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Cooperative Formation Control of Quadrotors with Obstacle Avoidance and Self Collisions Based on a Hierarchical MPC Approach

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***Abstract*— This paper presents a hierarchical model predictive control (MPC) approach for cooperative formation flight control and stabilization of a team of quadrotors under constraints. The proposed control structure is a hierarchical scheme with two layers. At the top layer, a linear MPC controller is developed to generate the optimized reference trajectory to be used in the lower-layer with an MPC attitude and altitude controllers. The MPC controller units in the lower-layer are considered separately for each quadrotor. At the top-layer, data transfer between the quadrotors is implemented cooperatively. Three cost functions are considered for keeping the quadrotors in a desired formation while avoiding obstacles and inner-vehicle collisions. The simulation results show that the overall hierarchical MPC approach effectively tracks the quadrotor reference trajectory while avoiding self and obstacle collisions.**

Keywords—cooperative control; formation control; model predictive control; leader follower; UAV

I. INTRODUCTION

Cooperative control problems with a multi-vehicle system have attracted a lot of attention from many researchers in recent years [1], specifically the unmanned aerial vehicles (UAVs) which have been widely applied in military fields and civil industries such as intelligence, surveillance, exploration, search and rescue, transportation, monitoring, and so on [2].

In this paper unmanned quadrotor rotor craft is considered as a type of UAV. This UAV is based on vertical take-off and landing (VTOL) concepts and it offers a set of advantages including hovering and extreme maneuvering.

Distributed formation refers to the group behavior that all of the UAVs form a predesigned geometrical configuration through local interactions with or without a common reference [3]. Multiple formation control structure has been developed, such as virtual structure, behavior-based and leader-follower method. In the leader-follower, one UAV is considered as the leader and the others are assighned as followers. This follower keeps track of the leader and maintains desired distance. A huge problem in the leader- follower is that leader crashed. For this problem multi-leader methods are adopted [4].

Due to the nonlinear model of quadrotor and several constraints, linear and nonlinear control schemes have been

developed for quadrotor formation control. In linear quadrotor formation control, the algorithm is easy to synthesize but it cannot handle constraints directly. In nonlinear control strategy, a big problem is the excessive computation time for cost function optimization in the presence of constraints. Therefore, according to the above mentioned, a proper strategy has been developed for solving it.

Several control methods have been developed for quadrotors formation control scenario. In [5] a PID controller is designed for a group of UAVs. In [6], [7] a consensus-based approach is considerd for change geometric configuration and time-varying UAVs formation problem, respictively. In reference [8], [9] artificial potential field approach is proposed for the planner for motion of multiple UAVs. In addition, a sliding mode based heading rate controller is proposed for the leader UAV in order to move and source seeking [10]. In [11] a formation guidance method for UAVs based on feedback linearization control is presented. Also fuzzy logic controller is designed in [12]. And so on, in [13], [14], [15], [16], [17], [18], MPC controllers methods are designed for explained scenario.

In MPC (or receding horizon control) controller, a model is used to predict the system output on a horizon through calculation of a control sequence minimizing such as cost function. It can be used to control variety of processes, from simple dynamics to more complex ones and multivariable case.in addition, it very usefull when further references are known. In MPC controller strategy, constraints can be handled easily as well as other controllers strategies [19]. Furthermore MPC is a very efficient method for handling constraints and complex problems such as stall velocity of fixed wing UAV, angular turn, rate constraints, and control input saturation constraints.

In [14], [15], two layers of linear-MPC controller are in [16], a nonlinear MPC controller is proposed for collision avoidance and formation control based on leader following.

In this paper, the control scheme is considered with two layers of MPC controller to stabilize a quadrotor and an online

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path planning for autonomous navigation with obstacle avoidance of a quadrotors team. At the upper layer a linear MPC controller minimizing cost functions in order to generate the optimized collision-free state and reference trajectory. Also, three cost functions are considered, for keeping quadrotors in a desired formation, obstacle avoidance and inner-vehicle collision avoidance.

At the lower-layer, a linear MPC controller is developed in order to stabilize quadrotors and follow the reference trajectory that was generated in the upper-layer. For stabilization, two controllers are being considered, an MPC controller for translational motion and another one for rotational quadrotor motion. Simulation result will show this control strategy is useful for formation control and reference trajectory tracking.

This paper is organized as follows. In Section II the nonlinear quadrotor dynamic based on Lagrange-Euler method is presented. In Section III control strategy is explained. In Section IV a controller based on linear MPC for stabilization and reference trajectory tracking is obtained. Section V explains the upper layer of hierarchical MPC controller in order to generate the reference trajectory in the presence of relevant constraints. Section VI presents the simulation results to validate the proposed approach and finally conclusion remarks are stated in Section VII.

II. Quadrotor Dynamics formulation

The quadrotor motion dynamics can be represented by twelve state-space equations that are obtained from Lagrange-Euler method based on the kinetic and potential energy concept. Let the translation movement vector of the quadrotor's center of mass be $\zeta = [x, y, z]^T$ and the rotation vector with respect to the earth frame be $\eta = [\varphi, \theta, \psi]^T$, then in [20] an extended formulation can be found, then the quadrotor dynamics can be formulated as:

$$\begin{bmatrix} \dot{\varphi} \\ \ddot{\varphi} \\ \dot{\theta} \\ \ddot{\theta} \\ \dot{\psi} \\ \ddot{\psi} \\ \dot{z} \\ \ddot{z} \\ \dot{x} \\ \ddot{x} \\ \dot{y} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} \dot{\varphi} \\ \dot{\theta}\dot{\psi}\dfrac{I_{yy} - I_{zz}}{I_{xx}} + \dot{\theta}\dfrac{J_r}{I_{xx}}\Omega_r + \dfrac{l_a}{I_{xx}}U_2 \\ \dot{\theta} \\ \dot{\varphi}\dot{\psi}\dfrac{I_{zz} - I_{xx}}{I_{yy}} - \dot{\theta}\dfrac{J_r}{I_{yy}}\Omega_r + \dfrac{l_a}{I_{yy}}U_3 \\ \dot{\psi} \\ \dot{\theta}\dot{\varphi}\dfrac{I_{xx} - I_{yy}}{I_{zz}} + \dfrac{1}{I_{zz}}U_4 \\ \dot{z} \\ g - (\cos\varphi \cos\theta)\dfrac{U_1}{m_s} \\ \dot{x} \\ u_x U_1 / m_s \\ \dot{y} \\ u_y U_1 / m_s \end{bmatrix} + \begin{bmatrix} 0 \\ \tilde{w}_1 \\ 0 \\ \tilde{w}_2 \\ 0 \\ \tilde{w}_3 \\ 0 \\ \tilde{w}_4 \\ 0 \\ \tilde{w}_5 \\ 0 \\ \tilde{w}_6 \end{bmatrix} \quad (1)$$

where $w_i, i = 1, 2, ..., 6$ are additive disturbances from quadrotor's blades wind on the rotation and translational motions. The $U_i, i = 1, ..., 4$ represents the control inputs from four motor's angular velocities $\Omega_i, i = 1, ..., 4$; that the U vector is defined as:

$$\boldsymbol{U} = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ \Omega_r \end{bmatrix} = \begin{bmatrix} b\left(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2\right) \\ b\left(-\Omega_2^2 + \Omega_4^2\right) \\ b\left(\Omega_1^2 - \Omega_3^2\right) \\ d\left(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2\right) \\ -\Omega_1 + \Omega_2 - \Omega_3 + \Omega_4 \end{bmatrix} \quad (2)$$

$$\begin{bmatrix} u_x \\ u_y \end{bmatrix} = \begin{bmatrix} \cos\varphi \sin\theta \cos\psi + \sin\varphi \sin\psi \\ \cos\varphi \sin\theta \sin\psi - \sin\varphi \cos\psi \end{bmatrix}$$

The control input U_1 is the total thrust and U_2, U_3, U_4 are related to the rotation of the quadrotor about x , y , z axis and Ω_r is the overall residual propeller angular speed. and u_x, u_y can be considered as the directions of U_1 that cause the movement through the x and y axis, respectively. The rest of the parameters are represented in TABLE I.

TABLE I
Quadrotor Model Parameters [21]

Symbol	Description
I_{xx}	Moment of inertia about E_x axes
I_{yy}	Moment of inertia about E_y axes
I_{zz}	Moment of inertia about E_z axes
l_a	Quadrotor Arm length
b	Thrust coefficient
d	Drag coefficient
J_m	Moment of inertia of the motor about its axis of rotation
J_p	Moment of inertia of the propeller about its axis of rotation
$J_r = J_m + J_p / 4$	Moment of inertia of the rotor about its axis of rotation
m_s	Mass of the quadrotors
g	Gravity

III. Control Strategy

The control strategy is a hierarchical scheme with two layers with linear MPC and nonlinear MPC in lower-layer and top layer respectively, shown in fig. 1. In detail, in top-layer, the nonlinear MPC controller generates an optimized reference trajectory by optimization of a cost function that consists formation flight, obstacle and collision avoidance problem. The

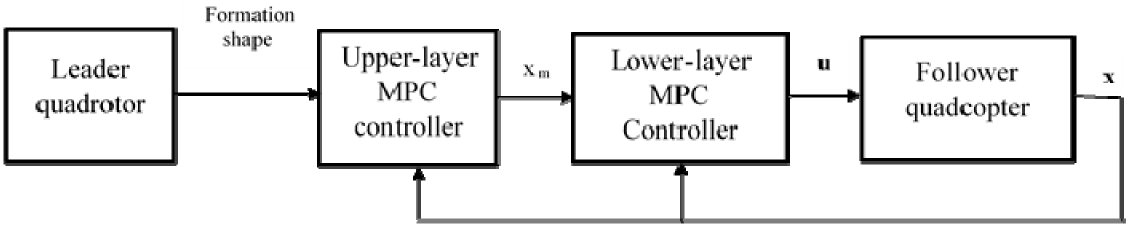


Fig. 1. MPC controller hierarchical scheme [14]

lower-layer with linear MPC controller following desired trajectory that is generated in top layer.

IV. LOWER-LAYER STABILIZATION CONTROL BASED ON LINEAR MPC

In this section a controller based on linear MPC for trajectory tracking and quadrotor's stabilization is designed. Given the equation (1), the first six equations describe rotational quadrotor dynamic which are independent of translational equation i.e. second six equation of equation (1), but the translational equation is dependent on translational equation due to u_x, u_y control signals. Thus, control laws can be designed for each dynamics separately.

In lower-layer a linear state-space MPC strategy is considered. In this structure the reference trajectory signal, which is generated from top layer (will be express in Subsection **V**), is track by translational motions MPC controller unit. Then, the reference of roll and pitch (i.e. φ_r, θ_r respectively) that is generated from translational motions unit, will be tracked by rotational motions MPC controller unit to perform the quadrotor stabilization.

Finally the control signal U is applied to nonlinear quadrotor dynamic for its stabilization and for tracking the reference trajectory.

A. Translational motions MPC controller unit

To design this controller unit, a linear state space MPC strategy based on the error model is performed. By linearization of the translational motion dynamic at equilibrium point of the quadrotor, the linear model can be defined as:

$$\dot{\boldsymbol{X}}_\zeta(t) = \boldsymbol{A}_\zeta(t)\boldsymbol{X}_\zeta(t) + \boldsymbol{B}_\zeta(t)\boldsymbol{U}_\zeta(t) \tag{3}$$

for reducing error position due to the presence of disturbance an integral position error is added to the state vector. Thus, the state vector and control action are defined as below:

$$\begin{aligned} \boldsymbol{X}_\zeta &= [x, \dot{x}, \textstyle\int x, y, \dot{y}, \int y, z, \dot{z}, \int z]^T \\ \boldsymbol{U}_\zeta &= [u_x, u_y, U_1]^T \end{aligned} \tag{4}$$

The translational controller unit can be separated into two subsystems; i.e. two systems with z error vector and x, y error vector. According to the above expression and discretization quadrotor dynamic model by using Euler's method, each subsystem is the following:

$$\begin{aligned} \boldsymbol{X}_z(k+1) &= \boldsymbol{A}_z \boldsymbol{X}_z(k) + \boldsymbol{B}_z(k)\tilde{\boldsymbol{U}}_z(k) \\ \boldsymbol{X}_{xy}(k+1) &= \boldsymbol{A}_{xy} \boldsymbol{X}_{xy}(k) + \boldsymbol{B}_{xy}(k)\tilde{\boldsymbol{U}}_{xy}(k) \end{aligned} \tag{5}$$

where $\tilde{\boldsymbol{U}}_z(k) = U_1, \tilde{\boldsymbol{U}}_{xy}(k) = [u_x, u_y]^T$.

The objective of the MPC controller design is to obtain a control signal in a way that the quadrotor can follow the reference trajectory and minimize a cost function over a control and prediction horizon. Then, the prediction state can be obtained as follows:

$$\begin{aligned} \hat{\boldsymbol{X}}_z(k+N_{12_z}\,|\,k) &= \boldsymbol{F}_{N_{12_z}}(k\,|\,k)\boldsymbol{X}_z(k\,|\,k) + \boldsymbol{H}_{N_{123_z}}(k\,|\,k)\hat{\hat{\boldsymbol{u}}}_z(k+1\,|\,k), \\ \hat{\boldsymbol{X}}_{xy}(k+N_{12_{xy}}\,|\,k) &= \boldsymbol{F}_{N_{12_{xy}}}(k\,|\,k)\boldsymbol{X}_{xy}(k\,|\,k) + \boldsymbol{H}_{N_{123_{xy}}}(k\,|\,k)\hat{\hat{\boldsymbol{u}}}_{xy}(k+1\,|\,k) \end{aligned} \tag{6}$$

where $\boldsymbol{H}_{N_{123_z}}, \boldsymbol{H}_{N_{123_{xy}}}$ are block lower triangular matrix $\boldsymbol{F}_{N_{12_z}}, \boldsymbol{F}_{N_{12_{xy}}}$ are the system free responses. $N_{1_z} \le N_{12_z} \le N_{1_z}, N_{1_{xy}} \le N_{12_{xy}} \le N_{1_{xy}}$ are prediction horizons and $N_{3_z}, N_{3_{xy}}$ are control horizons.

The equations are composed as two terms: the first one depends on the current state and therefore is known in instant k while the second depends on the vector of future control actions. The cost functions are presented as below:

$$\begin{aligned} J_z &= (\boldsymbol{H}_{N_{123_z}}\boldsymbol{u}_{N_{3_z}} + \boldsymbol{F}_{N_{12_z}}\hat{\boldsymbol{x}}_z - \boldsymbol{W}_z)^T \bar{\boldsymbol{R}}_z(\boldsymbol{H}_{N_{123_z}}\boldsymbol{u}_{N_{3_z}} + \boldsymbol{F}_{N_{12_z}}\hat{\boldsymbol{x}}_z - \boldsymbol{W}_z)^T \\ &\quad + u_{N_{3_z}}^T \bar{Q}_z u_{N_{3_z}}, \\ J_{xy} &= (\boldsymbol{H}_{N_{123_{xy}}}\boldsymbol{u}_{N_{3_{xy}}} + \boldsymbol{F}_{N_{12_{xy}}}\hat{\boldsymbol{x}}_{xy} - \boldsymbol{W}_{xy})^T \bar{\boldsymbol{R}}_{xy}(\boldsymbol{H}_{N_{123_{xy}}}\boldsymbol{u}_{N_{3_{xy}}} + \boldsymbol{F}_{N_{12_{xy}}}\hat{\boldsymbol{x}}_{xy} \\ &\quad - \boldsymbol{W}_{xy})^T + \boldsymbol{u}_{N_{3_{xy}}}^T \bar{\boldsymbol{Q}}_{xy}\boldsymbol{u}_{N_{3_{xy}}} \end{aligned} \tag{7}$$

where $\bar{\boldsymbol{R}}_z, \bar{\boldsymbol{Q}}_z, \bar{\boldsymbol{R}}_{xy}, \bar{\boldsymbol{Q}}_{xy}$ are diagonal positive definite weighting matrices and obtained with manipulation. Furthermore $\boldsymbol{W}_z, \boldsymbol{W}_{xy}$ are set-points that are generated from top layer.

If the constraints are not considered, the control actions can be obtained as:

$$\begin{aligned} \hat{\boldsymbol{u}}_z &= [(\boldsymbol{H}_{N_{123_z}}^T \bar{\boldsymbol{R}}\boldsymbol{H}_{N_{123_z}}) + \bar{\boldsymbol{Q}}_z]^{-1}\boldsymbol{H}_{N_{123_z}}^T \bar{\boldsymbol{R}}_z(\boldsymbol{W}_z - \boldsymbol{F}_{N_{12_z}}\hat{\boldsymbol{x}}_z(t)), \\ \hat{\boldsymbol{u}}_{xy} &= [(\boldsymbol{H}_{N_{123_{xy}}}^T \bar{\boldsymbol{R}}\boldsymbol{H}_{N_{123_{xy}}}) + \bar{\boldsymbol{Q}}_{xy}]^{-1}\boldsymbol{H}_{N_{123_{xy}}}^T \bar{\boldsymbol{R}}_{xy}(\boldsymbol{W}_{xy} - \boldsymbol{F}_{N_{12_{xy}}}\hat{\boldsymbol{x}}_{xy}(t)). \end{aligned} \tag{8}$$

The following control action is applied to the quadrotor:

$$U_1(k)=\hat{u}_z(k\,|\,k)+U_{1r} \tag{9}$$

The reference of roll and pitch in order to apply rotational controller unit is obtained as follows:

$$T=-m_s\sqrt{\ddot{x}^2+\ddot{y}^2+(\ddot{z}-g)^2},\theta_r=cos^{-1}\left(\frac{m_s\ddot{z}-m_s g}{T\,cos(\varphi)}\right),$$
$$\varphi_r=sin^{-1}\left(\frac{m_s\ddot{x}}{T}sin(\psi)-\frac{m_s\ddot{y}}{T}cos(\psi)\right). \tag{10}$$

B. Rotational motions MPC controller unit

According to subsection ***A***, by linearization the rotational motion dynamic at equilibrium point of the quadrotor, the linear model for following θ_r, φ_r can be defined as:

$$\dot{\boldsymbol{X}}_\eta(t)=\boldsymbol{A}_\eta(t)\boldsymbol{X}_\eta(t)+\boldsymbol{B}_\eta(t)\boldsymbol{U}_\eta(t) \tag{11}$$

thus, the state vector and control actions defined as below:

$$\boldsymbol{X}_\eta=[\varphi,\dot{\varphi},\int\varphi,\theta,\dot{\theta},\int\theta,\psi,\dot{\psi},\int\psi]^T$$
$$\boldsymbol{U}_\eta=[U_1,U_2,U_3,U_4,\Omega_r]^T \tag{12}$$

By discretization of the quadrotor rotational dynamic model, the discrete model is:

$$\boldsymbol{X}_\eta(k+1)=\boldsymbol{A}_\eta\boldsymbol{X}_\eta(k)+\boldsymbol{B}_\eta(k)\boldsymbol{U}_\eta(k) \tag{13}$$

The cost function for rotational motions is presented as below:

$$J_\eta=(\boldsymbol{H}_{N_{123_\eta}}\boldsymbol{u}_{N_{3_\eta}}+\boldsymbol{F}_{N_{12_\eta}}\hat{\boldsymbol{x}}_\eta-\boldsymbol{W}_\eta)^T\bar{\boldsymbol{R}}_\eta(\boldsymbol{H}_{N_{123_\eta}}\boldsymbol{u}_{N_{3_\eta}}+\boldsymbol{F}_{N_{12_\eta}}\hat{\boldsymbol{x}}_\eta-\boldsymbol{W}_\eta)^T$$
$$+\boldsymbol{u}^T_{N_{3_\eta}}\bar{\boldsymbol{Q}}_\eta\boldsymbol{u}_{N_{3_\eta}} \tag{14}$$

If the constraints are not considered, the control actions are:

$$\hat{\boldsymbol{u}}_\eta=[(\boldsymbol{H}^T_{N_{123_\eta}}\bar{\boldsymbol{R}}\boldsymbol{H}_{N_{123_\eta}})+\bar{\boldsymbol{Q}}_\eta]^{-1}\boldsymbol{H}^T_{N_{123_\eta}}\bar{\boldsymbol{R}}_\eta(\boldsymbol{W}_\eta-\boldsymbol{F}_{N_{12_\eta}}\hat{\boldsymbol{x}}_\eta(t)) \tag{15}$$

and finally the following control action is applied to the quadrotor:

$$\hat{\boldsymbol{u}}_\eta=\boldsymbol{U}_\eta=[U_1,U_2,U_3,U_4,\Omega_r]^T \tag{16}$$

V. TOP-LAYER REFERENCE TRAJECTORY GENERATION, BASED ON NONLINEAR MPC

In this layer, by optimizing the cost function of obstacle avoidance and collision-free vehicles formation**,** the reference trajectory is generated for lower-layer. For this expression, the discrete linearization translational dynamic for each vehicle $i=1,2,...,N$ is a double integrator:

$$\boldsymbol{p}_i(k+1)=\boldsymbol{p}_i(k)+\Delta t_h.\boldsymbol{v}_i(k)$$
$$\boldsymbol{v}_i(k+1)=\boldsymbol{v}_i(k)+\Delta t_h.\boldsymbol{a}_i(k) \tag{17}$$

The state $\boldsymbol{x}_i(k)=[\boldsymbol{p}_i(k)^T\ \boldsymbol{v}_i(k)^T]^T$ of a vehicle i is considered for its position $\boldsymbol{p}_i(k)=[p_i^x(k)\ p_i^y(k)\ p_i^z(k)]^T$ and its velocity $\boldsymbol{v}_i(k)=[v_i^x(k)\ v_i^y(k)\ v_i^z(k)]^T$ along x, y, z axes. The control input $\boldsymbol{a}_i(k)=[a_i^x(k)\ a_i^y(k)\ a_i^z(k)]^T$ is given for each vehicle i.

The MPC approach for this layer split up in to three part. Formation flight problem, obstacle avoidance and inner-collision avoidance.

A. Formation flight problem

In order to keep a desired formation for vehicles and leader following, the cost function for leader and followers is:

$$J_{l,N}(p_{l,e},u_l,k)=\sum_{s=1}^{Np}(\boldsymbol{p}_{l,e}^T(k+s\,|\,k)Q\,\boldsymbol{p}_{l,e}(k+s\,|\,k))$$
$$+\sum_{m=1}^{Nc}(\boldsymbol{a}_l^T(k+m\,|\,k)R\,\boldsymbol{a}_l(k+m\,|\,k))$$
$$J_{i,N}(p_{i,e},u_i,k)=\sum_{s=1}^{Np}(\boldsymbol{p}_{i,e}^T(k+s\,|\,k)Q\,\boldsymbol{p}_{i,e}(k+s\,|\,k))$$
$$+\sum_{l=1}^{Nc}(\boldsymbol{a}_i^T(k+l\,|\,k)R\,\boldsymbol{a}_i(k+l\,|\,k)) \tag{18}$$

where $J_{l,N}$ and $J_{i,N}$ are leader and followers cost function respectively. The leader tracking the reference trajectory p_w by defining $\boldsymbol{p}_{l,e}=\boldsymbol{p}_w-\boldsymbol{p}_l$. Also $\boldsymbol{p}_{i,e}=\boldsymbol{p}_l-\boldsymbol{D}-\boldsymbol{p}_i$ and $\boldsymbol{p}_l$ is the leader state and D is the relative distance between leader and followers.

B. Obstacle avoidance

In order to avoid obstacle, inequality constraints are considered for cost function optimization. In this way, is difficult to solve the optimization problem because of the non-convex optimization problem. In [16] a simple method is proposed to guarantee the obstacle avoidance scenario. In this procedure, two edges are considered as depicted in fig 2. The first edge l_d is the dangerous zone, when the vehicles approach the obstacle and enter in this zone, then a cost function is minimized to obstacle avoidance as well as reducing the distance between them to second edge i.e. l_m. The purposed cost function is [16]:

$$L_o(\boldsymbol{p}_i,k)=\begin{cases}0 & l_i^s(k)\geq l_D\\ \sum_{s=1}^{N_p}-a\left(\boldsymbol{l}_i^s(k+s\,|\,k)-l_M\right) & l_i^s(k)<l_D\end{cases} \tag{19}$$

Where

$$l_i^s(k)=\sqrt{(p_i^x(k)-x_o)^2+(p_i^y(k)-y_o)^2+(p_i^z(k)-z_o)^2}-R_o$$
$$\boldsymbol{l}_i^s(k+s\,|\,k)=$$
$$\sqrt{(\boldsymbol{p}_i^x(k+s\,|\,k)-x_o)^2+(\boldsymbol{p}_i^y(k+s\,|\,s)-y_o)^2+(\boldsymbol{p}_i^z(k+s\,|\,s)-z_o)^2}-R_o \tag{20}$$

α is a positive parameter and $(x_0,y_0,z_0),R_0$ are the Cartesian coordinates and the radius of obstacle, respectively.

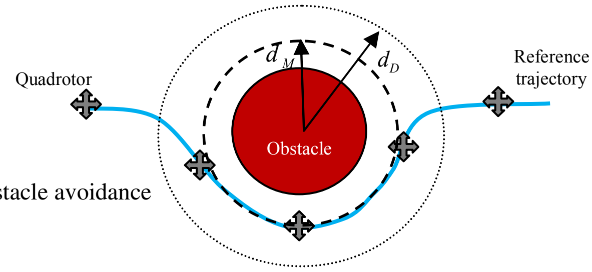


Fig. 2. Obstacle avoidance

C. Inner-collision avoidance

Due to the previous subsection, consider a protected zone with radius d_D and circular collision zone with radius d_M. If the distance between vehicles i and j is less than d_D, by minimizing a cost function, the vehicle i avoids to approach in to d_M zone. The proposed cost function is [16]:

$$L_P(\boldsymbol{p}_i, k) = \begin{cases} 0 & if\ d_{ij}(k) \geq 2d_D \\ \sum_{s=1}^{N_P} -b\left(d_{ij}(k+s \mid k) - 2d_M\right) & if\ d_{ij}(k) < 2d_D \end{cases} \tag{21}$$

where

$$d_{ij}(k) = \sqrt{\begin{array}{r}(p_i^x(k) - p_j^x(k))^2 + (p_i^y(k) - p_j^y(k))^2 \\ + (p_i^z(k) - p_j^z(k))^2\end{array}}$$
$$d_{ij}(k+s \mid k) = \sqrt{\begin{array}{l}(p_i^x(k+s \mid k) - (p_j^x(k+s \mid k))^2 \\ +(p_i^y(k+s \mid s) - (p_j^y(k+s \mid k))^2 \\ +(p_i^z(k+s \mid s) - (p_j^z(k+s \mid k))^2\end{array}} \tag{22}$$

and b is a positive parameter.

D. Cost functions optimization

Finally, to achieve collision-free formation flight at time k, the following cost function is minimized.

$$\boldsymbol{a}^* = \min_a \left(J_N(p,k) + L_o(p,k) + L_p(p,k)\right) \tag{23}$$

VI. SIMULATION

The simulation result is presented in this section. A scenario is developed to show that three quadrotors with leader-follower structure had a formation with triangular shape, in which quadrotors are moving from initial state to a final state while static obstacles are in their trajectory. Thus, the hierarchical MPC controller that is presented in the last sections, is designed for this scenario.

For lower-layer stabilization MPC controller, the parameters are presented as below:

$$N_{2_z} = N_{3_z} = 3 \quad , |Q_z| = 10, R_z = 10^4, N_{2_{xy}} = N_{3_{xy}} = 10,$$
$$|Q_{xy}| = 10, |R_z| = 10^4, N_{2_\eta} = N_{3_\eta} = 10, |Q_\eta| = 1, R_\eta = 10^4 .$$
$$m_s = 0.74kg\,, l = 0.21\,m\,, g = 9.81\frac{m}{s^2}, \Delta t_l = 0.09s$$
$$I_{xx} = I_{yy} = 0.004\,kgm^2\,, I_{zz} = 0.0084\,kgm^2.$$

In addition, for top-layer MPC controller, the parameters are: $N_P = 5, N_C = 5, \quad |Q| = 1, |R| = 10, a = 10^3, b = 500, \quad \Delta t_h = 0.35s$

The follower quadrotors are constrained to maintain a relative distance from the leader of *3m*. The l_d dangerous zone for obstacle avoidance is *4m* and the l_m zone is *2m*. Two obstacle are considered that are located in *(10, 10, 1)m* and *(30, 30, 2.5)m*.

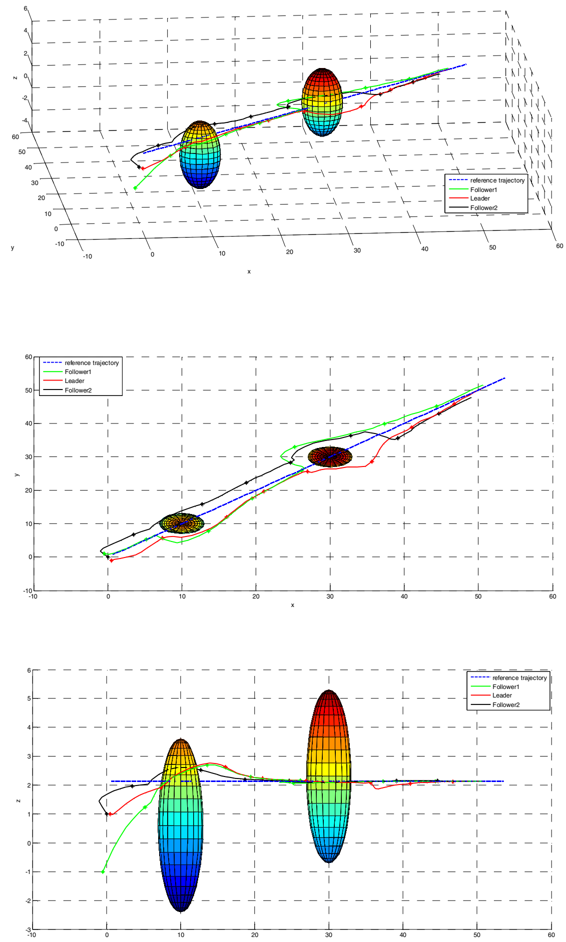
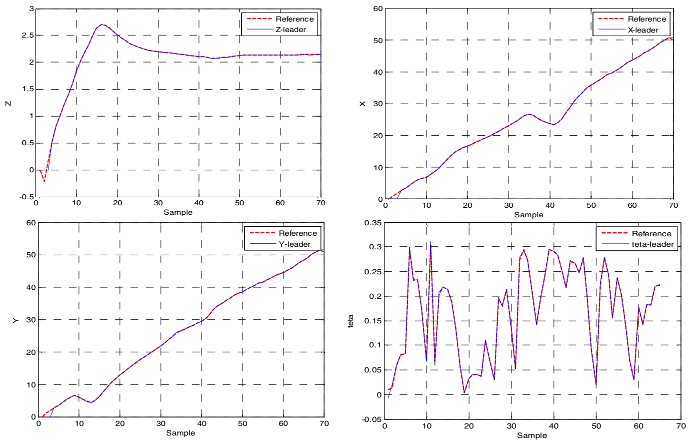


Fig.3. Formation control base on leader-follower and obstacle avoidance

The radius of first and second obstacle is *3m*. In fig. 3. The quadrotors are avoiding the obstacles in a triangular leader-follower formation and moving to a desired position. The x, y, z position of leader quadrotor tracking the reference trajectory which is generated by minimization of total cost functions, is depicted in Fig. 3.

Because of the symmetry in the shape of quadrotor, it is assumed that the yaw of the quadrotor is zero. The roll and pitch of leader quadrotor, following the set-points with linear translational MPC controller, are shown in Fig. 4.



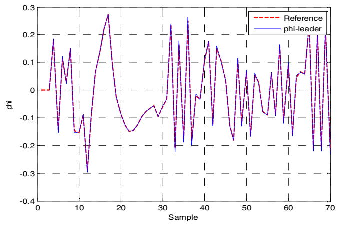


Fig. 4. Rotational and translational motion parameters

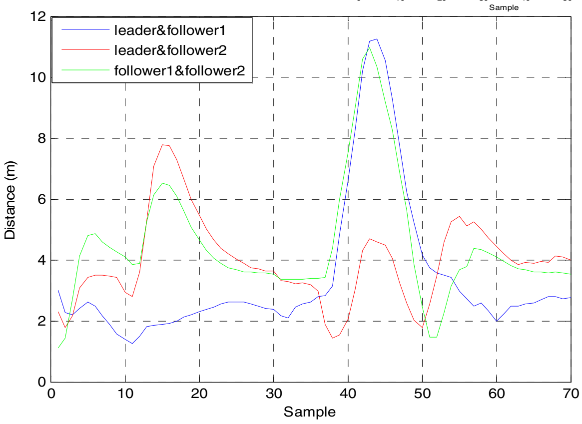


Fig. 5. Distance between quadrotors

VII. CONCLUSION

In this paper, first, a nonlinear dynamic model of quadrotor is derived by Lagrange-Euler formalism based on the kinetic and potential energy concept. A hierarchical model predictive control approach of two layers is explained. In lower-layer, an MPC controller approach is designed for translational and rotational dynamics of quadrotor in for attitude control and reference trajectory tracking. Also in the upper-layer, an MPC controller scheme is used to obtain an optimized control action with minimizing cost function and then to generate a desired reference trajectory to be applied in the lower-layer. Also, three cost functions are presented for keeping quadrotors in a desired formation while avoiding obstacles and inner-vehicle collisions. Finally, simulation results show the efficiency of proposed hierarchical MPC approach for obstacle avoidance and attitude stability control of the quadrotor in reference trajectory tracking and 3D space, but in [16] the proposed controller is designed for 2D space.

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