ADA Mini hw 4

Consider the following variant of knapsack problem:

Given n items, where the i-th item has value v_i and weighs w_i respectively. Additionally, the i-th item has c_i copies, i.e. there is a limited number of supply for each item compared to UKP.

Please give an $O(nW \log C)$ algorithm to find the max value with W capacity, where $C = \max_{1 \leq i \leq n} c_i$.
Conciseness makes life easier. Just **briefly** explain your algorithm and why it meets the time complexity requirement, and you will get full credit. Make your answer clear and concise.
Hint: reduce the problem to 0-1 knapsack.

Solution

Keyword: 二進位拆分、binary representation

We know that each item has c_i copies. Observe that for any positive integer m, we have

$$\{0, 1, \ldots, 2^m - 1\} = \{a_0 + a_1 2^1 + \ldots + a_{m-1} 2^{m-1} \mid a_i \in \{0, 1\}\}$$

Take the largest $k \in \mathbb{N}$ such that $2^{k+1} - 1 \leq c_i$. We can split each c_i into $1, 2, 4, \ldots, 2^k$ (binary part) and $c_i - 2^{k+1} + 1$ (the remainder), and regard those as new items. Obviously, the number of new items has a upper bound $O(n \log C)$.

After computing the corresponding values and weights of the new items, we reduced the problem to 0-1 knapsack. It follows that the time complexity is $O(nW \log C)$.

Note

Some of you get full credit in this problem, but there are some slight errors in your solution. Checking the details may help you in midterm.