We are going to stick to 2D, as we hadn't came up with any why to do 3D without making it hard for player to play the game from the perspective of a computer, instead of a human. We want player to be able to access all nesessary information easily, without having to rely on good control.

The game is played on the surface of a sphere with radius 1. The coord stored is stored as a 3d vector, with norm 1.

normal distribution:

$$\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{v^2}{2\sigma^2}}$$

2d:

$$(2\pi\sigma^2)^{-1} \cdot 2\pi v \cdot e^{-\frac{v^2}{2\sigma^2}}$$

3d:

$$(2\pi\sigma^2)^{-\frac{3}{2}} \cdot 4\pi v^2 \cdot e^{-\frac{v^2}{2\sigma^2}}$$

Expected $|v|$:

1d:

$$\sigma\sqrt{\frac{2}{\pi}}$$

2d:

$$\sigma\sqrt{\frac{\pi}{2}}$$

3d:

$$\sigma\sqrt{\frac{8}{\pi}}$$

Expected v^2:

1d:

$$\sigma$$

2d:

$$2\sigma$$

3d:

$$3\sigma$$

$$p = a + b\mathbf{i} + c\mathbf{j} + d\mathbf{k}$$
$$(a + b\mathbf{i} + c\mathbf{j} + d\mathbf{k}) * (a - b\mathbf{i} - c\mathbf{j} - d\mathbf{k}) = a^2 + b^2 + c^2 + d^2$$

$$\begin{aligned}
p^{-1} &= (a + b\mathbf{i} + c\mathbf{j} + d\mathbf{k})^{-1} \\
&= \frac{a - b\mathbf{i} - c\mathbf{j} - d\mathbf{k}}{a^2 + b^2 + c^2 + d^2}
\end{aligned}$$

$$\begin{aligned}
pqp^-1 =& \frac{(a_1 + b_1\mathbf{i} + c_1\mathbf{j} + d_1\mathbf{k}) \cdot (a_2 + b_2\mathbf{i} + c_2\mathbf{j} + d_2\mathbf{k})}{a_1^2 + b_1^2 + c_1^2 + d_1^2} \\
&\cdot \frac{(a_1 - b_1\mathbf{i} - c_1\mathbf{j} - d_1\mathbf{k})}{a_1^2 + b_1^2 + c_1^2 + d_1^2} \\
=& (\frac{(a_1a_2 - b_1b_2 - c_1c_2 - d_1d_2) + (a_1b_2 + b_1a_2 + c_1d_2 - d_1c_2)\mathbf{i}}{a_1^2 + b_1^2 + c_1^2 + d_1^2} \\
&+ \frac{(a_1c_2 - b_1d_2 + c_1a_2 + d_1c_2)\mathbf{j} + (a_1d_2 + b_1c_2 - c_1b_2 + d_1a_2)\mathbf{k}}{a_1^2 + b_1^2 + c_1^2 + d_1^2}) \\
&\cdot \frac{(a_1 - b_1\mathbf{i} - c_1\mathbf{j} - d_1\mathbf{k})}{a_1^2 + b_1^2 + c_1^2 + d_1^2} \\
=& \frac{1}{a_1^2 + b_1^2 + c_1^2 + d_1^2} \cdot ((a_1^2a_2 - a_1b_1b_2 - a_1c_1c_2 - a_1d_1d_2 + a_1b_1b_2 + b_1^2a_2 + b_1c_1d_2 \\
&- b_1d_1c_2 + a_1c_1c_2 - b_1c_1d_2 + c_1^2a_2 + c_1d_1b_2 + a_1d_1d_2 + b_1d_1c_2 - c_1d_1b_2 + d_1^2a_2) \\
&+ (-a_1b_1a_2 + b_1^2b_2 + b_1c_1c_2 + b_1d_1d_2 + a_1^2b_2 + a_1b_1a_2 + a_1c_1d_2 - a_1d_1c_2 \\
&- a_1d_1c_2 + b_1d_1d_2 - c_1d_1a_2 - d_1^2b_2 + a_1c_1d_2 + b_1c_1c_2 - c_1^2b_2 + c_1d_1a_2) \\
&+ (-a_1c_1a_2 + b_1c_1b_2 + c_1^2c_2 + c_1d_1d_2 + a_1d_1b_2 + b_1d_1a_2 + c_1d_1d_2 - d_1^2c_2 \\
&+ a_1^2c_2 - a_1b_1d_2 + a_1c_1a_2 + a_1d_1b_2 - a_1b_1d_2 - b_1^2c_2 + b_1c_1b_2 - b_1d_1a_2) \\
&+ (-a_1d_1a_2 + b_1d_1b_2 + c_1d_1c_2 + d_1^2d_2 - a_1c_1b_2 - b_1c_1a_2 - c_1^2d_2 + c_1d_1c_2 \\
&+ a_1b_1c_2 - b_1^2d_2 + b_1c_1a_2 + b_1d_1b_2 + a_1^2d_2 + a_1b_1c_2 - a_1c_1b_2 + a_1d_1a_2)) \\
=& \frac{1}{a_1^2 + b_1^2 + c_1^2 + d_1^2} \cdot ((a_1^2a_2 + b_1^2a_2 + c_1^2a_2 + d_1^2a_2) \\
&+ (b_1^2b_2 + 2b_1c_1c_2 + 2b_1d_1d_2 + a_1^2b_2 + 2a_1c_1d_2 - 2a_1d_1c_2 - d_1^2b_2 - c_1^2b_2) \\
&+ (2b_1c_1b_2 + c_1^2c_2 + 2c_1d_1d_2 + 2a_1d_1b_2 - d_1^2c_2 + a_1^2c_2 - 2a_1b_1d_2 - b_1^2c_2) \\
&+ (2b_1d_1b_2 + 2c_1d_1c_2 + d_1^2d_2 - 2a_1c_1b_2 - c_1^2d_2 + 2a_1b_1c_2 - b_1^2d_2 + a_1^2d_2)) \\
=& \frac{1}{a_1^2 + b_1^2 + c_1^2 + d_1^2} \cdot ((a_1^2a_2 + b_1^2a_2 + c_1^2a_2 + d_1^2a_2) \\
&+ ((a_1^2 + b_1^2 - c_1^2 - d_1^2)b_2 + 2b_1c_1c_2 - 2a_1d_1c_2 + 2b_1d_1d_2 + 2a_1c_1d_2) \\
&+ ((a_1^2 - b_1^2 + c_1^2 - d_1^2)c_2 + 2c_1d_1d_2 - 2a_1b_1d_2 + 2b_1c_1b_2 + 2a_1d_1b_2) \\
&+ ((a_1^2 - b_1^2 - c_1^2 + d_1^2)d_2 + 2b_1d_1b_2 - 2a_1c_1b_2 + 2a_1b_1c_2 + 2c_1d_1c_2))
\end{aligned}$$