

MULTIPLE LINEAL REGRESSION

Ejercicio 1: Se realizó un estudio a 12 estudiantes para ver cómo influyen las calificaciones del examen y el número de clases que los estudiantes pierden en la calificación de la materia de estadística. Los datos son:

Estudiante	Calificación de estadística (y)	Calificación del examen (x_1)	Clases perdidas (x_2)	x_1^2	x_2^2	$x_1 \cdot x_2$	$x_1 \cdot y$	$x_2 \cdot y$
1	85	65	1	4225	1	65	5.525	85
2	74	50	7	2.500	49	350	3.700	518
3	76	55	5	3.025	25	275	4.180	380
4	90	65	2	4.225	4	130	5.850	180
5	85	55	6	3.025	36	330	4.675	510
6	87	70	3	4.900	9	210	6.090	261
7	94	65	2	4.225	4	130	6.110	188
8	98	70	5	4.900	25	350	6.860	490
9	81	55	4	3.025	16	220	4.455	324
10	91	70	3	4.900	9	210	6.370	273
11	76	50	1	2.500	1	50	3.800	76
12	74	55	4	3.025	16	220	4.070	296
	1.011	725	43	44.475	195	2.540	61.685	3.581

Método matricial:
$$\begin{pmatrix} n & \sum x_1 & \sum x_2 \\ \sum x_1 & \sum x_1^2 & \sum x_1 \cdot x_2 \\ \sum x_2 & \sum x_1 \cdot x_2 & \sum x_2^2 \end{pmatrix} = \begin{pmatrix} n & \sum x_1 & \sum x_2 \\ \sum x_1 & \sum x_1^2 & \sum x_1 \cdot x_2 \\ \sum x_2 & \sum x_1 \cdot x_2 & \sum x_2^2 \end{pmatrix} = \begin{matrix} \text{Row 1:} \\ \text{Row 2:} \\ \text{Row 3:} \end{matrix} \begin{pmatrix} 12 & 725 & 43 \\ 725 & 44.475 & 2.540 \\ 43 & 2.540 & 195 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

COLUMN 1

Row 1 / 12:
$$\begin{pmatrix} 1 & \frac{725}{12} & \frac{43}{12} \\ 725 & 44.475 & 2.540 \\ 43 & 2.540 & 195 \end{pmatrix} = \begin{pmatrix} \frac{1}{12} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow \text{Row}_1 \cdot (-725) + \text{Row}_2: \begin{pmatrix} 1 & \frac{725}{12} & \frac{43}{12} \\ 0 & \frac{8075}{12} & \frac{-695}{12} \\ 43 & 2.540 & 195 \end{pmatrix} = \begin{pmatrix} \frac{1}{12} & 0 & 0 \\ -\frac{725}{12} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow \text{Row}_1 \cdot (-43) + \text{Row}_3: \begin{pmatrix} 1 & \frac{725}{12} & \frac{43}{12} \\ 0 & \frac{8075}{12} & \frac{-695}{12} \\ 0 & \frac{-695}{12} & \frac{491}{12} \end{pmatrix} = \begin{pmatrix} \frac{1}{12} & 0 & 0 \\ -\frac{725}{12} & 1 & 0 \\ -\frac{43}{12} & 0 & 1 \end{pmatrix}$$

COLUMN 2

Row 2 / $(\frac{8075}{12})$:
$$\begin{pmatrix} 1 & \frac{725}{12} & \frac{43}{12} \\ 0 & 1 & \frac{-139}{1615} \\ 0 & \frac{-695}{12} & \frac{491}{12} \end{pmatrix} = \begin{pmatrix} \frac{1}{12} & 0 & 0 \\ -\frac{29}{323} & \frac{12}{8075} & 0 \\ -\frac{43}{12} & 0 & 1 \end{pmatrix} \Rightarrow \text{Row}_2 \cdot (-\frac{725}{12}) + \text{Row}_3: \begin{pmatrix} 1 & 0 & \frac{2837}{323} \\ 0 & 1 & \frac{-139}{1615} \\ 0 & \frac{-695}{12} & \frac{491}{12} \end{pmatrix} = \begin{pmatrix} \frac{1779}{323} & \frac{-29}{323} & 0 \\ -\frac{29}{323} & \frac{12}{8075} & 0 \\ -\frac{43}{12} & 0 & 1 \end{pmatrix} \Rightarrow \text{Row}_2 \cdot (\frac{695}{12}) + \text{Row}_3: \begin{pmatrix} 1 & 0 & \frac{2837}{323} \\ 0 & 1 & \frac{-139}{1615} \\ 0 & 0 & \frac{11606}{323} \end{pmatrix} = \begin{pmatrix} \frac{1779}{323} & \frac{-29}{323} & 0 \\ -\frac{29}{323} & \frac{12}{8075} & 0 \\ -\frac{2837}{323} & \frac{139}{1615} & 1 \end{pmatrix}$$

COLUMN 3

Row 3 / $(\frac{11606}{323})$:
$$\begin{pmatrix} 1 & 0 & \frac{2837}{323} \\ 0 & 1 & \frac{-139}{1615} \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1779}{323} & \frac{-29}{323} & 0 \\ -\frac{29}{323} & \frac{12}{8075} & 0 \\ -\frac{2837}{11606} & \frac{139}{58030} & \frac{323}{11606} \end{pmatrix} \Rightarrow \text{Row}_3 \cdot (-\frac{2837}{323}) + \text{Row}_1: \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{-139}{1615} \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{765474}{58030} & \frac{-6431}{58030} & \frac{-2837}{11606} \\ -\frac{29}{323} & \frac{12}{8075} & 0 \\ -\frac{2837}{11606} & \frac{139}{58030} & \frac{323}{11606} \end{pmatrix} \Rightarrow \text{Row}_3 \cdot (\frac{139}{1615}) + \text{Row}_2: \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{765474}{58030} & \frac{-6431}{58030} & \frac{-2837}{11606} \\ -\frac{6437}{58030} & \frac{491}{290150} & \frac{139}{58030} \\ -\frac{2837}{11606} & \frac{139}{58030} & \frac{323}{11606} \end{pmatrix}$$

$$\begin{pmatrix} 7,65474 & -0,421 & -11606 \\ -6431 & 58030 & 491 \\ -2837 & 11606 & 139 \end{pmatrix} \cdot \begin{pmatrix} \sum y = 1.011 \\ \sum x_1 \cdot y = 61.685 \\ \sum x_2 \cdot y = 3.581 \end{pmatrix} \Rightarrow \begin{pmatrix} -112,0410305 & 104,3850939 & 8,577615027 \\ -247,13113976 & 147,7548682 & 99,66077891 \end{pmatrix} \Rightarrow \begin{cases} \beta_0 = 27,5467001 \\ \beta_1 = 0,921678427 \\ \beta_2 = 0,2842495075 \end{cases}$$

• Ecuación múltiple: $y = \beta_0 + \beta_1 \cdot x_1 + \dots + \beta_n \cdot x_n \rightarrow y = 27,5467001 + 0,921678427 x_1 + 0,2842495075 x_2$

• Estimación $\begin{cases} x_1 = 60 \\ x_2 = 4 \end{cases} \rightarrow y = 27,5467001 + 0,921678427 \cdot 60 + 0,2842495075 \cdot 4 = 83,9844$

Estudiante	Calificación de estadística (y)	Calificación del examen (x ₁)	Clases perdidas (x ₂)	$\hat{y} = 27,54 + 0,92x_1 + 0,28x_2$	$e = y - \hat{y}$	e^2	$(\hat{y} - \bar{y})^2$
1	85	65	1	87,74004736	-2,74004736	7,507859535	12,18043058
2	74	50	7	75,620368	-1,620368	2,625592455	74,47054846
3	76	55	5	79,66026112	-3,66026112	13,39751147	21,065570299
4	90	65	2	88,02429687	+1,97570313	3,903402858	14,24531686
5	85	55	6	79,94451063	+5,05548937	25,5579277	18,53723872
6	87	70	3	92,91693851	-5,91693851	35,01046133	75,11582314
7	94	65	2	88,02429687	+5,97570313	35,7090279	14,24531686
8	98	70	5	93,48543753	+4,51456247	20,3812743	85,29330637
9	81	55	4	79,37601162	+1,62398838	2,637338258	23,75576273
10	91	70	3	92,91693851	-1,91693851	3,674653251	75,11582314
11	76	50	1	73,91487096	+2,08512904	4,347763113	106,8148923
12	74	55	4	79,37601162	-5,37601162	28,90150094	23,75576273
	1.011	725	43			SCE = $\sum e^2 = 183,6540$	SCR = $\sum = 544,5959$

$\hookrightarrow n = 12 = n^{\circ} \text{ obs. muestra} \parallel k = 2 = n^{\circ} \text{ variables independientes} \parallel \bar{y} = \frac{\sum y}{n} = \frac{1.011}{12} = 84,25 = \text{Media}$

Error Estándar de la estimación Múltiple $(S_{y_n \dots k}) = \sqrt{\frac{SCE}{n - (k + 1)}} = \sqrt{\frac{183,6540}{12 - (2 + 1)}} = 4,517300794$

Suma Total de Cuadrados (STC) = SCE + SCR = 183,6540 + 544,5959 = 728,2499831

Coefficiente de determinación múltiple $(R^2) = \frac{SCR}{STC} = \frac{544,5959}{728,2499} = 0,747814 \approx 74,78 \%$

Coefficiente ajustado de determinación múltiple $(R^2_{\text{ajustado}}) = 1 - \frac{(1 - R^2) \cdot (n - 1)}{n - k - 1} = 1 - \frac{(1 - 0,747814) \cdot (12 - 1)}{12 - 2 - 1} = 0,6917733246 \approx 69,18 \%$

Coefficiente de correlación múltiple $(R) = \sqrt{R^2} = \sqrt{0,7478145383} = 0,8647627064 \leftarrow \text{Cerca de } +1 = \text{correlación positiva fuerte} = \text{estrecha relación entre variables}$

• SCE = Suma de cuadrados del error
• SCR = Suma de cuadrados de la regresión