^2 + (P2.y - P1.y)^2}$

Obs: Punctele s-au mișcat în jurul axei y, deci nu și-au modificat coordonata .y.

$\boxed{\varphi = \text{acos}\left(\frac{D_1}{D_2}\right)}$

! Sunt doar 2 cazuri pt că am adus punctele pe un z pozitiv.

$\cos\varphi = \frac{D_1}{D_2}$

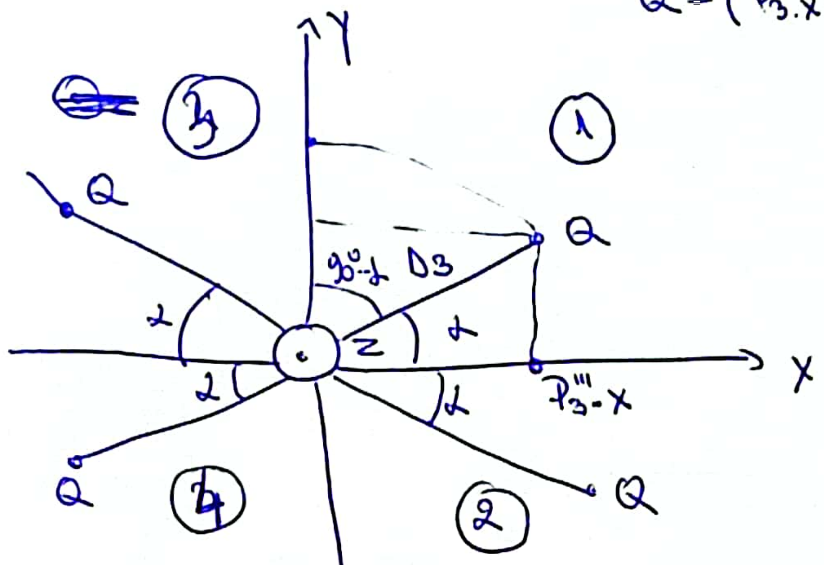
Rotația se face (unghi, 1, 0, 0)

1 unghi $= \varphi$

2 unghi $= -\varphi$

IV Rotație Oz a.i. $P_1'''P_3''' \in Oyz$

$Q = (P_3'''.x, P_3'''.y, 0)$



$D_3 = |P_1''Q| = \sqrt{P_3'''.x^2 + P_3'''.y^2}$

Cum aflu P_3'''? Fac înmulțiri cu mat de rotație

$$P_3' = \begin{pmatrix} x_3 - x_1 \\ y_3 - x_1 \\ z_3 - z_1 \\ 1 \end{pmatrix}$$

$$P_3'' = \begin{pmatrix} \frac{z_2-z_1}{D_1} & 0 & -\frac{x_2-x_1}{D_1} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{x_2-x_1}{D_1} & 0 & \frac{z_2-z_1}{D_1} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_3-x_1 \\ y_3-y_1 \\ z_3-z_1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{(z_2-z_1)(x_3-x_1)}{D_1} - \frac{(z_3-z_1)(x_2-x_1)}{D_1} \\ y_3-y_1 \\ \frac{(x_2-x_1)(x_3-x_1)}{D_1} + \frac{(z_2-z_1)(z_3-z_1)}{D_1} \\ 1 \end{pmatrix}$$

$$P_3''' = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{D_1}{D_2} & -\frac{y_2-y_1}{D_2} & 0 \\ 0 & \frac{y_2-y_1}{D_2} & \frac{D_1}{D_2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot$$

$$= \begin{pmatrix} \frac{(z_2-z_1)(x_3-x_1)-(z_3-z_1)(x_2-x_1)}{D_1} \\ \frac{(y_3-y_1)D_1}{D_2} - \frac{(y_2-y_1)((x_2-x_1)(x_3-x_1)+(z_2-z_1)(z_3-z_1))}{D_1 D_2} \\ \frac{(y_2-y_1)(y_3-y_1)}{D_2} + \frac{D_1}{D_2} \cdot \frac{(x_2-x_1)(x_3-x_1)+(z_2-z_1)(z_3-z_1)}{D_1} \\ 1 \end{pmatrix}$$

Am nevo doar de $P_3'''.x$, $P_3'''.y$

$\cos\alpha = \frac{P_3'''.x}{D_3}$

$\alpha = \operatorname{acos} \frac{P_3'''.x}{D_3}$

1) unghi $= 90° - \alpha$
2) unghi $= 90° + \alpha$
3) unghi $= -(90° - \alpha)$
4) unghi $= -(90° + \alpha)$