# **All Current Undergraduate courses**

#### MAT100 Precalculus/Prestatistics

Rigorous and intensive treatment of standard topics from algebra and trigonometry as a preparation for further courses in calculus or statistics. Fulfills the QR requirement. Fall Only.

#### **MAT103 Calculus I**

First semester of the standard 3-semester calculus sequence 103/104/201 for science, engineering and finance. Topics include limits, continuity, derivatives and their applications, introduction to the definite integral. Emphasizes concrete computations over more theoretical considerations. Offered both Fall and Spring in AY 2014-2015. Prerequisite: MAT100 or equivalent.

## MAT104 Calculus II (One Variable, Continued from 103)

Second semester of the standard 3-semester calculus sequence 103/104/201 for science, engineering and finance. Topics include techniques and applications of integration, convergence of infinite series and improper integrals, Taylor's theorem, introduction to differential equations and complex numbers. Emphasizes concrete computations over more theoretical considerations. Offered both Fall and Spring. Prerequisite: MAT103 or equivalent.

#### MAT175 Basic Multivariable Calculus for Economics & Life Sciences

Survey of topics from multivariable calculus as preparation for future course work in economics or life sciences. Topics include basic techniques of integration, average value, vectors, partial derivatives, gradient, optimization of multivariable functions and constrained optimization with Lagrange multipliers. Although examples from economics and biology will be used freely, the main focus is on the underlying mathematics. Offered Fall and Spring. Prerequisite: MAT103 or equivalent.

## **MAT189 Number, Shape and Symmetry**

Calculus, while very important to scientists and engineers, is but one part of modern mathematics, and the technicality of the subject often obscures the underlying mathematical principles. In response, this course is meant to be an alternative to a first semester calculus course for non-scientists, requiring less mathematical background, but of similar depth. In particular, the course will assume only the standard material from high school algebra and geometry. Math 189 attempts to give students an understanding of what mathematics is, what mathematicians do, and the subject's history. The course emphasizes the understanding of ideas and the ability to express them through mathematical arguments. Offered every other Spring, if staffing permits.

#### **MAT190 The Magic of Numbers**

This course will explore some of the intriguing and beautiful mathematics that underlie the arts, technology, and everyday life. Developed and taught by Professor Manjul Bhargava. (Last offered Spring 2010)

## MAT191 An Integrated Introduction to Engineering, Mathematics, Physics

Taken concurrently with EGR/MAT/PHY 192, this course offers an integrated presentation of the material from PHY 103 (General Physics: Mechanics and Thermodynamics) and MAT 201 (Multivariable Calculus) with an emphasis on applications to engineering. Physics topics include: mechanics with applications to fluid mechanics; wave phenomena; and thermodynamics. Open only to BSE freshmen and adminstered by the Keller Center.

# MAT192 An Integrated Introduction to Engineering, Mathematics, Physics

Taken concurrently with EGR/MAT/PHY 191, this course offers an integrated presentation of the material from PHY 103 (General Physics: Mechanics and Thermodynamics) and MAT 201 (Multivariable Calculus) with an emphasis on applications to engineering. Open only to BSE freshmen and administered by the Keller Center.

#### MAT198 Useful Fictions: How and why mathematics is developed and then changes the world

This course is geared toward students with no prior university math experience. It aims to provide a view of mathematics as a living, growing, creative human endeavor that classifies as both a science and an art, to give a feeling for, and some mastery of, the mathematical way of thinking (including "doing mathematics") as well as an awareness of some of the many applications of mathematics in today's world. Active class participation is an essential component of the course, required along with participation in Professor Keith Devlin's (free) Stanford MOOC "Introduction to Mathematical Thinking." We expect to offer this again in Fall 2015.

## **MAT199 Math Alive**

Mathematics has profoundly changed our world, from the way we communicate with each other and listen to music, to banking and computers. This course is designed for those without college mathematics who want to understand the mathematical concepts behind important modern applications. The course consists of individual modules, each focusing on a particular application (e.g., digital music, sending secure emails, and using statistics to explain,

or hide, facts). The emphasis is on ideas, not on sophisticated mathematical techniques, but there will be substantial problem-set requirements. Students will learn by doing simple examples.

## **MAT201 Calculus III (Multivariable Calculus)**

A continuation of MAT103/104, the third semester in the calculus sequence gives a thorough introduction to multivariable calculus. Topics include limits, continuity and differentiability in several variables, extrema, Lagrange multipliers, Taylor's theorem, multiple integrals, integration on curves and surfaces, Green's theorem, Stokes' theorem, divergence theorem. Emphasizes concrete computations over more theoretical considerations. Offered both Fall and Spring. Prerequisite: MAT104 or equivalent.

## **MAT202 Introduction to Linear Algebra**

Linear Algebra, mostly in real n-space. Companion course to 201. Main topics are matrices, linear transformations, linear independence and dimension, bases and coordinates, determinants, orthogonal projection, least squares, eigenvalues, eigenvectors and their applications to quadratic forms and dynamical systems. Offered Fall and Spring.

## **MAT203 Advanced Multivariable Calculus**

Advanced multivariable calculus. More theoretical treatment of limits, continuity, differentiation and integration for functions of several variables than that found in MAT201, but less theoretical than MAT218. A course for those with a strong mathematical background and interest. Recommended for physics majors and students interested in applied math. Offered Fall only.

#### **MAT204 Advanced Linear Algebra with Applications**

Advanced linear algebra. More theoretical treatment of vector spaces and matrices than that found in MAT202, but more concrete than that of MAT217. A course for those with a strong mathematical background and interest. Recommended for physics majors. Offered Spring only.

## **MAT214 Numbers, Equations, and Proofs**

Rigorous, proof-based introduction to classical number theory, to prepare for higher-level courses in the department. Topics include Pythagorean triples and sums of squares, unique factorization, Chinese remainder theorem, arithmetic of Gaussian integers, finite fields and cryptography, arithmetic functions and quadratic reciprocity. There will be a topic, chosen by the instructor, from more advanced or more applied number theory: possibilities include p-adic numbers, cryptography, and Fermat's Last Theorem. Suitable both for students preparing to enter the Mathematics Department and for non-majors interested in exposure to higher mathematics. Fall Only.

# **MAT215 Honors Analysis in a Single Variable**

Rigorous proof-based introduction to analysis. Topics include the rigorous epsilon-delta treatment of limits; convergence and uniform convergence of sequences and series; continuity, uniform continuity, and differentiability of functions; The Heine-Borel theorem; the Riemann integral; conditions for integrability of functions and term-by-term differentiation and integration of series of functions; Taylor's theorem.

# **MAT216 Accelerated Honors Analysis I**

Rigorous theoretical introduction to the foundations of analysis in one and several variables, including basic linear algebra, for students who already have substantial experience with formal mathematical proofs and a good knowledge of one-variable calculus. Covers basic set theory, vector spaces, metric and topological spaces, continuous and differential mappings between n-dimensional real vector spaces. Offered Fall Only. Normally followed by MAT218. (New in Fall 2014.)

# **MAT217 Honors Linear Algebra**

Continuation of MAT215. Rigorous course in linear algebra. Topics include vector spaces, linear transformations, inner product spaces, determinants, eigenvalues, the Cayley-Hamilton theorem, Jordan form, linear systems of differential equations, the spectral theorem for normal transformations, bilinear and quadratic forms. Spring Only.

#### **MAT218 Accelerated Honors Analysis II**

Continues the rigorous theoretical introduction to analysis in several variables, including basic linear algebra, begun in MAT216. Offered Spring Only. New in Spring 2014.

### MAT301 Mathematics in Engineering I

A treatment of the theory of differential equations. The objective is to provide the student with an ability to solve problems in this field. Administered and staffed by the Mechanical Engineering Department (MAE).

#### MAT302 Mathematics in Engineering II

This course provides an introduction to partial differential equations, covering PDEs of relevant interest in engineering and science problems.

## **MAT320 Introduction to Real Analysis**

Introduction to real analysis, including the theory of Lebesgue measure and integration on the line and n-dimensional space, and the theory of Fourier series. Prerequisite: MAT201 and MAT202 or equivalent. (Replaces MAT314 beginning AY2012-13)

#### **MAT321 Numerical Methods**

Introduction to numerical methods with emphasis on algorithms, applications and computer implementation issues. Topics covered include solution of nonlinear equations; numerical differentiation, integration and interpolation; direct and iterative methods for solving linear systems; numerical solutions of differential equations; two-point boundary value problems; and approximation theory. Lectures are supplemented with numerical examples using MATLAB. Prerequisites: MAT201 and MAT202; or MAT203 and MAT204; or equivalent.

## **MAT322 Introduction to Differential Equations**

An introduction to differential equations, covering both applications and fundamental theory.

## MAT323 Topics in Mathematical Modeling - Mathematical Neuroscience

Draws problems from the sciences & engineering for which mathematical models have been developed and analyzed to describe, understand and predict natural and man-made phenomena. Emphasizes model building strategies, analytical and computational methods, and how scientific problems motivate new mathematics. This interdisciplinary course in collaboration with Molecular Biology, Psychology and the Program in Neuroscience is directed toward upperclass undergraduate students and first-year graduate students with knowledge of linear algebra and differential equations.

#### MAT325 Analysis I: Fourier Series and Partial Differential Equations

Fourier series, Fourier transforms, and applications to the classical partial differential equations. Prerequisites: MAT215 or MAT218 or consent of instructor. (Replaces MAT330 beginning AY 2012-13)

## **MAT330 Complex Analysis with Applications**

Calculus of functions of one complex variable, power series expansions, residues, and conformal mapping. (Replaces MAT317 beginning AY 2012-13)

# **MAT335 Analysis II: Complex Analysis**

Study of functions of a complex variable, with emphasis on interrelations with other parts of mathematics. Cauchy's theorems, singularities, contour integration, power series, infinite products. The gamma and zeta functions and the prime number theorem. Elliptic functions, theta functions, Jacobi's triple product and combinatorics. An overall view of Special Functions via the hypergeometric series. The second course in a four-semester sequence, but may be taken independently. (Replaces MAT331 beginning AY 2012-13)

# **MAT340 Applied Algebra**

An applied algebra course that integrates the basics of theory and modern applications for students in mathematics, applied mathematics, physics, chemistry, computer science and electrical engineering. Intended for students who have taken a semester of linear algebra and are interested in the structures, properties and applications of groups, rings and fields. Applications and algorithmic aspects of algebra will be emphasized throughout. New in Fall 2014.

# MAT345 Algebra I

Covers the basics of symmetry and group theory, with applications. Topics include the fundamental theorem of finitely generated abelian groups, Sylow theorems, group actions and the representation theory of finite groups. Prerequisites: MAT202 or MAT204 or MAT217. (Replaces MAT322 beginning AY 2012-13)

## MAT346 Algebra II

Continuation of Algebra I.

Topics to be covered include: differentiation and integration in multiple dimensions; smooth manifolds; vector fields and differential forms; Stokes' theorem; de Rham cohomology; Frobenius' theorem on integrability of plane fields. This course will serve as a preparation for advanced courses in differential geometry and topology.

#### **MAT355 Introduction to Differential Geometry**

Riemannian geometry of surfaces. Surfaces in Euclidan space, second fundamental form, minimal surfaces, geodesics, Gauss curvature, Gauss-Bonnet Theorem, uniformization of surfaces. Prerequisites: MAT218 or equivalent. (Replaces MAT327 beginning AY 2012-13)

## **MAT365 Topology**

An introduction to point set topology, the fundamental group, covering spaces, methods of calculation and applications. Prerequisites: MAT202 or MAT204 or MAT218 or equivalent. (Replaces MAT325 beginning AY 2012-13)

## **MAT375 Introduction to Graph Theory**

This course will cover the fundamental theorems and algorithms of graph theory. Topics include: connectivity, mathchings, graph coloring, planarity, the four-color theorem, extremal problems, network flows, and related algorithms. Prerequisite: MAT202 or MAT204 or MAT217, or equivalent. (Replaces MAT306 beginning AY 2012-13)

#### **MAT377 Combinatorial Mathematics**

Combinatorics is the study of enumeration and structure of discrete objects. These structures are widespread throughout mathematics, including geometry, topology and algebra, as well as computer science, physics, and optimization. This course will give an introduction to modern techniques in the field, and how they relate to objects such as polytopes, permutations, and hyperplane arrangements. Current work and open problems will also be discussed.

## **MAT378 Theory of Games**

The mathematical concept of a game is an abstraction which encompasses conflict-cooperation situations in which strategy (not just chance) plays a role. Games in extensive form, pure and behavioral strategies; normal form, mixed strategies, equilibrium points; coalitions, characteristic-function form, imputations, solution concepts; related topics and applications. Prerequisites: MAT202 or 204 or 217 or equivalent. MAT215 or equivalent is recommended. (Replaces MAT308 beginning AY 2012-13)

## **MAT385 Probability Theory**

Sequences of independent trials, applications to number theory and analysis, Monte Carlo method. Markov chains, ergodic theorem for Markov chains, Entropy and McMillan theorem. Random walks, recurrence and non-recurrence; connection with linear difference equations. Strong laws of large numbers, random series and products. Weak convergence of probability measures, weak Helly theorems, Fourier transforms of distributions. Limit theorems of probability theory. Prerequisites: MAT203 or MAT218 or equivalent. (Replaces MAT390 beginning AY 2012-13)

# **MAT415 Analytic Number Theory**

An introduction to classical results in analytic number theory, presenting fundamental theorems with detailed proofs and highlighting the tight connections between them. Topics covered might include: the prime number theorem, Dirichlet L-functions, zero-free regions, sieve methods, representation by quadratic forms, and Gauss sums. Prerequisites: MAT335 (Complex Analysis) and MAT345 (Algebra I).

## **MAT419 Topics in Number Theory: Algebraic Number Theory**

Course on algebraic number theory. Topics covered include number fields and their integer rings, class groups, zeta and L-functions.

# MAT425 Analysis III: Integration Theory and Hilbert Spaces

The theory of Lebesgue integration in n-dimensional space. Differentiation theory. Hilbert space theory and applications to Fourier transforms, and partial differential equations. Introduction to fractals. The third semester of a four-semester sequence, but may be taken independently. Prerequisites: MAT215 or MAT218 or equivalent. (Replaces MAT332 beginning AY 2012-13)

#### **MAT427 Ordinary Differential Equations**

The course concerns explicit solution of simple differential equations. Methods of proving that one has found all the solutions are discussed. For this purpose, a brief review of foundational concepts in real analysis is provided. The second part concerns explicit solutions of simultaneous linear differential equations with constant coefficients, a topic closely connected with linear algebra (assumed prerequisite knowledge). The third part concerns the proof of the basic existence and uniqueness theorem for ordinary differential equations. Students will do simple proofs. (Replaces MAT303 beginning AY 2012-13)

## **MAT429 Advanced Topics in Analysis**

## **MAT443 Cryptography**

An introduction to modern cryptography with an emphasis on the fundamental ideas.

## **MAT447 Commutative Algebra**

This course will cover the standard material in a first course on commutative algebra. Topics include: ideals in and modules over commutative rings, localization, primary decomposition, integral dependence, Noetherian rings and chain conditions, discrete valuation rings and Dedekind domains, completion; dimension theory. Prerequisites: Algebra I & II, MAT345-6.

#### **MAT449 Topics in Algebra: Representation Theory**

An introduction to representation theory of Lie groups and Lie algebras. The goal is to cover roughly the first half of Knapp's book.

#### **MAT455 Advanced Topics in Geometry - Lie Theory**

Lie algebras and Lie groups are important in many areas of mathematics as well as theoretical physics. The course gives an introduction to the topic.

#### **MAT457 Algebraic Geometry**

Introduction to affine and projective algebraic varieties over fields.

#### **MAT469 Advanced Topology**

The course will target the following topics: The definition of knots in the 3-sphere, first invariants; algebraic knots and links in the 3-sphere; classification of algebraic knots, Puiseux pairs, iterated torus knots; fibred links, monodromy, the case of algebraic links; higher dimensional algebraic knots, Milnor theory of complex isolated hypersurface singularities.

# **MAT486 Random Processes**

Wiener measure, Stochastic differential equations, Markov diffusion processes, Linear theory of stationary processes, Ergodicity, mixing, central limit theorem for stationary processes, Gibbs random field. If time permits, the theory of products of random matrices and PDE's with random coefficients will be discussed. Prerequisite: MAT390 in the old numbering system or MAT385 in the new system. (Replaces MAT391 beginning AY 2012-13)

## **MAT520 Functional Analysis**

The course is intended as a basic introductory course to the modern methods of Analysis. Specific applications of these methods to problems in other fields, such as Partial Differential Equations, Probability, and Number Theory, will also be presented. Topics will include Lp spaces, tempered distributions and the Fourier transform, the Riesz interpolation theorem, the Hardy–Littlewood maximal function, Calderon–Zygmund theory, the spaces H1 and BMO, oscillatory integrals, almost orthogonality, restriction theorems and applications to dispersive equations, the law of large numbers and ergodic theory. We will also discuss applications of Fourier methods to discrete counting problems, using the Poisson summation formula.

#### **MAT522 Introduction to Partial Differential Equations**

Introduction to the techniques necessary for the formulation and solution of problems involving partial differential equations in the natural sciences and engineering, with detailed study of the heat and wave equations. Topics include method of eigenfunction expansions, Fourier series, the Fourier transform, inhomogeneous problems, the method of variation of parameters. Prerequisite MAT202 or MAT204 or MAT218.

# **MAT523 Advanced Analysis**

The course covers the essentials of the first eleven chapters of the textbook, "Analysis" by Lieb and Loss. Topics include Lebesque integrals, Measure Theory, L^p Spaces, Fourier Transforms, Distributions, Potential Theory, and some illustrative examples of applications of these topics.

## **MAT531 Riemann Surfaces**

#### **MAT982 Junior Seminar**

Random Walks - Junior Seminar with Michael Damron

# **Graduate courses and seminars**

#### MAT500 Effective Mathematical Communication

## MAT509 Topics in Logic and Foundations: Computational Complexity

The focus of the course will be consistency proofs. Discussion of why Gödel's second incompleteness theorem does not preclude a finitary consistency proof for Peano Arithmetic (PA). A finitary consistency proof for PA. Critique of finitism. Construction of Polynomial Time Arithmetic (PTA), weaker than Primitive Recursive Arithmetic. Some polynomial time consistency proofs.

## MAT515 Topics in Analytic Number Theory: Spectal theory of automorphic forms

We will discuss the proof of analytic continuation of Eisenstein Series, first in rank 1 cases and then in general. Various applications will be highlighted.

## MAT516 Topics in Algebraic Number Theory: Galois Representations

This course will cover the basic properties, examples, and deformation theory of I-adic representations. Time permitting, the following will also be covered: the relationship with automorphic forms, particularly, automorphic lifting theorems and potential automorphy.

#### MAT517 Topics in Arithmetic Geometry: Arithmetic Gan-Gross-Prasad Conjecture

I will start with geometry and arithmetic of abelian varieties, including the moduli spaces, Mordell-Weil theorem, the Birch and Swinnerton-Dyer conjecture, and their conjectured high codimension analogues by Beilinson and Bloch. Then, I will introduce Gross-Zagier type formulae for Shimura curves and their triple products, high codimensional analogues by Gan-Gross-Prasad for unitary Shimura varieties, and a relative trace formula approach by Wei Zhang.

## MAT518 Topics in Automorphic Forms: Philosophy of cusp forms

In this course, we will focus on some aspects of the foundation of automorphic forms. A general organization principle is that cusp forms should be the fundamental building blocks of the theory, which has many implications both to arithmetic and to local representation theory. The first part of the course will be a rigorous introduction to this circle of ideas. The aim is to give precise definitions in arbitrary dimension. The second part will be concerned with the structure of representations of real Lie groups, notably the Langlands classification, cohomological representations, the tempered dual and Vogan theory.

# **MAT520 Functional Analysis**

The course is intended as a basic introductory course to the modern methods of Analysis. Specific applications of these methods to problems in other fields, such as Partial Differential Equations, Probability, and Number Theory, will also be presented. Topics will include Lp spaces, tempered distributions and the Fourier transform, the Riesz interpolation theorem, the Hardy–Littlewood maximal function, Calderon–Zygmund theory, the spaces H1 and BMO, oscillatory integrals, almost orthogonality, restriction theorems and applications to dispersive equations, the law of large numbers and ergodic theory. We will also discuss applications of Fourier methods to discrete counting problems, using the Poisson summation formula.

## **MAT522 Introduction to Partial Differential Equations**

Introduction to the techniques necessary for the formulation and solution of problems involving partial differential equations in the natural sciences and engineering, with detailed study of the heat and wave equations. Topics include method of eigenfunction expansions, Fourier series, the Fourier transform, inhomogeneous problems, the method of variation of parameters. Prerequisite MAT202 or MAT204 or MAT218.

# **MAT523 Advanced Analysis**

The course covers the essentials of the first eleven chapters of the textbook, "Analysis" by Lieb and Loss. Topics include Lebesque integrals, Measure Theory, L^p Spaces, Fourier Transforms, Distributions, Potential Theory, and some illustrative examples of applications of these topics.

## MAT526 Topics in Geometric Analysis and Relativity: Introduction to general relativity

This is a fast moving introductory course in General Relativity with the goal of presenting some recent important results in the field. The following will be covered: quick introduction to general relativity, break-down criterion, a discussion of bounded L2 curvature conjecture, formation of trapped surfaces, and uniqueness of black holes.

# MAT527 Topics in Differential Equations: Global solutions of nonlinear evolutions

We will discuss the question of existence of global smooth solutions of certain quasilinear evolution equations. These equations include plasma models and the water wave problem. The main techniques include energy methods and semilinear harmonic analysis.

MAT528 Topics in Nonlinear Analysis: The rigidity, stability, and formation of black holes

MAT529 Topics in Analysis: Fluid dynamics and related equations

#### **MAT531 Riemann Surfaces**

#### MAT539 Topics in Complex Analysis: Brill-Noether theory for Riemann surfaces

The course will deal primarily with Brill-Noether theory on compact Riemann surfaces, the question of the number of meromorphic functions with at most some specified singularities. That has been discussed extensively on generic (or primitive) Riemann surfaces, as in the book Geometry of Algebraic Curves I by Arbarello, Cornalba, Griffiths and Harris. I will focus more on special curves, in particular on relations between discrete invariants of the analytic structures of curves such as the maximal sequence and the Luroth sequence, and mostly in terms of line bundles. If there is time and interest perhaps some discussion of the less complete case of vector bundles, or some possibilities in higher dimensions. I will try to make the course fairly self-contained, so initially not assuming much detailed knowledge of function theory on compact Riemann surfaces; but I will not attempt to prove the basic background results.

#### MAT547 Topics in Algebraic Geometry: Arithmetic algebraic geometry (continuation)

\*A continuation from Fall 2012\* We will discuss equidistribution questions about L functions, or rather "families" of L functions, in various finite field contexts.

MAT549 Topics in Algebra: Moduli of varieties of general type

**MAT550 Differential Geometry** 

MAT555 Topics in Differential Geometry: Kahler-Einstein Metrics

MAT558 Topics in Conformal and Cauchy-Riemann Geometry

MAT559 MAT559 Topics in Geometry: Conformally covariant operators and their associated Q-curvatures

# **MAT560 Algebraic Topology**

Singular homology, cellular complexes, Poincare duality, Lefschetz fixed point theorem. Prerequisite: MAT323 in the old numbering system or MAT345 in the new system; in other words, abstract algebra. (Replaces MAT326 beginning AY 2012-13)

MAT566 Topics in Differential Topology: Topical Invariants for Knots and Three-Dimensional Manifolds

MAT567 Topics in Low Dimensional Topology: Symplectic techniques in low-dimensional topology

#### MAT568 Topics in Knot Theory: Knot Floer homology

Knot theory involves the study of smoothly embedded circles in three-dimensional manifolds. There are lots of different techniques to study knots: combinatorial invariants, algebraic topology, hyperbolic geometry, Khovanov homology and gauge theory. This course will cover some of the modern techniques and recent developments in the field.

MAT569 Topics in Topology: Classical high dimensional manifold theory

# **MAT572 Introduction to Combinatorial Optimization**

This course will survey the theory of combinatorial optimization. We will cover: -The elementary min-max theorems of graph theory, such as Konig's theorems and Tutte's matching theorem -Network flows (Menger's theorem, the max-flow min-cut theorem, multicommodity flows) -Linear programming

and polyhedral optimization -Hypergraph packing and covering problems -Perfect graphs -Polyhedral methods to prove min-max theorems -Packing directed cuts, the Lucchesi-Younger theorem -Packing T-cuts, T-joins and circuits -Edmonds' matching polytope theorem -Relations with the four-colour theorem -Lehman's results on ideal clutters -Various further topics (the ellipsoid method, the matching lattice, the theta-function) as time permits.

#### MAT576 Advanced Topics in Computer Science: Arithmetic circuits

One the fundamental goals in computational complexity is to understand which functions are easy and which are hard to compute. This course will focus on this question from an algebraic perspective by studying the complexity of computing polynomials over a field using basic arithmetic operations. The basic model of computation is an arithmetic circuit, which is a circuit that takes variables and field constants as inputs and, at each gate, computes a sum or product of previously computed polynomials. Understanding the complexity of even the simplest polynomials in this model is a challenging (and mostly open) problem. However, a rich theory has evolved around this question and there are many beautiful and highly non trivial results known. A partial list of topics includes a sample of some nontrivial arithmetic circuits for problems such as matrix multiplication, Fourier transform, polynomial evaluation etc. Structural results for circuits (homogenization, depth reduction, elimination of division gates). Lower bounds for circuits, formulas and for restricted models. Valiant's complexity classes VP and VNP and the algebraic version of the P vs. NP problem. Approaches for proving lower bounds (matrix rigidity, elusive mappings, tensor rank). Polynomial identity testing.

## MAT579 Topics in Discrete Mathematics: Coloring and induced subgraphs

There are several well-known open questions about the coloring properties of graphs with certain induced subgraphs forbidden, for instance: 1. Erdos-Hajnal conjecture - for every graph H, there exists c>0 such that every n-vertex graph not containing H as an induced subgraph has a clique or stable set with at least O(n^c) vertices. 2. Gjarfas conjecture - for every integer k>0, there exists c>0 such that every graph with no clique of size k and no odd cycle of length >3 as an induced subgraph has chromatic number at most c. 3. Gyarfas-Sumner conjecture - for every integer k>0 and every forest F, there exists c such that every graph with no clique of size k and not cointaining F as an induced subgraph has chromatic number at most c. The aim of this course is to survey what is known about these conjectures, and discuss some analogous results and conjectures for tournaments.

#### **MAT582 Dynamical Systems**

This course will concentrate on chaos in dynamical systems. We will discuss Anosov (globally hyperbolic) diffeomorphisms and flows. I intend to prove their topological stability and then discuss their stable and unstable foliations. Anosov flows and diffeomorphisms are purely chaotic dynamical systems. Ergodic theory provides a means to study chaos, and I intend to discuss the beginning of this study as it pertains to Anosov systems—Markov partitions and Gibbs measures. Time permitting, I will discuss a bit about chaos in systems where random and stable motions exist side-by-side.

## MAT585/APC520 Mathematical Analysis of Massive Data Sets

This course focuses on spectral methods useful in the analysis of big data sets. Spectral methods involve the construction of matrices (or linear operators) directly from the data and the computation of a few leading eigenvectors and eigenvalues for information extraction. Examples include the singular value decomposition and the closely related principal component analysis; the PageRank algorithm of Google for ranking web sites; and spectral clustering methods that use eigenvectors of the graph Laplacian.

## MAT596 Mathematical Methods in Physics: Intro to the Calculus of Variations and Spectral Theory

The credo 'every effect in nature follows a maximum or minimum rule' is what Euler wrote in 1744 and which since then has driven the development of the mathematical field of Calculus of Variations. We will give an introduction into this area focusing both on problems from pure mathematics and from applications in atomic physics. Symmetries will play a major role in this analysis. Topics include the isoperimetric inequality, the concentration compactness principle, rearrangements and the moving plane method, Sobolev and Lieb-Thirring inequalities and the semi-classical limit of the Schrödinger equation.

# MATxxx Geometric Measure Theory: Regularity theory for area-minimizing currents

After a brief introduction to the theory of integral currents, I will focus on the regularity theory for area-minimizing ones. The aim is to prove the partial regularity theorem of Almgren, which shows that such objects are regular submanifolds except for a closed singular set of codimension at most two. In the course, I will follow my recent work with Emanuele Spadaro where we give a simpler approach to Almgren's result.

**SEMINAR Ergodic Theory and Statistical Mechanics**