# Delegation with Unawareness*

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**Abstract**

We incorporate unawareness into the delegation problem between a financial expert and an investor, and study their pre-delegation communication. The expert has superior awareness of the possible states of the world, and decides whether to reveal some of them to the investor. We find that the expert reveals all the possible states to the investor if the investor is initially aware of a large set of possible states, but only reveals partially or nothing otherwise. An investor of higher degree of unawareness tends to delegate a larger set of projects to the expert, giving rise to a higher incentive for the expert to keep her unaware.

*Keywords:* Delegation, Unawareness, Financial Advice
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# 1 Introduction

Financial advisors play important roles for retail investors. Experienced retail investors may have the ability to choose self-directed investments. Nevertheless, most retail investors need professional financial advice to make investment decisions. In America, the revenue of financial planning and advice industry keeps growing over the past five years and is estimated at $55 billion in 2018.[1] In the United Kingdom, it is found that people with low wealth would pay on average £258 for advice on investing an inheritance of £60,000.[2]

Demand for financial advice is strong as most retail investors are unfamiliar with, or even *unaware* of the potential risk of the available investment choices. However, conflicts of interest between retail investors and financial advisors might make services of the latter curses for the investors. Despite its prevalence, financial advice industry is often perceived as a dishonest and least trustworthy profession.[3] Both theoretical and empirical literature find that the combination of information asymmetry and conflicts of interest can turn financial consultancy to detriments of investors.

This paper focuses on the investors with limited understanding of factors determining the returns of available investment choices and attempts to study how the degree of an investor's financial illiteracy affects the quality of an expert's financial service. Specifically, when can experts benefit from withholding information regarding potential risks from the clients? How will the conflicts of interest and the investor's limited knowledge jointly determine the quality of the expert's financial advice, and the investor's reliance on the expert's discretion for making his financial decisions?

To answer these questions, we incorporate the concept of unawareness to capture an investor's limited understanding of the economic and financial environment, and investigate how the investor's degree of unawareness affects the quality of the expert's financial services. We model the investor-expert interaction in the framework of a

[1] https://www.ibisworld.com/industry-trends/market-research-reports/finance-insurance/securities-commodity-contracts-other-financial-investments-related-activities/financial-planning-advice.html

[2] See page 21 of the final report of Financial Advice Market Review in 2016. (https://www.fca.org.uk/publication/corporate/famr-final-report.pdf)

[3] See Egan et al. (2017) for an exposition on this issue.

delegation problem following an information-revelation stage. Prior to delegation, the expert might be willing to reveal eye-opening information to the investor to induce a more favorable delegation set of investment decisions. Both the investor's and the expert's payoffs depend on the implemented investment decision and the realized state of the world.

In the presence of an investor who is fully aware of the whole state space, standard delegation theory shows that the optimal delegation set for the expert is an interval, under some regularity conditions on the distribution of states.[4] In the presence of an unaware investor, several interesting findings emerge. First, in terms of the revelation of possible states contained in the expert's advice, the expert tends to reveal less eye-opening information to the investor as the investor's degree of unawareness increases. Second, full revelation, partial revelation and no revelation all may appear depending on the states of which the investor is initially aware . Third, as for the delegation outcome, an investor of a higher degree of unawareness tends to choose a larger delegation set.

The following of the paper is organized as below. Section 2 reviews the related literature. Section 3 introduces the setting of the model. Section 4 analyzes the model and summarizes our key results. The last section concludes. All proofs are relegated in the appendix.

# 2 Literature

This paper contributes to a growing literature on contract theory and unawareness. This literature can be roughly divided into two major streams: players are unaware of possible *actions* (von Thadden and Zhao, 2012; Von Thadden and Zhao, 2014; Auster and Pavoni, 2018) and players are unaware of possible *states* (Zhao, 2011; Filiz-Ozbay, 2012; Auster, 2013; Ma and Schipper, 2018). Our work is mostly related to Auster and Pavoni (2018) in which they consider a similar delegation framework with the principal unaware of possible actions. In contrast, in our paper, the principal is unaware of possible states.

[4] See Martimort and Semenov (2006).

This paper is also related to the literature on optimal delegation starting from Holmstrom (1977, 1982). Alonso and Matouschek (2008) characterize the optimal delegation set when the feasible delegation sets are compact and the players' preferences take a generalized quadratic form. Kováč and Mylovanov (2009) show that the optimal mechanism can be stochastic in a quadratic preferences setting, and provide a sufficient condition for the optimal mechanism to be deterministic. Variants of the delegation model have been used in political economy (Bendor et al., 2001; Krishna and Morgan, 2001), organization and regulation (Aghion and Tirole, 1997; Baron and Myerson, 1982), and trade (Amador and Bagwell, 2013), while the application in financial advice to unaware investors is still in its infancy.

Lastly, this paper is related to the literature on financial advice. These works, including Loewenstein et al. (2011), Inderst and Ottaviani (2012) and Lusardi and Mitchell (2014) among others, focus on how conflicts of interest and investors' limited financial knowledge lead to welfare loss and the necessity of ex-post regulation. One great value of this research stream is that it provides guidance for public policy to deal with financial illiteracy. For example, Gui et al. (2018) conduct experiments and surveys to investigate the financial fraud problem and financial awareness of Chinese residents, and find that financial education targeted at risk-averse investors is more effective in preventing financial frauds.

# 3 Model

An investor (she) seeks financial advice from an expert (he) and then delegates the investment choice to him. Factors that affect the return of available investment options are summarized by a one-dimensional random variable $\theta$. We refer to $\theta$ as a state of the world. The state space $\Theta$ is assumed to be $[0, 1]$. The set of available investment options is $Y = [\,\underline{y}, \overline{y}\,]$.

The investor cannot observe the state of the world. Moreover, she is only aware of a subset, $[\theta_1, \theta_2]$, of the whole state space. We refer to $[\theta_1, \theta_2]$ as the investor's *awareness set*. In contrast, the expert is aware of the whole state space. Before delegation, the expert is able to strategically expand the investor's awareness set

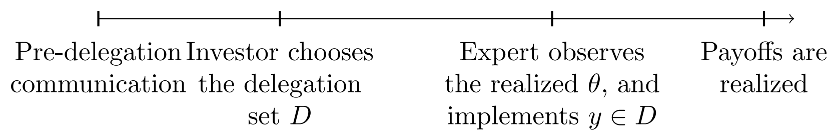


Figure 1: Timeline

(the *revelation phase*). Then the investor delegates a set of investment decisions to the expert (the *delegation phase*). After that, the expert privately observes the realized state of the world and implements his most desirable investment decision in the delegation set (the *investment phase*). Finally, the players receive the payoffs depending on the realized state and the chosen investment decision. More formally, the timing of the model is described as follows, and is also depicted in Figure 1.

1. Revelation Phase: the expert strategically expands the investor's awareness set from $[\theta_1, \theta_2]$ to $[\underline{\theta}, \overline{\theta}]$ with $[\theta_1, \theta_2] \subseteq [\underline{\theta}, \overline{\theta}] \subseteq [0, 1]$.

2. Delegation Phase: Given her interim awareness set $[\underline{\theta}, \overline{\theta}]$, the investor chooses a compact delegation set $D \subseteq Y$.

3. Investment Phase: The expert privately observes the realized state $\theta$ and then chooses some investment option $y$ from the delegation set.

The investor's utility function $u^I$ and the expert's utility function $u^E$ are given by

$$u^I(y, \theta) = -\frac{1}{2}(y - \theta - b)^2, \tag{1}$$

$$u^E(y, \theta) = -\frac{1}{2}(y - \theta)^2. \tag{2}$$

Assume $b > 0$, representing the conflicting interests between the expert and the investor. Given a realized state $\theta$, the expert's most preferred option is $y^E(\theta) = \theta$ while the investor's most preferred option is $y^I(\theta) = \theta + b$. We interpret a higher value of $\theta$ as a riskier economic environment, and a higher level of $y$ as a more defensive investment strategy involving a conservative plan of portfolio allocation aimed at minimizing the risk of losing principal. Both the investor and the expert prefer a more defensive investment decision in a riskier state. Moreover, the optimal investment option of the investor is more defensive than that of the expert in that

$y^I(\theta) - y^E(\theta) = b > 0$ for all $\theta$. The parameter $b$ thus captures the difference between the investor's and the expert's tolerance of risk.

As in the standard delegation model, we rule out contingent monetary transfers. Our model departs from the earlier works in that the investor is unaware of some possible states and that the expert can make the investor aware of additional states by, for example, providing a professional report on the possibility of an economic boom driven by technological change in high-tech industries such as artificial intelligence.

We impose the following assumptions on the players' difference in the attitudes toward risks, the available investment decisions and the players' beliefs throughout the paper unless otherwise specified.

A1. $b < 1/4$.

A2. $\underline{y} < -\frac{1}{2} - b$ and $\bar{y} > 1 + b$.

A3. The expert's belief distribution is the uniform distribution on $\Theta$, and the investor's belief distribution is the uniform distribution on her awareness set $[\underline{\theta}, \overline{\theta}\,]$.

A4. The expert can observe the investor's awareness set, while the investor perceives the support of the expert's distribution same as her awareness set.

For the benchmark case in which the investor's awareness set is $\Theta$, Assumption A1 guarantees that delegation is valuable for the investor. As in the standard delegation model, we assume a relatively larger action set in Assumption A2 to avoid tedious calculations. Assumptions A3 deals with each player's belief distribution of the state. If the expert's belief of the state is correctly specified, then in Assumption A3 our restriction regarding the investor's evolution of beliefs with growing awareness is aligned with the "reverse Bayesianism" as in Karni and Vierø (2013, 2015). Assumption A4 deals with each player's belief about the other's distribution support. It is assumed that the expert can perfectly identify the support of the investor's belief distribution, while the investor is unaware of her unawareness.[5] Another implicit assumption is

[5]Relatedly, Galperti (2018) studies a persuasion model in which the support of the agent's belief distribution is larger than that of the principal's belief distribution. However, in contrast to our model, the two players agree to disagree. His setting applies to people's opinions on "controversial"

that the expert has superior awareness and thus can choose the investor's new awareness set arbitrarily as long as the enlarged set is an interval contained in the state space. Dubious as it may seem, this assumption is consistent with certain empirical findings: retail investors with limited financial literacy tend to be more dependent on the experts' advice in making investments. Hackethal et al. (2010) use data from a large bank and demonstrate that less-educated investors were more likely to report relying on the advisor's investment advice. Georgarakos and Inderst (2014) find that financial advice significantly affects the likelihood that less-educated households will hold risky assets.

## 3.1 Solution Concept

The widely-used concept of Nash equilibrium does not apply in the presence of an unaware player[6]. Halpern and Rêgo (2014) define a general solution concept for extensive games with unaware players. Intuitively, it requires that each agent chooses their best move given their local belief of the whole game at their respective node. Our solution concept follows the generalized Nash equilibrium proposed by Halpern and Rêgo (2014) with the modification that we solve for the generalized Nash equilibrium (and henceforth, *equilibrium*) by backward induction to eliminate the inappropriate solutions.

Generally, the investor's optimal delegation set and the expert's revelation choice need not be unique. Following Auster and Pavoni (2018), for equilibrium selection, we focus on the expert's choice of the largest awareness set and the investor's choice of the largest delegation set. Specifically, we assume that (i) if the expert is indifferent between two enlarged awareness sets $\Theta_1$ and $\Theta_2$ with $\Theta_1 \subseteq \Theta_2$, he will choose $\Theta_2$ in the revelation phase and (ii) if the investor is indifferent between two delegation sets $D_1$ and $D_2$ with $D_1 \subseteq D_2$, she will choose $D_2$ in the delegation phase. In the experimental studies of delegation, Falk and Kosfeld (2006) show that the principal's control demotivates the agent's productive activity. Similarly, Charness et al. (2012)

topics such as political issues. Our assumption of an unaware principal fits more naturally in the study of professional consultancy such as financial advice in which the expert is usually more knowledgeable.

[6]See, for example, Section 5 in Ozbay (2007).

suggest that given more authority, agents perform better due to the nonstrategic motivation caused by a sense of enhanced responsibility. Their findings partly support our equilibrium selection here.

Let $\mathcal{C}(\Theta)$ be the set of all closed intervals in $\Theta$ and $T(Y)$ the set of all compact subsets of $Y$. Let the expert's revelation strategy be $\sigma : \mathcal{C}(\Theta) \to \mathcal{C}(\Theta)$, the investor's delegation choice $D^* : \mathcal{C}(\Theta) \to T(Y)$ and the expert's investment choice $y^* : T(Y) \times \Theta \to Y$. We call $(\sigma, D^*, y^*)$ a *generalized strategy profile* as in Halpern and Rêgo (2014).

**Definition 1.** A generalized strategy profile $(\sigma, D^*, y^*)$ is an *equilibrium* if

1. The expert chooses the investment option that maximizes his utility given the delegation set $D$ and the state of the world $\theta$:

$$y^*(D, \theta) \in \arg\max_{y \in D} u^E(y, \theta) \tag{3}$$

2. Given the expert's investment strategy $y^*$ and the investor's interim awareness set $\hat{\Theta}$, the investor chooses the delegation set that maximizes her interim expected utility:

$$D^*(\hat{\Theta}) \in \arg\max_{D \in T(Y)} \mathbb{E} u^I \big( y^*(D, \theta), \theta \,\big|\, \theta \in \hat{\Theta} \big) \tag{4}$$

   Specifically, if the investor is indifferent between $D_1^*(\hat{\Theta})$ and $D_2^*(\hat{\Theta})$ and, for all feasible $\hat{\Theta}$, $D_1^*(\hat{\Theta}) \subseteq D_2^*(\hat{\Theta})$, she will choose $D_2^*(\hat{\Theta})$.

3. Given the expert's investment strategy $y^*$, the investor's delegation strategy $D^*$ and the investor's initial awareness set $\Theta_0$, the expert chooses the revelation set $\sigma(\Theta_0)$ to maximize his expected utility:

$$\sigma(\Theta_0) \in \arg\max_{\hat{\Theta} \in \mathcal{C}(\Theta)} \mathbb{E} u^E \big[ y^* \big( D^*(\hat{\Theta}), \theta \big), \theta \,\big|\, \theta \in \Theta \big] \tag{5}$$

   such that (i) $\Theta_0 \subseteq \sigma(\Theta_0) \subseteq \Theta$ and (ii) $\sigma(\Theta_0) \in \mathcal{C}(\Theta)$. Specifically, if the expert is indifferent between $\sigma(\Theta_0)$ and $\sigma'(\Theta_0)$ and, for all feasible $\Theta_0$, $\sigma(\Theta_0) \subseteq \sigma'(\Theta_0)$, he will choose $\sigma'(\Theta_0)$.

# 4 Analysis

Given our solution concept, we solve the model backwards.

## 4.1 Investment choice

The expert's optimal investment strategy is given by

$$y^*(D,\theta) \in \arg\min_{y\in D} |y-\theta| \,. \tag{6}$$

The choice function $y^*$ in Equation 6 is well-defined due to the compactness of $D$.

## 4.2 Delegation choice

By Assumptions A3, the investor's belief distribution is the uniform distribution on her interim awareness set $[\underline{\theta}, \overline{\theta}]$. Intuitively, the investor would delegate those options above some threshold to the expert because $b > 0$. In the uniform-quadratic setting, the threshold is $\underline{\theta} + \min\left\{2b, \frac{\overline{\theta}-\underline{\theta}}{2} + b\right\}$. Moreover, the investor would also delegate those options close to $\underline{y}$ because she is unaware of states in $[0, \underline{\theta}]$.

By standard delegation theory, when $b < \left(\overline{\theta} - \underline{\theta}\right)/2$, the delegation is *valuable* for the investor and the minimal optimal delegation set would be $\left[\underline{\theta} + 2b, \overline{\theta} + b\right]$; when $b \geq (\overline{\theta} - \underline{\theta})/2$, the delegation is not valuable and the delegation set would be a singleton; that is, $\left\{ \frac{\overline{\theta}+\underline{\theta}}{2} + b \right\}$. By adding options that the investor believes would never be chosen by the expert, we get the maximal optimal delegation set in both cases.

**Proposition 1.** *If in the delegation phase the investor's awareness set is $[\underline{\theta}, \overline{\theta}]$, then the investor's optimal delegation strategy is:*

$$D^*([\underline{\theta}, \overline{\theta}]) = Y \big\backslash \left(\underline{\theta} - \Delta, \underline{\theta} + \Delta\right), \text{ where } \Delta = \min\left\{2b, \frac{\overline{\theta} - \underline{\theta}}{2} + b\right\}. \tag{7}$$

By Proposition 1, the delegation choice of the investor is characterized by a gap $\Delta$ given her awareness set $\hat{\Theta}(= [\underline{\theta}, \overline{\theta}])$, and the undelegated options take the form of $(\underline{\theta} - \Delta, \underline{\theta} + \Delta)$. The delegation choice depends on both the conflict of interests ($b$) and the investor's awareness set ($\underline{\theta}$ and $\overline{\theta}$). Figure 2 depicts the optimal delegation

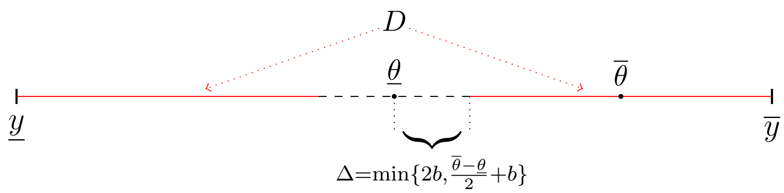


Figure 2: Delegation Choice

choice of an unaware investor with the awareness set being $[\underline{\theta},\overline{\theta}]$. An investor of a higher degree of unawareness (or equivalently, a smaller awareness set) tends to delegate more in equilibrium.

It is worth emphasizing that the investor is willing to delegate options below $\underline{\theta}-\Delta$ because she is unaware of the states $[0,\underline{\theta}]$ and then from her perspective the expert would never implement those lower options. In other words, the investor is naively too pessimistic about the economic environment and finds no reason for the expert to choose these extremely aggressive investment options. Thus, delegating these aggressive options to the expert involves no loss from the investor's perspective.

### 4.3 Revelation choice

To solve for the expert's revelation choice in equilibrium, we firstly focus on the simplest case that the expert could manipulate the investor's awareness set arbitrarily; that is, we ignore the constraint that $\sigma(\Theta_0) \supseteq \Theta_0$ at this moment but still require the awareness set be a closed interval or a singleton in $\Theta$. Note that from Proposition 1, the expert always prefers $(\overline{\theta}-\underline{\theta})/2 \leq b$ if possible.

**Lemma 2.** *If the expert could choose any closed interval or singleton in $\Theta$ as the investor's awareness set, then the expert would make the investor only aware of either of the two extreme states; that is, $\hat{\Theta}=\{0\}$ or $\hat{\Theta}=\{1\}$.*

We illustrate the proof of Lemma 2 in Figure 3. By Proposition 1, the gap $\Delta$ is minimized if and only if $\underline{\theta}=\overline{\theta}$. Therefore, the expert would make the investor only aware of a singleton. Compare the two revelation choices of $\theta=0$ and $\theta' \in (0,1)$. Their corresponding undelegated projects are represented by the two dashed segments

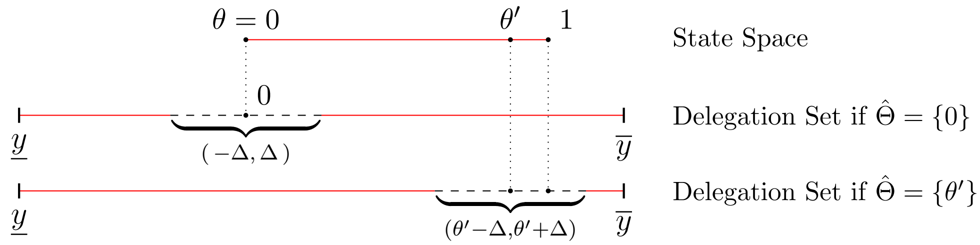


Figure 3: An Illutration of Lemma 2

respectively. To maximize his expected utility, the expert wants to induce as many projects in $[0, 1]$ to be delegated as possible. If the investor is aware of either of the extreme states, say $\theta = 0$, the measure of the undelegated projects in $[0, 1]$ would be only $\Delta$. By contrast, if the investor is aware of some $\theta'$ between the two extreme states, the measure of the undelegated projects in $[0, 1]$ will be greater than $\Delta$. Thus the expert prefers to reveal $\theta = 0$ to induce a more favorable delegation set. Therefore, the expert prefers to interact with either the most optimistic investor with $\theta = 0$ believing that the economic environment is certainly the safest, or the most pessimistic investor with $\theta = 1$.

Now we turn to the expert's revelation strategy with the constraint that $\sigma(\Theta_0) \supseteq \Theta_0$. A direct corollary of Lemma 2 is that if the investor finds the delegation valuable before revelation, the expert would make the investor aware of the whole state space.

**Proposition 3.** *If the investor's awareness set* $\Theta_0 (= [\theta_1, \theta_2])$ *satisfies* $\theta_2 - \theta_1 \geq 2b$, *then the expert's optimal revelation strategy is full revelation; that is,* $\sigma(\Theta_0) = [0, 1]$.

Here we briefly sketch the proof of Proposition 3. When delegation is valuable for the investor, the delegation gap would be $2b$. Similarly to the argument in Lemma 2, the expert would expand the investor's awareness set to the lower bound $\underline{\theta} = 0$. Note that the expert is indifferent between whether expanding the upper bound of the investor's awareness set or not. Therefore, the expert's revelation strategy would be full revelation.

It is interesting to consider whether the expert is willing to reveal more information when delegation is not valuable. In the case of $\overline{\theta} - \underline{\theta} < 2b$, the set of undelegated projects take the form of $(\underline{\theta} - \Delta^*, \underline{\theta} + \Delta^*)$ where $\Delta^* = \frac{\overline{\theta} - \underline{\theta}}{2} + b$. The expert would

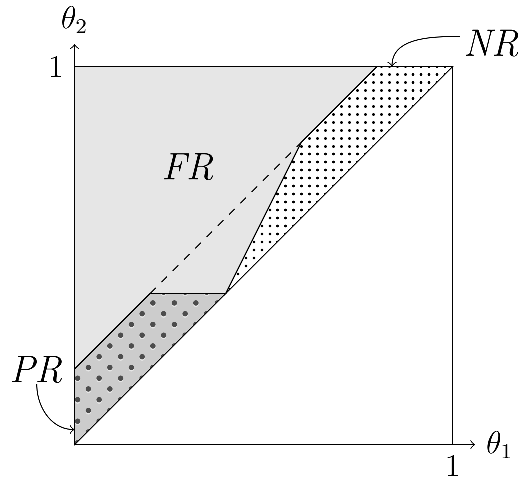


Figure 4: Revelation Choice

not increase the upper bound of the awareness set because it strictly shrinks the delegation set. Nevertheless, the expert faces trade-offs between whether informing the investor of those low states or not. On the one hand, revealing lower states would increase the measure of the undelegated projects in $[0, 1]$, which might harm the expert's welfare. On the other hand, decreasing the lower bound of the awareness set would also decrease the upper bound of the set of undelegated projects. The latter strictly benefits the expert when the lower bound of the undelegated projects ($\underline{\theta} - \Delta^*$) is much lower than 0.

**Proposition 4.** *If the investor's initial awareness set $[\theta_1, \theta_2]$ satisfies $\theta_2 - \theta_1 < 2b$, then the expert's optimal revelation strategy is:*

$$\sigma(\Theta_0) = \begin{cases} [\theta_1, \theta_2] & \text{if } b < \frac{3}{2}\theta_1 - \frac{\theta_2}{2}; \\ [0, \theta_2] & \text{if } b \geq \frac{3}{2}\theta_1 - \frac{\theta_2}{2} \text{ and } \theta_2 < 2b; \\ [0, 1] & \text{otherwise.} \end{cases}$$

By Proposition 4, full revelation, partial revelation and no revelation all may appear in equilibrium. Note that in partial revelation, the expert would reveal all the lower states ($\theta < \theta_1$) while keep the investor unaware of the riskier states ($\theta > \theta_2$).

Combining Proposition 3 and 4, we conclude that when the investor is aware of a large set of possible states ($\theta_2 - \theta_1 \geq 2b$), the expert would choose full revelation;

when the investor is aware of a small set of possible states ($\theta_2 - \theta_1 < 2b$), the expert could choose partial revelation or no revelation at all.

**Example 1.** Let $b = 0.1$. Figure 4 shows the revelation strategy of the expert characterized in Proposition 3 and 4. We depict three revelation outcomes, given different awareness set $\Theta_0(= [\theta_1, \theta_2])$, where $FR$, $NR$, and $PR$ stand for full revelation, no revelation, and partial revelation respectively.[7]

## 4.4 Summary

We have fully characterized the equilibrium results in the analysis above. Next we summarize the possible delegation and revelation outcomes.

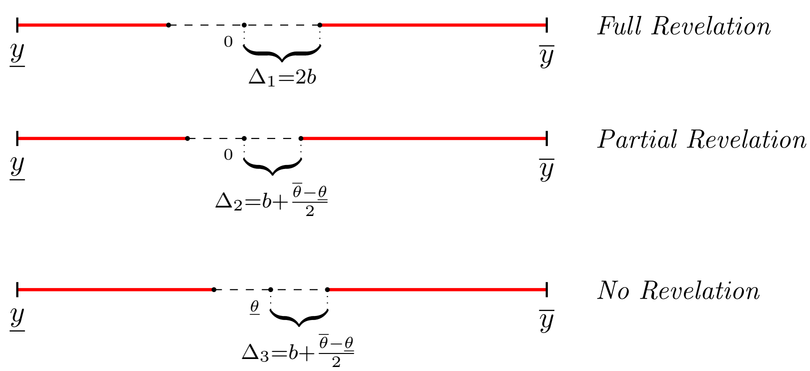


Figure 5: Delegation and Revelation Outcomes

Figure 5 depicts the delegation outcomes and the corresponding revelation choices (full, partial or no revelation). By Proposition 4, an investor of higher degree of unawareness tends to delegate more in equilibrium. Specifically, if the investor has a larger awareness set and the conflicts of interest is relatively small (that is, $b < (\theta_2 - \theta_1)/2$), the expert would choose full revelation and hence the investor's welfare would be the same as in the benchmark in which the investor is aware of the whole state space. In this case, financial advice *benefits* the investor. However, when the investor has a small awareness set and the unforeseen states are lower, the expert would choose

[7]The triangle, which lies in $FR$ and between $PR$ and $NR$ areas, exists due to our focus on the maximal awareness set. If the awareness set lies in the triangle, the expert is indifferent between full revelation and partial revelation.

no revelation and financial consultancy would not benefit the investor. Our result is largely supported in the empirical literature such as Calcagno and Monticone (2015) in that experts disclose their superior information only to the most knowledgeable investors.

# 5 Conclusion

This paper analyzes the delegation problem in which the expert has a superior awareness of the states of the world under the uniform-quadratic setting, and shows to what extent it is not in the expert's interests to voluntarily disclose information that the investor is unaware of, thereby possibly leading to an adverse delegation outcome for the investor.

The financial expert chooses full revelation when the investor is initially highly aware, but no revelation when the investor is sufficiently unaware and does not realize the existence of those lower states involving safer economic environment. Thus, investors with a higher degree of awareness benefit more from the expert's advice. This echoes the existing studies showing that investors with more education and higher financial literacy are more likely to seek additional financial advice (Hackethal et al., 2012; van Rooij et al., 2011).

On the other hand, financial advice from the experts may not be a sufficient instrument to cure the problem of financial illiteracy. In our model, the expert may not provide any information in equilibrium at all, or the expert may only make the investor aware of those safer states to induce the delegation of those more aggressive investments that might be suboptimal for the investor. In the European financial market, many consumers frequently receive advice from agents, but do not understand the potential impact of inducements and other incentives of those experts (Chater et al., 2010). Our analysis illustrates how investors' unawareness can be exploited through the expert's strategic information disclosure, suggesting the demand of policy interventions such as imposing mandatory disclosure in financial advice.

# References


Aghion, P. and Tirole, J. (1997). Formal and Real Authority in Organizations. *Journal of Political Economy*, 105(1):1–29.

Alonso, R. and Matouschek, N. (2008). Optimal Delegation. *Review of Economic Studies*, 75(1):259–293.

Amador, M. and Bagwell, K. (2013). The Theory of Optimal Delegation With an Application to Tariff Caps. *Econometrica*, 81(4):1541–1599.

Auster, S. (2013). Asymmetric Awareness and Moral Hazard. *Games and Economic Behavior*, 82:503–521.

Auster, S. and Pavoni, N. (2018). Optimal Delegation and Limited Awareness. *Working Paper*.

Baron, D. P. and Myerson, R. B. (1982). Regulating a Monopolist with Unknown Costs. *Econometrica*, 50(4):911–930.

Bendor, J., Glazer, A., and Hammond, T. (2001). Theories of Delegation. *Annual Review of Political Science*, 4(1):235–269.

Calcagno, R. and Monticone, C. (2015). Financial Literacy and the Demand for Financial Advice. *Journal of Banking & Finance*, 50:363–380.

Charness, G., Cobo-Reyes, R., Jiménez, N., Lacomba, J. A., and Lagos, F. (2012). The Hidden Advantage of Delegation: Pareto Improvements in a Gift Exchange Game. *American Economic Review*, 102(5):2358–79.

Chater, N., Huck, S., and Inderst, R. (2010). Consumer Decision-making in Retail Investment Services: A Behavioural Economics Perspective. *Report to the European Commission/SANCO*.

Egan, M., Matvos, G., and Seru, A. (2017). The Market for Financial Adviser Misconduct. *Journal of Political Economy*, forthcoming.

Falk, A. and Kosfeld, M. (2006). The Hidden Costs of Control. *American Economic Review*, 96(5):1611–1630.

Filiz-Ozbay, E. (2012). Incorporating Unawareness into Contract Theory. *Games and Economic Behavior*, 76(1):181–194.

Galperti, S. (2018). Persuasion: The Art of Changing Worldviews. *American Economic Review*, forthcoming.

Georgarakos, D. and Inderst, R. (2014). Financial Advice and Stock Market Participation. CEPR Discussion Paper No. DP9922.

Gui, Z., Huang, Y., and Zhao, X. (2018). Financial Fraud and Investor Awareness. *Working Paper*.

Hackethal, A., Haliassos, M., and Jappelli, T. (2012). Financial Advisors: A Case of Babysitters? *Journal of Banking & Finance*, 36(2):509–524.

Hackethal, A., Inderst, R., and Meyer, S. (2010). Trading on advice. CEPR Discussion Paper No. DP8091.

Halpern, J. Y. and Rêgo, L. C. (2014). Extensive Games with Possibly Unaware Players. *Mathematical Social Sciences*, 70:42–58.

Holmstrom, B. (1977). On Incentives and Control in Organizations. Ph.D. thesis, Stanford University.

Holmstrom, B. (1982). *On the Theory of Delegation*. Northwestern University.

Inderst, R. and Ottaviani, M. (2012). How (Not) To Pay For Advice: A Framework For Consumer Financial Protection. *Journal of Financial Economics*, 105(2):393–411.

Karni, E. and Vierø, M.-L. (2013). "Reverse Bayesianism": A Choice-Based Theory of Growing Awareness. *American Economic Review*, 103(7):2790–2810.

Karni, E. and Vierø, M.-L. (2015). Probabilistic Sophistication and Reverse Bayesianism. *Journal of Risk and Uncertainty*, 50(3):189–208.

Kováč, E. and Mylovanov, T. (2009). Stochastic Mechanisms in Settings Without Monetary Transfers: The Regular Case. *Journal of Economic Theory*, 144(4):1373–1395.

Krishna, V. and Morgan, J. (2001). A Model of Expertise. *Quarterly Journal of Economics*, 116(2):747–775.

Loewenstein, G., Cain, D. M., and Sah, S. (2011). The Limits of Transparency: Pitfalls and Potential of Disclosing Conflicts of Interest. *American Economic Review*, 101(3):423–428.

Lusardi, A. and Mitchell, O. S. (2014). The Economic Importance of Financial Literacy: Theory and Evidence. *Journal of Economic Literature*, 52(1):5–44.

Ma, W. and Schipper, B. (2018). Indescribable Contingencies versus Unawareness in Incomplete Contracting. *Working Paper*.

Martimort, D. and Semenov, A. (2006). Continuity in Mechanism Design Without Transfers. *Economics Letters*, 93(2):182–189.

Ozbay, E. Y. (2007). Unawareness and Strategic Announcements in Games with Uncertainty. In *Proceedings of the 11th Conference on Theoretical Aspects of Rationality and Knowledge*, pages 231–238, New York, NY, USA. ACM.

van Rooij, M., Lusardi, A., and Alessie, R. (2011). Financial Literacy and Stock Market Participation. *Journal of Financial Economics*, 101(2):449–472.

von Thadden, E.-L. and Zhao, X. (2012). Incentives for Unaware Agents. *Review of Economic Studies*, 79(3):1151.

Von Thadden, E.-L. and Zhao, X. (2014). Multi-task agency with unawareness. *Theory and Decision*, 77(2):197–222.

Zhao, X. (2011). Framing Contingencies in Contracts. *Mathematical Social Sciences*, 61(1):31–40.

# Appendix. Proofs

In this appendix, we provide proofs of the lemma and propositions.

## A.1: Proof of Proposition 1

When $b < (\overline{\theta} - \underline{\theta})/2$, by Alonso and Matouschek (2008) the delegation is valuable and the (minimal) optimal delegation set would be $[\underline{\theta} + 2b, \overline{\theta} + b]$. When $b > (\overline{\theta} - \underline{\theta})/2$, the investor would choose to delegate only $\{\frac{\overline{\theta}+\underline{\theta}}{2} + b\}$ in the (minimal) optimal delegation set. The upper threshold is

$$y_H = \underline{\theta} + \min\{2b, \frac{\overline{\theta} - \underline{\theta}}{2} + b\}.$$

Besides those projects above the threshold, the investor would also delegate those states below the threshold $y_L$ such that

$$-(y_H - \underline{\theta} - b)^2 = -(y_L - \underline{\theta} - b)^2.$$

The lower cutoff is given by $y_L = \underline{\theta} - \min\{2b, \frac{\overline{\theta}-\underline{\theta}}{2} + b\}$. Hence the investor's optimal delegation choice is: $D^*(\hat{\Theta}) = Y \backslash (\underline{\theta} - \min\{2b, \frac{\overline{\theta}-\underline{\theta}}{2} + b\}, \underline{\theta} + \min\{2b, \frac{\overline{\theta}-\underline{\theta}}{2} + b\})$.

## A.2: Proof of Lemma 2

By Proposition 1, the delegation set is characterized by the gap $\Delta = \min\left\{2b, \frac{\overline{\theta}-\underline{\theta}}{2} + b\right\}$. So the expert would make the investor aware of one singleton to minimize the measure of the undelegated projects; that is, $\underline{\theta} = \overline{\theta}$.

Since the expert's expected utility depends on the interception of the set of undelegated options and his preferred options, $[0, 1]$. Suppose that the expert's revelation choice is $\theta' = 0$. Then the expert's expected utility would be

$$\mathbb{E}_\theta u^E(D', \theta) = -2\int_0^{b/2} x^2 dx + 0 = -b^3/12.$$

Suppose that the revelation choice is $\theta'' \in (0, 1)$. Then the intersection would be $(\theta'' - b, \theta'' + b) \cap [0, 1]$. When $b \leq \theta'' \leq 1 - b$, the expert's expected utility would be

$$\mathbb{E}_\theta u^E(D'', \theta) = -2\int_0^b x^2 dx + 0 = -2b^3/3 < -b^3/12.$$

When $\theta'' < b$, the expert's expected utility would be

$$\mathbb{E}_\theta u^E(D'', \theta) = -2\int_0^{\frac{\theta''+b}{2}} x^2 dx + 0 = -(\theta''+b)^3/12 < -b^3/12.$$

The same case holds for $\theta'' > 1-b$. Therefore, the expert's optimal revelation choice would be $\hat{\Theta} = \{0\}$ or symmetrically $\hat{\Theta} = \{1\}$.

## A.3: Proof of Proposition 3

If $\theta_2 - \theta_1 \geq 2b$, the gap $\Delta^* = \min\left\{2b, \frac{\overline{\theta}-\underline{\theta}}{2} + b\right\} = 2b$ as $\frac{\overline{\theta}-\underline{\theta}}{2} \geq \frac{\theta_2-\theta_1}{2} = b$. Let the new awareness set be $[\underline{\theta}, \overline{\theta}]$. The set of undelegated options is $(\underline{\theta} - 2b, \underline{\theta} + 2b)$.

First, the expert would choose $\overline{\theta} = 1$ as we focus on the maximal awareness set. Second, a lower $\underline{\theta}$ benefits the expert as more of his preferred options are delegated. The optimal choice is hence $\underline{\theta} = 0$.

## A.4: Proof of Proposition 4

Let the awareness set before revelation be $[\theta_1, \theta_2]$. The expert expands the awareness set to $\hat{\Theta} = [\underline{\theta}, \overline{\theta}]$ with $L \leq \theta_1 \leq \theta_2 \leq \overline{\theta}$. Denote the expert's choice by $[\underline{\theta}^*, \overline{\theta}^*]$.

By Proposition 3, if in the solution $\overline{\theta}^* - \underline{\theta}^* \geq 2b$ then $[\underline{\theta}^*, \overline{\theta}^*] = [0,1]$. Assume that $\overline{\theta} - \underline{\theta} < 2b$. Then the set of undelegated options take the form of $(\underline{\theta} - \Delta^*, \underline{\theta} + \Delta^*)$ where $\Delta^* = \frac{\overline{\theta}-\underline{\theta}}{2} + b$. Clearly, the set of undelegated options expands when $\overline{\theta}$ increases. Therefore, in the solution $\overline{\theta}^* = \theta_2$ given that the expert does not choose full revelation.

On the other hand, decreasing $\underline{\theta}$ by one unit, the agent decreases the lower bound and upper bound of the set $(\underline{\theta} - \Delta^*, \underline{\theta} + \Delta^*)$ by 3/2 and 1/2 units respectively. As result, lowering $\underline{\theta}$ strictly benefits the expert when $\underline{\theta}$ is close to 0, and he would choose $\underline{\theta}^* = 0$ in that situation. When revealing more information is beneficial to the expert, he would choose $\hat{\Theta} = [0, \theta_2]$.

We identify the conditions under which the expert is willing to reveal extra information by comparing his expected utility in the two cases: $[\underline{\theta}, \overline{\theta}] = [0, \theta_2]$ and $[\underline{\theta}, \overline{\theta}] = [\theta_1, \theta_2]$. Denote by $\ell$ the length of $(\underline{\theta} - \Delta^*, \underline{\theta} + \Delta^*) \cap [0,1]$. Due to the symmetrical form of the utility function, the expert is better off if and only if $\ell$ is smaller.

1. Suppose $\theta_1 > \frac{\theta_2-\theta_1}{2}+b$. Without information revelation, $\ell = \theta_2 - \theta_1 + 2b$. With partial revelation, $\ell = \theta_2/2 + b$. The expert would reveal no information if and only if $\theta_2/2 + b > \theta_2 - \theta_1 + 2b \Leftrightarrow \theta_1 + \theta_2/2 < 3b$.

2. Suppose $\theta_1 < \frac{\theta_2-\theta_1}{2}+b$. Without information revelation, $\ell = \theta_1 + \frac{\theta_2-\theta_1}{2} + b > \frac{\theta_2}{2} + b$. The expert is always willing to reveal more information in this case.

Last, note that the expert would choose full revelation ($\hat{\Theta} = [0,1]$) instead of partial revelation ($\hat{\Theta} = [0,\theta_2]$) if $\theta_2 \geq 2b$. To sum up, if $\theta_2 - \theta_1 < 2b$, the expert's optimal revelation strategy is:

$$\sigma(\Theta_0) = \begin{cases} [\theta_1, \theta_2] & \text{if } b < \frac{3}{2}\theta_1 - \frac{\theta_2}{2}; \\ [0, \theta_2] & \text{if } b \geq \frac{3}{2}\theta_1 - \frac{\theta_2}{2} \text{ and } \theta_2 < 2b; \\ [0,1] & \text{otherwise.} \end{cases}$$